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## A Mathematical Portrayal of a Christmas Carol for Nigerian Basic Education: The Mathematics in the *Twelve Days of Christmas*

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### ABSTRACT

The full adoption of Western Christmas traditions is gradually establishing a new social pattern for many educated families in Nigeria. Among the several Christmas carols prevalent in the country, *Twelve Days of Christmas* stand out for children and appears to be a daunting challenge they are often very proud to master. This article explores the cultural richness of this carol along with its mathematical significance for basic education in Nigeria. Evidently, when poetry-based artifacts such as *Twelve Days of Christmas* are deployed in instructional contexts, it allows team building, aids recall, reinforces mathematical thinking process, connects to history and the real world, and humanize mathematics for students at all levels.

**Keywords:** Poetry-based Mathematics, *Twelve Days of Christmas*; realistic Mathematics Education, cultural mathematics; triangular numbers.

On the first day of Christmas,  
My true love sent to me  
A partridge in a pear tree

On the second day of Christmas  
My true love sent to me  
Two turtle doves  
And a partridge in a pear tree

On the third day of Christmas  
My true love sent to me  
Three French hens  
Two turtle doves  
And a partridge in a pear tree

On the fourth day of Christmas,  
My true love sent to me  
Four calling birds,  
Three French hens  
Two turtle doves  
And a partridge in a pear tree

On the fifth day of Christmas  
My true love sent to me  
Five golden rings  
Four calling birds  
Three French hens  
Two turtle doves  
And a partridge in a pear tree

On the sixth day of Christmas,  
My true love sent to me  
Six geese a-laying  
Five golden rings  
Four calling birds  
Three French hens  
Two turtle doves  
And a partridge in a pear tree  
On the seventh day of Christmas,  
My true love sent to me  
Seven swans a-swimming  
Six geese a-laying  
Five golden rings  
Four calling birds,  
Three French hens  
Two turtle doves  
And a partridge in a pear tree

On the eight day of Christmas,  
My true love sent to me

Eight maids a-milking,  
Seven swans a-swimming  
Six geese a-laying  
Five golden rings  
Four calling birds,  
Three French hens  
Two turtle doves  
And a partridge in a pear tree

On the ninth day of Christmas,  
My true love sent to me  
Nine ladies dancing  
Eight maids a-milking  
Seven swans a-swimming  
Six geese a laying  
Five golden rings  
Four calling birds  
Three French hens  
Two turtle doves  
And a partridge in a pear tree

On the tenth day of Christmas  
My true love sent to me  
Ten lords a-leaping  
Nine ladies dancing  
Eight maids a-milking  
Seven swans a-swimming  
Six geese a-laying  
Six geese a-laying  
Five golden rings  
Four calling birds  
Three French hens  
Two turtle doves  
And a partridge in a peer tree

On the eleventh day of Christmas  
My true love sent to me,  
Eleven pipers piping,  
Ten lords a-leaping  
Nine ladies dancing  
Eight maids a-milking  
Seven Swans a-swimming  
Six geese a-laying  
Five golden rings  
Four calling birds  
Three French hens  
Two turtle doves  
And a partridge in a pear tree

On the twelfth day of Christmas  
My true love sent to me,

Twelve drummers drumming  
Eleven pipers piping,  
Ten lords a-leaping  
Nine ladies dancing  
Eight maids a-milking  
Seven Swans a-swimming  
Six geese a-laying  
Five golden rings  
Four calling birds  
Three French hens  
Two turtle doves  
And a partridge in a pear tree

