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*DE BROGLIE WAVES AND QUANTIFICATION OF KEPLERIAN ORBITS
IN A PLATONIC QUADRIDIMENSIONAL SPACE*

*ONDE DE PHASE DE DE BROGLIE
ET QUANTIFICATION DES ORBITES KÉPLÉRIENNES
DANS UN ESPACE QUADRIDIMENSIONNEL PLATONICIEN*

Alain Jégat

Abstract

The properties of the platonic quadridimensional space, the modeling of De Broglie waves and the resulting concept of mass (see hal-01165196; hal-01207447; hal-01213447) seem to allow a quantum approach to the laws of gravitation by a postulate of quantified declination.

The quantized declination postulate proposed in the article hal-01584918 is here supplemented by a ***second quantized declination postulate*** concerning, this time, the direction of the absolute velocities of the moving bodies.

These absolutes quanta are ***independent of the chosen reference frames and of the rest masses of the considered bodies***. They are related to the De Broglie waves generated by the massive bodies in gravitational interaction and lead, surprisingly, to the well-known characteristics of Keplerian orbits.

We propose here an introduction, restricted to the relatively simple case where the guiding angle of the observation frame is equal to $\frac{\pi}{2}$.

Résumé

Les propriétés relativistes de l'espace quadridimensionnel platonicien, la modélisation de l'onde de phase et le concept de masse qui en découlent (cf. les documents hal-01081576; hal-01205805; hal-01213062) semblent permettre une approche quantique des lois de la gravitation.

Le postulat de déclinaison quantifiée proposé dans l'article hal-01577669 est ici enrichi d'un ***second postulat de déclinaison quantifiée*** concernant, cette fois, la direction des vitesses absolues des corps en mouvement.

Ces quanta absolus sont ***indépendants des référentiels d'observation choisis et des masses au repos des corps considérés***. Ils sont liés aux ondes de phase de De Broglie générées par les corps massifs en interaction gravitationnelle et conduisent rapidement, de façon surprenante, aux caractéristiques bien connues des orbites képlériennes.

Nous en proposons ici une introduction, restreinte au cas relativement simple où l'angle directeur du référentiel d'observation est égal à $\frac{\pi}{2}$.

1. The geometrical framework

This modeling is based on the Platonic space outlined in the following articles:

« UN MODÈLE PLATONICIEN (EUCLIDIEN-PROJECTIF) POUR LA THÉORIE DE LA RELATIVITÉ RESTREINTE » (pré-publication hal-01081576, version 1).

« A PLATONIC (EUCLIDEAN-PROJECTIVE) MODEL FOR THE SPECIAL THEORY OF RELATIVITY » (pre-publication hal-01165196, version 1).

$(O, \vec{i}, \vec{j}, \vec{k}, \vec{h})$ is a frame for the four-dimensional Euclidean space whose axes are denoted $(OX), (OY), (OZ), (OW)$; the direction of the projection is that of the vector \vec{h} .

Following the hal-01207447 v1 and hal-01213447 v1 articles, the notion of relativistic mass of a particle is described here as a result of its interaction with a stratification of the four-dimensional Platonic space by a sequence of hyperplanes $H^{(n)}$ which are orthogonal to the direction of the projection \vec{h} , regularly spaced by a distance $\Delta w_0 > 0$.

This distance Δw_0 is equal to the **Compton wavelength** of the particle in question (for example,

for an electron, $\Delta w_e = \frac{h}{m_e c} \approx 2,426.10^{-12}$, where m_e denotes the rest mass of the electron).

These concepts are detailed in the HAL articles below:

hal-01165196, v1 : A platonic (euclidean-projective) model for the special theory of relativity.

hal-01207447, v1 : Towards a modeling of De Broglie waves in a platonic quadridimensional space.

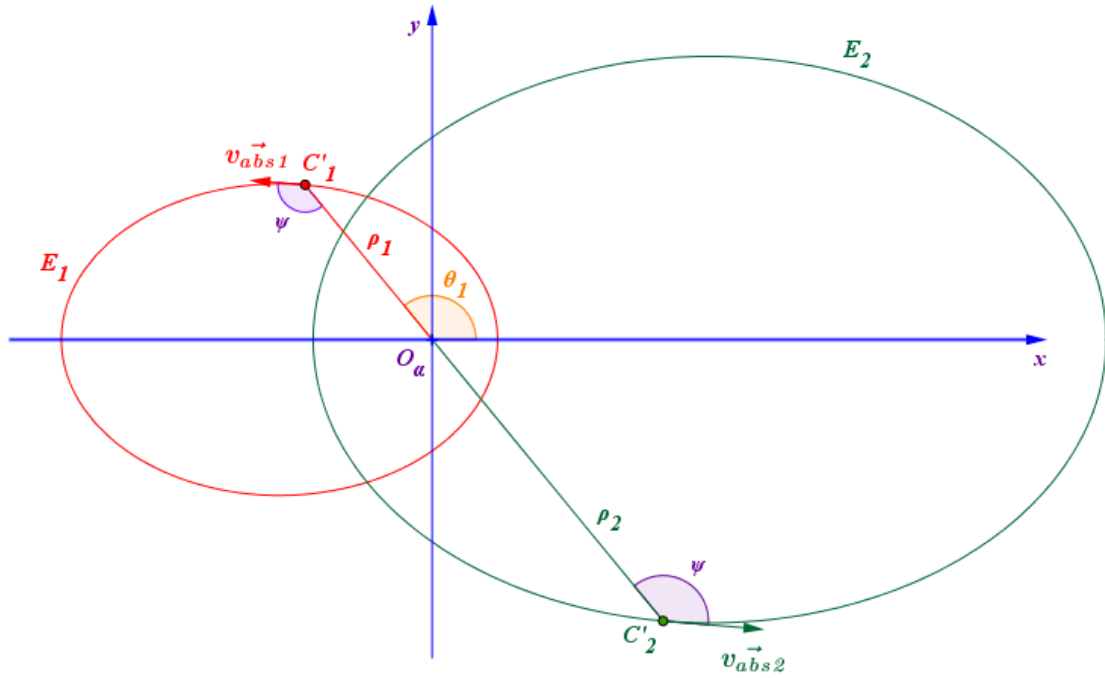
hal-01213447, v1 : An idea of the mass of a particle in a platonic quadridimensional space.

hal-01340134, v1 : One-dimensional elastic collisions in a platonic quadridimensional space.

hal-01378215, v1 : About time measurement in a platonic quadridimensional space.

hal-01584918, v1 : A quantified approach to the laws of gravitation in a platonic quadridimensional space.

2. Presentation of the object of this study



Observers of a reference frame R_α study the movement of two bodies C'_1 and C'_2 in gravitational interaction.

These two objects are in fact the projections of two bodies C_1 and C_2 moving in the Platonic space (see also the diagram in paragraph 3).

Let us denote by β_1 and β_2 the measures of the angles formed by the velocities \vec{v}_1 and \vec{v}_2 of these two bodies with the hyperplanes of respective equations $W = W(C_1)$ and $W = W(C_2)$; by ε_1 and ε_2 the measures of the angles formed by the vector \vec{i} and the absolute velocities \vec{v}_{abs1} and \vec{v}_{abs2} of the observed bodies C'_1 and C'_2 : $\varepsilon_1 = (\vec{i}, \vec{v}_{abs1})$ and $\varepsilon_2 = (\vec{i}, \vec{v}_{abs2})$.

We shall introduce a **double postulate of quantified declinations** of these angles (generated by the De Broglie waves linked to their masses), which, by generating the mutual accelerations of these two bodies in the Platonic space, will show us that the **elliptic trajectories** of C'_1 and C'_2 here observed in the reference frame R_α are the **projections of the quantified quadridimensional trajectories** thus obtained from the bodies C_1 and C_2 .

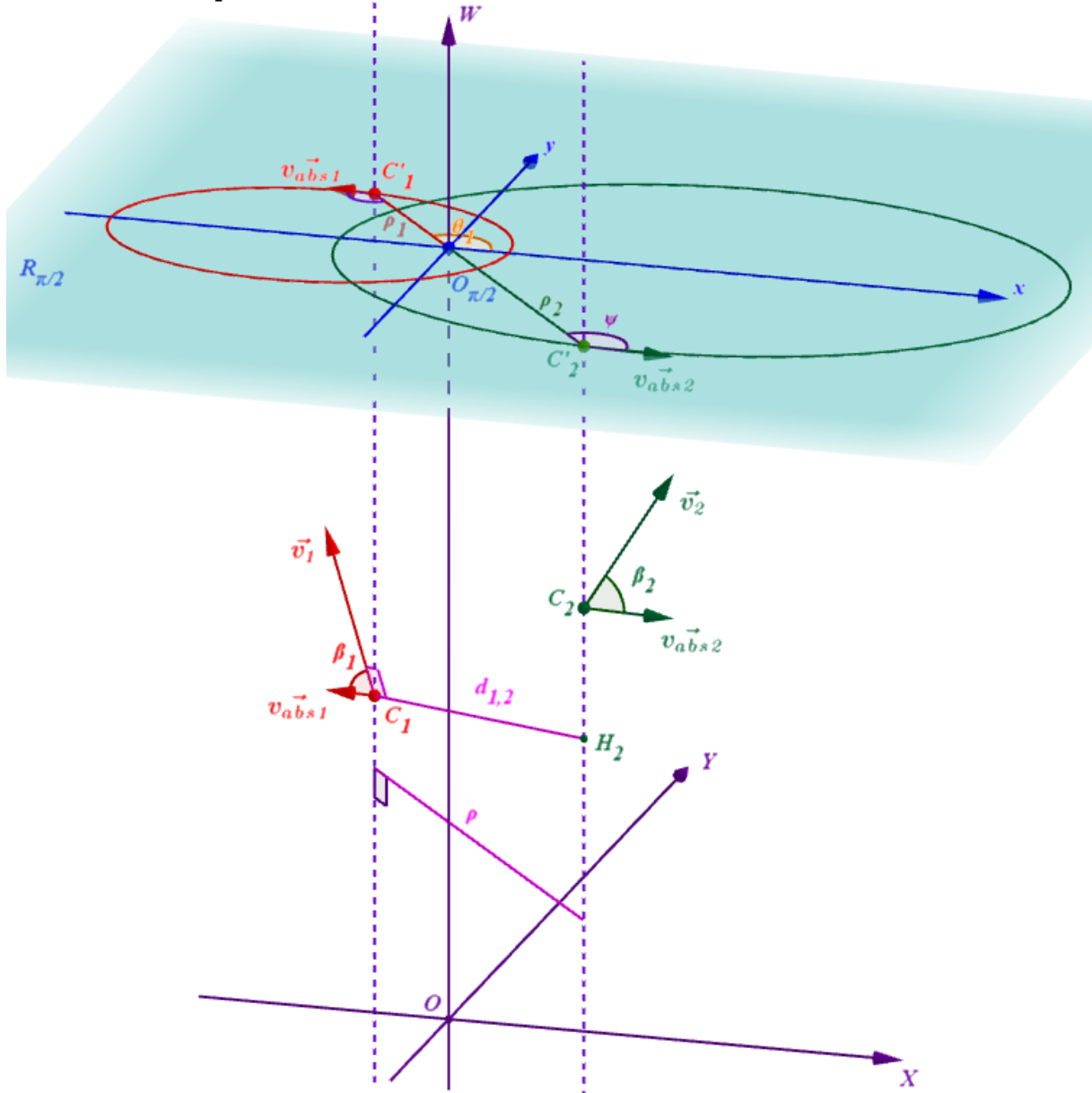
To simplify the calculations and the presentation of this quantum gravitational approach, the observation frame chosen is $R_{\pi/2}$, the orbits of C'_1 and C'_2 in this reference frame are in a plane whose equation is $z = z_0$, the axes of $R_{\pi/2}$ are positioned so as to have their origin $O_{\pi/2}$ on the axis (OW) and their guiding vectors $\vec{i}_{\pi/2}$, $\vec{j}_{\pi/2}$, $\vec{k}_{\pi/2}$ coincide with the guiding vectors \vec{i} , \vec{j} , \vec{k} of the axes (OX) , (OY) , (OZ) .

