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# Leader-follower simultaneous tracking-agreement formation control of nonholonomic vehicles

Mohamed Maghenem   Antonio Loría   Elena Panteley <sup>\*†</sup>

**Abstract**— We present a unified decentralized force-controller that solves both, the leader-follower tracking and agreement formation control problems for a group nonholonomic mobile vehicles interconnected via a directed spanning-tree graph topology. The control strategy is inspired by and extends the main results in [20] which apply only in the one-leader-one-follower scenario. We assume that the leader velocities may be persistently exciting (in the tracking case) or vanishing (in the agreement scenario). As it is common in the literature our controller is composed of an inner loop at the velocity kinematics level and an outer-loop at the dynamics level. We establish robustness of the kinematics closed-loop system in the sense that the convergence of the formation errors is preserved, under the action of *any* velocity-controller that guarantees that the velocity errors are square integrable.

## I. INTRODUCTION

Designing a universal controller that solves the leader-follower control problem for the case of one nonholonomic mobile robot under different scenarios of the leader's velocities is a very challenging problem. Indeed, to the best of our knowledge the leader-follower *tracking-stabilization* problem for *one* leader and *one* follower nonholonomic vehicle has only been studied in [9], [18], [5], [6], [20]. The general idea is to design a unified velocity or torque controller for the follower robot in order to track the trajectories of the leader robot asymptotically (or practically) under different scenarios of the leader's velocities. Possible scenarios include the case in which the leader vehicle describes a generic time-varying trajectory (*tracking* scenario) —see *e.g.* [4], [7], [13], as well as the stabilization scenario in which the leader converges to a set point (*parking* scenario) or, more generally, the case in which the leader's velocities converge to zero (*robust stabilization*) —see *e.g.* [9], [15].

Some controllers are designed to apply in distinct scenarios. In [9] a saturated time-varying velocity controller is proposed to track the leader's trajectories under different scenarios of the leader's velocities that include the *robust stabilization* scenario and a particular form of *tracking* scenario. In [5] a unified force controller is proposed and, under all possible behaviors of the leader's vehicle, practical convergence of the tracking errors is established. In [18], the problem of tracking a general trajectory has been addressed using the concept of transverse functions, practical convergence of the tracking errors is guaranteed. In [6] and

[20], a unified torque controller is proposed in order to make the tracking error converging to the origin under the *tracking* and the *parking* scenarios. In [20], a keen idea has been used which consists in combining the *tracking* controller with a *stabilization* controller via a smooth supervisor that depends on the leader's velocities and promotes each controller depending on the current scenario.

The elegant transverse-functions control approach of [17] was used in [19] to solve the leader-follower formation problem, guaranteeing ultimate boundedness of the error trajectories. This is done under little restrictive conditions on the leader's velocities, using a full force-controlled model, and assuming that the interconnections graph is directed. It is also assumed, however, that some of the leader's coordinates are accessible to *all* the agents in the network. To the best of our knowledge, designing a unified controller that applies in both mutually exclusive formation control scenarios, *tracking and agreement*, *i.e.*, guaranteeing the convergence of the tracking errors to zero, remains an open problem.

Following the approach of [20] for the case of one leader and one follower, we propose a decentralized controller that covers more general scenarios relative to the nature of the leader's velocities. In contrast to previously cited articles, in the *tracking* scenario and for the kinematics control loop, we establish uniform global asymptotic stability (UGAS). This property cannot be overestimated, only UGAS implies local input to state stability. In addition, in the *robust stabilization* scenario, we establish strong integral Input-to-State Stability (strong iISS) [2], [3] with respect to the leader's velocities. For the overall dynamics we guarantee that the leader-follower error trajectories converge to zero provided that so do the velocity errors and that these are square integrable. Finally, in the multi-agent case, assuming that the vehicles are interconnected via a spanning-tree graph and based on the robustness properties established for the one-follower case, we extend our results to that of leader-follower formation control.

In the next section we formulate the tracking and agreement control problems; in Section III we present our main statements for a pair of vehicles. In Section IV we solve the formation control problems. Some numerical simulations are presented in Section V and we wrap up the paper with Concluding remarks in Section VI.

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<sup>\*</sup> M. Maghenem is with Univ Paris Saclay. A. Loria and E. Panteley are with the CNRS. LSS-CentraleSupélec, 3 Rue Joliot Curie, 91192, France. E. Panteley is also with ITMO University, Kronverkskiy av. 49, Saint Petersburg, 197101, Russia.

