Distributed Power Control with Partial Channel State Information: Performance Characterization and Design

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A Framework for Distributed Power Control with Partial Channel State Information

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***Abstract*—One of the goals of this paper is to contribute to finding distributed power control strategies which exploit, as well as possible, the information available about the global channel state; which may be local or noisy. A suited way of measuring the global efficiency of a distributed power control scheme is to use the long-term utility region. First, we provide the utility region characterization for any power control problem for which the utility takes the form under consideration, the channel state is i.i.d., and the observation structure is memoryless. Second, the corresponding theorem is exploited to construct an iterative algorithm which provides memoryless and stationary power control strategies. The performance of the proposed algorithm is assessed for energy-efficient utility functions and shown to perform much better than closest state-of-the-art solutions, with the additional advantage of being applicable even in the presence of arbitrary observation structures such as noisy channel gain estimates.**

I. Introduction

Many modern wireless networks tend to become distributed. This is already the case of Wifi networks which are distributed decision-wise; for example, each access point performs channel or band selection without the assistance of a central or coordinating node. As another example, small cells networks, which are envisioned to constitute one of the key components to implement the ambitious roadmap set for 5G networks [2] [3] [4], will need to be largely distributed; distributedness is one way of dealing with complexity and signalling issues induced by the large number of small base stations and mobile stations. In this paper, we consider wireless interference networks that are distributed both decision-wise and information-wise. More specifically, each transmitter has to perform a power control or, more generally, a radio resource allocation task by itself and by having only access to partial information of the network state.

When inspecting the literature on distributed power control (see e.g., [5] [6] [7]), it appears that the derived power control schemes are effectively distributed decision-wise and information-wise but almost always globally inefficient. A natural and important question arises. Is this because the considered power control scheme is not good enough or does it stem from intrinsic limitations such as limited information availability? To the authors' knowledge, this question has not been addressed formally. One of the goals of the this paper is precisely to provide a framework that allows one to derive the limiting performance of power control with partial information and therefore to be able to measure the efficiency of a given power control scheme. To reach this goal we resort to recent results that bridge the gap between decision theory and information theory [8]. We exploit these results to characterize the limiting performance in terms of long-term utility region[1], each transmitter being assumed to have its own utility function. The performance characterization is then exploited in a constructive manner to determine power control strategies and more specifically one-shot decision functions, which allow the transmitter to choose its power based on a single observation about the global channel state. The practical interest in designing one-shot decision power control functions is very well motivated in the literature (see e.g., [9] [10]); in particular, it allows the transmitter to take quick decisions, which do not generate extra delay (e.g., due to backhauling or non-direct inter-transmitter exchanges). To be more concrete, if Transmitter i knows an estimate $\widehat{g}_{ii}$ of the channel gain of the link between Transmitter i and Receiver i, the decision function writes under the form $f_i(\widehat{g}_{ii})$. For example, in the pioneering work on energy-efficient power control [11] and more advanced works such as [12] [13] the obtained distributed decision function is of the form of a channel inversion formula $f_i(x) = \frac{1}{x}$. Remarkably, our approach allows one to obtain decision functions which may perform much better globally e.g., when measured in terms of sum energy-efficiency. And more importantly, our approach allows one to generate decision functions which take any partial information about the channel as an argument, which is clearly not possible by using the state-of-the-art approaches. To provide a concrete limitation of the state-of-the-art there are many relevant works such as [14] where the energy-efficient power control strategies are computed from the knowledge of the SINR (signal-noise-plus interference ratio) feedback but there is no clue about what should be done if some **arbitrary partial information** e.g., the individual channel gain g_{ii} would be available, which shows the strong interest in the framework proposed in this paper.

The developed approach mainly relies on two key assumptions: the global channel state is assumed to be i.i.d., which is a quite common assumption; the transmit power and channel state are assumed to be discrete. The latter assumption is less common and is supported by several strong arguments. First, assuming the transmit power to be discrete is of practical interest since there exist wireless communication standards

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The material in this paper was partly presented in [1].