The orbits chosen for C'_1 and C'_2 in this reference frame are ellipses (the absolute velocities considered being small with respect to the speed of light).

See diagram above and the following diagrams (axes $(O_{\pi/2}z)$ and (OZ) are not shown).

3. Quantified declination postulates

3.1. First postulate



Referring to the diagram above, consider the De Broglie mass wave generated by the body C_i and perceived by the body C_j . Let us note $d_{i,j}$ the distance $C_i H_j$, where H_j designates the projection of C_j on the hyperplane H_{p_i} associated with C_i (that is to say the hyperplane orthogonal to \vec{v}_i and passing through C_i).

We shall adopt the following assumption:

*in the Platonic space, the **angular variation** $\Delta\beta_{i,j}$ corresponding to the change of the trajectory undergone by the body C_j **perceiving an occurrence of the mass wave** emitted by the body C_i is **independent of the reference frame R_α and of the rest mass of the considered body** (see also paragraph 8) and has the following value:*

$$\Delta\beta_{i,j} = \frac{Gh}{c^3 (d_{i,j})^2} \sin \beta_j \cos \psi . \quad [1]$$

Note that if one sets $d_{i,j} = n_{i,j} \times l_p$, where $l_p = \sqrt{\frac{Gh}{2\pi c^3}}$ denotes the **Planck length**, then

the relation [1] becomes:
$$\Delta\beta_{i,j} = \frac{2\pi}{(n_{i,j})^2} \sin \beta_j \cos \psi \quad (\text{with, in fact, } n_{i,j} = E \left(\frac{d_{i,j}}{l_p} \right)).$$

Moreover, we have:

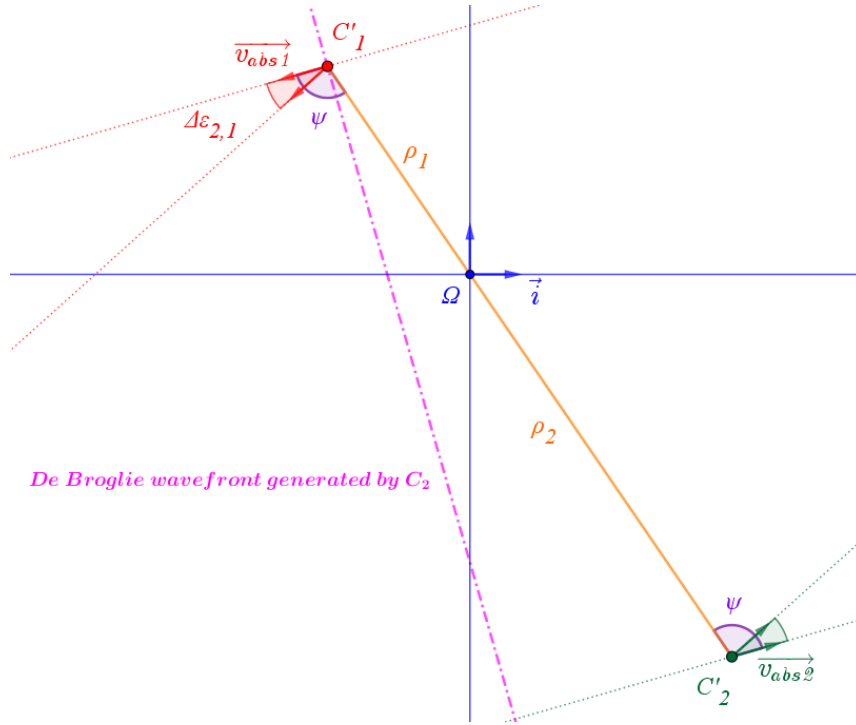
$$(d_{i,j})^2 = \rho^2 \frac{\cos^2 \beta_i \cos^2 \psi + \sin^2 \beta_i}{\sin^2 \beta_i}, \quad [1\text{bis}]$$

(see details in paragraph 8.1),

it comes therefore:

$$\Delta\beta_{i,j} = \frac{Gh}{c^3 \rho^2} \frac{\sin^2 \beta_i \sin \beta_j \cos \psi}{\sin^2 \beta_i + \cos^2 \beta_i \cos^2 \psi}. \quad [1\text{ter}]$$

3.2. Second postulate



We will introduce in this article a second postulate of quantified declination, which completes the article hal-01577669. This assumption concerns angles $\varepsilon_1 = (\vec{i}, \vec{v}_{abs1})$ and $\varepsilon_2 = (\vec{i}, \vec{v}_{abs2})$:

when the body C_j perceives an occurrence of the mass wave generated by the body C_i , the direction of the vector $\vec{v}_{abs j}$ is modified by a quantum $\Delta\varepsilon_{i,j}$, **independent of the reference frame R_α and of the rest mass of the body considered**, which has for value:

$$\Delta\varepsilon_{i,j} = \frac{Gh}{c^3 (d_{i,j})^2} \cdot \frac{\sin \psi}{\cos \beta_i + \cos \beta_j}. \quad [2]$$

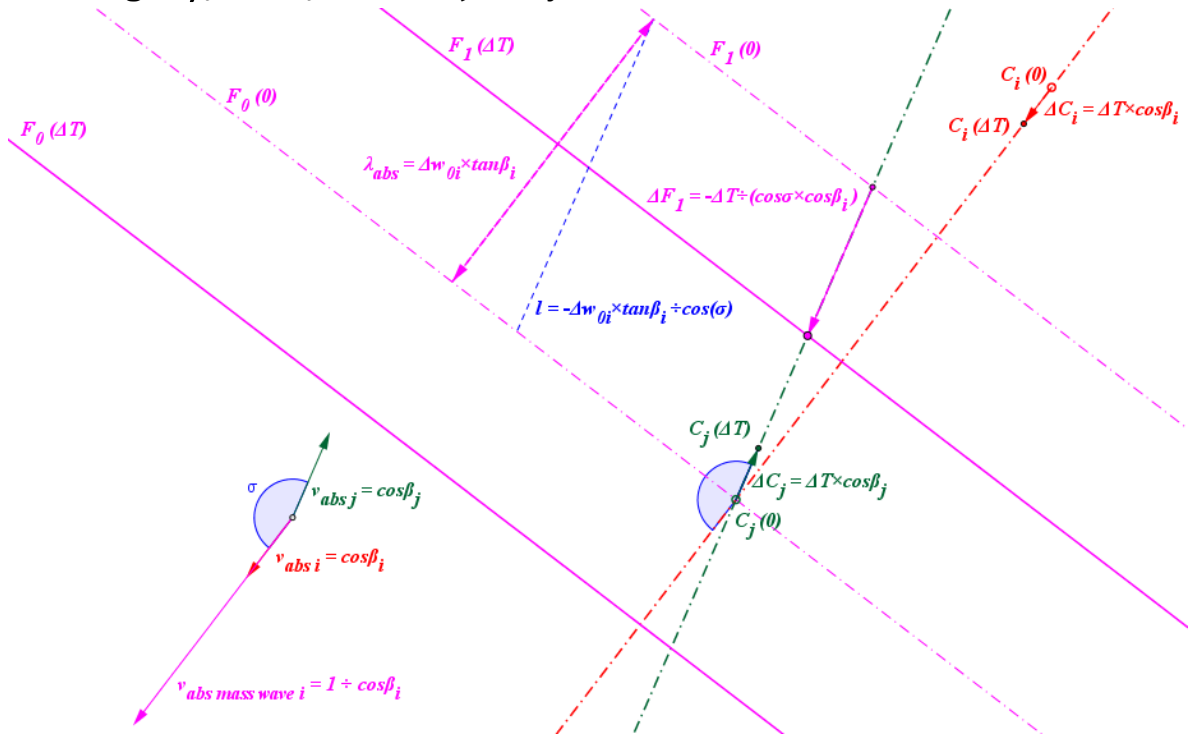
Like before, if one sets $d_{i,j} = n_{i,j} \times l_p$, where $l_p = \sqrt{\frac{Gh}{2\pi c^3}}$ denotes the **Planck length**, then

the relation [2] becomes:
$$\Delta\varepsilon_{i,j} = \frac{2\pi}{(n_{i,j})^2} \frac{\sin \psi}{\cos \beta_i + \cos \beta_j}.$$

Moreover, with:
$$(d_{i,j})^2 = \rho^2 \frac{\cos^2 \beta_i \cos^2 \psi + \sin^2 \beta_i}{\sin^2 \beta_i}, \quad (\text{cf. Annex 8.1})$$

we obtain :
$$\Delta\varepsilon_{i,j} = \frac{Gh}{c^3 \rho^2} \frac{\sin^2 \beta_i}{\sin^2 \beta_i + \cos^2 \beta_i \cos^2 \psi} \cdot \frac{\sin \psi}{\cos \beta_i + \cos \beta_j} \quad [2^{\text{bis}}]$$

4. Frequency of the mass wave perceived by the body C_j and variation of the angles β_j and ε_j of its trajectory



Referring to the diagram above, consider the mass wave generated by the body C_i and perceived by the body C_j .

Let us denote by σ a measure of the angle $(\vec{v}_{abs i}, \vec{v}_{abs j})$.

A first wavefront $F_0(0)$ is perceived by C_j when it is at $C_j(0)$.

The next wavefront F_1 is then located at $F_1(0)$, separated by the absolute wavelength $\lambda_{abs} = \Delta w_{0i} \tan \beta_i$ (in addition, see the diagram in paragraph 8).

After an absolute time ΔT , the bodies C_i and C_j are at $C_i(\Delta T)$ and $C_j(\Delta T)$ and the wavefront F_1 is at $F_1(\Delta T)$, with the relations:

$$\Delta F_1 = \frac{-\Delta T}{\cos \beta_i \cos \sigma} \quad \text{and} \quad \Delta C_j = \Delta T \cos \beta_j.$$

Thus, the body C_j perceives the wavefront F_1 when the absolute duration ΔT verifies the equation: $l = \Delta C_j + \Delta F_1$, i.e. $-\frac{\Delta w_{0i} \tan \beta_i}{\cos \sigma} = \frac{-\Delta T}{\cos \beta_i \cos \sigma} + \Delta T \cos \beta_j$,

$$\text{i.e.: } \Delta T = \frac{\Delta w_{0i} \sin \beta_i}{1 - \cos \beta_i \cos \beta_j \cos \sigma} \quad (\text{absolute period of the mass wave}).$$

It follows that the **absolute frequency** $f_{i,j}$ of this mass wave is given by:

$$f_{i,j} = \frac{1}{\Delta T} = \frac{1 - \cos \beta_i \cos \beta_j \cos \sigma}{\Delta w_{0i} \sin \beta_i}.$$

With the equality $\sigma = \pi$ in the particular case studied in these first paragraphs, the absolute frequency $f_{i,j}$ of this phase wave becomes:

$$f_{i,j} = \frac{1 + \cos \beta_i \cos \beta_j}{\Delta w_{0i} \sin \beta_i}. \quad [3]$$

- This result allows us to estimate the derivative of the direction angle β_j of the trajectory followed by the body C_j with respect to the absolute time T (although the mass waves have high frequencies, the term "derivative" seems unsuitable for quantifying quantities):

$$\frac{d\beta_j}{dT} = f_{i,j} \times \Delta\beta_{i,j} = \frac{Gh}{c^3 \rho^2 \Delta w_{0i}} \frac{1 + \cos \beta_i \cos \beta_j}{\sin^2 \beta_i + \cos^2 \beta_i \cos^2 \psi} \sin \beta_i \sin \beta_j \cos \psi. \quad [4]$$

- The estimation of the variation of the angles ε_1 and ε_2 will appeal, for its part, to a **complementary hypothesis**:
during the perception by the body C_j of a mass wavefront generated by the body C_i , the angle ε_j is modified by a quanta $\Delta\varepsilon_{i,j}$ (cf.3.2.) and, concomitantly, the angle ε_i is also modified from this same quanta.