## II. PROBLEM FORMULATION

We consider force-controlled autonomous vehicles modeled by the generic equations

$$\begin{cases} \dot{x} = v \cos \theta \\ \dot{y} = v \sin \theta \\ \dot{\theta} = \omega \end{cases} \quad (1)$$

$$\begin{cases} \dot{v} = f_1(v, \omega, q) + g_1(v, \omega, q)u_1 \\ \dot{\omega} = f_2(v, \omega, q) + g_2(v, \omega, q)u_2 \end{cases} \quad (2)$$

The variables  $v$  and  $\omega$  denote the forward and angular velocities respectively, the first two elements of the vector  $q := [x \ y \ \theta]^\top$  correspond to the Cartesian coordinates of a point on the robot with respect to a fixed reference frame, and  $\theta$  denotes the robot's orientation with respect to the same frame. The two control inputs are the torques  $u_1, u_2$ . The functions  $f_i$  and  $g_i$  are assumed to be locally Lipschitz.

Generally speaking, the control strategy consists in designing virtual control laws at the kinematics level, i.e., considering  $v$  and  $\omega$  as control inputs. Then, we design  $u_1$  and  $u_2$  to steer  $v$  and  $\omega$  toward the ideal control laws  $v^*$  and  $\omega^*$ . That is, we show that if  $v \equiv v^*$  and  $\omega \equiv \omega^*$ , the origin of the closed-loop system, for the kinematics equations is uniformly globally asymptotically stable. Moreover, for (1), we establish robustness statements in the sense of input-to-state stability hence, our statements are valid for *any* controller that guarantees the stabilization of the origin at the force level —Equations (2).

### A. Single follower case

For clarity of exposition, we start by describing the most elementary scenario, that of leader-follower tracking control, as defined in [8]. Such problem consists in making the robot to follow a fictitious reference vehicle modeled by

$$\dot{x}_r = v_r \cos \theta_r \quad (3a)$$

$$\dot{y}_r = v_r \sin \theta_r \quad (3b)$$

$$\dot{\theta}_r = \omega_r, \quad (3c)$$

and which moves about with reference velocities  $v_r(t)$  and  $\omega_r(t)$ . More precisely, it is desired to steer the differences between the Cartesian coordinates to some values  $d_x, d_y$ , and to zero the orientation angles and the velocities of the two robots, that is, the quantities

$$p_\theta = \theta_r - \theta, \quad p_x = x_r - x - d_x, \quad p_y = y_r - y - d_y.$$

The distances  $d_x, d_y$  define the position of the robot with respect to the (virtual) leader. In general, these may be functions that depend on time and the state or may be assumed to be constant, depending on the desired path to be followed. In our study, we consider these distances to be defined as piece-wise constant functions —cf. [11].

Then, as it is customary, we transform the error coordinates  $[p_\theta, p_x, p_y]$  of the leader robot from the global coordinate frame to local coordinates fixed on the robot, that is, we define

$$\begin{bmatrix} e_\theta \\ e_x \\ e_y \end{bmatrix} := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} p_\theta \\ p_x \\ p_y \end{bmatrix}. \quad (4)$$

In these new coordinates, the error dynamics between the virtual reference vehicle and the follower becomes

$$\dot{e}_\theta = \omega_r(t) - \omega \quad (5a)$$

$$\dot{e}_x = \omega e_y - v + v_r(t) \cos(e_\theta) \quad (5b)$$

$$\dot{e}_y = -\omega e_x + v_r(t) \sin(e_\theta) \quad (5c)$$

which is to be completed with Eqs (2).

Hence, the control problem reduces to steering the trajectories of (5) to zero via the inputs  $u_1$  and  $u_2$  in (2), i.e.,  $\lim_{t \rightarrow \infty} e(t) = 0$ . As we mentioned, a natural method consists in designing virtual control laws at the kinematic level, that is,  $w^*$  and  $v^*$ , and control inputs  $u_1$  and  $u_2$ , depending on the latter, such that the origin  $(e, \tilde{v}, \tilde{\omega}) = (0, 0, 0)$  with

$$\tilde{v} := v - v^*, \quad \tilde{\omega} := \omega - \omega^*, \quad e = [e_\theta \ e_x \ e_y]^\top, \quad (6)$$

is uniformly globally asymptotically stable.

### B. Multiple followers case

The previous setting naturally extends to the case in which a swarm of  $n$  robots is required to follow a virtual leader, advancing in formation. This may be achieved in a variety of manners; in this paper we assume that the  $i$ th robot follows a leader, indexed  $i - 1$ , thereby forming a spanning-tree graph communication topology.