[1]The utility region can be seen as a counterpart of the capacity region. Instead of considering the ultimate performance of a code, one considers the ultimate performance of a power control strategy.

in which the power can only be decreased or increased by step and in which quantized wireless channel state information (CSI) is used (see e.g., [15] [16]). Additionally, if the transmitter task is to perform band or channel selection, which is a special instance of power control, the transmitter action set is again intrinsically discrete. Second, the argument is mathematical; it is well known from the coding theorem literature [17] that the performance characterization for the continuous case follows as a special case of the discrete case. Third, quite remarkably, imposing the transmitters to use a reduced action space may be beneficial both for the network and individual performance; simulations provided in this paper show that the very interesting result obtained in [18] is in fact more general; in [18], the authors show that binary power control may be optimal or generate a very small performance loss compared to the continuous case. In this respect, the authors have shown in [9] that using one-shot decision functions which are step functions may be optimal when the utility function is chosen to be the Shannon sum-rate. One of the important contributions of the present work can be seen as a generalization of such a result to **arbitrary utility functions** (namely, functions that have the quite general form assumed in this paper). At last, concerning the discretization of the channel gains, it has to be noticed that the actual channel gains are assumed to be continuous but the version of the algorithm proposed in this paper that generates decision functions uses quantized channel gains; this technical point is discussed further in Sec. V.

This paper is structured as follows. In Sec. II, we provide the proposed general formulation of the problem of power control under partial information. In Sec. III, we derive the characterization of the best achievable performance of power control under partial information, which amounts to characterizing the long-term utility region; the latter being fully characterized by its Pareto frontier. In Sec. IV, we propose one possible way of constructing good or efficient power control strategies under partial information. Sec. V corresponds to a detailed numerical analysis; to facilitate comparisons, several simulation scenarios have been chosen to mimic the closest state-of-the-art papers as much as possible. The paper is concluded by Sec. VI, which not only recaps some attractive features of our approach but also some its weaknesses.

II. Problem statement

The wireless system under consideration comprises $K \geqslant 2$ pairs of interfering transmitters and receivers which can operate over $B \geqslant 1$ non-overlapping bands. The power Transmitter $i \in \{1,...,K\}$ allocates to band $b \in \{1,...,B\}$ is denoted by a_i^b, a_i^b being subject to classical power limitations: $a_i^b \leqslant P_{\max}$ and $\sum_{b=1}^{B} a_i^b \leqslant P$, with $P_{\max} \leqslant P$. In the setup under study, the quantities of interest for Transmitter i to control its *power vector*

$$a_i = (a_i^1, ..., a_i^B) \tag{1}$$

are given by the channel gains of the different links between the transmitters and receivers. The channel gain of the link between Transmitter $i \in \{1,...,K\}$ and Receiver $j \in \{1,...,K\}$ for band $b \in \{1,...,B\}$ is denoted by $g_{ij}^b = |h_{ij}^b|^2 \in \mathcal{G}$. The *global channel state* is then given by the following K^2-dimensional vector which comprises all channel gains:

$$a_0 = (g_{11}^1, ..., g_{11}^B, g_{12}^1, ..., g_{12}^B, ..., g_{KK}^1, ..., g_{KK}^B) \tag{2}$$

and is assumed to follow a given probability distribution which is denoted by ρ_0.

Transmitter i, $i \in \{1,...,K\}$, can update its power vector a_i from block to block. To update its power, each transmitter has a certain knowledge of the global channel state, which is called the *partial information* available to Transmitter i and is represented by the signal s_i. Before defining s_i, it has to be mentioned that for all the analytical and algorithmic results provided in this paper, the key quantities such as the power vector, the global channel state, and the partial information are assumed to be discrete (this assumption has been discussed in the introduction section). This means that: $\forall i \in \{0,1,...,K\}, a_i \in \mathcal{A}_i$ with $|\mathcal{A}_i| < \infty$; $\forall i \in \{1,...,K\}, s_i \in \mathcal{S}_i$ with $|\mathcal{S}_i| < \infty$. More specifically, the signal s_i is assumed to be the output of a discrete memoryless channel $\mathrm{P}(S_i = s_i | A_0 = a_0) = \daleth_i(s_i|a_0)$ [17], where A_0 and S_i represent the random variables used to model the channel state variations and the partial information available to Transmitter i respectively[2]. The full or perfect global CSI at Transmitter i corresponds to $s_i = a_0$. The case where only perfect individual CSI is available is given by $s_i = (g_{ii}^1, ..., g_{ii}^B)$. The signal s_i may also be a noisy estimate of $(g_{ii}^1, ..., g_{ii}^B)$: $s_i = (\widehat{g}_{ii}^1, ..., \widehat{g}_{ii}^B)$. Note that in the numerical performance analysis, the proposed power control strategy is effectively implemented by using discrete quantities but is tested over continuous channels (namely, channel gains correspond to realizations of complex Gaussian random variables i.e., Rayleigh fading is assumed). At last, each channel gain is assumed to obey a classical block-fading variation.