By reciprocity, this hypothesis thus leads us to the common value of these derivatives:

$$\frac{d\varepsilon_1}{dT} = \frac{d\varepsilon_2}{dT} = f_{1,2} \Delta\varepsilon_{1,2} + f_{2,1} \Delta\varepsilon_{2,1} \quad [5]$$

i.e., noting $\frac{d\varepsilon}{dT}$ this common value:

$$\frac{d\varepsilon}{dT} = \frac{Gh}{c^3 \rho^2} \cdot \frac{1 + \cos \beta_1 \cos \beta_2}{\cos \beta_1 + \cos \beta_2} \cdot \sin \psi \cdot \left[\sum_{i=1}^{i=2} \frac{\sin \beta_i}{\Delta w_{0i} (\sin^2 \beta_i + \cos^2 \beta_i \cos^2 \psi)} \right]. \quad [5\text{bis}]$$

5. Absolute energy, absolute momentum and absolute angular momentum

5.1. Recall of definitions

We will retain the definitions adopted in the article hal-01584918:

absolute velocity of a particle (without unit): $v_{abs} = \cos \beta$, (cf. **note** in § 8.1.)

absolute mass of a particle (in kg): $m_{abs} = \frac{h}{c\Delta w_0 \sin \beta}$,

absolute energy of a particle (in J): $E_{abs} = m_{abs}c^2 = \frac{hc}{\Delta w_0 \sin \beta}$.

By choosing for **absolute potential energy of the system** the quantity:

$$E_{Pabs} = -\frac{Gm_{abs1}m_{abs2}}{\rho} = -\frac{G}{\rho} \frac{h^2}{c^2\Delta w_{01}\Delta w_{02} \sin \beta_1 \sin \beta_2},$$

the **conservation of energy** in the two-body problem becomes:

$$\frac{hc}{\Delta w_{01} \sin \beta_1} + \frac{hc}{\Delta w_{02} \sin \beta_2} - \frac{G}{\rho} \frac{h^2}{c^2\Delta w_{01}\Delta w_{02} \sin \beta_1 \sin \beta_2} = k_1 \quad [6]^*.$$

Similarly, by setting $\rho = C_1'C_2'$, $\rho_1 = \Omega C_1'$ and $\rho_2 = \Omega C_2'$, where Ω denotes the barycenter of the points C_1' and C_2' respectively assigned to the coefficients m_{abs1} and m_{abs2} , the **conservation of the absolute angular momentum of the system** leads to:

$$\rho_1 \frac{h}{c\Delta w_{01} \tan \beta_1} \sin \psi + \rho_2 \frac{h}{c\Delta w_{02} \tan \beta_2} \sin \psi = k_2 \quad [7]^*$$

with the relations $\rho = \rho_1 + \rho_2$; $\rho_1 = \rho \frac{m_{abs2}}{m_{abs1} + m_{abs2}} = \rho \frac{\Delta w_{01} \sin \beta_1}{\Delta w_{01} \sin \beta_1 + \Delta w_{02} \sin \beta_2}$;

$$\rho_2 = \rho \frac{m_{abs1}}{m_{abs1} + m_{abs2}} = \rho \frac{\Delta w_{02} \sin \beta_2}{\Delta w_{01} \sin \beta_1 + \Delta w_{02} \sin \beta_2} \quad \text{and} \quad \rho_2 = \frac{m_{abs1}}{m_{abs2}} \rho_1.$$

And the consideration of **absolute momentum** leads to :

$$\frac{h}{c\Delta w_{01} \tan \beta_1} - \frac{h}{c\Delta w_{02} \tan \beta_2} = 0 \quad [8].$$

(**N.B.** : to avoid factors $\tan \beta_i$, prefer [8^{bis}] : $\frac{h \cos \beta_1}{c\Delta w_{01} \sin \beta_1} - \frac{h \cos \beta_2}{c\Delta w_{02} \sin \beta_2} = 0$.)

*(k_1 and k_2 are two constants).

5.2. Link between the second postulate and the conservation of the absolute angular momentum of the system

A profound link is to be emphasized between the two quantities quoted in this paragraph:

*it can be shown that **the second postulate** of quantified declination, relative to the variation of the angles ε_i , **is equivalent to the principle of conservation of the absolute angular momentum of the system.***

This demonstration is attached as Annex 8.2.

5.3. Note about the absolute energy of the system

This quantity, used in the article hal-01584918, made it possible to establish the trajectories of the bodies in gravitational interaction in the four-dimensional Platonic space and to thus find, according to the classical expectations, those of their projections. In this article, it should be emphasized that the study of these trajectories will be carried out according to a different process (detailed in paragraph 7.) which does not make use of this concept (the study is here based on the two postulates of quantified declinations and some basic geometrical considerations), but that **the results obtained here remain fully consistent with this concept of absolute energy of the system.**

As an indication, a comparative study was conducted for the Sun-Earth system with these two approaches.

Results using the relationships [P1] and [P2]:

$$\frac{d\rho}{dT} = \left[\frac{\cos \beta_1 \sin \beta_2 \left(\sin \beta_2 - \frac{Gh}{c^3 \rho \Delta w_{02}} \right)}{\sin^2 \beta_2 + \cos^2 \beta_2 \cos^2 \psi} + \frac{\cos \beta_2 \sin \beta_1 \left(\sin \beta_1 - \frac{Gh}{c^3 \rho \Delta w_{01}} \right)}{\sin^2 \beta_1 + \cos^2 \beta_1 \cos^2 \psi} \right] (1 + \cos \beta_1 \cos \beta_2) \cos \psi$$

and :

$$\frac{d\rho_i}{dT} = \frac{\rho \Delta w_{01} \Delta w_{02} \left(\cos \beta_i \sin \beta_j \frac{d\beta_i}{dT} - \sin \beta_i \cos \beta_j \frac{d\beta_j}{dT} \right) + \frac{d\rho}{dT} (\Delta w_{01} \Delta w_{02} \sin \beta_1 \sin \beta_2 + \Delta w_{0i}^2 \sin^2 \beta_i)}{(\Delta w_{01} \sin \beta_1 + \Delta w_{02} \sin \beta_2)^2}$$

from potential energy (cf. hal-01584918), are discussed in paragraph 8.4.6 and those

based on [G6^{bis}] : $\frac{d\rho}{dT} = (\cos \beta_1 + \cos \beta_2) \cos \psi$ (cf. § 6.) are discussed in paragraph

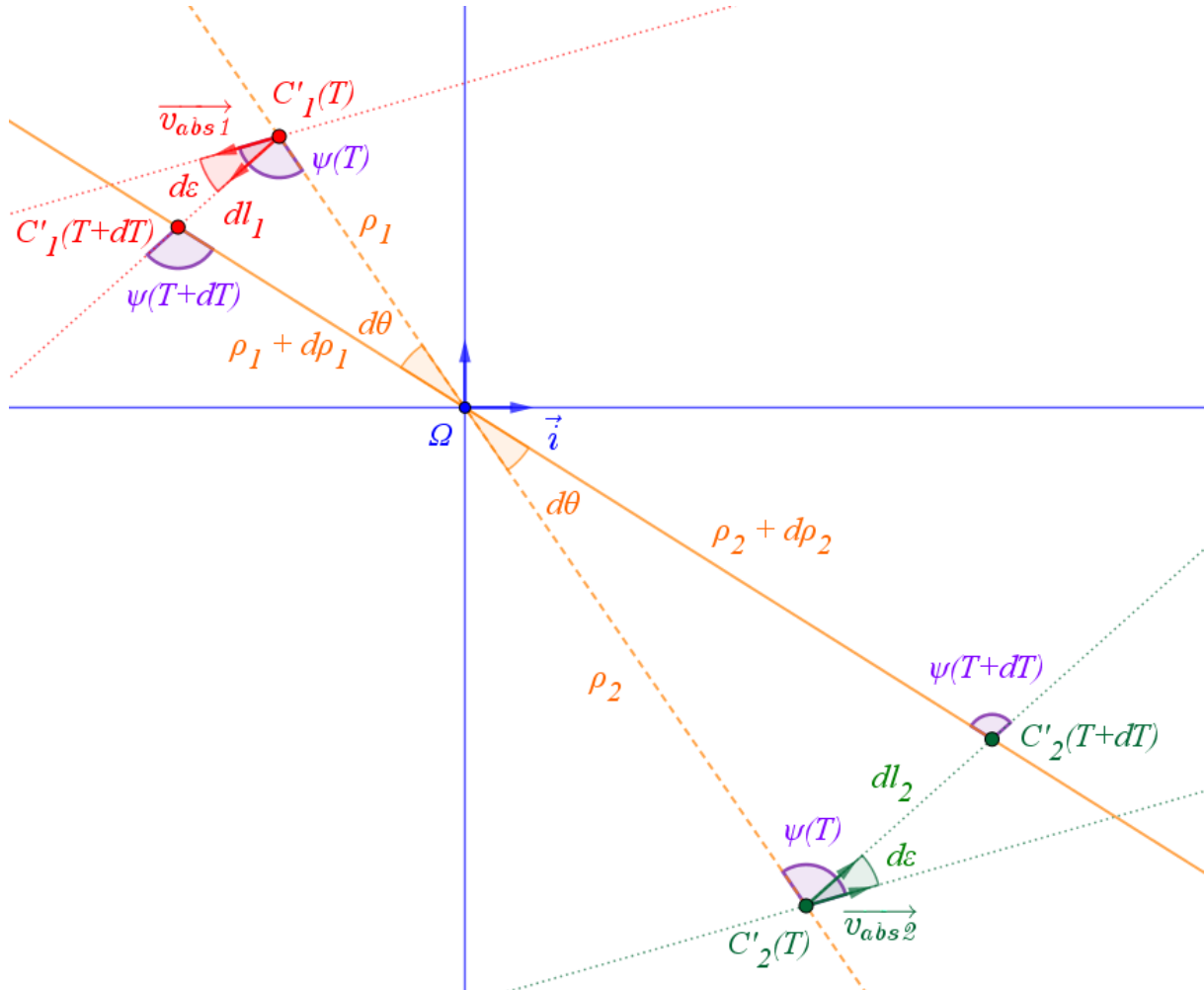
8.4.1.

As can be seen, **these results are consistent.**

6. Some preliminary geometric considerations

Before detailing, in the following paragraph, the iterative process chosen for the study of the movement of the C_1 and C_2 bodies, it will be useful to establish some elementary geometric relations between the different quantities to be measured.

