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1 **Attractor dimension of time-averaged climate**
2 **observables: insights from a low-order**
3 **ocean-atmosphere model**

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11 **Abstract**

12 The ocean and atmosphere have very different characteristic timescales and display a rich
13 range of interactions. Here, we investigate the sensitivity of the dynamical properties of
14 the coupled atmosphere-ocean system to time-averaging. We base our analysis on a con-
15 ceptual model of the atmosphere-ocean dynamics which allows us to compute the attrac-
16 tor properties for different coupling coefficients and averaging time-scales. When the av-
17 eraging time is increased, the local dimension shows a non-monotonic behaviour for short
18 averaging times, but ultimately decreases for windows longer than 1 year. The analy-
19 sis of daily, monthly and annual instrumental and reconstructed indices of oceanic and
20 atmospheric circulation supports our results. This has important implications for the anal-
21 ysis and interpretation of long climate timeseries with a low temporal resolution, but also
22 on the possible convergence of climate observables for long time-averages toward attrac-
23 tors close to hyperbolicity.

24 **1 Introduction**

25 The climate system is a complex system characterised by turbulent dynamics. The
26 time-energy spectra of instrumental and proxy climate data show a rich structure with
27 energy cascades from timescales of millions of years to a few seconds and no spectral gaps
28 [*Lovejoy et al.*, 2001]. Moreover, atmospheric and oceanic motions feature specific char-
29 acteristics which differentiate them from the homogeneous and isotropic turbulence of
30 Kolmogorov [*Pouquet and Marino*, 2013]. Indeed, the rotation and stratification effects
31 allow for an inverse energy cascade contributing to large-scale motions, such as the at-
32 mospheric planetary waves and ocean currents. The different components of the climate
33 system - each with their own complex dynamics - further show a broad range of inter-
34 actions. In this study we will specifically focus on the interplay between the ocean and
35 atmosphere. The former has slow characteristics timescales (up to thousands of years),
36 while the latter has a swifter temporal evolution, with synoptic-scale features typically
37 evolving over periods of days [*Pedlosky*, 2013]. These fast timescales limit our ability to
38 predict the future evolution of atmospheric dynamics: indeed, *Lorenz* [1969, 1982] and
39 *Dalcher and Kalnay* [1987] postulated a limit of mid-latitude weather predictability at
40 10-15 days. However, the ocean's slow variability provides a possible predictability path-
41 way beyond this range [*Palmer and Anderson*, 1994; *Baehr et al.*, 2015; *Vannitsem and*
42 *Ghil*, 2017]. This makes the study of the ocean's low frequency variability (LFV) and

43 its coupling with the atmosphere a topic of considerable scientific and practical inter-
44 est.

45 The most famous example of ocean-led predictability is the alternance of El-Nino
46 and La-Nina events and their effects on large-scale precipitation and temperatures. This
47 phenomenon has provided some of the earliest indications of the feasibility of annual and
48 longer forecasts [*Cane et al.*, 1986]. However, extracting the full predictability potential
49 inherent to LFV features on longer timescales remains a challenge. Long instrumental
50 time-series are scarce, and even reanalysis products only provide well-constrained data
51 over the past few decades. Long-term reconstructions of coupled ocean/atmosphere vari-
52 ability must therefore rely on model simulations, documentary evidence or proxy data.
53 The latter typically provide a time series representative of some feature of oceanic and/or
54 atmospheric circulation on a regional or larger scale, with a time resolution of seasons
55 to decades or longer [*Bond et al.*, 2001; *Vinther et al.*, 2010]. This type of data is essen-
56 tial to verify that the coupled dynamics generated by climate models are compatible with
57 those found in real world.

58 An important question is whether it is possible to quantify the impact of the av-
59 eraging procedure implicit in proxy records when performing such comparisons. In this
60 paper, we address this question from a theoretical angle by using a conceptual coupled
61 ocean-atmosphere model and investigating its dynamical properties. We apply dynam-
62 ical systems theory to measure the dimensionality of the system, and compare the re-
63 sults for model output with a high temporal resolution versus a degraded dataset where
64 the system is known only through long-term averages. This allows us to objectively quan-
65 tify the modifications induced by the averaging. We conclude by applying our approach
66 to a number of instrumental and reconstructed indices of large-scale climate modes and
67 discussing the implications of our results for the general analysis of climate data.