By denoting t as the block index, the purpose of Transmitter i is to tune the power vector $a_i(t)$ for block t by exploiting its knowledge about the channel state that is, the signal $s_i(t)$. More precisely, we assume that Transmitter knows s_i at time t but also the past realizations of it, namely $s_i(1), ..., s_i(t-1)$, the transmission being assumed to start at block $t = 1$ and to stop at block $t = T$. In its general form, the *power control strategy* of Transmitter i is a sequence of functions which is denoted by $f_i = (f_{i,t})_{1 \leqslant t \leqslant T}$ and defined by:

$$\begin{array}{cccc} f_{i,t} & : & \mathcal{S}_i^t & \longrightarrow & \mathcal{A}_i \\ & & (s_i(1), s_i(2), ..., s_i(t)) & \longmapsto & a_i(t). \end{array} \tag{3}$$

The typical power control scenario is that Transmitter i has to implement a power control strategy which aims at maximizing a certain performance metric called utility function in this paper and denoted by u_i. In full generality, we assume that this maximization has to be performed in presence of constraints (e.g., quality-of-service constraints) which are represented by the *constraint functions* $\gamma_1, ..., \gamma_M$, M being the number of constraints.

[2]To avoid any ambiguity where there is any, we use capital letters to refer to random processes or variables. In particular, A_i is used to represent the random process of a_i.

The two main issues addressed in this paper are as follows. First, we characterize the achievable performance in terms of long-term utility region when the block or *instantaneous utility* is a function of the form $u_i(a_0, a_1, ..., a_K)$. The *long-term utility* of Transmitter i is defined by:

$$U_i(f_1, ..., f_K) = \lim_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\big[u_i(a_0(t), a_1(t), ..., a_K(t))\big] \quad (4)$$

whenever the above limit exists. The *long-term utility region* therefore formally corresponds to all the points $(U_1, ..., U_K) \in \mathbb{R}^K$ that can be reached by considering all possible power control strategies as defined by (3). The presence of the expectation operator is required in general (it can be omitted when a law of large numbers is applicable) since the channel is random and every power vector is a function of it. In general the channel is a random process $A_0(1), ..., A_0(T)$ but since we assume the channel gains to be i.i.d., the notation can be simplified by only using a single random variable A_0. The corresponding probability distribution is the *global channel state distribution* (as already mentioned, it is denoted by ρ_0). The second issue we want to address in this paper is to determine power control strategies which only use the available local information while performing as well as possible in terms of a global utility e.g., in terms of sum-utility $\sum_{i=1}^{K} U_i$ with $u_i = \log(1 + \mathrm{SINR}_i)$ (see [18]).

III. Limiting performance characterization of power control with partial information

While many power control schemes using partial CSI are available in the literature, very often it is not possible to know whether the available information is exploited optimally by the considered power control scheme. While the problem of optimality is in general a very important and challenging problem, it turns out to be solvable in important scenarios such as the scenario under investigation in this paper. Indeed, an important message of the present work is that, under the made assumptions, information theory tools can be used to fully characterize the limiting theoretical performance of the power control strategies. The two key assumptions which are made for this are as follows: (i) The channel state $a_0(t)$ is i.i.d.; (ii) The observation structure which defines the partial observation s_i is memoryless. Assuming (i) and (ii), the following theorem provides the utility region characterization for any power control problem under the form specified by [8]. For the sake of clarity we will use the following notations: $a = (a_0, a_1, ..., a_K)$ and $s = (s_1, ..., s_K)$; $\daleth$ stands for the conditional probability $\mathrm{P}_{S|A_0}$, $S = (S_1, ..., S_K)$ being the random variable used to model the vector of individual signals available to the transmitters; V is an *auxiliary variable* as used in coding theorems [17] and its operational meaning will be interpreted a little further; the notation Δ_K will refer to the *unit simplex* of dimension K: $\Delta_K = \left\{(x_1, ..., x_K) \in \mathbb{R}^K : \forall i \in \{1, ..., K\}, x_i \geqslant 0; \sum_{i=1}^{K} x_i = 1\right\}$. At last, to state the theorem which follows, we will use the following notations: the function $w_\lambda = \sum_{i=1}^{K}\lambda_i u_i$ represents the *weighted utility* with $\lambda = (\lambda_1, ..., \lambda_K) \in \Delta_K$; the function W_λ represents the expected version of the function w_λ i.e., $W_\lambda = \mathbb{E}(w_\lambda)$; the function Γ_m represents the expected version of the constraint function γ_m i.e., $\Gamma_m = \mathbb{E}(\gamma_m)$.