The diagram below illustrates the positions of the observed bodies C'_1 and C'_2 at two absolute moments T and $T + dT$.



First, remember that $v_{abs1} = \cos \beta_1$ and $v_{abs2} = \cos \beta_2$ (see 8.1); hence: $dl_i = \cos \beta_i dT$.

Therefore, in the $\Omega C'_i(T)C'_i(T + dT)$ triangles, we have:

$$\frac{dl_i}{d\theta} = \frac{\rho_i}{\sin \psi} \Rightarrow \frac{\cos \beta_i dT}{d\theta} = \frac{\rho_i}{\sin \psi}.$$

Then, successively:

$$\frac{d\theta}{dT} = \frac{\cos \beta_i}{\rho_i} \sin \psi ; \quad [G1]$$

$$\frac{\cos \beta_1}{\rho_1} = \frac{\cos \beta_2}{\rho_2}. \quad [G2]$$

And, with $\rho = \rho_1 + \rho_2$:

$$\frac{d\theta}{dT} = \frac{\cos \beta_1 + \cos \beta_2}{\rho} \sin \psi ; \quad [\text{G3}]$$

$$\rho_1 = \rho \frac{\cos \beta_1}{\cos \beta_1 + \cos \beta_2} ; \quad \rho_2 = \rho \frac{\cos \beta_2}{\cos \beta_1 + \cos \beta_2} . \quad [\text{G3bis}]$$

With $\frac{\rho_1}{\rho_2} = \frac{\cos \beta_1}{\cos \beta_2}$ and $\Delta w_{01} \tan \beta_1 = \Delta w_{02} \tan \beta_2$ (cf. [8]), we obtain :

$$\frac{\rho_1}{\rho_2} = \frac{\Delta w_{01} \sin \beta_1}{\Delta w_{02} \sin \beta_2} ; \quad \text{hence, with } m_{absi} = \frac{h}{c \Delta w_{0i} \sin \beta_i}, \text{ the relation : } \frac{\rho_1}{\rho_2} = \frac{m_{abs2}}{m_{abs1}} .$$

Important note:

the relation [G3] will be used in place of the relation $\frac{d\theta}{dT} = \frac{\cos \beta_i}{\rho_i}$ which had been

retained in the article hal-01584918, page 9.

Indeed, this last relation is only suitable for quasi-circular trajectories and was at the origin of the underlined defects for trajectories with high eccentricities.

Link between increases $d\varepsilon$, $d\psi$ and $d\theta$ and calculation of $\frac{d\psi}{dT}$:

This study being limited to Keplerian orbits, for which $\psi_1 = \psi_2$, put $\psi = (\overrightarrow{C'_i}, \overrightarrow{\Omega}, \overrightarrow{v_{absi}})$.

With $d\theta = (\overrightarrow{\Omega C'_i}(T), \overrightarrow{\Omega C'_i}(T+dT))$, $d\varepsilon = (\overrightarrow{v_{absi}}(T), \overrightarrow{v_{absi}}(T+dT))$

and $d\psi = \psi(t+dT) - \psi(T)$, we get easily: $d\varepsilon = d\psi + d\theta$. [G4]

This relation allows us, by using [G3] and [5], to obtain successively:

$$\frac{d\psi}{dT} = \frac{d\varepsilon}{dT} - \frac{d\theta}{dT}, \text{ then : } \boxed{\frac{d\psi}{dT} = f_{1,2} \Delta \varepsilon_{1,2} + f_{2,1} \Delta \varepsilon_{2,1} - \frac{\cos \beta_1 + \cos \beta_2}{\rho} \sin \psi .} \quad [\text{G5}]$$

Finally, we will reuse the relation [G6]: $\frac{d\rho_i}{dT} = \cos \beta_i \cos \psi$ (see hal-01584918 pages 8

and 9) to deduce, from $\frac{d\rho}{dT} = \frac{d\rho_1}{dT} + \frac{d\rho_2}{dT}$, the relation:

$$\boxed{\frac{d\rho}{dT} = (\cos \beta_1 + \cos \beta_2) \cos \psi .} \quad [\text{G6bis}]$$

7. Study of the movement

From the previous relations, we will be able to set up an **iterative process** to study the motion of the two bodies in interaction as a function of the absolute time T . As indicated in paragraph 5.3, this process will be significantly different from the one adopted in article hal-01584918 because it involves only the quantified assumptions [1] and [2], the conservation of the absolute momentum [5] of the system and the considerations detailed in paragraph 6.

Insofar as the mass differences chosen are large and the speeds involved are small compared with the speed of light, **the position of the barycenter Ω of the system does not vary very much** (its fluctuations may initially be neglected relative to the distance considered in the examples in paragraph 8).

Therefore, the postulates [1] and [2], and the results [1^{bis}], [3], [4], [5], [G3],[G5] and [G6] can already allow us to set up an **iterative process** to study the motion of two bodies in interaction as a function of the absolute time T .

We will choose Ω as the origin of the reference frame.

- The masses of the bodies C_i and C_j are in a first approximation equal to the sums of the masses of the particles which constitute them.
The stratification distances Δw_{0i} and Δw_{0j} taken into account in the calculations on the basis of the relation $\Delta w_0 = \frac{h}{cm_{abs} \sin \beta}$ are fictitious distances which make it possible to generate the occurrences of the mass waves corresponding to the accumulations of the occurrences generated by all of these particles.
The masses of these bodies are to be indicated in **cells C4 and E4**.
- The initial value of ρ is the distance at the periapsis (to be indicated in **cell G5**).
- The eccentricity e of the elliptic trajectories is to be indicated in the **cell G4**.
- The initial values of the angles β_1 and β_2 are estimated from the velocity v_{2p} at the periapsis of the body C'_2 and from the relations:

$$v_{abs\ i} = \cos \beta_i = \frac{v_{ip}}{c}.$$

- The initial value of θ is 0 and that of ψ is $\frac{\pi}{2}$.
- The coordinates W_1 and W_2 of the bodies C_1 and C_2 are implemented by the increments $dW_1 = dT \sin \beta_1$ and $dW_2 = dT \sin \beta_2$.

This, beyond the situation of these bodies in the Platonic space, makes it possible to evaluate the proper durations Δt_1 and Δt_2 elapsed for C_1 and C_2 , from the relation

$$\Delta t_i = \frac{\Delta W_i}{c}.$$

The initial values of W_1 and W_2 are chosen null, arbitrarily.

- In order for these tables to give usable results, **the absolute time step dT of the cell J5 must be adjusted** so that the measurement of the angle θ_{\max} appearing in the **cell I17** is close to 2π .
This setting requires some successive tests (... and a little patience ...).

The iterative process has been applied to the following situations (see paragraph 8):

- ✓ C'_1 : Sun and C'_2 : Earth,
- ✓ C'_1 : Sun and C'_2 : Mercury,
- ✓ C'_1 : Sun and C'_2 : Neptune,
- ✓ C'_1 : Earth and C'_2 : Moon,
- ✓ the Pulsar PSR B1913+16.

These situations were first calculated using the classical laws of gravitation (the formulas used are given in Appendix 8); which gives in the Excel tables of paragraph 8 the quantities referred to as "theoretical" (columns AN to AQ).

They were then simulated as indicated in this paragraph.

Finally, the results of these simulations and the "theoretical" data were compared.

At the margins of errors due to the numerous iterations (here 20 000) and the software used (which only retains 15 significant digits for each calculation), this process allows us to find the main characteristics of each of the different orbits established according to the laws of classical mechanics.

By way of illustration, the maximum errors recorded for the Sun-Earth system are of the order of 0,004 % for the speeds, $5,6 \cdot 10^{-5}$ % for the distances, $2 \cdot 10^{-5}$ % for the period of revolution and 0,075% for the eccentricity.

In the article hal-01584918, it was found that, as the eccentricity increases, the maximum errors on velocities and distances remain low, but that the error on the period of revolution increases. The relation [G3^{bis}] (cf §6) corrects this anomaly significantly:

for e close to 0.2, the relation used previously led to an error close to 0.8%, it becomes here close to 0.0036%.

The "Excel" file corresponding to this iterative process is attached.

The numbers of the formulas used are specified on line 17 of this file.

As the software used retains only 15 significant digits, it has also been noticed that **large mass differences** induce a significant increase **in relative errors for the observed body velocities C'_1 and C'_2** .

For the calculation of the angle β_1 (closely related to the estimate of the speed of the body C'_1), **it is therefore preferable, in these situations**, to replace the iterative

method proposed above by the relation $\beta_1 = \arctan\left(\frac{\Delta w_{02}}{\Delta w_{01}} \tan \beta_2\right)$ (from conservation of the absolute momentum). **This correction is very effective**: for the Sun-Mercury system, for example, the average of the errors observed for the speed of the Sun (observed body C'_1) has thus decreased from 0.26% to 0.0012%.

8. Related documents

8.1. Calculation of the distance $d_{1,2}$

The quantities used in this paragraph refer to the diagrams in paragraphs 2 and 3.

Since the movements considered here are assumed to take place with a third constant coordinate ($Z = Z_0$) in the space $(O, \vec{i}, \vec{j}, \vec{k}, \vec{h})$, the velocity vector of the body C_1 is given, with β_1 not multiple of π , by:

$$\vec{v}_1 = \begin{pmatrix} -\cos \beta_1 \cos(\psi - \theta) \\ \cos \beta_1 \sin(\psi - \theta) \\ 0 \\ \sin \beta_1 \end{pmatrix} \quad (\text{cf. note below}).$$

Let H_{C_1} be the hyperplane associated with $C_1(X_1, Y_1, Z_1, W_1)$ (hyperplane orthogonal to \vec{v}_1 passing through C_1) and let H_2 be the projection of C_2 onto H_{C_1} .

We thus have the equivalence: $M(X, Y, Z, W) \in H_{C_1}$ if and only if

$$-\cos \beta_1 \cos(\psi - \theta)(X - X_1) + \cos \beta_1 \sin(\psi - \theta)(Y - Y_1) + \sin \beta_1(W - W_1) = 0.$$

As $H_2(X_2, Y_2, Z_2, W_2) \in H_{C_1}$, we have :

$$-\cos \beta_1 \cos(\psi - \theta)(X_2 - X_1) + \cos \beta_1 \sin(\psi - \theta)(Y_2 - Y_1) + \sin \beta_1(W_2 - W_1) = 0,$$

from which emerges: $W_2 - W_1 = -\frac{\cos \beta_1}{\sin \beta_1} [\cos(\psi - \theta)(X_2 - X_1) - \sin(\psi - \theta)(Y_2 - Y_1)]$.

On the other hand, by definition, we have:

$$(d_{1,2})^2 = (X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (W_2 - W_1)^2 = \rho^2 + (W_2 - W_1)^2,$$

with: $X_2 - X_1 = \rho \cos \theta$ and $Y_2 - Y_1 = \rho \sin \theta$.

