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First Predictions of the Angular Power Spectrum of the Astrophysical Gravitational Wave Background

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We present the first predictions for the angular power spectrum of the astrophysical gravitational wave background constituted of the radiation emitted by all resolved and unresolved astrophysical sources. Its shape and amplitude depend on both the astrophysical properties on galactic scales and on cosmological properties. We show that the angular power spectrum behaves as $C_\ell \propto 1/\ell$ on large scales and that relative fluctuations of the signal are of order 30% at 100 Hz. We also present the correlations of the astrophysical gravitational wave background with weak lensing and galaxy distribution. These numerical results pave the way to the study of a new observable at the crossroad between general relativity, astrophysics, and cosmology.

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Introduction.—The detection of the first gravitational wave (GW) source [1] by the Advanced Laser Interferometric Gravitational-wave Observatory (LIGO) has triggered the birth of a new window in astronomy. Gravitational-wave astronomy has so far mostly focused on the study of resolved sources. However, complementary information may come also from the study of the superposition of signals from unresolved astrophysical sources.

Indeed, astronomical observations include diffuse stochastic backgrounds of radiations due to the superposition of the signals from all, resolved and unresolved, sources. The electromagnetic backgrounds include the cosmic microwave background (CMB) with a blackbody spectrum [2] and the extragalactic background made up of all the electromagnetic radiation emitted by stars, galaxies, galaxy clusters, etc., since their formation [3–5]. Similarly, there exists a neutrino background [6] and indeed a GW background.

This latter has several components contributing at different frequencies and with different statistical properties [7,8]; its spectrum is defined by the dimensionless density parameter

$$\bar{\Omega}_{\text{GW}}(\nu_O) = \frac{\nu_O}{\rho_c} \frac{d\rho_{\text{GW}}}{d\nu_O}, \quad (1)$$

where $\rho_c = 3H_0^2/8\pi G$ is the critical density of the Universe and ν_O is the frequency measured by the observer. In standard cosmology, one expects a primordial component

of GW produced during inflation but also during preheating [9]. Many other more speculative sources have been considered; see, e.g., Refs. [10,11]. In addition, the astrophysical GW background (AGWB) contribution arises from the superposition of a large number of unresolved astrophysical sources since the beginning of stellar activity.

This Letter focuses on this last component of the AGWB to which many sources may contribute such as merging binary black holes (BH) and neutron stars (NS) [12–17], supermassive binary BH mergers [18–22], rotating NS [23–25], and stellar core collapses [26,27]. Its properties depend on (i) local astrophysics through the energy spectrum of each kind of sources, (ii) the host galaxies through the abundance of these systems and their evolution, and (iii) cosmology through the distribution of the large scale structure and history of merging of galaxies and clusters. It follows that the AGWB depends on the direction of observation \mathbf{e} ,

$$\Omega_{\text{GW}} \equiv \frac{\nu_O}{\rho_c} \frac{d^3\rho_{\text{GW}}(\nu_O, \mathbf{e})}{d\nu_O d^2\mathbf{e}} = \frac{\bar{\Omega}_{\text{GW}}}{4\pi} + \delta\Omega_{\text{GW}}(\mathbf{e}, \nu_O). \quad (2)$$

Our goal is to present the properties of the anisotropic part of the AGWB. In the formalism we introduced in Refs. [28,29], each galaxy is described by a GW luminosity $\mathcal{L}_G(\eta, \nu_G, \theta_G)$ that depends on time (η), on the frequency in the galaxy rest frame (ν_G), and on the properties of the galaxy (θ_G , that includes, e.g., its mass, metallicity). This

quantity depends on all the GW sources and their galactic distribution, and therefore it represents a great source of new astrophysical information. The observed signal in a given direction is the “sum” of the contributions over all galaxies in that direction. As it depends on the galaxy distribution, it is related to cosmology. $\delta\Omega_{\text{GW}}(\mathbf{e}, \nu_O)$ depends on the history of the large scale structure such as the mergers of galaxies and clusters but also on the initial power spectrum inherited from inflation. It is also correlated to other cosmological probes. While the theory predicts the total GW energy density in a given direction, GW detectors are sensitive to the GW strain. The link between them has been fully clarified in our former work [29]. Our formalism, fully developed in Refs. [28,29], has been applied to the case of cosmic strings in Ref. [30].