Theorem III.1. *Define the $(K+1)-$uplet of probability distributions $\left(Q^\lambda_{A_1|S_1,V}, ..., Q^\lambda_{A_K|S_K,V}, Q^\lambda_V\right)$ as a solution of the following optimization problem*

$$\max_{P_{A_1|S_1,V}, ..., P_{A_K|S_K,V}, P_V} W_\lambda\left(P_{A_1|S_1,V}, ..., P_{A_K|S_K,V}, P_V\right) \quad (5)$$

$$\text{s.t. } \forall m \in \{1, \dots, M\},\ \Gamma^{(m)}\left(P_{A_1|S_1,V}, ..., P_{A_K|S_K,V}, P_V\right) \geqslant 0 \quad (6)$$

where

$$W_\lambda\left(P_{A_1|S_1,V}, ..., P_{A_K|S_K,V}, P_V\right) = \sum_{a,s,v}\rho_0(a_0)\daleth(s|a_0)P_V(v)\prod_{i=1}^{K}P_{A_i|S_i,V}(a_i|s_i,v)w_\lambda(a_0, a_1, ..., a_K) \quad (7)$$

$$\Gamma^{(m)}\left(P_{A_1|S_1,V}, ..., P_{A_K|S_K,V}, P_V\right) = \sum_{a,s,v}\rho_0(a_0)\daleth(s|a_0)P_V(v)\prod_{i=1}^{K}P_{A_i|S_i,V}(a_i|s_i,v)\gamma_m(a_0, a_1, ..., a_K) \quad (8)$$

P_V being the distribution of some auxiliary variable $V \in \mathcal{V}$ verifying the Markov chain $V - (A_0, A_1, ..., A_K) - (S_1, ..., S_K)$.
Then, when $T \to +\infty$, the Pareto frontier $\overline{\mathcal{U}}$ of the long-term utility region associated with the constraints is given by:

$$\overline{\mathcal{U}} = \left\{(U_1, ..., U_K) \in \mathbb{R}^K : U_i = \sum_{a,s,v}Q^\lambda(a,s,v)u_i(a_0, a_1, ..., a_K)\right\} \quad (9)$$

with $Q^\lambda(a,s,v) = \rho_0(a_0)\daleth(s|a_0)Q^\lambda_V(v)\prod_{i=1}^{K}Q^\lambda_{A_i|S_i,V}(a_i|s_i,v)$ and $\lambda \in \Delta_K$.

Proof: First of all, we show that the power control strategies of the different users $f_1, ..., f_K$ intervene in the long-term utility only through the joint probability over $\mathcal{A}_0 \times ... \times \mathcal{A}_K$. Therefore, characterizing the long-term utility region is equivalent to characterizing the set of achievable or implementable joint probability distributions. We have that:

$$U_i(f_1, ..., f_K) \quad (10)$$

$$= \lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\left[u_i(A_0(t), A_1(t), ..., A_K(t))\right] \quad (11)$$

$$= \lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_{a_0,\dots,a_K}P_t(a_0, ..., a_K)u_i(a_0, ...a_K) \quad (12)$$

$$= \sum_{a_0,\dots,a_K}u_i(a_0, ..., a_K)\lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}P_t(a_0, ..., a_K) \quad (13)$$

where $P_t(a_0, ..., a_K)$ is the joint probability distribution induced by the power control strategy profile $f_1, ..., f_K$ at time t. Again, we denote the random process $A_i(t)$ by capital letters to distinguish it from its realization, denoted by a_i. Therefore, a utility μ_i is achievable if and only if it can be written as

$$\mu_i = \sum_{a_0,a_1,...,a_K} Q(a_0, a_1, ..., a_K) u_i(a_0, a_1, ..., a_K) \quad (14)$$

and there exists a power control strategy profile $(f_1, ..., f_K)$ such that

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} P_t(a_0, ..., a_K) = Q(a_0, ..., a_K). \quad (15)$$