We thus obtain: $(W_2 - W_1)^2 = \rho^2 \left(\frac{\cos \beta_1}{\sin \beta_1} \right)^2 [\cos(\psi - \theta) \cos \theta - \sin(\psi - \theta) \sin \theta]^2$,

$$\text{i.e. } (W_2 - W_1)^2 = \rho^2 \left(\frac{\cos \beta_1}{\sin \beta_1} \right)^2 \cos^2 \psi.$$

Finally, we arrive at: $(d_{1,2})^2 = \rho^2 + (W_2 - W_1)^2 = \rho^2 \left(1 + \frac{\cos^2 \beta_1 \cos^2 \psi}{\sin^2 \beta_1} \right)$,

$$\text{i.e. } (d_{1,2})^2 = \rho^2 \frac{\cos^2 \beta_1 \cos^2 \psi + \sin^2 \beta_1}{\sin^2 \beta_1}.$$

Note on the concept of **velocity** of a punctual object M in the Platonic space :

given the definition of the absolute time T (in m), the **norm of the speed vector** of all the mobiles **is equal to 1** and the **velocity vector** $\vec{v} = \frac{d\vec{M}}{dT}$ of any mobile M is given by:

$$\vec{v} = \begin{pmatrix} \cos \beta \cos \gamma \cos \varphi \\ \cos \beta \cos \gamma \sin \varphi \\ \cos \beta \sin \gamma \\ \sin \beta \end{pmatrix},$$

with any φ ,

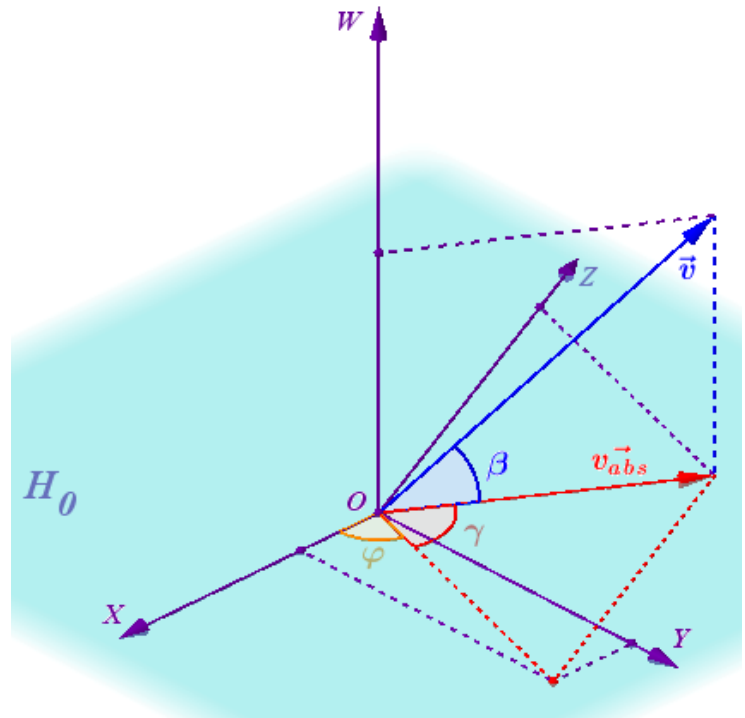
$$\gamma \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \text{ and } \beta \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right].$$

Its **absolute speed** is given by:

$$\overline{v}_{abs} = \begin{pmatrix} \cos \beta \cos \gamma \cos \varphi \\ \cos \beta \cos \gamma \sin \varphi \\ \cos \beta \sin \gamma \\ 0 \end{pmatrix},$$

whose **norm is equal to** $\cos \beta$.

For the vector \vec{v}_1 in this paragraph, we have $\gamma = 0$ and $\varphi = \theta + \pi - \psi$.



8.2. Equivalence between the second quantified declination postulate and the conservation of the absolute angular momentum of the system

To conduct this proof, we will call on the following relations:

$$[G3] : \frac{d\theta}{dT} = \frac{\cos \beta_1 + \cos \beta_2}{\rho} \sin \psi$$

$$[G3^{\text{bis}}] : \rho_i = \rho \frac{\cos \beta_i}{\cos \beta_1 + \cos \beta_2}$$

$$[G5] : \frac{d\psi}{dT} = \frac{d\varepsilon}{dT} - \frac{d\theta}{dT}$$

$$[G6] : \frac{d\rho_i}{dT} = \cos \beta_i \cos \psi$$

$$[8] : \Delta w_{01} \tan \beta_1 = \Delta w_{02} \tan \beta_2$$

$$[2^{\text{ter}}] : \Delta \beta_{i,j} = \frac{\cos \psi}{\sin \psi} \sin \beta_j (\cos \beta_1 + \cos \beta_2) \Delta \varepsilon_{i,j} \quad (\text{established from [1] and [2]}).$$

Proof:

The relation [5] : $\frac{d\varepsilon}{dT} = f_{1,2} \Delta \varepsilon_{1,2} + f_{2,1} \Delta \varepsilon_{2,1}$ leads to :

$$\rho \frac{\cos \psi \cos \beta_1}{\Delta w_{01} \sin \beta_1} \frac{d\varepsilon}{dT} = \rho \frac{\cos \psi \cos \beta_1}{\Delta w_{01} \sin \beta_1} (f_{1,2} \Delta \varepsilon_{1,2} + f_{2,1} \Delta \varepsilon_{2,1}) ; \text{ from where, with [G5], we have :}$$

$$\frac{\cos \psi \cos \beta_1}{\Delta w_{01} \sin \beta_1} \left(\rho \frac{d\psi}{dT} + \rho \frac{d\theta}{dT} \right) = \frac{\cos \psi \cos \beta_1 (\cos \beta_1 + \cos \beta_2)}{\Delta w_{01} \sin \beta_1} \frac{\rho}{\cos \beta_1 + \cos \beta_2} (f_{1,2} \Delta \varepsilon_{1,2} + f_{2,1} \Delta \varepsilon_{2,1})$$

With $\rho = \rho_1 + \rho_2$, [8], [G3] et [G3^{bis}], we obtain:

$$\begin{aligned} (\rho_1 + \rho_2) \frac{\cos \psi \cos \beta_1}{\Delta w_{01} \sin \beta_1} \frac{d\psi}{dT} + \cos \psi \sin \psi \sum_{i=1}^{i=2} \frac{\cos^2 \beta_i}{\Delta w_{0i} \sin \beta_i} \\ = \frac{\cos \psi \cos \beta_1 (\cos \beta_1 + \cos \beta_2)}{\Delta w_{01} \sin \beta_1} \left(\frac{\rho_2}{\cos \beta_2} f_{1,2} \Delta \varepsilon_{1,2} + \frac{\rho_1}{\cos \beta_1} f_{2,1} \Delta \varepsilon_{2,1} \right). \end{aligned}$$

Then, with [G6] :

$$\sum_{i=1}^{i=2} \left(\rho_i \frac{\cos \psi \cos \beta_i}{\Delta w_{0i} \sin \beta_i} \frac{d\psi}{dT} + \frac{\sin \psi \cos \beta_i}{\Delta w_{0i} \sin \beta_i} \frac{d\rho_i}{dT} \right) \\ = \cos \psi (\cos \beta_1 + \cos \beta_2) \left(\frac{\rho_2}{\Delta w_{02} \sin \beta_2} f_{1,2} \Delta \varepsilon_{1,2} + \frac{\rho_1}{\Delta w_{01} \sin \beta_1} f_{2,1} \Delta \varepsilon_{2,1} \right)$$

The right-hand member R of this equality can then be written successively:

$$R = \frac{\rho_2}{\Delta w_{02} \sin \beta_2} f_{1,2} \Delta \varepsilon_{1,2} \cos \psi (\cos \beta_1 + \cos \beta_2) + \frac{\rho_1}{\Delta w_{01} \sin \beta_1} f_{2,1} \Delta \varepsilon_{2,1} \cos \psi (\cos \beta_1 + \cos \beta_2) ;$$

i.e., with [2^{ter}] :

$$R = \frac{\rho_2 \sin \psi}{\Delta w_{02} \sin^2 \beta_2} f_{1,2} \Delta \beta_{1,2} + \frac{\rho_1 \sin \psi}{\Delta w_{01} \sin^2 \beta_1} f_{2,1} \Delta \beta_{2,1}.$$

We arrive thus at :

$$\sum_{i=1}^{i=2} \left(\rho_i \frac{\cos \psi \cos \beta_i}{\Delta w_{0i} \sin \beta_i} \frac{d\psi}{dT} + \frac{\sin \psi \cos \beta_i}{\Delta w_{0i} \sin \beta_i} \frac{d\rho_i}{dT} - \frac{\rho_i \sin \psi}{\Delta w_{0i} \sin^2 \beta_i} \frac{d\beta_i}{dT} \right) = 0 ;$$

and finally, by integration :

$$\rho_1 \frac{h}{c \Delta w_{01} \tan \beta_1} \sin \psi + \rho_2 \frac{h}{c \Delta w_{02} \tan \beta_2} \sin \psi = k .$$

Relation that corresponds to the relation [7], expressing the conservation of the absolute angular momentum of the system.

The reciprocal, easy to establish from the previous calculation elements, thus shows ***the equivalence between the second quantified declination postulate and the conservation of the absolute angular momentum of the system.*** ■

8.3. Formulas used to obtain theoretical reference data in simulations

The results of the quantified simulations are compared to the theoretical elliptic trajectories of two bodies C'_1 and C'_2 in gravitational interaction. Given the velocities considered, which are low compared to the speed of light, the calculations are made from the laws of classical mechanics (the Excel file cells containing these formulas are shown below, in parentheses).

The data used are: the distance at the periapsis d_p , the common eccentricity e , the masses at rest m_1 and m_2 .

From these elements, we obtain:

- the speeds of the two bodies (columns AP and AQ):

$$v_1 = m_2 \sqrt{\frac{G(1+2e \cos \theta + e^2)}{d_p (m_1 + m_2)(1+e)}}, \quad v_2 = m_1 \sqrt{\frac{G(1+2e \cos \theta + e^2)}{d_p (m_1 + m_2)(1+e)}};$$

- the speeds of the two bodies at the periapsis (C6 and E6):

$$v_{1p} = m_2 \sqrt{\frac{G(1+e)}{d_p (m_1 + m_2)}}, \quad v_{2p} = m_1 \sqrt{\frac{G(1+e)}{d_p (m_1 + m_2)}};$$

- the speeds of the two bodies at the apoapsis (C7 and E7):

$$v_{1a} = m_2 (1-e) \sqrt{\frac{G}{d_p (m_1 + m_2)(1+e)}}, \quad v_{2a} = m_1 (1-e) \sqrt{\frac{G}{d_p (m_1 + m_2)(1+e)}};$$

- the distances $\rho_1 = \Omega C'_1$ and $\rho_2 = \Omega C'_2$ (columns AN and AO):

$$\rho_1 = \frac{m_2}{m_1 + m_2} d_p \frac{1+e}{1+e \cos \theta}, \quad \rho_2 = \frac{m_1}{m_1 + m_2} d_p \frac{1+e}{1+e \cos \theta};$$

- the distance at the apoapsis (G6): $d_a = d_p \frac{1+e}{1-e}$;

- the period of revolution (in seconds) (X6): $p_r = 2\pi \sqrt{\frac{d_p^3}{G(m_1 + m_2)(1-e)^3}}$;

- the half-axes of the ellipses traveled by C'_1 and C'_2 (AD10 to AD13):

$$a_1 = \frac{m_2}{m_1 + m_2} \frac{d_p}{1-e} \quad \text{and} \quad b_1 = \frac{m_2}{m_1 + m_2} d_p \sqrt{\frac{1+e}{1-e}},$$

$$a_2 = \frac{m_1}{m_1 + m_2} \frac{d_p}{1-e} \quad \text{and} \quad b_2 = \frac{m_1}{m_1 + m_2} d_p \sqrt{\frac{1+e}{1-e}}.$$

The results of the simulations and the comparisons between the reference results and the results of the simulations are carried out in the following paragraph.