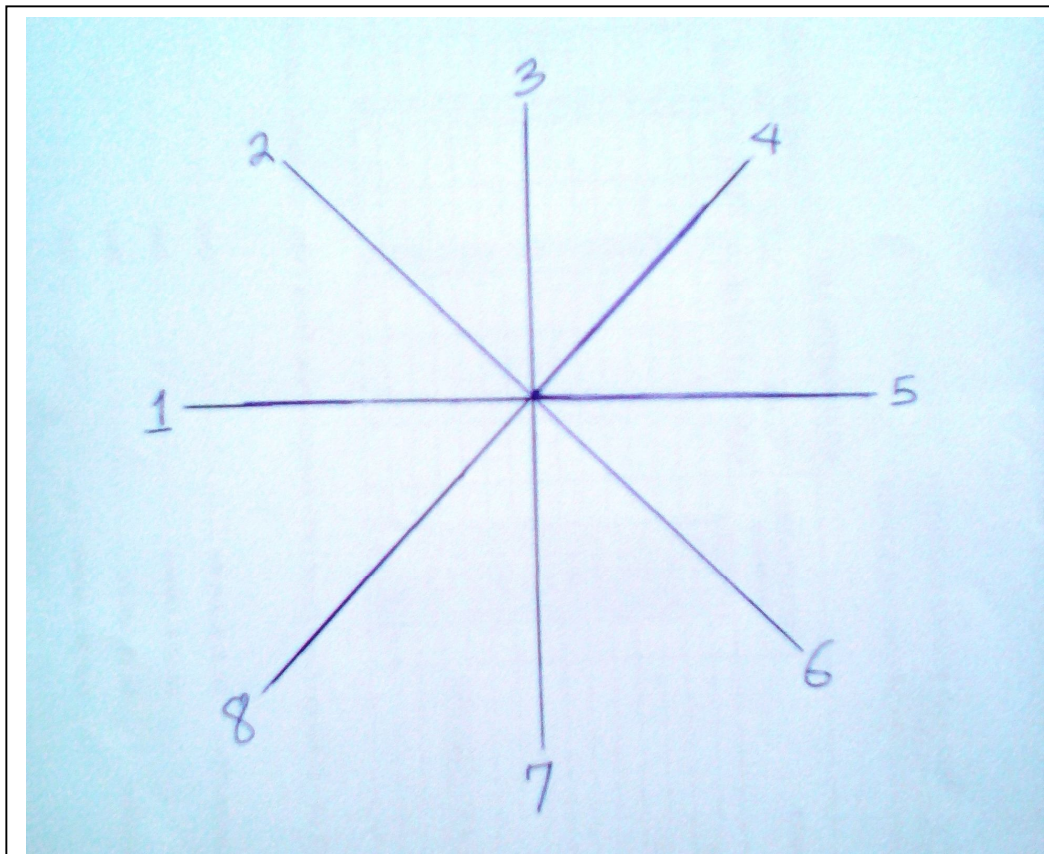
## 1. INTRODUCTION

What a thrilling song! A very long one at that, but amazingly, Nigeria children could memorize this old Christmas carol for presentation. The level of mastery of this hitherto complex Christmas carol lyrics is an indication of an increasingly rich Christmas tradition in Nigeria. Being the biggest holiday season in the country, many activities of families, basic schools and churches are being re-designed around Christmas, establishing a new social pattern for many literate homes. Apart from the traditional presentation of new clothing materials like shoes, cloths, sunshades, Santa-toned caps and badges to children, parents and guardians are beginning to see the need to encourage their wards to participate in Christmas programmes at school, in churches and in play groups within the neighborhood. Many basic schools now organize colorful end-of-first-term celebrations which feature the season's Nine Lessons and Carols, presentation of Christmas-themed gifts as prizes, and dancing with "Father Christmas". Children expectantly look forward to these events at school and there are never dull moments at the music rehearsals.

Dgreeting.Com (2017) reports that Nigerians celebrate Christmas with as much gusto as their American or European counterparts. Weeks before the day of Christmas, people buy lots of hens, turkeys, goats and cows. Children hover around the beasts, taunting and mostly gawking at them. People start elaborate preparations for the holiday when they travel and exchange gifts. According to Dgreetings.com (2017), Christmas carols are very popular and they can be heard right from the beginning of December in anticipation of Christmas, with Nigerians having some of the most beautiful, original Christmas carol-ever. The church choir may visit the church congregation in their homes to sing Christmas carols (Cooper, 2017). Among the several carols, *Twelve Days of Christmas* stand out for children. First of all, the song appears to be a daunting challenges waiting to be conquered. This explains why children are often proud to have mastered the lyrics, incrementally adding a new set of gifts on each day and re-iterating the addition of gifts for the previous day.

Obviously, the *Twelve Days of Christmas* holds more significance than the children and choir directors are aware of. The song has an amazing mathematical aspect with the lot of numbers involved. The numbers and the way they add up make for some interesting pattern that, coincidentally or not, resemble such seasonal icons as Christmas trees, stockings and stars (Spice, 2003). Analyzing this song has been something of an old chestnut among some mathematics educators, who use the song to discuss mathematics principles and history. Such analysis is relevant because mathematicians are justifiably proud of the elegance of their symbols, definitions and proofs (Huber & Karaali, 2017). At the end of the day, the basic notions of mathematics are described by words, and exploring the words of mathematics leads to the humanistic questions of understanding and interpretation that make the study of great literature, such as *Twelve Days of Christmas*, so open-ended.

There are many ways music can be used to motivate or facilitate the learning of mathematics. One type of connection between mathematics and music is to write or analyze songs whose lyrics connect with mathematics, as obtainable in *Twelve Days of Christmas*. In the view of Lesser (2015), while most examples of such songs seem to target elementary school students (e.g. helping children learn multiplication facts), there is also precedence for using mathematics songs with students in subsequent educational levels to increase students learning, recall and motivation as well as reduce stress or anxiety. Through songs, melodies and lyrics can provide memory cues for easy recall of mathematical facts (Governor, 2011). Music activates emotion and as such any instruction situated in emotions binds and improves learning. Governor (2011) opines that melodies used in science/mathematics-content songs can be used to carry science/mathematics-content information and because of the emotional response elicited by music, there is greater likelihood that the brain will encode it in long-term memory. Music is emotionally powerful and memories encoded in an emotional state are stronger and offer multiple paths for encoding and retrieval (Governor, 2015).



***Poet's instruction:***

Place each one of the 8 phrases on each of the 8 lines so that, read from 1 to 8, they form parallel statements

- 1 You at the centre
- 2 The sin's knife
- 3 Divides this octagon
- 4 Eyes nose hands tongue ears
- 5 East and West North and South
- 6 Cuts this bread
- 7 The abyss is at the centre
- 8 See smell touch taste hear

**Figure 1: Riddle in the shape of an octagon – Poet's version**

Mathematics and poetry are the extremes of language. If mathematics is, through its density and precision, the supreme form of science, poetry represents the supreme modality of concentrating and essentializing the language of suggestion, implying an intricate interplay between the two poles of language (Parpala&Afana, 2008). Kempthorne (2015) examined the complex and

varied nature of mathematics in the late nineteenth and early twentieth centuries, providing extensive literature review that draws on various strands of that richness, suggesting multiple connections with poetry through questions of meaning, truth, ambiguity, imagination, concision and language. At the classroom level, Glaz and Liang (2009) use poetry projects to ease the difficulties students have with the transition between word-problems representing natural phenomenon, and the corresponding mathematical models - the equations representing the phenomena. This particular intervention project also uses poetry to stimulate classroom participation, develop mathematical intuition, enhance number sense and introduce new concepts in an engaging way.