Second, we show that the long-term utility region is necessarily convex, whatever the instantaneous utility functions under consideration. As a consequence, as indicated by (9) the Pareto frontier can be obtained by maximizing the long-term weighted utility W_λ. Assume that there exists a power control strategy profile $(f_1, ..., f_K)$ which allows to reach a point $(\mu_1, ..., \mu_K)$ of the long-term utility region. Then, there exists a joint distribution Q which is implementable. Similarly, we consider another power control strategy profile $(f_1', ..., f_K')$ which ensures that $(\mu_1', ..., \mu_K')$ can be reached and that there exists an implementable Q'. By using 100α % of the time the strategy profile $(f_1, ..., f_K)$ and $100\alpha' = 100(1-\alpha)$ % of the time the strategy profile $(f_1', ..., f_K')$ it follows that the convex combination $Q'' = \alpha Q + \alpha' Q'$, $\alpha + \alpha' = 1$, $\alpha \geqslant 0$, $\alpha' \geqslant 0$, is also implementable. Therefore the point $(\mu_1", ..., \mu_K")$, $\mu_i" = \alpha\mu_i + \alpha\mu_i'$ can be attained. Note that this argument holds in presence of the constraints defined by (6)(8).

As the last step of the proof, we exploit the coding theorem of [8] which states that a joint probability distribution $Q(a_0, a_1, ..., a_K)$ is implementable if and only if it writes as:

$$Q(a) = \rho_0(a_0) \sum_{s,v} \daleth(s|a_0) P_V(v) \prod_{i=1}^{K} P_{A_i|S_i,V}(a_i|s_i, v) \quad (16)$$

where V is any random variable which verifies the Markov chain $V - (A_0, A_1, ..., A_K) - (S_1, ..., S_K)$. ■

To better understand Theorem III.1 and its proof, let us comment on it in detail.

The first comment which can be made is that the long-term utility region Pareto frontier characterization relies on the use of an auxiliary random variable V. The presence of such variables is very common in coding theorems. For example, the capacity region of degraded broadcast channels is parameterized by auxiliary variables; for one transmitter and two receivers, only one auxiliary variable suffices. In the latter case, the auxiliary variable can be interpreted for instance as a degree of freedom the transmitter has for allocating the available resource between the two receivers [17]. In general, auxiliary random variables have to be considered as parameters which allow one to describe a set of points and thus constitute, before all, a purely mathematical tool. Their operational meaning is generally given by the achievability part of the coding theorem. As far as Theorem III.1 is concerned, the achievability part mainly corresponds to the general coding theorem given in [8]. In a power control setting, V may be seen as a coordination random variable or a lottery which allows one to generate a coordination key. To be more concrete, consider a single-band interference channel with two transmitters and two receivers. The idea is to exchange a coordination key offline and which consists of a sequence of realizations $v(1), ..., v(T)$ of a (Bernouilli) binary random variable: $V \sim \mathcal{B}(\tau)$, $\tau \in [0, 1]$. Then, online, a possible rule for the transmitters might be as follows: if $v(t) = 1$, Transmitter 1 transmits and if $v(t) = 0$, Transmitter 2 transmits. We see that in this simple example, V would act as a time-sharing variable which would allow to manage interference even if the transmitters have no knowledge at all about the channel (i.e., $s_i = const.$). Then, by optimizing the Bernoulli probability τ, one can obtain better performance than transmitting at full (or constant) power. Note that the full power operation point would be obtained by applying the iterative water-filling algorithm (IWFA) (see e.g. [19] [20]) to a single-band interference network where each Transmitter i wants to maximize its utility $u_i = \log(1 + \text{SINR}_i)$, SINR_i being the SINR at Receiver i.

The second comment we would like to make on Theorem III.1 is that the achievable utility region can be described only by its Pareto frontier. This result follows from the fact that the long-term utility region is convex, as shown throughout the proof. This explains the presence of the vector λ. The vector allows one to move along the Pareto frontier $\overline{\mathcal{U}}$.

The third comment we will make here is that the power control strategy only intervenes in the long-term utility through its average behavior i.e., in terms of conditional probability $P_{A_i|S_i,U}$ that is, the (conditional) frequency at which a given power vector a_i is used. Optimality of a given power control strategy under partial information is only related to the frequencies at which the possible transmit power levels are used.

The fourth comment concerns the alphabet V lies in, namely $\mathcal{V}$. Indeed, it is possible to cover all the feasible utility region by choosing appropriately the possible range for $|\mathcal{V}|$ i.e., by following the next theorem. In general, to cover all the feasible utility region, the range for $|\mathcal{V}|$ has to vary in an interval which is specified in Theorem III.1. Considering larger values for $|\mathcal{V}|$ would not bring any performance gain.