68 **2 A dynamical systems approach**

69 Determining the attractor properties of complex systems has been a long-standing
70 challenge in the field of dynamical systems theory. However, recent theoretical advances
71 in our understanding of the limiting distribution of Poincaré recurrences now enable us
72 to compute both mean and instantaneous (in time, and hence local in phase space) dy-
73 namical properties of complex systems. The key finding is that, under suitable rescal-

74 ing, the probability p of entering a ball in phase space centred on ζ with a radius r for
 75 chaotic attractors obeys a generalized Pareto distribution [Freitas *et al.*, 2010; Faranda
 76 *et al.*, 2011; Lucarini *et al.*, 2012, 2016]. In order to compute such probability, we first
 77 calculate the series of distances $dist(\zeta, x(t))$ between the point on the attractor ζ and
 78 all other points $x(t)$ on the trajectory. We then put a logarithmic weight on the time se-
 79 ries of the distance to increase the discrimination of small values of $dist(\zeta, x(t))$, which
 80 correspond to large values of $g(x(t))$:

$$g(x(t)) = -\log(dist(\zeta, x(t))).$$

81 The probability of entering a ball of radius r centred on ζ can now be expressed
 82 as the probability p of exceeding a threshold q of the distribution of $g(x(t))$. In the limit
 83 of an infinitely long trajectory, such probability is the exponential member of the gen-
 84 eralized Pareto distribution:

$$p = \Pr(g(x(t)) > q, \zeta) \simeq \exp(-[x - \mu(\zeta)]/\beta(\zeta))$$

85 whose parameters μ and σ are a function of the point ζ chosen on the attractor.
 86 Remarkably, $\sigma = 1/d(\zeta)$, where $d(\zeta)$ is the local dimension around the point ζ . The
 87 attractor dimension $\langle d \rangle$ can then be obtained by averaging d for a sufficiently large sam-
 88 ple of points ζ_i on the attractor. Here, we use the quantile 0.98 of the series $g(x(t))$ to
 89 determine q . We have checked the stability of the results against reasonable changes in
 90 the quantile. The universality of the convergence law implies that the above is akin to
 91 a central limit theorem of Poincaré recurrences. For further details, the reader is referred
 92 to Lucarini *et al.* [2016]. The above approach has been successfully used to describe the
 93 evolution of sea-level pressure [Faranda *et al.*, 2017a] and geopotential height fields [Mes-
 94 sori *et al.*, 2017] over the North Atlantic, as well as sea-level pressure, temperature and
 95 precipitation fields at hemispheric scale [Faranda *et al.*, 2017b].

96 **3 Data and Model Specifications**

97 **3.1 A Conceptual Atmosphere-Ocean Coupled Model**

98 The coupled ocean-atmosphere model we use here is the same as described by Van-
 99 nitsem [2015]. The atmospheric component is based on the vorticity equations of a two-