8.4. Results of simulations and comparisons with reference results

The following screenshots were obtained from the attached Excel file.
 Only the data, final results, checks and calculation steps 0 and 1 are displayed here.
 The numbers in brackets ([1], [2], ...) on line 17 refer to the formulas used. The related comments are in paragraph 6.

8.4.1. Sun-Earth System (version A)

Data and Theoretical calculations										
Body C'1			Body C'2			System		Constants		Absolute Time dt (m)
Mass (kg) ("m1"):	1,9884E+30	Mass (kg) ("m2"):	5,9722E+24	Eccentricity ("e"):	0,01671022	G ("G"):	c ("c"):	4,73047E+11		
Δw01 (m) ("w1"):	1,11156E-72	Δw02 (m) ("w2"):	3,70085E-67	Periapsis (m) ("dp"):	1,47098E+11	6,67428E-11	299792458			
v1 periapsis (m/s) ("v1p"):	0,090966006	v2 periapsis (m/s) ("v2p"):	30286,46162	Apoapsis (m) ("da"):	1,52098E+11	h ("h"):	6,62607E-34			
v1 apoapsis (m/s) ("v1a"):	0,087975848	v2 apoapsis (m/s) ("v2a"):	29290,91062	Period rev. (days) ("pr"):	365,2578657	Observer				
_v1p/c:	3,0343E-10	_v2p/c:	0,000101025			α ("alp"):	dt (days) ("dt"):			
						1,570796327	0,01826289			
						θ max (6,28318531)				
						6,28318531				
Number of steps	Number of days	β1	β2	ρ	ρ1	ρ2	θ	ψ		
0	0	1,570796326	1,570695302	1,47098E+11	441810,7418	1,47098E+11	0	1,570796327		
1	0,01826289	1,570796326	1,570695302	1,47098E+11	441810,7418	1,47098E+11	0,000324882	1,570790987		

Simulation results checks											
System											
Simul. Eccent.	Theoretical distances (m)			Simulated distances (m)			Errors (Δ%)				
0,01672273	Periapsis (m)	1,47098E+11	ρ min =	1,47096E+11	-0,001705356						
Error (Δ%)	Apoapsis (m)	1,52098E+11	ρ max =	1,52099E+11	0,000797331						
0,074864322											
[3]	[3]	[1 ^{bis}]	[1 ^{bis}]	[2]	[2]	[5]	[1]	[1]	[4]	[4]	[6 ^{bis}]
f1,2	f2,1	(d1,2) ²	(d2,1) ²	Δε1,2	Δε2,1	dis	Δβ1,2	Δβ2,1	dβ1	dβ2	dp
8,99639E+71	2,70209E+66	2,16378E+22	2,16378E+22	7,50854E-88	7,50854E-88	0,000319543	4,6467E-108	4,6467E-108	5,93945E-30	1,9775E-24	2,92747E-09
8,99639E+71	2,70209E+66	2,16378E+22	2,16378E+22	7,50854E-88	7,50854E-88	0,000319543	4,05038E-97	4,05038E-97	5,17724E-19	1,72372E-13	255,1785384

Simulation results checks											
Bodies C'1 & C'2											
Theoretical Speeds (m/s)			Simulated Speeds (m/s)			Errors (Δ%)					
v1 periapsis(m/s) =	0,090966006	v1 max =	0,090967226	0,001341804							
v1 apoapsis(m/s) =	0,087975848	v1 min =	0,087968436	-0,008424891							
v2 periapsis(m/s) =	30286,46162	v2 max =	30286,96559	0,00166399							
v2 apoapsis(m/s) =	29290,91062	v2 min =	29290,67103	-0,000817976							
Theoretical half-axes (m)			Simulated half-axes (m)			Errors (Δ%)					
a1 =	449318,9605	a1 =	449317,0145	-0,00043102							
b1 =	449256,2241	b1 =	449254,1844	-0,00045402							
a2 =	1,49597E+11	a2 =	1,49597E+11	-0,00043102							
b2 =	1,49577E+11	b2 =	1,49576E+11	-0,00045402							
Proper Times											
Proper time Δt1 elapsed for C1 (days):					365,257790706762						
Proper time Δt2 elapsed for C2 (days):					365,257788904117						
[G6]	[G6]	[G3]	[G5]	v1 (m/s)	v2 (m/s)	X1	Y1	W1	X2	Y2	W2
dp1	dp2	dθ	dψ	30286,46162	441810,7418	0	0	-1,47098E+11	1,80216E-05	2,1237E-11	0
8,79269E-15	2,92746E-09	0,000324882	-5,33963E-06	0,090966028	30286,46162	441810,7185	143,5365004	4,73047E+11	-1,47098E+11	-47789420,54	4,73047E+11
0,000766432	255,1777719	0,000324882	-5,33963E-06	0,090966028	30286,46162	441810,7185	143,5365004	4,73047E+11	-1,47098E+11	-47789420,54	4,73047E+11

Simulation results checks											
Theoretical measurements according to θ (according to the laws of classical mechanics)											
					Average errors (Δ%)						
					ρ1	ρ2	v1	v2			
					-5,61641E-05	-2,07049E-05	-0,004092519	6,93439E-06			
					Errors (Δ%)						
m1 (kg)	m2 (kg)	dW1	dW2	ρ1	ρ2	v1 (m/s)	v2 (m/s)	ρ1	ρ2	v1	v2
1,9884E+30	5,9722E+24	4,73047E+11	4,73047E+11	441810,7418	1,47098E+11	0,090966006	30286,46162	-1,31748E-14	0	2,45965E-05	2,1237E-11
1,9884E+30	5,9722E+24	4,73047E+11	4,73047E+11	441810,7422	1,47098E+11	0,090966006	30286,4616	-8,67376E-08	-8,67375E-08	2,46818E-05	8,53332E-08

8.4.2. Sun-Mercury System

Data and Theoretical calculations										
Body C'1			Body C'2			System		Constants		Absolute Time dt (m)
Mass (kg) ("m1"):	1,9884E+30	Mass (kg) ("m2"):	3,3011E+23	Eccentricity ("e"):	0,20563069	G ("G"):	c ("c"):	1,13926E+11		
Δw01 (m) ("w1"):	1,11156E-72	Δw02 (m) ("w2"):	6,6954E-66	Periaapsis (m) ("dp"):	46001272000	6,67428E-11	299792458			
v1 periaapsis (m/s) ("v1p"):	0,009791094	v2 periaapsis (m/s) ("v2p"):	58976,13115	Apoapsis (m) ("da"):	69817079542	h ("h"):	6,62607E-34			
v1 apoapsis (m/s) ("v1a"):	0,006451183	v2 apoapsis (m/s) ("v2a"):	38858,3577	Period rev. (days) ("pr"):	87,96969311	Observer				
_v1p/c:	3,26596E-11	_v2p/c:	0,000196723			α ("alp"):	dt (days) ("dt"):			
							1,570796327	0,004398327		
							θ max (6,28318531)			
							6,28318531			
Number of steps	Number of days	β1	β2	ρ	ρ1	ρ2	θ	ψ		
0	0	1,570796327	1,570599604	46001272000	7637,033484	46001264363	0	1,570796327		
1	0,004398327	1,570796327	1,570599604	46001272000	7637,033484	46001264363	0,000487201	1,570713231		

Simulation results checks											
System											
Simul. Eccent.	Theoretical distances (m)			Simulated distances (m)			Errors (A%)				
0,205799455	Periaapsis (m)	46001272000	ρ min =	45987701885	-0,029499435						
Error (A%)	Apoapsis (m)	69817079542	ρ max =	69821087617	0,005740823						
0,082071761											
[3]	[3]	[1 ^{bit}]	[1 ^{bit}]	[2]	[2]	[5]	[1]	[1]	[4]	[4]	[6 ^{bit}]
f1,2	f2,1	(d1,2) ²	(d2,1) ²	Δε1,2	Δε2,1	dc	Δβ1,2	Δβ2,1	dβ1	dβ2	dp
8,99639E+71	1,49356E+65	2,11612E+21	2,11612E+21	3,94278E-87	3,94278E-87	0,000404104	4,7514E-107	4,7514E-107	8,08468E-31	4,86976E-24	1,37289E-09
8,99639E+71	1,49356E+65	2,11612E+21	2,11612E+21	3,94278E-87	3,94278E-87	0,000404104	6,44525E-95	6,44525E-95	1,09669E-18	6,60588E-12	1862,339892

Simulation results checks											
Bodies C'1 & C'2											
Period of revolution (days)			Theoretical Speeds (m/s)			Simulated Speeds (m/s)			Errors (A%)		
Theoretical	Simulated	Error (A%)	v1 periaapsis(m/s) =	0,009791094	v1 max =	0,009793333	0,02286766				
87,9696931052	87,9665331489	-0,003592097	v1 apoapsis(m/s) =	0,006451183	v1 min =	0,006450589	-0,009205435				
Error in seconds			-273,020225	v2 periaapsis(m/s) =	58976,13115	v2 max =	58989,63158	0,022891349			
			v2 apoapsis(m/s) =	38858,3577	v2 min =	38854,84307	-0,009044723				
			Theoretical half-axes (m)			Simulated half-axes (m)			Errors (A%)		
			a1 =	9613,958379	a1 =	9613,164644	-0,008256067				
			b1 =	9408,504858	b1 =	9407,387047	-0,011880859				
			a2 =	57909166157	a2 =	57904385138	-0,008256067				
			b2 =	56671627821	b2 =	56664894745	-0,011880859				
			Proper Times								
			Proper time Δt1 elapsed for C1 (days):			87,966533148891					
			Proper time Δt2 elapsed for C2 (days):			87,966532027345					
[G6]	[G6]	[G3]	[G5]	v1 (m/s)	v2 (m/s)	X1	Y1	W1	X2	Y2	W2
dp1	dp2	d0	dψ	2,27926E-16	1,37289E-09	0,000487201	-8,30962E-05	0,009791136	58976,13115	7637,033484	0
0,000309181	1862,339582	0,000487201	-8,30962E-05	0,009791069	58976,13115	7637,032577	3,720766457	1,13926E+11	-46001258903	5,63584E-06	
									-22411838,55	-1,13926E+11	0

Simulation results checks											
Average errors (A%)											
Theoretical measurements according to 0 (according to the laws of classical mechanics)					ρ1	ρ2	v1	v2			
					-0,001314966	-0,003448816	0,001219751	0,001220645			
					Errors (A%)						
m1 (kg)	m2 (kg)	dW1	dW2	ρ1	ρ2	v1 (m/s)	v2 (m/s)	ρ1	ρ2	v1	v2
1,9884E+30	3,3011E+23	1,13926E+11	1,13926E+11	7637,033484	46001264363	0,009791094	58976,13115	-1,1909E-14	0	0,000431751	-1,17943E-11
1,9884E+30	3,3011E+23	1,13926E+11	1,13926E+11	7637,033638	46001265294	0,009791094	58976,13016	-2,02423E-06	-2,02423E-06	-0,000246446	1,67897E-06