We generalize previous works [7,14] in which the sources were assumed to be homogeneously and isotropically distributed, hence focusing on the monopole of the distribution. Its amplitude has been bound by LIGO [31,32], $\bar{\Omega}_{\text{GW}} < 5.6 \times 10^{-6}$ for $\nu_O \in [41.5, 169.25]$ Hz. Analysis of data from the Pulsar Timing Array (PTA) leads to $\bar{\Omega}_{\text{GW}} < 1.3 \times 10^{-9}$ for $\nu_O = 2.8 \times 10^{-9}$ Hz [33]. The possibility of mapping the GW background is discussed in Refs. [34–39] and the reconstruction of an angular resolved map of the sky in Refs. [40,41]. Note that some early constraints on the anisotropy have already been obtained by the PTA [42,43]. This is a slowly emergent field for which predictions are necessary to guide its observational prospective.

Our Letter gives the first prediction of the angular power spectrum of the AGWB and its correlations with other cosmological probes. As such, this is an important prediction that opens a new field of research at the

crossroad between general relativity, astrophysics, and cosmology.

AGWB energy density.—In standard cosmology, the universe is described by a Friedmann-Lemaître spacetime with perturbations that describe the large scale structure. In Newtonian gauge, its metric is $ds^2 = a^2[-(1 + 2\Phi)d\eta^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j]$, where the scale factor a is a function of conformal time η and Φ and Ψ are the two gravitational potentials. In the theory of cosmological perturbations, any variable, $X(\eta, x^i)$ for instance, is a stochastic field. It can be decomposed in Fourier modes, $X(\eta, \mathbf{k})$, which can be split as the product of a transfer function and of the initial metric perturbation as $X(\eta, \mathbf{k}) = X_k(\eta)\Phi^P(\mathbf{k})$. The power spectrum of $\Phi^P(\mathbf{k})$ is predicted, e.g., from inflation and constrained from CMB analysis (we use Planck satellite [44] cosmological parameters). Linear transfer functions are obtained from CMBQUICK [45] and we use Halofit [46] to account for the nonlinearities in the matter power spectrum.

As any observable on the sphere, $\delta\Omega_{\text{GW}}(\mathbf{e}, \nu_O)$ can be decomposed in spherical harmonics. Its angular correlation function $\langle \delta\Omega_{\text{GW}}(\mathbf{e}_1, \nu_O)\delta\Omega_{\text{GW}}(\mathbf{e}_2, \nu_O) \rangle$ is, thanks to statistical isotropy, a function of $\cos\theta \equiv \mathbf{e}_1 \cdot \mathbf{e}_2$ and can be decomposed in Legendre polynomials to define the angular power spectrum $C_\ell(\nu_O)$ as

$$C_\ell(\nu_O) = \frac{2}{\pi} \int dk k^2 |\delta\Omega_\ell(k, \nu_O)|^2. \quad (3)$$

The term $\delta\Omega_\ell(k, \nu_O)$ has been derived in our previous analysis [28], where all the details can be found, as

$$\begin{aligned} \delta\Omega_\ell(k, \nu_O) = & \frac{\nu_O}{4\pi\rho_c} \int_{\eta_*}^{\eta_O} d\eta \mathcal{A}(\eta, \nu_O) \left[\left(4\Phi_k(\eta) + b\delta_{m,k}(\eta) + (b-1)3\mathcal{H}\frac{v_k(\eta)}{k} \right) j_\ell(k\Delta\eta) - 2kv_k(\eta)j'_\ell(k\Delta\eta) \right] \\ & + \frac{\nu_O}{4\pi\rho_c} \int_{\eta_*}^{\eta_O} d\eta \mathcal{B}(\eta, \nu_O) [-\Phi_k(\eta)j_\ell(k\Delta\eta) + kv_k(\eta)j'_\ell(k\Delta\eta)] \\ & + \frac{\nu_O}{4\pi\rho_c} \int_{\eta_*}^{\eta_O} d\eta [6\mathcal{A}(\eta, \nu_O) - 2\mathcal{B}(\eta, \nu_O)] \int_{\eta}^{\eta_O} d\tilde{\eta} \Phi'_k(\tilde{\eta}) j_\ell(k\Delta\tilde{\eta}). \end{aligned} \quad (4)$$

This expression involves three types of quantities. First, background quantities such as \mathcal{H} , the Hubble parameter in conformal time, and the look-back time, $\Delta\eta = \eta_O - \eta$. Then, the cosmological perturbations include the gravitational potential $\Psi_k \simeq \Phi_k$, the velocity field v_k , and the matter density δ_m . The bias b relates the galaxy overdensity to δ_m and is defined as their ratio in the comoving gauge, see, e.g., Ref. [47]. We spot in this expression the contribution from galaxy overdensity, Sachs-Wolfe-like, and Doppler-like terms. Terms in the last line come from the integrated Sachs-Wolfe effect. The j_ℓ are spherical