100 layer, quasi-geostrophic flow defined on a β -plane, supplemented with a thermodynamic
 101 equation for the temperature at the interface between the two atmospheric layers. The
 102 ocean component is based on the reduced-gravity, quasi-geostrophic shallow-water model
 103 with the same first order approximation of the Coriolis parameter. The oceanic temper-
 104 ature is considered as a passive scalar transported by the ocean currents, but it displays
 105 strong interactions with the atmospheric temperature through radiative and heat exchanges.
 106 A time-dependent radiative forcing mimicks the annual radiative input coming from the
 107 Sun at midlatitudes. A low-order model version is built based on truncating the Fourier
 108 expansion of the fields at the minimal number of modes that are believed to capture key
 109 features of the observed large scale dynamics of both the ocean and the atmosphere. The
 110 truncation leads to 20 ordinary differential equations for the atmospheric variables, 8 equa-
 111 tions for the ocean transport variables, 6 equations for the temperature anomaly within
 112 the ocean, and 2 additional equations for the spatially averaged temperatures in the at-
 113 mosphere and the ocean. The parameter values used are the same as in Figure 3 of *Van-*
 114 *nitsem* [2015], except that here we test a different range of values of the friction coef-
 115 ficient between the ocean and the atmosphere, namely C . Specifically, we consider 4 dif-
 116 ferent runs, coined CD0002, CD0005, CD0007 and CD0008, corresponding to 4 differ-
 117 ent values of the friction coefficient between the ocean and the atmosphere, namely $C=0.002$,
 118 0.005 , 0.007 and $0.008 \text{ kgm}^{-2}\text{s}^{-1}$, respectively. The last two runs display LFV, while the
 119 first two do not. The lengths of the series is such that we can retain 10000 time-steps
 120 for all averaging windows in what follows.

121 3.2 Data

122 In addition to output from an idealised model, we also analyse real-world timeseries
 123 at different temporal resolutions. Specifically, we use daily NINO3 data provided by the
 124 NOAA climate prediction center [*Barnston and Livezey, 1987; Reynolds et al., 2007*] over
 125 the period 01 Jan 1981 - 28 Feb 2018, monthly NINO3 data over 1854 to 2016 provided
 126 by [*Huang et al., 2017*] and a yearly NINO3 dataset over the period 1049-1995 provided
 127 by *Mann et al.* [2009]. We further analyse daily NAO data provided by the NOAA cli-
 128 mate prediction center [*Barnston and Livezey, 1987; Reynolds et al., 2007*] over the pe-
 129 riod 01 Jan 1981 - 28 Feb 2018, monthly NAO data over 1854 to 2016 provided by [*Jones*
 130 *et al., 1997*] and yearly data over the period 1049-1995 [*Trouet et al., 2009*].

4 Dynamical Implications of Time-Averaging

We begin by analysing the dependence of the phase portraits on the time-averaging of model output. In order to depict this we have to choose 3 of the 36 modes of the model to represent the attractor on a Poincaré section. We choose modes $\psi_{o,2}$, $\theta_{o,2}$ and $\psi_{a,1}$ (Figure 1). These three modes are the dominant modes of the coupled ocean-atmosphere dynamics as discussed in detail in *Vannitsem et al.* [2015]. We consider a run with no LFV ($C=0.002$, Figure 1 a, c, e, g) and a run with a marked LFV ($C=0.007$, Figure 1 b, d, f, h). The colourscales show the values of the local dimension d (for readability each panel has a different colourscale). The effect of averaging depends both on the chosen time-window and on the coupling. The daily portraits show quasi-periodic cycles, associated with the annual cycle present in the system, in both simulations (Figure 1 a, b). These are partly destroyed by the monthly averaging (Figure 1 c, d). Longer time-averaging rapidly smooths all structures in the phase portraits of the no-LFV run (Figure 1 e, g), so that the Poincaré section looks like that of a noisy fixed point in 3 dimensions. For the LFV run, the slow signal associated with the ocean dynamics survives the averaging procedures, and is still evident under an 8-year averaging. We further note that at sub-annual time scales (Figure 1 a, b, c, d), the local dimension is in general higher during the winter period, i.e. when $\psi_{a,1}$ – characterizing the amplitude of the atmospheric zonal flow – displays high values. We will come back to this point in the next section.

A more quantitative analysis of the changes in the attractor properties under averaging is reported in Figures 2 and 3, which present the mean values and distributions of d . The first remarkable feature is the non-monotonic behaviour of the dimension with the averaging window. A naive hypothesis would be that, independently of the coupling, one might observe a decrease of the dimension with increasing time-averaging. However, this is only true for averaging periods larger or equal to 1 year, for which the seasonal cycle is averaged out. Indeed, all four simulations analysed here show non-monotonic behaviour for shorter averaging times. This feature reveals that the filtering through averaging tends to modify the frequency of specific categories of local dimensions. The analysis of the distributions of d , shown in Figure 3, provides further insights on this behaviour. Taking $C=0.002$ as example, one can see a clear shift of the distribution toward larger values in going from daily to monthly values.