8.4.3. Sun-Neptune System

Data and Theoretical calculations										
Body C'1			Body C'2			System		Constants		Absolute Time dt (m) ("dT _a ")
Mass (kg) ("m ₁ "):	1,9884E+30	Mass (kg) ("m ₂ "):	1,0243E+26	Eccentricity ("e"):	0,00858587	G ("G"):	c ("c"):	7,78203E+13		
Δw01 (m) ("w ₁ "):	1,11156E-72	Δw02 (m) ("w ₂ "):	2,15778E-68	Periapsis (m) ("dp"):	4,45294E+12	6,67428E-11	299792458			
v1 periapsis (m/s) ("v1p"):	0,282422503	v2 periapsis (m/s) ("v2p"):	5482,465154	Apoapsis (m) ("da"):	4,53007E+12	h ("h"):	6,62607E-34			
v1 apoapsis (m/s) ("v1a"):	0,277614102	v2 apoapsis (m/s) ("v2a"):	5389,12311	Period rev. (days) ("pr"):	60088,11528	Observer				
_v1p/c:	9,4206E-10	_v2p/c:	1,82875E-05			α ("alp"):	dt (days) ("dt"):			
							1,570796327	3,004405632		
							θ max (6,28318531)			
							6,28318531			
Number of steps	Number of days	β1	β2	ρ	ρ1	ρ2	θ	ψ		
0	0	1,570796326	1,570778039	4,45294E+12	229375998,1	4,45271E+12	0	1,570796327		
1	3,004405632	1,570796326	1,570778039	4,45294E+12	229375998,1	4,45271E+12	0,000319612	1,570793606		

Simulation results checks											
System											
Simul. Eccent.	Theoretical distances (m)			Simulated distances (m)			Errors (Δ%)				
0,008592269	Periapsis (m)	4,45294E+12	ρ min =	4,4529E+12	-0,000862557						
Error (Δ%)	Apoapsis (m)	4,53007E+12	ρ max =	4,53009E+12	0,000417363						
0,074531058											
[3]	[3]	[1 ^{bh}]	[1 ^{bh}]	[2]	[2]	[5]	[1]	[1]	[4]	[4]	[6 ^{bh}]
f1,2	f2,1	(d1,2) ²	(d2,1) ²	Δε1,2	Δε2,1	dε	Δβ1,2	Δβ2,1	dβ1	dβ2	dρ
8,99639E+71	4,63438E+67	1,98287E+25	1,98287E+25	4,52613E-90	4,52613E-90	0,000316892	5,0706E-111	5,0706E-111	1,82872E-29	3,54997E-25	8,71825E-08
8,99639E+71	4,63438E+67	1,98287E+25	1,98287E+25	4,52613E-90	4,52613E-90	0,000316892	2,2522E-100	2,2522E-100	8,1224E-19	1,57674E-14	3872,269962

Simulation results checks											
Bodies C'1 & C'2											
Theoretical Speeds (m/s)			Simulated Speeds (m/s)			Errors (Δ%)					
v1 periapsis(m/s) =	0,282422503	v1 max =	0,282424494	0,000704852							
v1 apoapsis(m/s) =	0,277614102	v1 min =	0,277633512	0,00699193							
v2 periapsis(m/s) =	5482,465154	v2 max =	5482,511841	0,00085157							
v2 apoapsis(m/s) =	5389,12311	v2 min =	5389,100324	-0,000422824							
Theoretical half-axes (m)			Simulated half-axes (m)			Errors (Δ%)					
a1 =	231362445,9	a1 =	231361943,6	-0,000217102							
b1 =	231353918,1	b1 =	231353403,1	-0,00022599							
a2 =	4,49127E+12	a2 =	4,49126E+12	-0,000217102							
b2 =	4,49111E+12	b2 =	4,4911E+12	-0,00022599							
Proper Times											
Proper time Δt1 elapsed for C1 (days):							60088,112643957000				
Proper time Δt2 elapsed for C2 (days):							60088,112634079600				
[G6]	[G6]	[G3]	[G5]	[G6]	[G6]	[G3]	[G5]	[G6]	[G6]	[G3]	[G5]
dp1	dp2	dθ	dψ	v1 (m/s)	v2 (m/s)	X1	Y1	W1	X2	Y2	W2
4,49087E-12	8,7178E-08	0,000319612	-2,72079E-06	0,282422497	5482,465154	229375998,1	0	0	-4,45271E+12	0,000545523	0
0,199464983	3872,070497	0,000319612	-2,72079E-06	0,282422497	5482,465154	229375986,3	73311,4148	7,78203E+13	-4,45271E+12	-1423141826	7,78203E+13

Simulation results checks											
Average errors (Δ%)											
Theoretical measurements according to θ (according to the laws of classical mechanics)											
p1		p2		v1		v2					
1,68558E-05		-4,87011E-06		0,003707739		1,22558E-06					
Errors (Δ%)											
p1		p2		v1		v2					
1,9884E+30		1,0243E+26		7,78203E+13		7,78203E+13		229375998,1		4,45271E+12	
1,9884E+30		1,0243E+26		7,78203E+13		7,78203E+13		229375998,2		4,45271E+12	

8.4.4. Earth-Moon System

Data and Theoretical calculations										
Body C'1			Body C'2			System		Constants		Absolute Time dt (m)
Mass (kg) ("m1"):	5,9736E+24	Mass (kg) ("m2"):	7,3477E+22	Eccentricity ("e"):	0,0549	G ("G"):	c ("c"):	35304051050		
Δw01 (m) ("w1"):	3,69998E-67	Δw02 (m) ("w2"):	3,00804E-65	Periapsis (m) ("dp"):	363104000	6,67428E-11	299792458			
v1 periapsis (m/s) ("v1p"):	13,15742816	v2 periapsis (m/s) ("v2p"):	1069,684566	Apoapsis (m) ("da"):	405288762,7	h ("h"):	6,62607E-34			
v1 apoapsis (m/s) ("v1a"):	11,7879281	v2 apoapsis (m/s) ("v2a"):	958,3457043	Period rev. (days) ("pr"):	27,2596999	Observer				
_v1p/c:	4,38885E-08	_v2p/c:	3,56808E-06			α ("alp"):	dt (days) ("dt"):			
							1,570796327	0,001362982		
							θ max (6,28318531)			
							6,28318531			
Number of steps	Number of days	β1	β2	ρ	ρ1	ρ2	θ	ψ		
0	0	1,570796283	1,570792759	363104000	4412014,699	358691985,3	0	1,570796327		
1	0,001362982	1,570796283	1,570792759	363104000	4412014,699	358691985,3	0,000351187	1,57077805		

Simulation results checks											
System											
Simul. Eccent.	Theoretical distances (m)		Simulated distances (m)		Errors (A%)						
0,054942048	Periapsis (m)	363104000	ρ min =	363082131,7	-0,006022591						
Error (A%)	Apoapsis (m)	405288762,7	ρ max =	405298539,6	0,002412324						
0,076590388											
[3]	[3]	[1 ^{bih}]	[1 ^{bih}]	[2]	[2]	[5]	[1]	[1]	[4]	[4]	[6 ^{bih}]
f1,2	f2,1	(d1,2) ²	(d2,1) ²	Δε1,2	Δε2,1	dc	Δβ1,2	Δβ2,1	dβ1	dβ2	dp
2,70272E+66	3,32442E+64	1,31845E+17	1,31845E+17	3,44661E-81	3,44661E-81	0,00033291	7,626E-103	7,626E-103	8,95026E-28	7,27646E-26	7,81138E-12
2,70272E+66	3,32442E+64	1,31845E+17	1,31845E+17	3,44661E-81	3,44661E-81	0,00033291	2,27528E-91	2,27528E-91	2,6704E-16	2,171E-14	2,330600731

Simulation results checks															
Bodies C'1 & C'2															
Period of revolution (days)			Theoretical Speeds (m/s)							Simulated Speeds (m/s)		Errors (A%)			
Theoretical	Simulated	Error (A%)	v1 periapsis(m/s) =		13,15742816		v1 max =		13,15816348		0,00588642				
27,2596999009	27,2596385308	-0,000225131	v1 apoapsis(m/s) =		11,7879281		v1 min =		11,78761684		-0,0026405				
Error in seconds	-5,302377397		v2 periapsis(m/s) =		1069,684566		v2 max =		1069,744198		0,00574676				
			v2 apoapsis(m/s) =		958,3204405		v2 min =		958,3204405		-0,002636191				
			Theoretical half-axes (m)							Simulated half-axes (m)		Errors (A%)			
			a1 =		4668304,622		a1 =		4668231,162		-0,001573595				
			b1 =		4661264,155		b1 =		4661180,009		-0,001805222				
			a2 =		379528076,7		a2 =		379522104,5		-0,001573595				
			b2 =		378955694,4		b2 =		378948853,4		-0,001805222				
			Proper Times												
			Proper time Δt1 elapsed for C1 (days):							27,259638530839					
			Proper time Δt2 elapsed for C2 (days):							27,259638530679					
[G6]	[G6]	[G3]	[G5]	dp1	dp2	d0	dψ	v1 (m/s)	v2 (m/s)	X1	Y1	W1	X2	Y2	W2
				9,49147E-14	7,71646E-12	0,000351187	-1,82767E-05	13,15742818	1069,684566	4412014,699	0	0	-358691985,3	4,39451E-08	0
				0,028318732	2,302281999	0,000351187	-1,82767E-05	13,15742818	1069,684566	4412014,427	1549,440266	35304051050	-358691963,2	-125967,8046	35304051050

Simulation results checks											
Average errors (A%)											
Theoretical measurements according to θ (according to the laws of classical mechanics)											
ρ1		ρ2		v1		v2					
-0,000225189		-0,000224796		7,6312E-05		7,53621E-05					
Errors (A%)											
ρ1		ρ2		v1		v2					
2,11088E-14		0		1,46192E-07		-1,75044E-10					
m1 (kg)	m2 (kg)	dW1	dW2	ρ1	ρ2	v1 (m/s)	v2 (m/s)	ρ1	ρ2	v1	v2
5,9736E+24	7,3477E+22	35304051050	35304051050	4412014,699	358691985,3	13,15742816	1069,684566	2,11088E-14	0	1,46192E-07	-1,75044E-10
5,9736E+24	7,3477E+22	35304051050	35304051050	4412014,713	358691986,5	13,15742812	1069,684563	-3,20927E-07	-3,20927E-07	4,50417E-07	3,0405E-07

8.4.5. Pulsar PSR B1913+16

This last system has been simulated in order to show that the quantified modeling proposed in this article remains consistent with the classical gravitational laws for high velocities at the periapsis (close to $0.0015c$ for the body C'_2).

Given the **high eccentricity** of this system (close to 0.627) and to minimize the accumulation of rounding errors, its study was conducted over **half a period of revolution**.