Theorem III.2. *(Cardinality of $\mathcal{V}$) The set of implementable distributions Q (as defined per Theorem III.1) can be reached by considering the possible auxiliary random variables $V \in \mathcal{V}$ with:*

$$|\mathcal{V}| \leqslant |\mathcal{A}| \cdot |\mathcal{S}| - 1 \quad (17)$$

where $|\mathcal{A}| = \prod_{i=0}^{K} |\mathcal{A}_i|$ *and* $|\mathcal{S}| = \prod_{i=1}^{K} |\mathcal{S}_i|$.

Proof: The proof is based on the following lemma (see [21] [22]).

Support Lemma. Let $\mathcal{X}$ be a finite set and $\mathcal{V}$ be an arbitrary set. Let $\mathcal{P}$ be a connected compact subset of probability distributions on $\mathcal{X}$ and $p(x|v) \in \mathcal{P}$, indexed by $v \in \mathcal{V}$, be a collection of (conditional) pmfs on $\mathcal{X}$. Suppose that $\eta_j(\pi)$, $j = 1, ..., d$, are real-valued continuous functions of $\pi \in \mathcal{P}$. Then for every $V \sim F(v)$ defined on $\mathcal{V}$, there exists a random variable $V' \sim p(v')$ with $|\mathcal{V}'| \leqslant d$ and a collection of conditional probability distributions $p(x|v') \in \mathcal{P}$, indexed by $v' \in \mathcal{V}'$, such that for $j = 1, ..., d$,

$$\int_{\mathcal{V}} \eta_j(p(x|v))\mathrm{d}F(v) = \sum_{v' \in \mathcal{V}'} \eta_j(p(x|v'))p(v'). \tag{18}$$

We now show how this lemma is used to bound the cardinality of auxiliary random variables. Suppose $\mathcal{X} = \mathcal{A} \times \mathcal{S}$, which refers to the joint action and joint state (observation) profiles. The corresponding $\mathcal{P}$ will be a connected compact subset of probability distributions on $\mathcal{A} \times \mathcal{S}$ and $p(a_1, ..., a_K, s_1, ..., s_K|v) \in \mathcal{P}$, indexed by $v \in \mathcal{V}$, will be a collection of (conditional) pmfs on $\mathcal{A} \cdot \mathcal{S}$. Note that the product distribution $\prod_{i=1}^{K} P_{A_i|S_i,V}(p_i|s_i, v)$ constitutes a special form of the general probability $P_{A_1,...,A_K|S_1,...,S_K,V}(a_1, ..., a_K|s_1, ..., s_K, v)$, which itself can be rewritten as:

$$\begin{aligned} &P_{A_1,...,A_K|S_1,...,S_K,V}(a_1, ..., a_K|s_1, ..., s_K, v) \\ &= \frac{P_{A_1,...,A_K,S_1,...,S_K|V}(a_1, ..., a_K, s_1, ..., s_K|v)}{P_{S_1,...,S_K|V}(s_1, ..., s_K|v)} \end{aligned} \tag{19}$$

Hence, $\prod_{i=1}^{K} P_{A_i|S_i,V}(a_i|s_i, v)$ can be expressed by $\pi \in \mathcal{P}$. Denoting by j_q the ratio of j over K, consider the following $|\mathcal{A}| - 1$ continuous functions on $\mathcal{P}$:

$$\eta_j(\pi) = \frac{\pi(j)}{\sum_{i=j_q+1}^{i=j_q+K} \pi(i)} \quad j = 1, ..., |\mathcal{A}| \times |\mathcal{S}| - 1. \tag{20}$$

Clearly, these $|\mathcal{A}| \times |\mathcal{S}| - 1$ functions are continuous. According to the support lemma, for every $V \sim P_V(v)$ defined on $\mathcal{V}$, for the distribution $Q(a)$, there exist a $V' \sim P_{V'}(v')$ with $|\mathcal{V}'| \leqslant |\mathcal{A}| \cdot |\mathcal{S}| - 1$ such that

$$\begin{aligned} Q(a) &= \rho_0(a_0) \sum_{s,v} \daleth(s|a_0) P_V(v) \prod_{i=1}^{K} P_{A_i|S_i,V}(a_i|s_i, v) \\ &= \rho_0(a_0) \sum_{s,v'} \daleth(s|a_0) P_{V'}(v') \prod_{i=1}^{K} P_{A_i|S_i,V'}(a_i|s_i, v'). \end{aligned} \tag{21}$$

Remark. In general, the auxiliary random variable is required to describe the long-term utility region for the problem of power control under consideration. However, there are special cases where choosing $\mathcal{V}$ to be a singleton set does not induce any performance loss. For example, if there are no constraints and if one wants to operate at the Pareto frontier of the utility region, it can be checked that the aforementioned choice is optimal. But more generally, if there are communication constraints (such as QoS constraints) or if one considers feasible points which are also (Nash/correlated) equilibrium points, the auxiliary variable is required. This would be typically the case for selfish power control under a minimum communication rate constraint.