162 On the contrary, averaging beyond 1 year time-scales suppresses the extreme d val-
 163 ues in the tails of the distributions, which corresponds to a smoothing of the variabil-
 164 ity of the dimension, thus lowering $\langle d \rangle$. This is particularly evident for the case of $C=0.007$
 165 and $C=0.008$ (LFV runs) and should be expected since there is a smoothing of the vari-
 166 ability on the attractor (Figure 1 b, d, f, h). This smoothing removes specific frequen-
 167 cies in the dynamics, as discussed in details in *Nicolis and Nicolis* [1995]; *Vannitsem and*
 168 *Nicolis* [1995, 1998], and also reduces the local variability of the instability properties
 169 of the flow. Another interesting result is that at monthly and seasonal scales the distri-
 170 butions of d display a double peak for runs both with and without LFV (Figure 4). This
 171 double peak is associated with the seasonal variability; there is a dominance of large d
 172 in Winter and low d in Summer. For instance for $C=0.005$, there is a maximum around
 173 $d = 8$ for the winter conditions and $d = 4$ for summer conditions (Figure 4) . To in-
 174 terpret this feature one must recall that the large-scale winter dynamics in the mid-latitudes
 175 is driven by a larger gradient of equator-to-pole radiative input than in summer [*Goosse,*
 176 *2015; Vannitsem, 2015, 2017*]. This has strong implications for the instability proper-
 177 ties of the flow [*Buizza and Palmer, 1995*]. This is also a property of the coupled ocean-
 178 atmosphere model used here, which displays lower averaged local Lyapunov exponents
 179 (and averaged local Lyapunov dimensions) in Summer than in Winter [*Vannitsem, 2017*].
 180 The technique we adopt here succesfully captures this increase in the complexity of the
 181 dynamics. The distributions further highlight the fact that, in some cases, the average
 182 of d remains roughly constant but the positive tails of the distributions change radically.
 183 This suggests that a decrease in $\langle d \rangle$ due to averaging might change little in the system's
 184 ground state while altering the configurations with the largest number of degrees of free-
 185 dom. Table 1 reports the values of the first four moments of the distribution of d for all
 186 C and averaging times.

187 One can further wonder whether $\langle d \rangle$ are determined predominantly by the oceanic
 188 or the atmospheric modes. To do this, we compute the local dimensions of the oceanic
 189 and atmospheric components separately. The results are shown in Figure 5 for different
 190 averaging periods. Except for the $C=0.007$ case, the atmospheric modes alone return al-
 191 most the same value of the total dimension as the joint calculation. The analysis of the
 192 ocean variables instead gives a lower dimension. This interesting feature likely reflects
 193 the fact that, although the ocean variables are coupled to the atmosphere, they only re-
 194 tains part of the complex structure of the system, in particular for the low values of C .

195 In our view, this results from the fact that the dynamics in the ocean is only "weakly"
 196 driven by the chaotic variability present in the atmosphere for small values of C due to
 197 the large inertia of the ocean that integrates the atmospheric forcing on long time scales.
 198 For large values of C , LFV develops and this effect is considerably weakened; variables
 199 from both components then provide similar results. However, the behavior observed for
 200 $C = 0.007$ is still slightly non-monotonic because this run still gives a chaotic attrac-
 201 tor, whereas the run $C = 0.008$ provide a quasi-periodic flow.

202 **5 Implications for Ocean-Atmosphere Coupling and conclusions**

203 In the present study we have investigated the effects of time averaging on the ocean-
 204 atmosphere system as represented by a conceptual coupled model. The impact of aver-
 205 aging is quantified in terms of changes in the attractor properties of the system. When
 206 the averaging time is increased, the local dimension shows a non-monotonic behaviour
 207 for short averaging times, but ultimately decreases for windows longer than 1 year. For
 208 these averaging windows, the distribution of the local dimension becomes closer to Gaus-
 209 sian and the variability decreases. This corresponds to a progressive smoothing of the
 210 attractor. Time-averaging therefore has profound and sometimes counter-intuitive im-
 211 plications for the dynamical characteristics of climate data. Our results also suggest that,
 212 on longer time scales, the climate dynamics is smoother and closer to that of homoge-
 213 nous, hyperbolic systems.