Early use mathematics and poetry are sometimes depicted as pattern poetry. Pattern poetry is an art form that combines literary and visual elements to produce poems in which the text and the shape of the poem work together as an aesthetic whole (Glaz, 2012). Mathematical pattern poetry as a genre is further enriched by the graphing capabilities of modern computers and now entails a substantial mathematical components – including poems whose shape or content involve geometric figures, mathematical curves or other mathematical notions and symbols. Glaz(2012) provides an exposition on Riddle in the shape of an Octagon by the Mexican Nobel Laureate, Poet Octavio Paz. It involves 8 line segments radiating from a centre- an image that can be viewed as a division of an octagon. Following the poet's instructions one can recorder and place the 8 statements appearing under the geometric figure in the Octagon's "slices" to read an enhancing poem (Glaz, 2012). These activities can be perform as shown in Figure 1 and 2.

Similar use of poetry is also reported of the Nobel Laureate Wislawa Szymborska who gave voice to the historical uncertainty of the origin of the number 0 (Zero) in *A Poem in Honor of* (Glaz 2013). Outside these traditional contexts, mathematics written in poetic form appears regularly to this day, often in the service of mathematical pedagogy. For instance Civil (2007) reports empirical results that show that when mathematics and music were combined in the classroom the students exhibited higher achievement. The students in the experimental group had as much as a 60% growth as opposed to students in the control group whose scores were only 30% higher. The results show that using music was an effective way to teach because of the distinct similarities and uses of patterns in music and mathematics.

These outcomes and possibilities are, however, vague in the body of available literature from within Nigeria. Specifically, there seem to be a general lack-luster appeal of divergent instructional approaches among Nigerian mathematics teachers and mathematics educators as evidenced in the scarcity of research reports and expositions on culture-based pedagogy. To contribute to knowledge in this vital area of mathematics education, this exposition takes a holistic view of the opportunities for interesting mathematics pedagogy that can be harnessed from the popular Christmas carol *Twelve Days of Christmas*. The exposition first delved into the historical roots of *Twelve Days of Christmas*. This is followed by a consideration within the theoretical framework of realistic mathematics education. The core section on the mathematics in *Twelve Days of Christmas* is followed by a final deliberation on the educational implication of this particular Christmas carol for pupils at the basic education level in Nigeria.

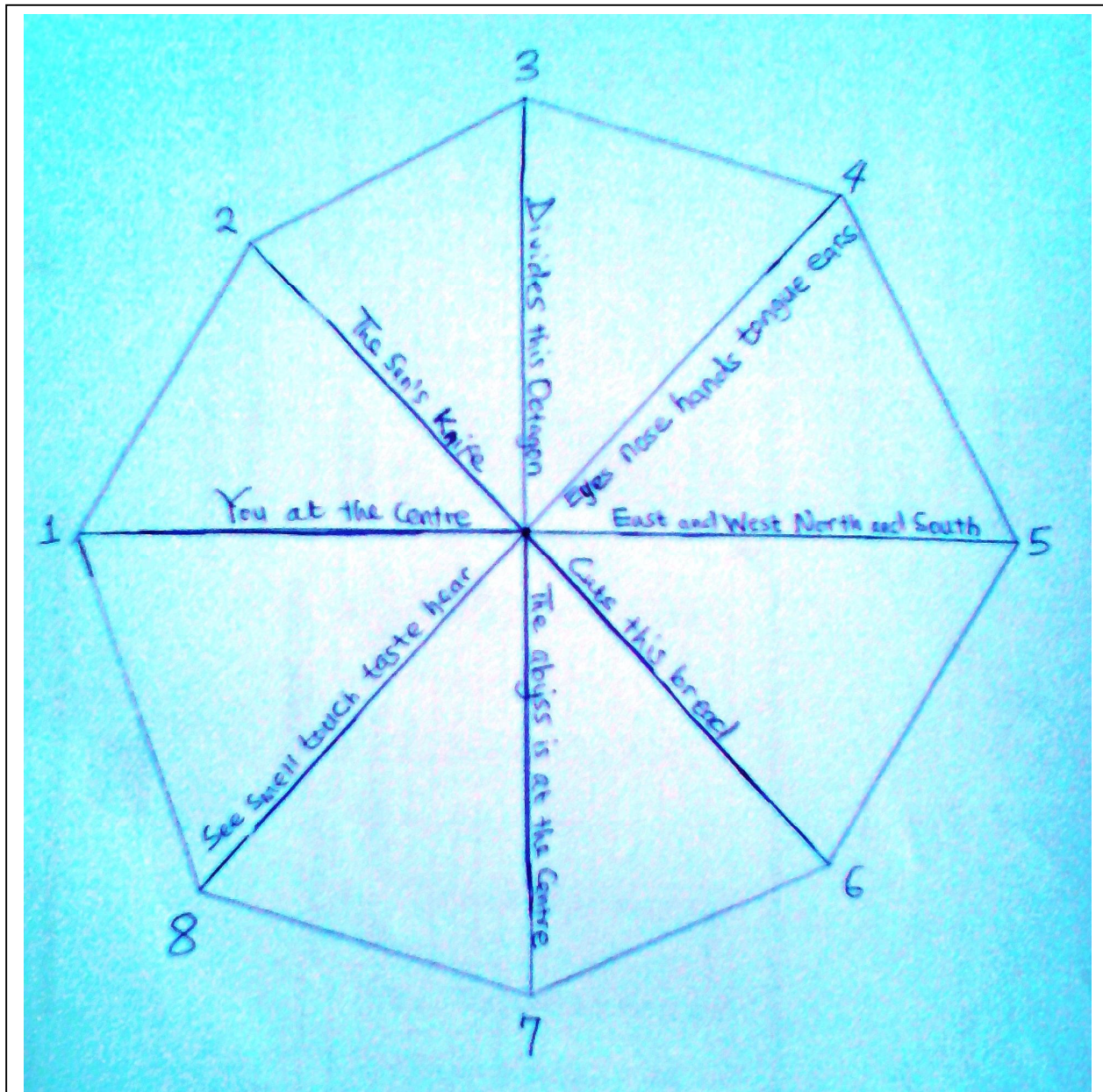
## 2. HISTORICAL BACKGROUND OF *TWELVE DAYS OF CHRISTMAS*