The geometry of the formation may be defined via the relative distances between any pair of leader-follower robots,  $d_{xi}, d_{yi}$  and it is independent of the communications graph (two robots may communicate independently of their relative positions). Then, the relative position error dynamics is given by a set of equations similar to (5), that is,

$$\dot{e}_{\theta i} = \omega_{i-1}(t) - \omega_i \quad (7a)$$

$$\dot{e}_{xi} = \omega_i e_{yi} - v_i + v_{i-1}(t) \cos(e_{\theta i}) \quad (7b)$$

$$\dot{e}_{yi} = -\omega_i e_{xi} + v_{i-1}(t) \sin(e_{\theta i}). \quad (7c)$$

For  $i = 1$  we recover the error dynamics for the case of one robot following a virtual leader that is, by definition,  $v_0 := v_r$  and  $\omega_0 := \omega_r$ . Then, we introduce the virtual controls  $(v_i^*, \omega_i^*)$  which, on one hand are designed to stabilize the reference trajectories for the kinematics equations and, on the other hand, serve as references for the actual controls  $u_{1i}$  and  $u_{2i}$  in

$$\dot{v}_i = f_{1i}(t, v_i, \omega_i, e_i) + g_{1i}(t, v_i, \omega_i, e_i)u_{1i} \quad (8a)$$

$$\dot{\omega}_i = f_{2i}(t, v_i, \omega_i, e_i) + g_{2i}(t, v_i, \omega_i, e_i)u_{2i}, \quad i \leq n \quad (8b)$$

As in the case of one follower, it is required to stabilize the origin of the closed-loop system. In particular, it is required that for all  $i \leq n$ ,

$$\lim_{t \rightarrow \infty} e_i(t) = 0 \quad (9)$$

and, as an intermediary step,  $u_{1i}$  and  $u_{2i}$  must guarantee that the velocity errors

$$\tilde{\omega}_i := \omega_i - \omega_i^*, \quad \tilde{v}_i := v_i - v_i^*$$

converge to zero sufficiently fast ( $\tilde{\omega}_i$  and  $\tilde{v}_i$  must be square integrable).

**Remark 1** Based on the transverse-functions approach of [17], in [19] the problem previously described is partially solved in the sense that only *practical* asymptotic stability is established, yet assuming that the vehicles communicate over a generic directed graph. Even though our results are established for spanning-tree communication topologies, they also apply if the communication graph is time-varying and uni-directional, provided that a distributed estimator that exponentially estimates the leader's trajectories is incorporated, *e.g.* based on [10] and [1]. In this case, the virtual controllers ( $v_i^*, \omega_i^*$ ), for all  $i \leq n$ , are the same as in the one-follower case ( $v_1^*, \omega_1^*$ ) in which we replace  $(v_r, \omega_r)$  by the estimated leader's velocities, and the errors  $e_i$  are expressed with respect to the estimated leader's trajectories that are generated by the  $i$ th agent internal estimator.

### III. CONTROL UNDER RELAXED CONDITIONS ON THE REFERENCE VELOCITIES

Following the keen idea proposed in [20] which consists in combining a tracking controller and a stabilization controller via a smooth supervisor that depends on the leader's velocities, in this section we solve the leader-follower control problem for the case of one follower and one leader, but under less restrictive assumptions on the leader's velocities. Technically, our results rely on and cover those obtained in [14] for the tracking control scenario, as well as those in [15] for the robust stabilization problem. These scenarios are defined as follows:

**Tracking scenario (S1).** It is assumed that the reference velocities are persistently exciting, *i.e.*, there exist  $T$  and  $\mu > 0$  such that

$$\int_t^{t+T} (|v_r(\tau)|^2 + |\omega_r(\tau)|^2) d\tau \geq \mu > 0, \quad \forall t \geq t_0. \quad (10)$$

**Robust stabilization scenario (S2).** It is assumed that the reference velocities vanish, *i.e.*,

$$\lim_{t \rightarrow \infty} v_r(t) = 0, \quad \lim_{t \rightarrow \infty} \omega_r(t) = 0 \quad (11)$$

The following kinematics controller solves the formation-tracking and agreement control problem, as formulated in Section II-A, under each one of the two mutually exclusive scenarios **S1** and **S2**:

$$v^* = v_r \cos(e_\theta) + k_x e_x \quad (12)$$

$$\omega^* = \omega_r + k_\theta e_\theta + k_y e_y v_r \phi(e_\theta) + \rho(t) k_y p(t) |e_{xy}|, \quad (13)$$

where  $e_{xy} := [e_x, e_y]^\top$ ,  $\phi(e_\theta) := \sin(e_\theta)/e_\theta$ ,

$$\rho(t) := \exp\left(-\int_{t_0}^t F(v_r(\tau), \omega_r(\tau)) d\tau\right), \quad (14)$$

and  $F : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$  is a piecewise continuous function that satisfies the following conditions:

1) If (11) holds, then,

$$\int_{t_0}^t F(v_r(\tau), \omega_r(\tau)) d\tau < \infty, \quad \forall t \geq 0$$

2) If, on the contrary, (10) holds, then there exist  $T_1$  and  $\mu_1$  such that

$$\int_t^{t+T_1} F(v_r(s), \omega_r(s))^2 ds \geq \mu_1, \quad \forall t \geq 0.$$

The first three terms in the definition of  $\omega^*$  ensure the achievement of the tracking control goal; the fourth, involving  $\rho(t)$ , makes it possible to stabilize converging trajectories. Thus, the function  $\rho$  favours the action of one control action or the other, depending on the nature of the reference trajectories. If  $\rho = 0$  the tracking control goal (of persistently exciting reference velocities) is enforced and, to that end,  $F$  is designed in a way that  $F(v_r(t), \omega_r(t))$  is also persistently exciting (PE). In the case that the agreement control goal is to be enforced,  $\rho$  is designed so that it remains separated from zero. Then, the last term in the right hand side of (13) ensures the achievement of the control objective.

An important relaxation with respect to the literature, in the stabilization scenario, is that reference velocities may vanish slowly and this is still sufficient. This is due to the enforced robustness guaranteed by our controller: strong integral-input-to-state stability —see Proposition 1 below.

In the following statement we provide a choice for the function  $F$  that satisfies the previous properties 1) and 2) above.

**Lemma 1** Let  $v_r$  and  $\omega_r$  be two scalar continuous functions and let  $\alpha(t) := \sqrt{v_r^2(t) + \omega_r^2(t)}$ . Assume that there exists  $\bar{\alpha} > 0$  such that  $|\alpha(t)|_\infty \leq \bar{\alpha}$ . Then, the function

$$F(v_r, \omega_r) := K(\alpha) := \begin{cases} 0 & \text{if } \alpha \in (0, \frac{\mu}{2T\bar{\alpha}}] \\ \alpha & \text{Otherwise} \end{cases} \quad (15)$$

satisfies the following properties

- 1)  $K(\alpha(t))$  is PE, if  $\alpha(t)$  is PE.
- 2)  $K(\alpha(t))$  is integrable, if  $\lim_{t \rightarrow \infty} \alpha(t) = 0$ .

*Proof:* For the second property observe that, after (11), there exists  $T_f < \infty$  such that  $\alpha(t) \in (0, \frac{\mu}{2T\bar{\alpha}}]$  so  $K(\alpha(t)) \equiv 0$  for all  $t \geq T_f$ . Therefore

$$\int_0^\infty K(\alpha(s)) ds \leq \int_0^{T_f} K(\alpha(s)) ds < \infty, \quad \forall t \geq 0.$$

To prove the first property, we use [12, Lemma 2] which states that, if  $\alpha(t)$  is PE *i.e.*, if

$$\int_t^{t+T} \alpha(\tau) d\tau \geq \mu \quad \forall t \geq 0$$

then for every  $t \geq 0$  there exists a non null measure interval

$$I_t := \{\tau \in [t, t+T] : |\alpha(\tau)| \geq a := \mu/(2T\bar{\alpha})\},$$

such that

$$\text{meas}(I_t) \geq b := T\mu/(2T\bar{\alpha}^2 - \mu).$$

Using this lemma we obtain

$$\int_t^{t+T} K^2(\alpha(s))ds \geq \int_{I_t} K^2(\alpha(s))ds \geq \int_{I_t} a^2 ds \geq a^2 b > 0,$$

so  $K(\alpha(s))$  is PE.  $\blacksquare$

**Remark 2** The definition of  $\rho$  in (14) covers that introduced in [20] since in the latter reference it is required that in addition to (11) the reference trajectories are integrable. The control law (13) covers the controllers from [15], [16] which apply only under scenario **S2** and, hence, the third term is absent. Moreover, in the controller proposed in [15]  $\rho(t) \equiv 1$  and in [16] the reference trajectories are required to be integrable. Another preliminary version of the control law (13) appears in [14], in which only uniform global asymptotic stability was established for the kinematics model and restricted to the tracking scenario (*i.e.*, with  $\rho \equiv 0$ ).