214 It is however necessary to verify whether the results from the idealised model pre-
 215 sented above find a match in real-world data. Here, we repeat our analysis for El Nino-
 216 Southern Oscillation Nino3 and North Atlantic Oscillation (NAO) indices. As a caveat,
 217 we note that this analysis has an important difference from that of the full coupled model.
 218 Indeed, the NAO and Nino3 indices do not represent the full climate attractor whereas
 219 they can be thought of as a projection (a special Poincaré section) of the full dynam-
 220 ics. In this sense the analysis can still inform us on numerous aspects of the system (see
 221 for example *Faranda et al.* [2017c] for a similar argument on the Von Karmann turbu-
 222 lent swirling flow). A separate problem to consider is the length of the time series, as
 223 our method for computing the local dimensions is dependent on processing a sufficiently
 224 long series. The shortest timeseries we analyse are the yearly ones, for which we only dis-
 225 pose of 947 years; we therefore perform two different computations of the dimension: i)
 226 for each dataset we use the complete timeseries, ii) for each dataset we only use 947 data

227 points. This provides some indication of the robustness of our conclusions. The results
 228 are reported in Figure 6. The top panels show the boxplot of the local dimension dis-
 229 tributions when all the data are considered, whereas the lower panel presents the aver-
 230 age dimension for the two cases described. The analysis of the boxplots suggest that the
 231 extremes of d change with the time scale considered. For the yearly time series, we ob-
 232 tain values of d up to 10. This may seem nonphysical since we are only analyzing two
 233 time series but, following again *Faranda et al.* [2017c], can be understood by consider-
 234 ing the role of small scale turbulence in increasing the effective dimension of the attrac-
 235 tor. Sampling issues may be discarded because both the full and reduced datasets show
 236 comparable relative changes between the different temporal resolutions. Finally the non-
 237 monotonic behavior of the average dimension for the climate indices follows the one found
 238 in the coupled model. We therefore conclude that the inferences drawn from the con-
 239 ceptual model provide valuable insights into the behaviour of real-world climate data and
 240 should be considered when performing dynamical analyses of data with low temporal res-
 241 olutions. It is worth performing further analysis in more sophisticated climate models
 242 in order to clarify in particular the increase of variability of the local dimension found
 243 in Figure 6. For that, very long (historical) runs should be considered in order to have
 244 enough data for the extreme value analysis, provided that climate models can correctly
 245 reproduce the internal variability of the climate system.