Bessel functions, and \mathcal{A} and \mathcal{B} are related to the luminosity function of a galaxy per unit of emitted frequency $[\nu_G \equiv (1+z)\nu_O]$

$$\mathcal{A}(\eta, \nu_O) \equiv a^4 \bar{n}_G(\eta) \int d\theta_G \mathcal{L}_G(\eta, \nu_G, \theta_G), \quad (5)$$

$$\mathcal{B}(\eta, \nu_O) \equiv a^3 \nu_O \bar{n}_G(\eta) \int d\theta_G \frac{\partial \mathcal{L}_G}{\partial \nu_G}(\eta, \nu_G, \theta_G). \quad (6)$$

Astrophysical model.—The computation of \mathcal{A} and \mathcal{B} requires (i) the luminosity $\mathcal{L}_G(z, \nu_G, M_G)$ of a galaxy with halo mass M_G at redshift z and (ii) to sum it over the entire galaxy population using the halo mass function $dn/dM(M, z)$ calibrated to numerical simulations [48]. Our computation follows the formalism we developed in Refs. [14,16,49]. It has three main steps: (i) the determination of the emitted energy spectrum $dE/d\nu$ of each type of GW sources, (ii) the evolution of the population of this type of sources in a galaxy of mass M_G , and (iii) the integration over the galaxy population.

Galaxies contain many sources of GW. We concentrate on the background from merging binary BHs which contribute to the range of frequencies between a few Hz to a few hundreds of Hz. We describe the emitted energy using the results of Ref. [50].

Let us turn to the computation of the number of BH mergers per unit time in a galaxy of mass M_G .

First, we calculate the formation rate of binary BHs, \mathcal{R}_f . It depends on the star formation rate (SFR) $\psi(M_G, t)$, given in units of M_\odot/yr and on the evolution and lifetime of these stars, which is determined by their initial mass and metallicity. We use the SFR of Ref. [51] to calculate the mass in stars of a galaxy. It provides a fit to a large number of observables, including the stellar mass function, the specific SFR and cosmic star formation rates from $z = 0$ to $z = 8$ and in the halo mass range of 10^9 – $10^{15} M_\odot$. Then, we use the Salpeter initial mass function (IMF) [52] to describe the number of stars per unit total stellar mass formed, $\phi = dN/dM_* dM_{\text{tot},*} \propto M_*^{-2.35}$, where M_* is the mass of the star at birth.

Second, we need to describe the evolution of a star of initial mass M_* . Its lifetime $\tau(M_*, Z_*)$ and the end point (BH or NS) of its evolution depend on both M_* and its metallicity. We use the stellar evolution model by Ref. [53] to obtain the function $m = g_s(M_*, Z)$ that gives the mass of the BH formed for a star with initial (M_*, Z) and Ref. [54] to calculate stellar lifetimes $\tau(M_*, Z_*)$. Typically, massive stars ($M_* \gtrsim 8 M_\odot$) explode as supernovae or collapse to form BH on a timescale of a few Myr. If we assume such short stellar lifetimes, the stellar metallicity tracks the metallicity of the interstellar medium (ISM) given by $Z = Z(M_G, z)$. We adopt the prescription of Ref. [55] for the ISM metallicity as a function of galaxy mass and redshift.

Under these assumptions, the instantaneous rate of BH formation at a given cosmic time t and for a galaxy M_G in units of events per unit BH mass m is given by $\mathcal{R}_1(m, t) = \psi[M_G, t - \tau(M_*)] \phi(M_*) dM_*/dm$, where $M_*(m)$ and dM_*/dm are deduced from the relation $m = g_s(M_*, Z)$. We then assume that only a fraction $\beta = 0.005$ of these BHs resides in binary systems so that the rate of formation of the latter is $\mathcal{R}_2(m, t) = \beta \mathcal{R}_1(m, t)$ (chosen to match the mean GW density in Refs. [14,16]). And, following Ref. [49], the birth rate of binaries with