**Proposition 1** Consider the system (5) in which we replace  $v$  with  $v^* + \tilde{v}$ , and  $\omega$  with  $\omega^* + \tilde{\omega}$ , where the virtual inputs  $(v^*, \omega^*)$  are given by (12)-(13). Let  $k_x, k_\theta$ , and  $k_y > 0$ ; let  $p$  and  $\dot{p}$  be bounded and persistently exciting functions, and assume that there exists a positive constant  $\beta$  such that

$$\max \{ |\omega_r|_\infty, |\dot{\omega}_r|_\infty, |v_r|_\infty, |\dot{v}_r|_\infty \} \leq \beta. \quad (16)$$

Then,

- 1) If the reference trajectories satisfy (10) (scenario **S1**), the closed-loop system is integral input-to-state stable with respect to  $\eta_1 := [\tilde{v} \ \tilde{\omega}]^\top$ . Moreover, if  $\eta_1 \rightarrow 0$  and is square integrable, then the closed-loop trajectories converge to the origin.
- 2) If, on the contrary, (11) holds (scenario **S2**), the closed-loop system is strongly integral input-to-state stable (see [2]) with respect to  $\eta_2 := [v_r \ \omega_r \ \tilde{v} \ \tilde{\omega}]^\top$ . As a consequence, if  $\eta_2$  converges to zero then the closed-loop trajectories also converge to zero.  $\square$

*Sketch of proof.* The complete proof is omitted due to space limitations. It relies on the observation that the closed-loop system may be written in the form of a time-varying system with a vanishing perturbation. For the case of scenario **S1** we have

$$\dot{e} = A_{v_r}(t, e)e + B_1(t, e)\rho(t) + B_2(e)\eta_1, \quad (17)$$

where

$$A_{v_r}(t, e) := \begin{bmatrix} -k_\theta & 0 & -v_r(t)k_y\phi(e_\theta) \\ 0 & -k_x & \omega^*(t, e) \\ v_r(t)\phi(e_\theta) & -\omega^*(t, e) & 0 \end{bmatrix},$$

$$B_1(t, e) := \begin{bmatrix} -k_y f(t, e_x, e_y) \\ k_y f(t, e_x, e_y) e_y \\ -k_y f(t, e_x, e_y) e_x \end{bmatrix},$$

$$B_2(e) := \begin{bmatrix} 0 & -1 \\ -1 & e_y \\ 0 & -e_x \end{bmatrix}, \quad f(t, e_x, e_y) := p(t)|e_{xy}|.$$

Writing the closed-loop dynamics as in (17) is convenient to stress that the ‘‘nominal’’ system  $\dot{e} = A_{v_r}(t, e)e$  has a

familiar structure encountered in model reference adaptive control. Then, we proceed according to the following steps:

- 1) we build a strict Lyapunov function  $V(t, e)$  for the nominal system  $\dot{e} = A_{v_r}(t, e)e$ ;
- 2) we construct a strict Lyapunov function  $W(t, e)$  for the perturbed system  $\dot{e} = A_{v_r}(t, e)e + B_1(t, e)\rho$ ;
- 3) we use  $W(t, e)$  to prove integral ISS of (17) with respect to  $\eta_1$  as well as the boundedness of the trajectories of (17) under the assumption that  $\eta_1 \in \mathcal{L}_2$ . This and the assumption that  $\eta_1 \rightarrow 0$  imply the convergence of the error trajectories generated by (17).

Under the case of scenario **S2** the closed-loop equation is

$$\dot{e} = A(t, e)e + B(e)\eta_2 \quad (18)$$

where  $p_1(t) := \rho(t)p(t)$ ,  $\eta_2 := [v_r, \omega_r, \tilde{v}, \tilde{\omega}]$ , and

$$A(\cdot) := \begin{bmatrix} -k_\theta & -k_y p_1(t) \frac{|e_{xy}|}{e_x} & -k_y p_1(t) \frac{|e_{xy}|}{e_y} \\ 0 & -k_x & \psi(t, e) \\ 0 & -\psi(t, e) & 0 \end{bmatrix},$$

$$\psi(\cdot) := k_\theta e_\theta + k_y p_1(t) |e_{xy}|,$$

$$B(e) := \begin{bmatrix} -k_y e_y \phi(e_\theta) & 0 & 0 & -1 \\ k_y e_y^2 \phi(e_\theta) & e_y & -1 & e_y \\ \sin(e_\theta) - k_y e_x e_y \phi(e_\theta) & -e_x & 0 & -e_x \end{bmatrix}.$$

In this case we establish the strong iISS property with respect to the vector  $\eta_2$  following the steps in [16].

#### IV. A LEADER-FOLLOWER FORMATION CASE

In this section we extend the statement of Proposition 1 to the case of multi-agent formation control. To the best of our knowledge, a unified controller which solves the leader-follower tracking and agreement formation problem simultaneously is considered only in [19], but in this reference only ultimate boundedness of the error-trajectories is established. Moreover, the controller proposed therein is centralized hence, it relies on the assumption that part of the leader’s coordinates is available to all the agents in the network. Here, we assume that at least (and possible only) one vehicle has access to the reference vehicle’s trajectories. For a swarm of vehicles interconnected forming a spanning-tree we rely on cascaded arguments and the input to state stability property for any pair of vehicles in closed-loop to establish the formation tracking and agreement control goals with a universal controller.