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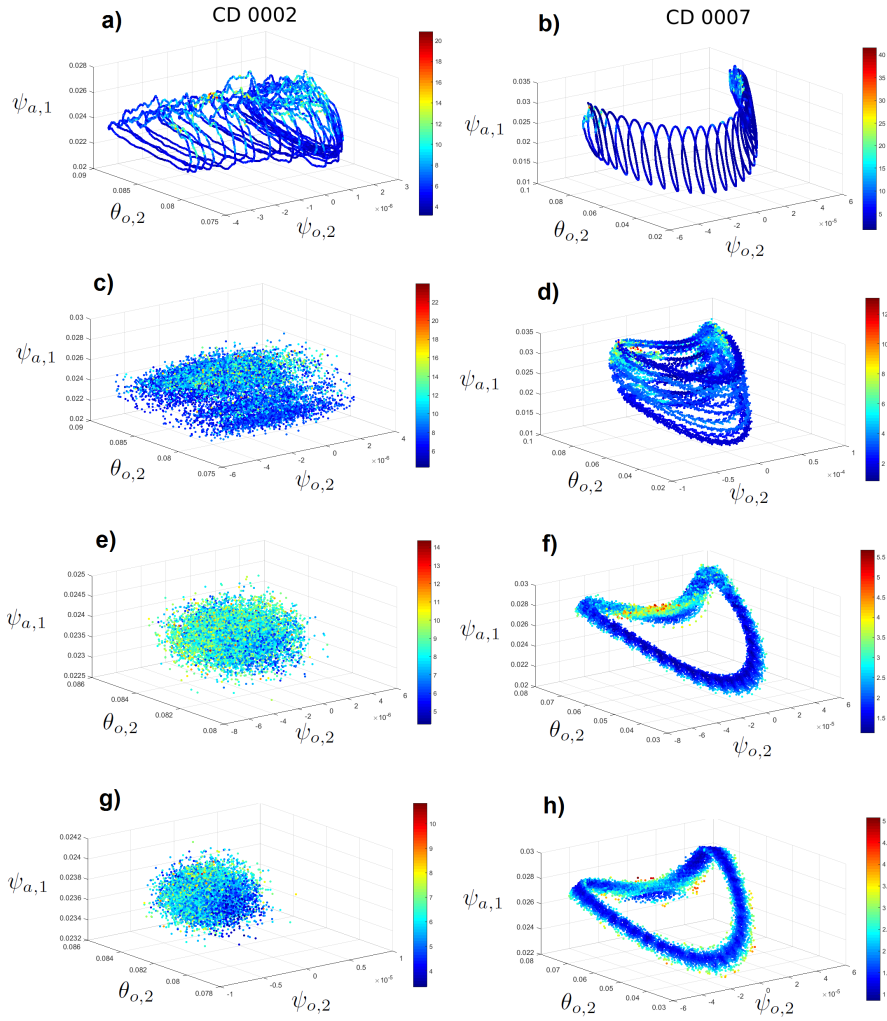
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350

Table 1. Moments of the distributions of d for different C and averaging times.

	$C=0.002$				$C=0.005$				$C=0.007$				$C=0.008$			
	year	seas	month	day	year	seas	month	day	year	seas	month	day	year	seas	month	day
median	8.10	7.50	8.09	5.78	7.81	6.04	6.50	5.57	1.98	1.87	2.95	3.56	0.92	2.04	2.81	2.42
std	1.19	2.33	2.33	1.84	1.27	1.53	2.97	2.47	0.62	0.78	1.49	3.8	0.11	0.64	0.99	1.16
skewness	0.31	0.88	1.19	1.55	0.28	0.50	0.59	2.5	1.19	1.68	1.47	2.67	3.09	1.66	1.10	2.21
kurtosis	3.31	3.98	4.87	6.61	3.45	3.95	3.41	12.34	4.67	7.71	5.98	12.55	23.27	13.24	5.43	10.38



351

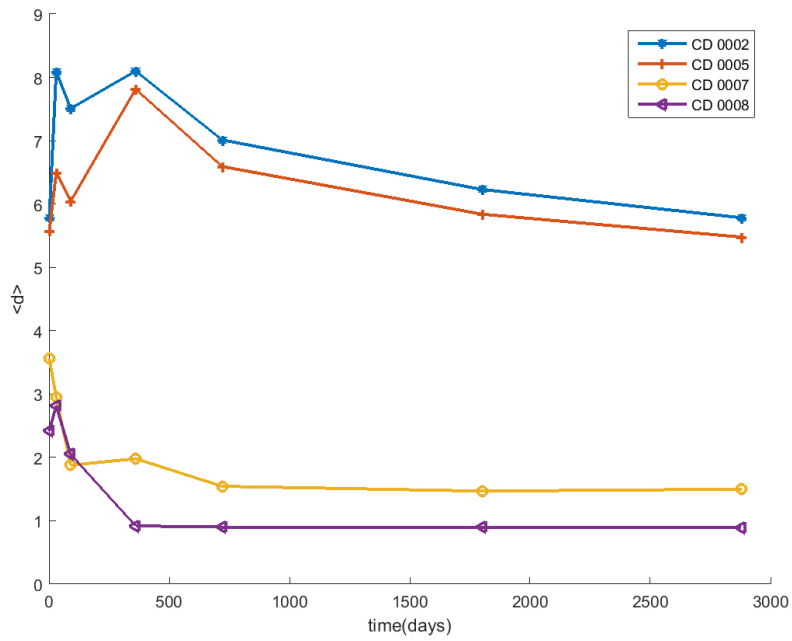
Figure 1. Poincaré sections for the modes 2,9,15. Different couplings a,c,e,g): $C=0.002$.