IV. Proposed power control strategies

Just as the problem of designing multiuser channel codes, knowing the capacity region, there is no general recipe to find power control schemes which allows one to operate arbitrarily close to a point of the utility region established through Theorem III.1. Therefore, to be able to provide practical power control schemes, we propose to focus on a special class of power control schemes. We will restrict our attention to memoryless and stationary power control strategies, which amounts to finding good one-shot decision functions. A strategy is *memoryless* in the sense that it does not exploit the past realizations of the signal s_i; it is therefore a sequence of functions which writes as $f_{i,t}(s_i(t))$. Additionally, we assume it is *stationary* which means that the function $f_{i,t}$ does not depend on time, which ultimately means that a power control strategy boils down to a single decision function say $\overline{f}_i$; the latter function will be referred to as a *decision function*. In fact, considering that the power level, vector, or matrix of a transmitter only depends on the current realization of the channel, and this in a stationary manner, is a very common and practical scenario in the wireless literature [9]. As advocated by recent works (see e.g., [23] for the MIMO case), the problem of finding one-shot decision functions with partial information and which perform well in terms of global performance is still a challenging problem. Remarkably, one of our observations is that Theorem III.1 can be exploited in a constructive way, that is, it can be exploited to find good decision functions. This is precisely the purpose of this section.

The key observation we make is as follows. The functional W_λ is a multilinear function of its arguments which are conditional probability distributions $P_{A_1|S_1,V}, ..., P_{A_K|S_K,V}, P_V$. When the constraints defined by (6)(8) are not active,W_λ is multilinear, its maximum points are on the vertices of the unit simplex [24]. The important consequence of this is that optimal condition probabilities boil down to functions $a_i = f_i(s_i, v)$, $i \in \{1, ..., K\}$. The key idea is to solve the corresponding optimization problem to determine these functions and use them as candidates for power control decision functions. This is why we will denote these functions by $\overline{f}_i$, $i \in \{1, ..., K\}$. Finding a low-complexity numerical technique to determine the optimal functions is left as a challenging extension of the present work. Instead, here we propose a suboptimal optimization technique which has a lower complexity and relies on the use of the sequential best-response dynamics (see e.g., [7] [25]).

To apply the sequential best-response dynamics to W_λ, we rewrite it by isolating the sum w.r.t. s_i i.e., the observation of Transmitter i:

$$\begin{aligned} &W_\lambda \\ &= \sum_{a_0,s,v} \rho_0(a_0) \daleth(s|a_0) P_V(v) w_\lambda(a_0, \overline{f}_1(s_1, v), ..., \overline{f}_K(s_K, v)) \\ &= \sum_{a_0,s_i,v} \rho_0(a_0) \daleth_i(s_i|a_0) P_V(v) \sum_{s_{-i}} \daleth_{-i}(s_{-i}|a_0) \times \\ &\qquad w_\lambda(a_0, \overline{f}_1(s_1, v), ..., \overline{f}_K(s_K, v)) \end{aligned} \tag{22}$$

where: $s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_K)$ represents the vector comprising all observations of the transmitters other than Transmitter i; the condition probability $\daleth_{-i}$ is given by

$$\daleth_{-i}(s_{-i}|a_0) = \sum_{s_i} \daleth(s|a_0). \tag{23}$$

To describe the proposed iterative algorithm, it is convenient to introduce the following auxiliary quantity:

$$\omega(s_i, a_i, v) = \sum_{a_0} \Bigg[\rho_0(a_0) \daleth_i(s_i|a_0) \sum_{s_{-i}} \daleth_{-i}(s_{-i}|a_0) \times w_\lambda(a_0, \overline{f}_1(s_1, v), ..., \overline{f}_{i-1}(s_{i-1}, v), a_i, \overline{f}_{i+1}(s_{i+1}, v), ..., \overline{f}_K(s_K, v))) \Bigg]. \tag{24}$$