Similarly to (12)-(20) we introduce the virtual controls

$$v_i^* := v_{i-1} \cos(e_{\theta i}) + k_{x i} e_{x i} \quad (19)$$

$$\omega_i^* := \omega_{i-1} + k_{\theta i} e_{\theta i} + k_{y i} e_{y i} v_{i-1} \phi(e_{\theta i}) + \rho_{i-1}(t) k_{y i} p(t) |e_{x y i}| \quad (20)$$

where,  $e_{x y i} := [e_{x i} \ e_{y i}]^\top$

$$\rho_{i-1}(t) := \exp \left( - \int_{t_0}^t F(v_{i-1}(\tau), \omega_{i-1}(\tau)) d\tau \right) \quad (21)$$

which at the dynamic level, serve as references for the actual controls  $u_{1i}$  and  $u_{2i}$  in

$$\dot{v}_i = f_{1i}(t, v_i, \omega_i, e_i) + g_{1i}(t, v_i, \omega_i, e_i) u_{1i} \quad (22a)$$

$$\dot{\omega}_i = f_{2i}(t, v_i, \omega_i, e_i) + g_{2i}(t, v_i, \omega_i, e_i)u_{2i}, \quad i \leq n. \quad (22b)$$

**Proposition 2** Consider the network-interconnected system composed of (7) with  $i \in \{1, \dots, n\}$  and the virtual-velocity controller (19)-(20). Let the control gains  $k_{x_i}$ ,  $k_{y_i}$ ,  $k_{\theta_i} > 0$ , let  $p_i$  and  $\dot{p}_i$  be bounded and persistently exciting, and assume that (16) holds. Then, all the error trajectories, *i.e.*  $e_i$  with  $i \in \{1, \dots, n\}$ , converge to zero provided that the leader's velocities satisfy either (10) or (11) and the velocities  $[\tilde{v}_1, \tilde{\omega}_1, \dots, \tilde{v}_n, \tilde{\omega}_n]$  are square integrable and converge to zero.  $\square$

*Proof:* Under the scenario S1, the closed-loop equations for each pair leader-follower is

$$\dot{e}_i = A_{v_{i-1}}(t, e_i)e_i + B_{1i}(t, e_i)\rho_i(t) + B_{2i}(e_i)\eta_{1i}, \quad (23)$$

where  $\eta_{1i} := [\tilde{v}_i, \tilde{\omega}_i]$ ,

$$A_{v_{i-1}}(\cdot) := \begin{bmatrix} -k_{\theta_i} & 0 & -v_{i-1}(t)k_{y_i}\phi(e_{\theta_i}) \\ 0 & -k_{x_i} & \omega_i^*(t, e_i) \\ v_{i-1}(t)\phi(e_{\theta_i}) & -\omega_i^*(t, e_i) & 0 \end{bmatrix},$$

$$B_{1i}(t, e_i) := \begin{bmatrix} -k_{y_i}p_i(t)e_{y_i} \\ k_{y_i}p_i(t)e_{y_i}^2 \\ -k_{y_i}p_i(t)e_{y_i}e_{x_i} \end{bmatrix}, \quad B_{2i}(e_i) := \begin{bmatrix} 0 & -1 \\ -1 & e_{y_i} \\ 0 & -e_{x_i} \end{bmatrix}.$$

The proof follows invoking recursively the statement of Proposition 1, by exploiting the cascaded structure of the system. Indeed, for the first follower the closed-loop system is reduced to (17), which, under the scenario **S1**, is integral Input-to-State Stable with respect to the vector  $\eta_{11} := [\tilde{v}_1, \tilde{\omega}_1]$ . As a result, using the square-integrability of  $\eta_{11}(t)$  and its convergence to zero, we obtain that  $e_1(t) \rightarrow 0$  and, consequently,

$$\lim_{t \rightarrow \infty} v_1(t) = v_r(t), \quad \lim_{t \rightarrow \infty} \omega_1(t) = \omega_r(t). \quad (24)$$

Moreover, there exists  $\bar{c}_1 > 0$  such that

$$\max\{v_1, \dot{v}_1, \omega_1, \dot{\omega}_1\} \leq \bar{c}_1. \quad (25)$$

For  $i = 2$  the closed-loop system (23) is equivalent to (17), if we replace  $v_r$  by  $v_1$  and  $\omega_r$  by  $\omega_1$ . Using (24), (25) we obtain that there exists  $t_1 > 0$  and  $\mu_1 > 0$  such that for all  $t \geq t_1$ , we have

$$\int_t^{t+T} [v_1^2(s) + \omega_1^2(s)] ds \geq \mu_1, \quad \forall t \geq t_1.$$