352

b,d,f,h) $C=0.007$. Different averaging windows: a,b) daily; c,d) monthly; e,f) yearly; g,h) 8 years

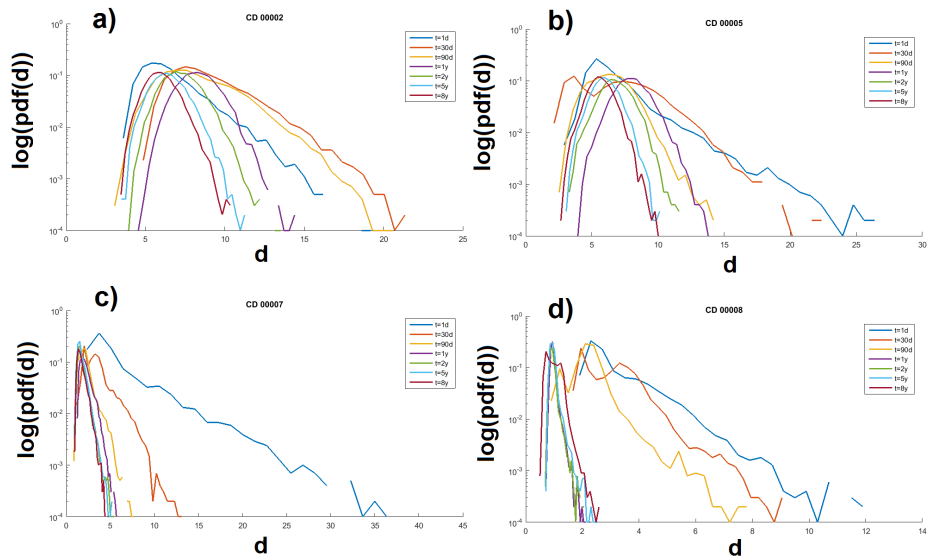
353

averages.



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Figure 2. Average local dimensions as a function of the coupling and the averaging window.