component masses $(m, m' \leq m)$ is $\mathcal{R}_{\text{bin}}(m', m) = \mathcal{R}_2(m) \mathcal{R}_2(m') P(m, m')$, where $P(m', m)$ is the 2-mass distribution of binaries. We assume $P(m, m') = \text{const}$ in the following. The merger rate is obtained once the time to coalescence of the binaries is known. This requires the distribution of the orbital parameters (a_f, e_f) at the time of formation. Since orbits circularize quickly, we assume zero eccentricity for all the binaries. We also assume that the distribution of the semimajor axis at formation is $f(a_f) \propto a_f^{-1}$ with cutoff at $a_{f,\text{min}} = 0.2 \text{ AU}$ and $a_{f,\text{max}} = 5000 \text{ AU}$ [14]. Hence, the birth rate of BH binaries (per unit mass squared per unit time and per units of a_f) is $\mathcal{R}_f[m, m', a_f, t] = \mathcal{R}_{\text{bin}}(m', m) f(a_f)$ from which we deduce that the merger rate at time t is $\mathcal{R}_m[m, m', a_f, t] = \mathcal{R}_f[m, m', a_f, t - \tau_m(m, m', a_f)]$ where $\tau_m(m, m', a_f)$ is the merger time of the system (m, m', a_f) .

The GW luminosity is the overall contribution of the mergers, i.e.,

$$\mathcal{L}_G = \int dmdm' da_f \frac{dE}{d\nu} \mathcal{R}_m[m, m', a_f, t]. \quad (7)$$

Its integration (5) on θ_G reduces to an integration on M_G , hence weighted by the halo mass function and gives \mathcal{A} presented on Fig. 1. We check that $\nu_O \int \mathcal{A}(\nu_O, \eta) d\eta / \rho_c$ matches with the mean GW density $\bar{\Omega}_{\text{GW}}(\nu_O)$ computed in Refs. [14,16], where the BH merger rate was normalized to the observed rates.

Angular power spectrum.—Figure 2 presents the C_ℓ 's for different frequency bands. We estimate in Ref. [56] that on large scales, we have approximately

$$\left(\ell + \frac{1}{2}\right) C_\ell(\nu_O) \simeq \left(\frac{\nu_O \mathcal{A}(\eta_O, \nu_O) b(\eta_O)}{4\pi\rho_c}\right)^2 \int_{k_{\text{min}}} P_\delta(k) dk, \quad (8)$$

where $P_\delta(k)$ is the matter power spectrum today, $k_{\text{min}} \equiv 1/\eta_O$ and we are considering a scale-independent bias

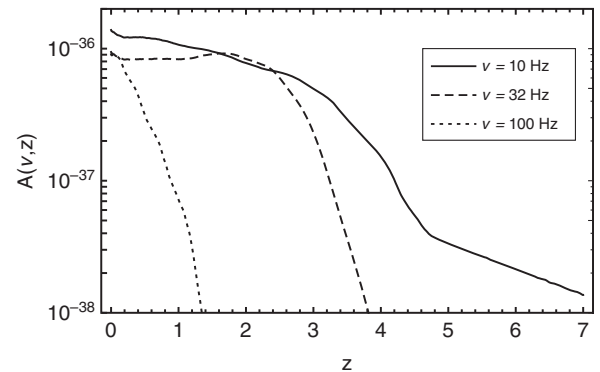


FIG. 1. The astrophysical source function $\mathcal{A}(\nu_O, z)$ as a function of redshift, for different frequency bands.

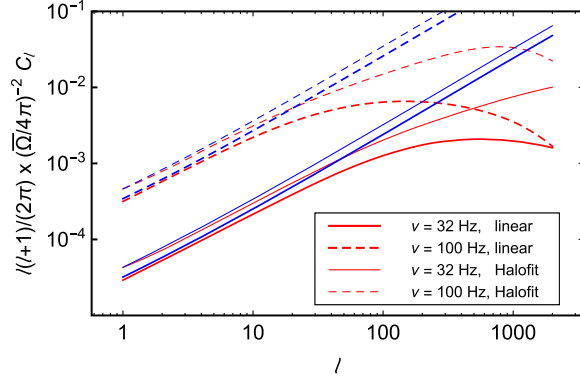


FIG. 2. Angular power spectrum of AGWB density fluctuation normalized to the monopole, for linear and nonlinear (Halofit) spectra of density fluctuations. The straight blue lines are the associated large scale approximations (8).

$b \propto \sqrt{1+z}$; see, e.g., Refs. [57,58]. Clearly, nonlinearities shift the amplitude but do not alter significantly the shape of the angular power spectrum. They affect both small and large angular scales because the function \mathcal{A} extends up to today, in contrast with what happens, e.g., for the CMB, or galaxy surveys when considering a given redshift bin. The variance of Ω_{GW} due to the distribution of the large scale structures is

$$\sigma_{\text{GW}}^2(\nu_O) \equiv \sum_{\ell} \frac{(2\ell+1)}{4\pi} C_{\ell}(\nu_O). \quad (9)$$

Considering multipoles up to $\ell_{\text{max}} = 2000$, we find $4\pi\sigma_{\text{GW}}/\bar{\Omega}_{\text{GW}} \simeq 0.14$ (0.32) for $\nu_O = 32$ Hz ($\nu_O = 100$ Hz), that is variations of the AGWB are typically of order 14% (32%) at 32 Hz (100 Hz). When using only the linear power spectrum, these values are approximately halved.