Since the Middle Ages, Christmas had been celebrated in much the same way as today. 25 December was the High Holy Day on which the birth of Christ was commemorated, and it kicked off an extended period of merriment, lasting until Twelfth Night on 5 January (Burton-Hill, 2014). Bratcher (2005) observes that the Twelve Days of Christmas is probably the most misunderstood part of the church year among Christians who are not part of liturgical church traditions. Contrary to much popular belief, these are not the twelve days before Christmas, but in the Western church are the twelve days from Christmas until the beginning of Epiphany on January 6<sup>th</sup>. Epiphany is traditionally celebrated as the time the three Wise Men or Magi arrived to present gifts to the young Jesus (Bratcher, 2005). The 12 days count from December 25<sup>th</sup> until January 5<sup>th</sup>.

The first "carols" had been heard in Europe thousand of years before the word was probably derived from the French "*Carole*" (Burton-Hill, 2014). Carols were folk songs; originally they were folk dances, for that's what "carol" meant: "a dance in a ring" (Forsyth, 2016). Historically, before the time of Edward White Benson, Bishop of Truon who later became Arch Bishop of Canterbury, Christmas carols had not been sung in the church but in the pub (Forsyth, 2016). That is why so many of them are really rather old for the piety of Christmas season. Carols tended to be pagan songs for events such as the Winter Solstice, until early Christians appropriated them (Burton-hill, 2014).

The Christmas carol *Twelve Days of Christmas* dates back to the 1700's, a time of severe religious persecution, with the song being written as a catechism song that could be sung without fear of persecution or arrest (Marlene, 2013). The rulership of the then English world was heavily influence by Puritanism, with Oliver Cromwell appointed the Puritan Lord Protector of England, Scotland and Ireland (Burton-Hill, 2014). According to the puritans, singing and related Christmas festivities were not only abhorrent but sinful, "popish" and wasteful tradition. In 1647, an Act of Parliament effectively banned the festival and in June 1647 the Long Parliament passed an Ordinance confirming the abolition of the feast of Christmas (Burton-Hill, 2014). For nearly two decades that

the ban on Christmas was in place, semi-clandestine religious services marking Christ's nativity continued to be held on 25 December and people continue to sing the carols in secret.



### **Full Poem**

You at the centre, east and West North and South  
 The sun's knife, cuts this bread  
 Divides this octagon, the Abyss is at the centre  
 Eyes nose, hands tongue ears, see smell touch taste hear.

**Figure 2: Riddle Solved and Full Poem**



Although this account has been questioned for its historical accuracy, the evidence to disprove it is not substantive either. Bratcher (2005) provides a concise review of the two opposing sides to the historical background of the *Twelve Days of Christmas*, concluding that “for the sake of historical accuracy, we need to acknowledge this uncertainty”.

The traditional (orthodox) church however, continues to propagate the richness of the symbology of the *Twelve Days of Christmas*. It is a known fact that many of the symbols of Christianity were not originally religious, but were appropriated from contemporary culture by the Christian faith as vehicles of worship and proclamation (Bratcher, 2005). According to Marlene (2013) on this ground, the religious symbolism of this carol can be summarized as:

- 1<sup>st</sup> Day: The Partridge in a pear tree is Christ Jesus upon the Cross
- 2<sup>nd</sup> day: The ‘two turtle doves’ represent the Old Testament and the New Testament
- 3<sup>rd</sup> Day: The “three French hens” symbolize faith, hope and love
- 4<sup>th</sup> Day: The “four calling birds” represent the four Gospels (Matthew, Mark, Luke and John)
- 5<sup>th</sup> Day: The “five golden rings” represent the first five books of the Old Testament, also known as the Hebrew Torah or the Pentateuch (Genesis, Exodus, Leviticus, Numbers and Deuteronomy).
- 6<sup>th</sup> Day: The “six geese a-laying” represent the six days of creation
- 7<sup>th</sup> Day: The ‘seven swans a-swimming’ represent the seven gifts of the Holy Spirit (prophesy, serving, teaching, exhortation, generosity, leading and compassion).
- 8<sup>th</sup> Day: The “eight maids a milking” represent the eight beatitudes of Matthew 5: 3-10.
- 9<sup>th</sup> Day: The “ladies dancing” represent the nine fruits of the Spirit (Love, Joy, Peace, Patience, Kindness, Goodness, Faithfulness, Gentleness and Self –control).
- 10<sup>th</sup> Day: The “ten lords a-leaping” are the Ten Commandments
- 11<sup>th</sup> Day: The ‘eleven pipers piping’ represent the eleven faithful Apostles
- 12<sup>th</sup> Day: The “twelve drummers drumming” symbolize the twelve points of the Apostle’s Creed

### 3. AN ARTIFACT FOR REALISTIC MATHEMATICS EDUCATION

Mathematics exists as a body of truths about relationships between abstract entities and structures. These abstract relationships are reflected or instantiated, in various forms and at different levels in the concrete structure of the physical world. Even in its most advanced form, the subject is rooted in reality with applications in everyday life (Abah, 2016). The significance of the interwoven relationship of the subject with real life contexts and settings is best represented by the theory of Realistic Mathematics Education (or RME).