Data and Theoretical calculations									
Body C'1		Body C'2		System		Constants		Absolute Time dt (m)	
Mass (kg) ("m1"):	2,86608E+30	Mass (kg) ("m2"):	2,75731E+30	Eccentricity ("e"):	0,62712	G ("G"):	c ("c"):	215908650	
Δw_01 (m) ("w1"):	7,71165E-73	Δw_02 (m) ("w2"):	8,01585E-73	Periapsis (m) ("dp"):	765270000	6,67428E-11	299792458		
v1 periapsis (m/s) ("v1p"):	438017,7101	v2 periapsis (m/s) ("v2p"):	455295,8299	Apoapsis (m) ("da"):	3339374926	h ("h"):	6,62607E-34		
v1 apoapsis (m/s) ("v1a"):	100378,6099	v2 apoapsis (m/s) ("v2a"):	104338,1613	Period rev. (days) ("pr"):	0,34900613	Observer			
v_{1p}/c :	0,00146107	v_{2p}/c :	0,001518703			α ("alp"):	dt (days) ("dt"):		
						1,570796327	8,72165E-06		
								θ max (3,14159265)	
								3,14159265	
Number of steps	Number of days	β_1	β_2	ρ	ρ_1	ρ_2	θ	ψ	
0	0	1,569335256	1,569277623	765270000	375234224	390035776	0	1,570796327	
1	8,72165E-06	1,569335256	1,569277623	765270000	375234224	390035776	0,000879633	1,570457302	

Simulation results checks											
System											
Simul. Eccent.	Theoretical distances (m)		Simulated distances (m)		Errors (A%)						
0,627003869	Periapsis (m)	765270000	ρ min =	765270000	0						
Error (A%)	Apoapsis (m)	3339374926	ρ max =	3338096965	-0,03826947						
-0,018518069											
[3]	[3]	[1 ^{bits}]	[1 ^{bits}]	[2]	[2]	[5]	[1]	[1]	[4]	[4]	[6 ^{bits}]
f1,2	f2,1	(d1,2) ²	(d2,1) ²	$\Delta e_{1,2}$	$\Delta e_{2,1}$	d_e	$\Delta \beta_{1,2}$	$\Delta \beta_{2,1}$	$d \beta $	$d \beta $	$d\rho$
1,29674E+72	1,24753E+72	5,85638E+17	5,85638E+17	9,40558E-85	9,40558E-85	0,000540608	1,7168E-103	1,7168E-103	4,83851E-23	5,02937E-23	4,12358E-11
1,29674E+72	1,24753E+72	5,85638E+17	5,85638E+17	9,40558E-85	9,40558E-85	0,000540608	9,50166E-91	9,50166E-91	2,67784E-10	2,78347E-10	228,2164696

Simulation results checks															
Bodies C'1 & C'2															
Half-period of revolution (days)		Theoretical		Simulated		Error (A%)									
		0,1745030648	0,1744329038	-0,040206169											
Error in seconds		-6,061910192													
		Theoretical Speeds (m/s)		Simulated Speeds (m/s)		Errors (A%)									
		v1 periapsis(m/s) =	438017,7101	v1 max =	438017,7101	2,47173E-12									
		v1 apoapsis(m/s) =	100378,6099	v1 min =	100389,262	0,010611953									
		v2 periapsis(m/s) =	455295,8299	v2 max =	455295,8299	2,33958E-12									
		v2 apoapsis(m/s) =	104338,1613	v2 min =	104349,2366	0,010614825									
		Theoretical half-axes (m)		Simulated half-axes (m)		Errors (A%)									
		a1 =	1006313624	a1 =	1006000313	-0,03113451									
		b1 =	783840481,6	b1 =	783690481,5	-0,019136566									
		a2 =	1046008839	a2 =	1045683169	-0,03113451									
		b2 =	814759984,3	b2 =	814604067,2	-0,019136566									
		Proper Times													
		Proper time Δt_1 elapsed for C1 (days):		0,174432861108											
		Proper time Δt_2 elapsed for C2 (days):		0,174432857675											
[G6]	[G6]	[G3]	[G5]	$d\rho_1$	$d\rho_2$	$d\theta$	$d\psi$	v1 (m/s)	v2 (m/s)	X1	Y1	W1	X2	Y2	W2
				2,02191E-11	2,10167E-11	0,000879633	-0,000339024	438017,7101	455295,8299	375234224	0	0	-390035776	4,77852E-08	0
				111,9011981	116,3152715	0,000879633	-0,000339024	438017,7101	455295,8299	375234078,9	330068,2661	225908408,9	-390035625,1	-343088,1941	225908389,5

Simulation results checks											
Theoretical measurements according to θ (according to the laws of classical mechanics)											
					Average errors (A%)						
					ρ_1	ρ_2	v_1	v_2			
					-0,027372565	-0,027376986	0,002677742	0,002674622			
					Errors (A%)						
m1 (kg)	m2 (kg)	dW1	dW2	ρ_1	ρ_2	v1 (m/s)	v2 (m/s)	ρ_1	ρ_2	v_1	v_2
2,86608E+30	2,75731E+30	225908408,9	225908389,5	375234224	390035776	438017,7101	455295,8299	0	1,52818E-14	2,48502E-12	2,35236E-12
2,86608E+30	2,75731E+30	225908408,9	225908389,5	375234280	390035834,1	438017,67	455295,7882	-1,49109E-05	-1,49109E-05	9,16399E-06	9,16399E-06

8.4.6. Sun-Earth system (version B, based on the concept of absolute potential energy of the system developed in the article hal-01584918)

Data and Theoretical calculations										
Body C'1			Body C'2			System		Constants		Absolute Time dT (m)
Mass (kg) ("m1") :	1,9884E+30	Mass (kg) ("m2") :	5,9722E+24	Eccentricity ("e") :	0,01671022	G ("G") :	c ("c") :	4,73047E+11		
Δw_01 (m) ("w1") :	1,11156E-72	Δw_02 (m) ("w2") :	3,70085E-67	Periapsis (m) ("dp") :	1,47098E+11	6,67428E-11	299792458			
v1 periapsis (m/s) ("v1p") :	0,090966006	v2 periapsis (m/s) ("v2p") :	30286,46162	Apoapsis (m) ("da") :	1,52098E+11	h ("h") :	6,62607E-34			
v1 apoapsis (m/s) ("v1a") :	0,087975848	v2 apoapsis (m/s) ("v2a") :	29290,91062	Period rev. (days) ("pr") :	365,2578657	Observer				
v1p/c :	3,0343E-10	v2p/c :	0,000101025			α ("alp") :	dt (days) ("dt") :			
							1,570796327	0,01826289		
							0 max (6,28318531)			
							6,28318531			
Number of steps	Number of days	β_1	β_2	ρ	ρ_1	ρ_2	0		ψ	
0	0	1,570796326	1,570695302	1,47098E+11	441810,7418	1,47098E+11	0		1,570796327	
1	0,01826289	1,570796326	1,570695302	1,47098E+11	441810,7418	1,47098E+11	0,000324882		1,570790987	

Simulation results checks											
System											
Simul. Eccent.	Theoretical distances (m)			Simulated distances (m)			Errors (%)				
0,01672273	Periapsis (m)	1,47098E+11	ρ min =	1,47096E+11	-0,001705368						
Error (%)	Apoapsis (m)	1,52098E+11	ρ max =	1,52099E+11	0,000797263						
0,074862643											
[3]	[3]	[1 ^{bit}]	[1 ^{bit}]	[2]	[2]	[5]	[1]	[1]	[4]	[4]	From the p
f1,2	f2,1	(d1,2) ²	(d2,1) ²	$\Delta e_{1,2}$	$\Delta e_{2,1}$	d _{is}	$\Delta \beta_{1,2}$	$\Delta \beta_{2,1}$	d β_1	d β_2	d ρ
8,99639E+71	2,70209E+66	2,16378E+22	2,16378E+22	7,50854E-88	7,50854E-88	0,000319543	4,6467E-108	4,6467E-108	5,93945E-30	1,9775E-24	2,92747E-09
8,99639E+71	2,70209E+66	2,16378E+22	2,16378E+22	7,50854E-88	7,50854E-88	0,000319543	4,05038E-97	4,05038E-97	5,17724E-19	1,72372E-13	255,1785358

Simulation results checks											
Bodies C'1 & C'2											
Period of revolution (days)			Theoretical Speeds (m/s)			Simulated Speeds (m/s)			Errors (%)		
Theoretical	Simulated	Error (%)	v1 periapsis(m/s) =	0,090966006	v1 max =	0,090967226	0,001341804				
365,257865626	365,2577906836	-2,05249E-05	v1 apoapsis(m/s) =	0,087975848	v1 min =	0,087968436	-0,008424891				
Error in seconds	-6,477317771		v2 periapsis(m/s) =	30286,46162	v2 max =	30286,96559	0,001663997				
			v2 apoapsis(m/s) =	29290,91062	v2 min =	29290,67104	-0,000817942				
			Theoretical half-axes (m)			Simulated half-axes (m)			Errors (%)		
a1 =	449318,9605	a1 =	449317,0143	-0,000433143							
b1 =	449256,2241	b1 =	449254,1842	-0,00045406							
a2 =	1,49597E+11	a2 =	1,49597E+11	-0,000433143							
b2 =	1,49577E+11	b2 =	1,49576E+11	-0,00045406							
						Proper Times					
Proper time Δt_1 elapsed for C1 (days):						365,257790683597					
Proper time Δt_2 elapsed for C2 (days):						365,257788880953					
Potential energy, relations [P1] and [P2]		[G3]	[G5]	v1 (m/s)	v2 (m/s)	X1	Y1	W1	X2	Y2	W2
dp1	dp2	d0	d ψ								
8,79268E-15	2,92746E-09	0,000324882	-5,33963E-06	0,090966028	30286,46162	441810,7418	0	0	-1,47098E+11	1,80216E-05	0
0,000766432	255,1777693	0,000324882	-5,33963E-06	0,090966028	30286,46162	441810,7185	143,5365004	4,73047E+11	-1,47098E+11	-47789420,54	4,73047E+11

Simulation results checks											
Theoretical measurements according to 0 (according to the laws of classical mechanics)											
					Average errors (%)						
					ρ_1	ρ_2	v1	v2			
					-2,07496E-05	-2,07496E-05	-0,004092521	6,9603E-06			
					Errors (%)						
m1 (kg)	m2 (kg)	dW1	dW2	ρ_1	ρ_2	v1 (m/s)	v2 (m/s)	ρ_1	ρ_2	v1	v2
1,9884E+30	5,9722E+24	4,73047E+11	4,73047E+11	441810,7418	1,47098E+11	0,090966006	30286,46162	-1,31748E-14	0	2,45965E-05	2,1237E-11
1,9884E+30	5,9722E+24	4,73047E+11	4,73047E+11	441810,7422	1,47098E+11	0,090966006	30286,4616	-8,67376E-08	-8,67375E-08	2,46818E-05	8,53332E-08

9. Conclusion

In addition to the article hal-01584918, this article offers a quantification of the angles ε_i measuring the orientation of the absolute speeds of the observed bodies.

In the relativistic Platonic space, **these quanta have an absolute character** (they do not depend on the reference frames of observation) and are **independent of the rest masses** of the considered bodies.

Moreover, it should be emphasized that they are intimately related to the **Planck length** (by the presence of the quantity $\frac{Gh}{c^3}$ in their expression).

This quantification is **equivalent** to the conservation of the **absolute angular momentum** of the system and is **consistent** with the concept of **potential energy of the system** proposed previously, in the article hal-01584918.

The two proposed postulates on which this study is based, associated with the conservation of the absolute momentum of the system, offer, in the framework of the Platonic model, a surprisingly well **quantified approach** to the classical laws of gravitation applied to **elliptic Keplerian orbits**.

As in the article hal-01584918, these results and their review can certainly be quickly refined using more efficient and more sophisticated computer tools than those used here. And the principles retained deserve to be enriched and deepened in order to propose, in a more general framework, a much richer and complete approach to a quantum theory of gravitation (available for any frame references, taking into account barycentric fluctuations in the case of higher absolute velocities of the interacting bodies, search for a coupling with the standard model of particle physics, etc.).

In any case, the original way proposed in this article seems, at the very least, to be able to favor the emergence of new and numerous questions, promising, in directions still unexplored, particularly with regard to the deep links that it suggests between the quantum effects of De Broglie waves, the laws of gravitation and the concepts of potential energy and absolute angular momentum of the considered systems.

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