Cross-correlations.—Since Eq. (4) depends on cosmological perturbations, it correlates with any other cosmological probe, as galaxy number counts and weak lensing convergence. The cross-correlation power spectra have been computed in Ref. [28],

$$B_{\ell}^X(\nu_O) \equiv \frac{2}{\pi} \int dk k^2 \frac{4\pi}{\bar{\Omega}_{\text{GW}}(\nu_O)} \delta\Omega_{\ell}^*(k, \nu_O) X_{\ell}(k), \quad (10)$$

with $X_{\ell} = \kappa_{\ell}$ for cosmic convergence given by Eq. (100) of Ref. [28] and $X_{\ell} = \Delta_{\ell}$ for galaxy number counts given by Eq. (129) of Ref. [28] in the Kaiser approximation, on which we also add the effect of weak-lensing convergence as in Eq. (13) of Ref. [47]. They are depicted in Fig. 3.

Conclusions and perspectives.—This Letter has presented the first numerical computation of the AGWB power spectrum and its correlations with galaxy number counts and weak lensing following the formalism we developed in Refs. [28,29]. These spectra depend on cosmology through the transfer functions and the initial power spectrum and on astrophysics through the merger history of galaxies, the SFR, IMF, and stellar evolution that determine the mass distribution of BH and NS. The GW luminosity function also depends on the distribution of the initial orbital parameters of the binaries, which influences their lifetime and on general relativistic models for the emitted energy of each type of sources. Indeed, many ingredients of this calculation [such as the distribution of the orbital parameter, the function $P(m, m')$, the contribution of other GW sources] come with an associated uncertainty and for this study we have adopted standard prescriptions. The effect of these choices on the monopole of the AGWB was discussed in Ref. [14] and a full analysis of their impact, as well as the one of the cosmological parameters and of the nonlinear regime, on the AGWB power spectra will be presented in a companion article [56].

This analysis has already determined the general shape of the power spectrum and the variance of $\delta\Omega_{\text{GW}}$. To get closer to observations, one will also need to design estimators of this spectra and evaluate their signal-to-noise ratio for forthcoming experiments. The observation of the AGWB and its anisotropies will convey to us information about both astrophysics and cosmology. In particular, comparing predictions to observations will allow us to put new types of constraints on astrophysical models. The study of the AGWB can impact astrophysics as much as CMB did for cosmology during the past decades.

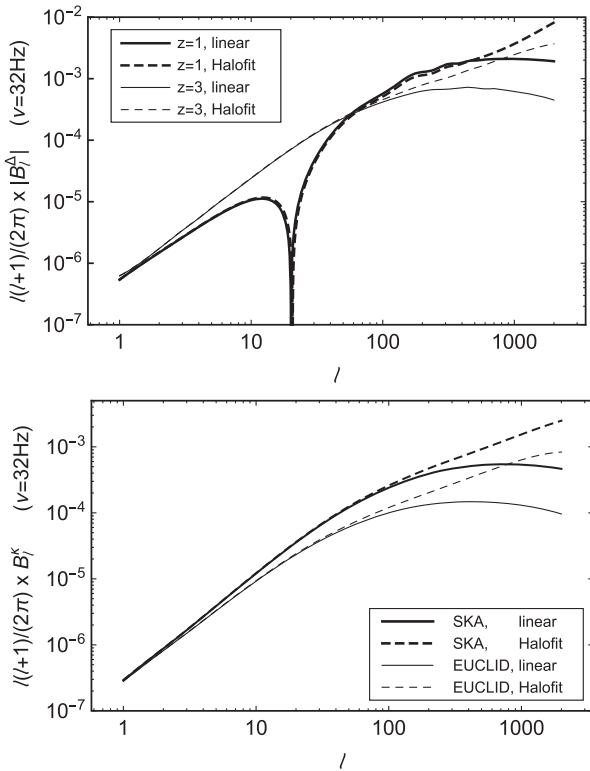


FIG. 3. Top: cross-correlation with galaxy number counts. Bottom: cross-correlations with convergence (κ) using the SKA [59] and Euclid [60] redshift distributions.

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