Essentially, Realistic Mathematics Education is a domain-specific theory of mathematics which stress that rich, “realistic” situations are given prominent position in the learning process of the subject. “Realistic” in RME means students are offered problem situations which they can imagine, implying that problems presented to students can come from the real world but also from the fantasy world of fairy tales or the formal world of mathematics, as long as the problems are experientially real in the student’s mind (Vanden Heuvel-Punhuizen & Drijvers, 2014).

In RME, realistic situations serve as source for initiating the development of mathematical concepts, tools and procedures and as a context in which students can in a later stage apply their mathematical knowledge, which then gradually has become more formal and general and less context specific (Vanden Heuvel-Punhuizen & Drijvers, 2014). The two focal points of the RME paradigm are that mathematics must be connected to reality and the conception of mathematics as human activity. Mathematics education therefore, must be organized as a process of guided reinvention, where students can experience a similar process compared to the process by which mathematics was invented (Zulkardi, 2010). It is obvious throughout history, that mathematics has been used in activities concerned with the immediate environment and material reality of practitioners (Sepulcre, 2017).

The ideas generated from real life attachments to mathematical concepts can truly create conflictive situations in which students are encouraged to reflect upon the rules that define their action when dealing with the concepts (Bernardes & Roque, 2015 in Abah 2016). This approach results in both sensitive and historical thinking, mediated by bodies, signs, artifacts and cultural meanings (Guillemette, 2015 in Abah 2016), giving rise to a non-mentalist conception of thought (Abah, 2016). By adopting realistic mathematics education as a theoretical framework for classroom instruction, several researchers have been able to affirm enhanced mathematical literacy, increased psychological motivation, and linguistic and transverse competencies. A Malaysian study for instance, reports a significant difference in favour of RME approach in terms of achievement (Zakaria & Syamaun, 2017). Similarly, in a study carried at determining first year university students’ reflections when Fibonacci tiling, the ancient Chinese Fang Cheng procedures, and the ancient Indian Meru Prastara recursions were introduced as historical snippets in an adventure pedagogy for basic mathematics, Abah (2017) reports the artifacts aid in the concretizing of concepts, spurring of behavioural engagement in

learners, adding of aesthetic value to mathematics, sustaining of students attention, computational ease and effective recall of mathematical procedures.

Studies rooted in realistic mathematics continue to establish that everybody's mathematical education follows the path taken by our ancestors. Glaz (2010) expounds that it starts with counting in nursery rhymes which teach about numbers by playing with rhyming words, utilizing the power of poetry to engage learner's attention and enhance retention of abstract concepts. The *Twelve Days of Christmas* falls into this intriguing category of songs which can be used in the mathematics classroom to shape instructional content, to facilitate integration of material and to ease the transition from theory to application. This carol can shape mathematics course content by focusing attention on a particular aspect of the material taught in class and acting as a springboard to initiate class-wide or small group discussions, assignments or projects based on the poem's content.

Exemplars (2005) provide an easy to follow task guide for deploying the carol *Twelve Days of Christmas*. The song is a project that engages students around the holidays. Students who know the song are astounded that so many gifts are received. Most think that only 12 gifts are given (Exemplars, 2005). Tasks based on the song provide students with real challenges. They can recognize how important it is to read problems closely and observe patterns of numbers that emerge. By thinking through what the song says very carefully, students can set up a "system" to solve the problem and recognize patterns and symmetries (Exemplars, 2005). There is also the possibility of emphasizing interdisciplinary links from the usage of the song. Exemplars (2005) observed that it can also be integrated with history and mythology (for example the symbolic meaning of the presents). Students might want to think through the problems and set up and discuss solutions. At the appropriate levels, a different type of project may require students to compose their own poems about mathematical techniques or concepts (Glaz, 2010).

#### 4. THE MATHEMATICS IN *TWELVE DAYS OF CHRISTMAS*

*"Pure Mathematics is, in its way, the poetry of logical ideas"*

Albert Einstein

When The New York Times failed to publish an Obituary following the death of poet and Algebraist Amalie "Emmy" Noether, the most accomplished mathematicians of all time, Einstein, wrote a letter to the Editor to correct the omission. In the letter published in The Times on May 5, 1935, Albert Einstein noted Noether's accomplishments by making the astounding analogy in the opening quote of this section (Growney, 2017). The Nobel Prizewinner indeed reflects the compositional element of many mathematics' works and their singular ways of seeing and thinking (Watkinson, 2017).