As a result, Proposition 1 applies for all  $t \geq t_1$ . Since  $\eta_{12} := [\tilde{v}_2, \tilde{\omega}_2]$  converges and is square integrable, we conclude that

$$\lim_{t \rightarrow \infty} |e_2(t)| = 0, \quad \lim_{t \rightarrow \infty} v_2(t) = v_r(t), \quad \lim_{t \rightarrow \infty} \omega_2(t) = \omega_r(t) \quad (26)$$

—*cf.* [3]. Moreover, there exists  $\bar{c}_2 > 0$  such that

$$\max\{v_2, \dot{v}_2, \omega_2, \dot{\omega}_2\} \leq \bar{c}_2. \quad (27)$$

The result follows by induction.

Under the case of scenario **S2** the closed-loop equation for each  $i \in \{1, \dots, n\}$  becomes

$$\dot{e}_i = A_i(t, e_i)e_i + B(e_i)\eta_{2i} \quad (28)$$

where  $p_{1i}(t) := \rho_i(t)p_i(t)$ ,  $\eta_{2i} := [v_{i-1}, \omega_{i-1}, \tilde{v}_i, \tilde{\omega}_i]$ , and

$$A_i(\cdot) := \begin{bmatrix} -k_{\theta_i} & -k_{y_i}p_{1i}(t)\frac{|e_{x_{y_i}}|}{e_{x_i}} & -k_{y_i}p_{1i}(t)\frac{|e_{x_{y_i}}|}{e_{y_i}} \\ 0 & -k_{x_i} & \psi_i(t, e_i) \\ 0 & -\psi_i(t, e_i) & 0 \end{bmatrix},$$

$$\psi_i(\cdot) := k_{\theta_i}e_{\theta_i} + k_{y_i}p_{1i}(t)|e_{x_{y_i}}|,$$

$$B(e_i) := \begin{bmatrix} -k_{y_i}e_{y_i}\phi(e_{\theta_i}) & 0 & 0 & -1 \\ k_{y_i}e_{y_i}^2\phi(e_{\theta_i}) & e_{y_i} & -1 & e_{y_i} \\ \sin(e_{\theta_i}) - k_{y_i}e_{x_i}e_{y_i}\phi(e_{\theta_i}) & -e_{x_i} & 0 & -e_{x_i} \end{bmatrix}.$$

As before, we invoke Proposition 1 recursively. For  $i = 1$ , the system (28) reduces to (18) so, by Proposition 1, it is strong iISS with respect to  $\eta_{21} := [v_r, \omega_r, \tilde{v}_1, \tilde{\omega}_1]$ . Consequently, when  $\eta_{21} \rightarrow 0$ , we have

$$e_1 \rightarrow 0, \quad v_1 \rightarrow 0, \quad \omega_1 \rightarrow 0.$$

For  $i = 2$ , the convergence of  $[v_1, \omega_1]$  implies that the closed-loop (17) is strong iISS with respect to  $\eta_{22} := [v_1, \omega_1, \tilde{v}_2, \tilde{\omega}_2]$ . Consequently

$$e_2 \rightarrow 0, \quad v_2 \rightarrow 0, \quad \omega_2 \rightarrow 0.$$

The result follows by induction.  $\blacksquare$

## V. SIMULATION RESULTS

We consider a group of four mobile robots following a virtual leader in formation (diamond shape). The tracking scenario (**S1**) is defined by persistently-exciting reference velocities  $w_r(t) = \beta_1(t)\alpha_1(t)$  and  $v_r(t) = \beta_2(t)\alpha_2(t)$  where  $\alpha_1(t)$  and  $\alpha_2(t)$  are sinusoids of respective periods  $T_1 = 10.8s$  and  $T_2 = 10.5s$ ,  $\beta_1(t)$  and  $\beta_2(t)$  are periodic “on-off” signals taking values in  $\{0, 1\}$  with duty-cycles of 50% and 33%, and periods of  $1.5T_1$  and  $T_2$  respectively. For the robust stabilization scenario (**S2**) we set  $w_r = 0$  and  $v_r$  as solution of  $\dot{v}_r = -100v_r^3$  that is, a non-integrable function.