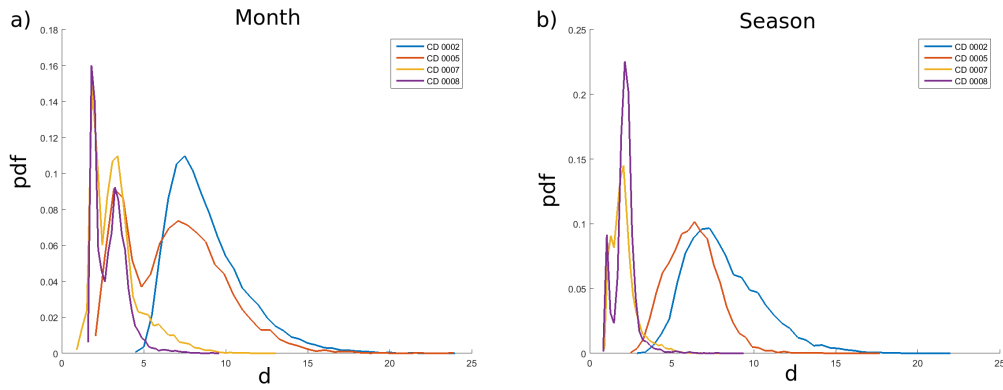


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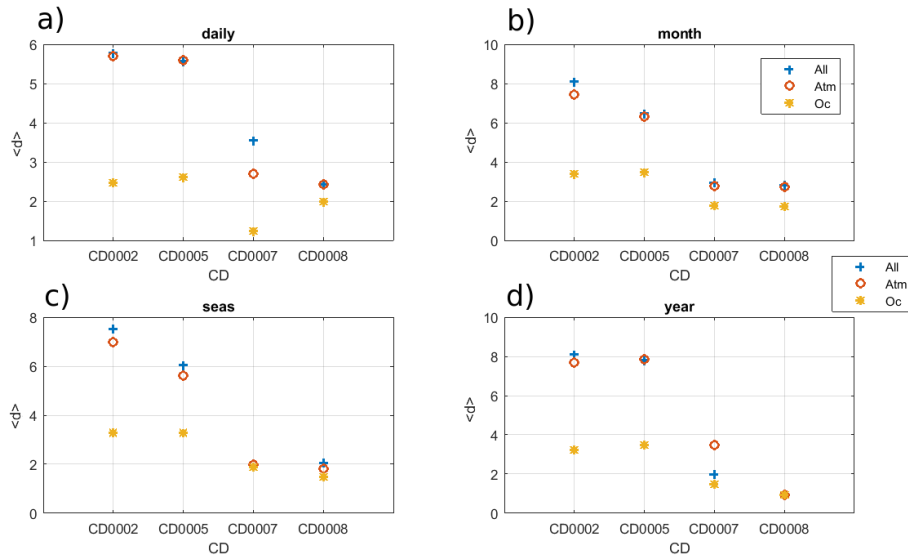
Figure 3. Attractor dimensions' distributions as a function of the coupling and the averaging

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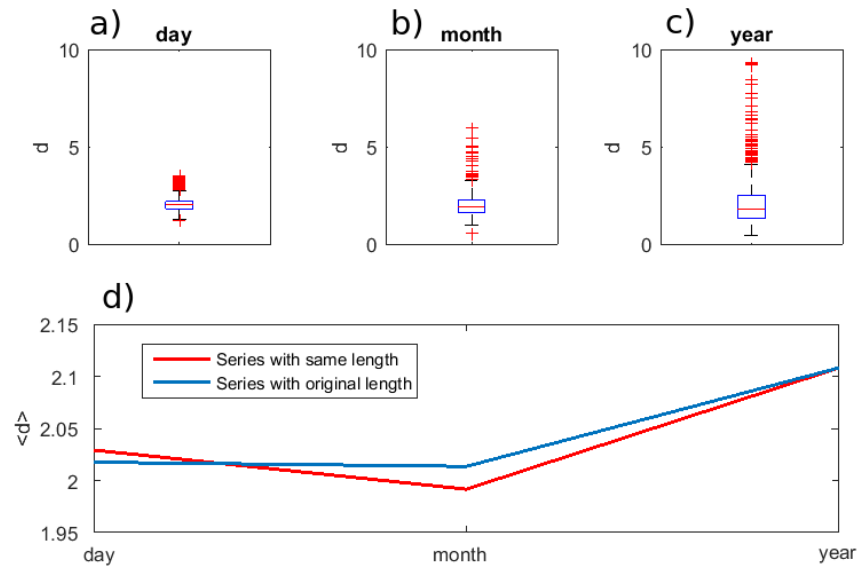
window. a) $C=0.002$, b) $C=0.005$, c) $C=0.007$, d) $C=0.008$



357 **Figure 4.** Attractor dimensions' distributions as a function of the coupling for monthly (a)
 358 and seasonal (b) averaging time.



359 **Figure 5.** Average dimensions $\langle d \rangle$ computed selecting all modes (blue stars) atmospheric
 360 modes only (red crosses), oceanic modes only (yellow triangles). a) daily, b) monthly, c) seasonal
 361 and d) yearly data.



362 **Figure 6.** a,b,c): boxplots of local dimensions d for NAO and Nino3 indices at different time
 363 scales. On each box, the central mark is the median, the edges of the box are the 25th and 75th
 364 percentiles, the whiskers extend to the most extreme data points not considered outliers, and
 365 outliers are plotted individually. a) daily, b) monthly and c) yearly data. d): average dimension
 366 $\langle d \rangle$ computed with the full length time-series (blue) and only 947 time steps (red).