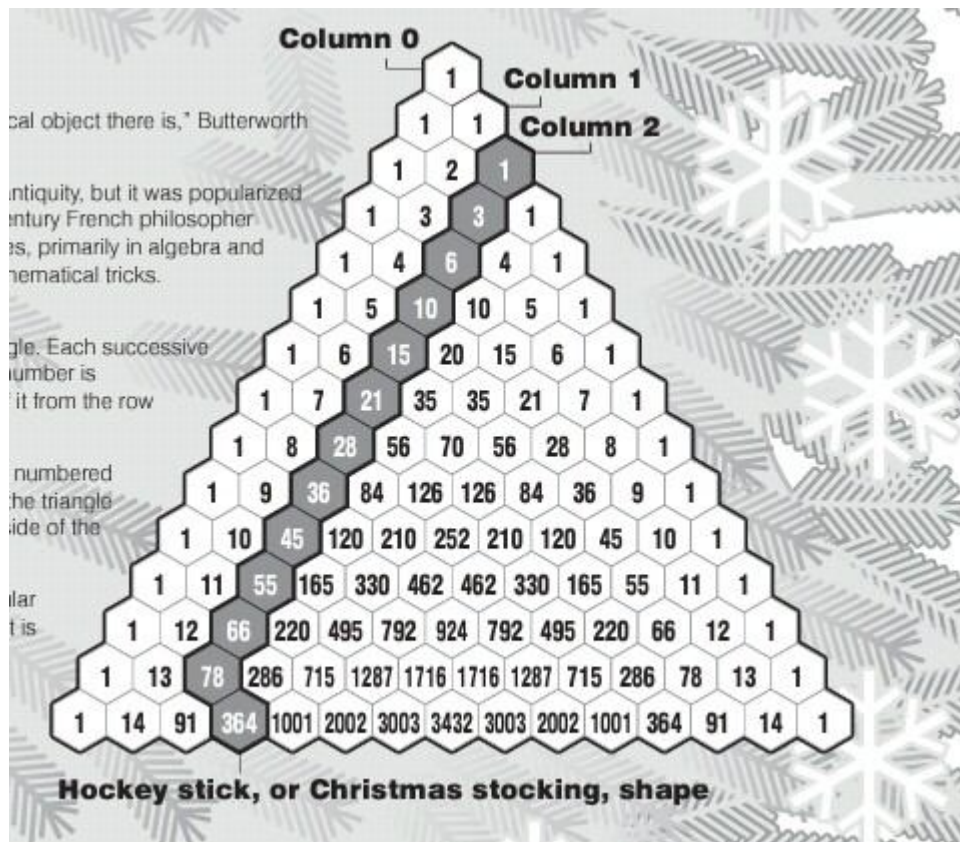
The poetry of *Twelve Days of Christmas* reveals the singer receives the gift of a partridge in a pear tree on one day, two turtle doves and a partridge in a pear tree on the second, three French hens, two turtle doves ..., which at any rate mounts to a lot of stuff by the twelfth day (Spice, 2008). So, the first mathematics that comes to mind from the song is in the question: how many gifts? The easiest, though longest, method to calculate the total is to simply add them up.

**Table 1** Adding up the gifts in Twelfth Days of Christmas

Day of Song	Number or Gifts
One	1 +
Two	(1+2) +
Three	(1+2+3)+
Four	(1+2+3+4)+
Five	(1+2+3+4+5)+
Six	(1+2+3+4+5+6)+
Seven	(1+2+3+4+5+6+7)+
Eight	(1+2+3+4+5+6+7+8)+
Nine	(1+2+3+4+5+6+7+8+9)+
Ten	(1+2+3+4+5+6+7+8+9+10)+
Eleven	(1+2+3+4+5+6+7+8+9+10+11)+
Twelve	(1+2+3+4+5+6+7+8+9+10+11+12) =
<b>Total</b>	<b>364</b>

Spice (2003) observes that the totals for each day, i.e., 1,3,6, 10, etc are what the ancient Greeks called “triangular numbers”, because they can be arranged in a compact triangular pattern. Triangular numbers have a number of interesting properties. In the case of calculating gifts in *Twelve Days of Christmas*, they can shorten the work. It is easy to see how many gifts are given on each day simply by scanning down the sequence of triangular numbers. The 10<sup>th</sup> triangular number (i.e.  $a_{10}$  in the sequence) for instance, is 55, so one immediately knows that 55 gifts were given on the 10<sup>th</sup> Day. To get a total for the entire 12 Days, the student just need to add up the first 12 triangular numbers.

Interestingly, the Pascal's Triangle relies on a Christmas tree-like pattern. Abah (2017) in emphasizing the ancient Indian MeruPrastararecursions provides a detailed overview of the combinatorics of the Pascal's Triangle. The triangle has numerous uses, primarily in algebra and in probabilities, and includes a number of odd mathematical tricks (Spice, 2003). For instance, if one sight down column two (remember the columns begin with column 0 down the right slant side of the triangle), the triangular numbers can be seen along the diagonal column, bending inwards after 78 to form a hockey-stick shape at 364 (see Figure 3 for a screenshot of this amazing number play as provided by Spice (2003)).



**Figure 3: Hockey-stick shape formed from the Pascal's Triangle**

This Hockey Stick Theorem works the same for any column within the Pascal's Triangle. It will be captivating for basic education pupils if they move their little fingers down any column and stop at any number they choose; then move their fingers to the next row numbers and move one column toward the centre of the triangle; that number will be the sum of the numbers in the initial column.

Another interesting activity that can be carried out by basic education pupils using the Christmas carol is to find out what it will cost each pupil to give his or her true love the gifts of the *Twelve Days of Christmas*. Fulton (2013) demonstrated this activity on a spreadsheet computer package, showing it could be a source of great fun determining present day costs of the bountiful gifts. In the absence of digital software, children can easily prepare a handwritten budget and analysis using the format in Table 2. Teachers

can provide a tentative price list of the items in the song and supervise how students prepare their cost per day and for the entire 12 days.