The initial conditions are set to  $[x_r(0), y_r(0), \theta_r(0)] = [0, 0, 0]$ ,  $[x_1(0), y_1(0), \theta_1(0)] = [1, 3, 4]$ ,  $[x_2(0), y_2(0), \theta_2(0)] = [0, 2, 2]$ ,  $[x_3(0), y_3(0), \theta_3(0)] = [0, 4, 1]$  and  $[x_4(0), y_4(0), \theta_4(0)] = [0, 3, 1]$ ; the velocity-control gains are set to  $k_{x_i} = k_{y_i} = k_{\theta_i} = 1$  and the function  $p(t) = 20 \sin(0.5t)$ , which has a persistently exciting time-derivative, as required. Furthermore, we define

$$F(a, b) := K(\sqrt{a^2 + b^2}) := \begin{cases} \sqrt{a^2 + b^2} & \forall \alpha \geq 0.1 \\ 0 & \text{Otherwise.} \end{cases}$$

A diamond-shaped formation is obtained by setting a certain desired distance between the robots and all desired orientation offsets set to zero:  $[d_{x_{r,1}}, d_{y_{r,1}}] = [0, 0]$ ,  $[d_{x_{1,2}}, d_{y_{1,2}}] = [-1, 0]$  and  $[d_{x_{2,3}}, d_{y_{2,3}}] = [1/2, -1/2]$  and  $[d_{x_{3,4}}, d_{y_{3,4}}] = [0, 1]$ ; see Figures 4 and 2 for illustration. At the dynamics level, we use the adaptive controller from [15] to guarantee that the error velocities converge with finite  $\mathcal{L}_2$  norm; the torque controller parameters are set to  $(\gamma, k_d) = (10^{-5}, 15)$ , and the initial estimates  $\hat{\Theta}(0) = (\hat{m}_1, \hat{m}_2, \hat{c}) = (0, 0, 0)$ .

The simulation results with PE reference trajectories (scenario **S1**) are showed in Figures 1 and 2; with vanishing references (scenario **S2**), they are depicted in Figures 3–4. In both cases, the tracking errors converge to zero and the formation follows the prescribed path, which includes straight lines and turns before converging to a set-point.

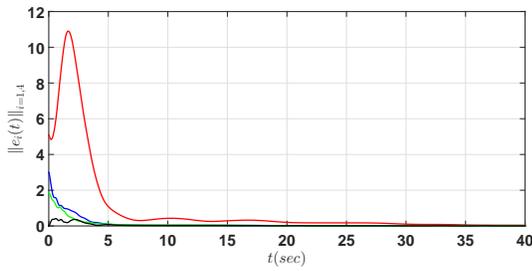


Fig. 1. Relative errors (in norm) for each pair leader-follower under S1

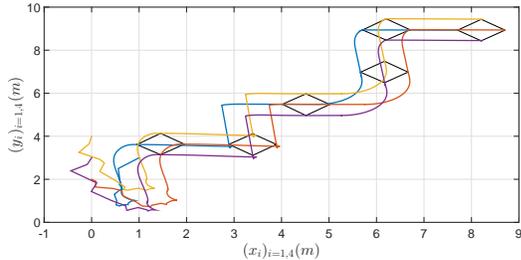


Fig. 2. Illustration of the path-tracking in formation under S1

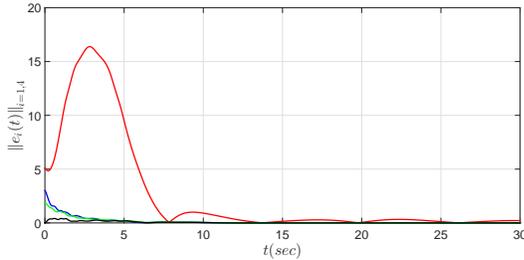


Fig. 3. Relative errors (in norm) for each pair leader-follower under S2

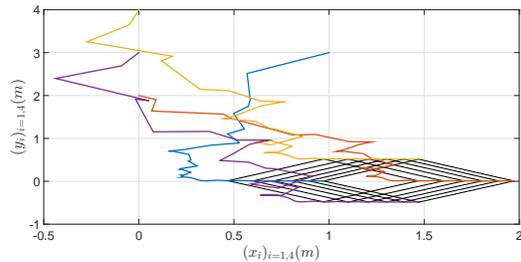


Fig. 4. Illustration of the path-tracking in formation under S2

## VI. CONCLUSION

We studied the decentralized leader-follower tracking and agreement formation control problem for a group of nonholonomic vehicles. Under a directed spanning tree communication topology, we proposed a unified kinematic level controller that solves the two problems at the kinematic level. On the dynamical level, the virtual kinematic controller serves as a reference for the torque-control design. For the global closed-loop system we proved that any torque controller that

ensures the tracking of the desired velocities with finite  $\mathcal{L}_2$  norm solves the leader-follower formation problem. Further research is being carried out to incorporate other aspects such as changing topologies and obstacle avoidance.

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