**Table 2** Costing the Gifts in Twelve Days of Christmas

Day	Gifts	Cost
First Day	1 partridge in a pear tree	
Second Day	2 turtle doves + 1 partridge in a pear tree	
Third Day	3 French hens + 2 turtle doves + 1 partridge in a pear tree	
Fourth Day	4 calling birds + 3 French Hens + 2 turtle doves + 1 partridge in a pear tree	
Fifth Day	5 gold rings + 4 calling birds + 3 French Hens + 2 turtle doves + 1 Partridge in a pear tree	
Sixth Day	6 geese a-laying + 5 gold rings + 4 calling birds + 3 French Hens + 2 turtle doves + 1 Partridge in a pear tree	
Seventh Day	7 Swans a -swimming + 6 geese a -laying 5 gold rings + 4 calling birds + 3 French Hens + 2 turtle doves + 1 Partridge in a pear tree	
Eighth Day	8 maids a-milking + 7 Swans a-swimming + 6 geese a-laying + 5 gold rings + 4 calling birds + 3 French Hens + 2 turtle doves + 1 Partridge in a pear tree	
Ninth Day	9 ladies dancing + 8 maids a-milking + 7 Swans a-swimming + 6 geese a-laying + 5 gold rings + 4 calling birds + 3 French Hens + 2 turtle doves + 1 Partridge in a pear tree	
Tenth Day	10 lords a-leaping + 9 ladies dancing + 8 maids a-milking + 7 Swans a-swimming + 6 geese a-laying + 5 gold rings + 4 calling birds + 3 French Hens + 2 turtle doves + 1 Partridge in a pear tree	
Eleventh Day	11 pipers + 10 lords a-leaping + 9 ladies dancing + 8 maids a-milking + 7 Swans a-swimming + 6 geese a -laying + 5 gold rings + 4 calling birds + 3 French Hens + 2 turtle doves + 1 Partridge in a pear tree	
Twelfth Day	12 drummers drumming + 11 pipers piping + 10 lords a-leaping + 9 ladies dancing + 8 maids a-milking + 7 Swans a-swimming + 6 geese a-laying + 5 gold rings + 4 calling birds + 3 French Hens + 2 turtle doves + 1 Partridge in a pear tree	
	<b>Total</b>	

There are online platforms and educational organizations which provide, as their special service, annual price list of the items on this carol. Tracey (2017) reports the PNC in 2017 celebrates 34<sup>th</sup> year of tallying the cost (in dollars) of the gifts in the classic holiday song. The PNC Christmas price Index predicts true loves may need to get ready to spend \$34,558.65 on the gifts in *Twelve Days of Christmas* for the year 2017. Mathematics teachers at the basic education level in Nigeria may find such table of price index handy for introducing pupil to Business Mathematics.

The Mathematics and the Environment (2013) research team at University of Exeter's Penryn Campus in Cornwall took the analysis of the mathematics in *Twelve Days of Christmas* a bit further by finding a formula for the number of presents for  $N$  days of Christmas. According to the computations suitable for undergraduates taking Mathematical Analysis course, the number of presents on each  $n$ -th day of Christmas is equal to the sum of the first  $n$  integers, known by the formula

$$1 + 2 + 3 + \dots + n = \sum_{k=1}^n k = \frac{n(n+1)}{2}$$

This formula can be proven by mathematical induction. So the sum of all presents from until the  $N$ -th day of Christmas is the sum of those sums from the preceding formula.

$$\sum_{n=1}^N (1 + 2 + 3 + \dots + n) = \sum_{n=1}^N \frac{n(n+1)}{2}$$

The product in the sum can be expanded to give

$$\sum_{n=1}^N (1 + 2 + 3 + \dots + n) = \sum_{n=1}^N \left( \frac{n^2}{2} + \frac{n}{2} \right)$$

which is half the sum of the squares of the first  $N$  integers plus half the sum of the first  $N$  integers. This implies that

$$\sum_{n=1}^N (1 + 2 + 3 + \dots + n) = \frac{1}{2} \sum_{n=1}^N n^2 + \frac{1}{2} \sum_{n=1}^N n = \frac{1}{2} \sum_{n=1}^N n^2 + \frac{1}{2} \frac{N(N+1)}{2}$$

It can also be proven by induction that the sum of the first  $n$  squares is

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

and so,

$$\sum_{n=1}^N (1 + 2 + 3 + \dots + n) = \frac{1}{2} ((2N^3 + 3N^2 + N) + 3(N^2 + N)) = \frac{1}{6} (N^3 + 3N^2 + 2N)$$

With this general formula, it is easy to calculate the number of gifts across any consecutive number of days. For  $N=12$ , this makes

$$\frac{1}{6} (12^3 + 3 * 12^2 + 2 * 12) = 364$$

Students can hence compute for  $N=13$ , even  $N=365$  days (i.e. one year round gifts in the *Twelve Days of Christmas* pattern). For  $N=365$ , total number of gifts

$$\frac{1}{6} (365^3 + 3 * 365^2 + 2 * 365) = 8,171,255 \text{ presents}$$

Another unique perspective may assume each person and each bird in the carol has two legs and asks what day of Christmas is it if there are now 200 legs in the home of the receiver (not including the legs of the receiver). MathCounts (2012) toes this line to investigate the day with 200 legs as follows:

1<sup>st</sup> Day: 2legs

2<sup>nd</sup> Day: The two turtle doves and new partridge bring another 6 legs to add to the 2 legs of the first day.  $2 + 6 = 8$  legs

This pattern continues until the 8<sup>th</sup> Day

8<sup>th</sup> Day: 8 maids and more swans, geese, calling birds, hens, turtle doves and another partridge join the previous group of legs with an additional 62 legs.

The total now is  $138 + 62 = 200$  legs.

Therefore, it must be the eighth day of Christmas.

If we refer to the formula for the sum of all presents from the first until the  $n$ -th day of Christmas, the total number of gifts as at the eight day will be

$$\frac{1}{6} (8^3 + 3 * 8^2 + 2 * 8) = 120 \text{ presents}$$

This can easily be picked out from Figures 3 (stopping at 36 and moving one column to the centre on the next row). Within these eight days of Christmas, every gift, but the five golden rings, brings an additional pair of legs, so it is necessary to determine how many of the 120 gifts are golden rings. The rings were given on days 5, 6, 7, and 8 for a total of  $5(4) = 20$  golden rings. If 20 presents are deducted from the total of 120 presents on the eighth day there are 100 gifts, each with a pair of legs.

From all the foregoing mathematical possibilities based on the carol, *Twelve Days of Christmas*, it appears the ingenuity of the teacher is what is paramount in harnessing captivating mathematical lessons from the song. This, of course, holds great implications for basic education in Nigeria.

## 5. EDUCATIONAL IMPLICATION OF *TWELVE DAYS OF CHRISTMAS*

Augmenting routine classroom instruction with relevant and related mathematical facts raises the aesthetic value of the subject. The target of this style of mathematics instruction is to stimulate curiosity and sustain interests in students, while enriching the repertoire of the mathematics teachers by enriching their pedagogical content knowledge (Abah, 2017). When poetry-based materials are employed to persuade students to think about instructional content in mathematics, they will be more capable of applying and extending what they learned to new situations, with the instructor facilitating both understanding and integration of subject matter (Glaz & Liang, 2009). Such alternative discourse in which learners could explore mathematical ideas strengthens students' cognitive understanding of mathematical concepts and simultaneously bolsters students' confidence in carrying out mathematical operations.

Poems and songs such as the *Twelve Days of Christmas* when deployed in instructional context allow group and individual learning, team building, reinforce teaching concepts, and expose students of all ages to singing. Additionally, mathematical lyrics aid recall, reinforce mathematical thinking processes, connect to history, connect to the real world and humanize mathematics (Lesser, 2014). Lesson from the use of the carol in the classroom may spur an innovative basic school into organizing mathematics poetry competitions.

In catering for students who are not deemed kinesthetic, an opportunity could be created where students are allowed to express themselves in poetic form (Ministry of Education 2010). In this type of competition, students will write poems to reflect different mathematical concepts. The best poem will be identified according to how much it embodies a particular mathematical concept as well as creativity. Poems could be written at the end of a topic. The writer of the best poem could be given prizes or highlighted on the school's notice board (Ministry of Education, 2011).

The *Twelve Days of Christmas* can be used at the Upper Basic education level in Nigeria to create extra challenges for students. Active Learning in Maths (n. d.) for instance, suggests group activities in which students can create a page of a catalogue, advertising each present and be asked how they would persuade people to buy them. Students can be made to re-write their own versions of the song with more modern presents and provide an accompanying price list.

For students taking advance courses in computer science, the pattern revealed in *Twelve Days of Christmas* holds further significance. The carol fall into a class of folk songs and holiday songs in which each verse consists of the previous verse, with one extra line added on. The song tends to last a long time, despite having relatively short scripts. According to Kleinberg and Tardos (2006) one can convey the words plus instructions for the song by specifying just the new line that is added to each verse without having to write all the previous line each time. This asymptotic property can be deployed in algorithm development to encode such a song using a script that has length  $f(n)$  which grows as slowly as possible and runs for  $n$  words total (Kleinberg & Tardos, 2006)

Furthermore, Abah (2017) recommends that mathematics educators in particular and teachers in general consider the epistemological and cultural approach in the design and delivery of instruction. Introducing ideas from a wide cultural background into classroom teaching, as emphasized for the *Twelve Days of Christmas*, not only demystify the abstract concepts, but foster a sense of admiration of past contributors to knowledge and build openness and global intellectual relationship in learners. The visual and musical appeal of the Christmas carol lend it as tools for aiding the teaching of students known to have mathematics learning difficulty.

Evidently, only a high level of co-ordination on the part of the mathematics teacher can prevent mathematics instruction from being reduced to mere data transmission, a process in which there is no sharing of excitement and wonder (Abah, 2016). The teacher of mathematics is expected to first develop himself or herself at the personal level in the art of building openness and honest intellectual relationships with students by leveraging on the connecting power of rudimentary mathematical artifact like the *Twelve Days of Christmas* in instruction delivery. Abah (2016) emphasizes that every mathematics class should be loaded with rich learning experiences based on the teacher's ability to connect the past to the present and reorganize information without prejudice to distance from the current era. The *Twelve Days of Christmas* offers such a loaded opportunity to the innovative mathematics teacher, be it at the basic, secondary or tertiary education levels.

## 6. CONCLUSION

This exploratory review has attempted to project the mathematical significance of the Christmas Carol, *Twelve Days of Christmas*. The song was projected as being more prevalent in the Nigerian Christmas culture now than ever. As an artifact rooted in the fantasies of children and grown-ups alike, the carol can be deployed in the paradigm of Realistic Mathematics Education to use the transition from theory to application and create different workable projects. The interesting mathematics present in the children song holds exciting significance for education at all levels in Nigeria.

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