The sequential best-response dynamics procedure consists in updating one variable at a time, the variables being the decision functions here. Denoting an algorithm iteration as iter, the auxiliary quantity ω at iteration iter writes as:

$$\omega^{\mathrm{iter}}(s_i, a_i, v) = \sum_{a_0} \Bigg[\rho_0(a_0) \daleth_i(s_i|a_0) \sum_{s_{-i}} \daleth_{-i}(s_{-i}|a_0) \times w_\lambda(a_0, \overline{f}_1^{\mathrm{iter}}(s_1, v), ..., \overline{f}_{i-1}^{\mathrm{iter}}(s_{i-1}, v), a_i, \overline{f}_{i+1}^{\mathrm{iter}-1}(s_{i+1}, v), ..., \overline{f}_K^{\mathrm{iter}-1}(s_K, v))) \Bigg]. \tag{25}$$

By assuming the knowledge of the utility function w_λ, the alphabets $\mathcal{A}_0, \mathcal{A}_1, ..., \mathcal{A}_K$, $\mathcal{S}_1, ..., \mathcal{S}_K$, the probability distribution of the channel ρ_0, the observed signals $\daleth$, and an initial choice for the decision functions $\overline{f}_1^{\mathrm{init}}, ..., \overline{f}_K^{\mathrm{init}}$, Algorithm 1 can be implemented offline. The proposed algorithm would typically be implemented offline, whereas the obtained decision functions are designed to be exploited online. Therefore, even though the decision function determination operation requires the knowledge of the different alphabets, the channel statistics, the observation signal statistics, and the initial decision functions, Transmitter i only needs s_i and possibly v to tune (online) its power vector. Typically, the former operation might be performed offline by a base station while the online operations would be executed by the transmitters.

A classical issue is to know whether this iterative algorithm converges. For clarity, we state the following convergence result under the form of a proposition.

Proposition IV.1. *Algorithm 1 always converges.*

Proof: The result can be proved by induction or by calling for an exact potential game property [26]. Indeed, since the underlying game is a strategic-form game with a common utility W_λ, it is trivially an exact potential game, which ensures convergence.■

Obviously, there is no guarantee for global optimality and only local maximum points for W_λ are reached in general by implementing Algorithm 1. Quantifying the optimality gap is known to be a non-trivial issue related to the problem of determining a tight bound of the price of anarchy [7] [27]. Two comments can be made. First, if the algorithm is initialized by the best state-of-the-art decision functions, then it will lead to new decision functions which perform at least as well as the initial functions. Second, many simulations performed for a large variety of scenarios have shown that the optimality gap seems to be relatively small for classical utility functions used in the power control literature.

inputs : $\forall i \in \{0, ..., K\},\ \mathcal{A}_i;\ \forall i \in \{1, ..., K\},\ \mathcal{S}_i$
$w_\lambda,\ \rho_0,\ \daleth$
$\forall i \in \{1, ..., K\},\ \overline{f}_i^{\mathrm{init}}$
outputs: $\forall i \in \{1, ..., K\},\ \overline{f}_i^\star$

Initialization: $\overline{f}_i^0 = \overline{f}_i^{\mathrm{init}}, \quad \mathrm{iter} = 0$

while $\exists i : \overline{f}_i^{\mathrm{iter}-1} - \overline{f}_i^{\mathrm{iter}} \geqslant \epsilon$ *AND* $\mathrm{iter} \leqslant \mathrm{iter}_{\max}$ *OR* $\mathrm{iter} = 0$ **do**
  $\mathrm{iter} = \mathrm{iter} + 1$;
  foreach $i \in \{1, \dots, K\}$ **do**
    foreach $s_i \in \mathcal{S}_i$ **do**
      $\overline{f}_i^{\mathrm{iter}}(s_i, v) = \arg\max_{a_i} \omega_i^{\mathrm{iter}}(s_i, a_i, v)$ using (25);
    end
  end
end
Final update: $\forall i \in \{1, ..., K\},\ \overline{f}_i^\star = \overline{f}_i^{\mathrm{iter}}$

Algorithm 1: Proposed decentralized algorithm for finding decision functions for the transmitters

V. NUMERICAL PERFORMANCE ANALYSIS

In this section, unless explicitly mentioned otherwise, our attention will be dedicated to energy-efficient power control. The reason for this is twofold. First, the problem of rate-efficient power control (for which it is typically assumed that $u_i = \log(1 + \mathrm{SNR}_i)$) has been largely addressed in the literature, albeit almost always in presence of perfect individual CSI. Second, designing energy-efficient communications is becoming a more and more important issue in real wireless systems. The assumed sum-utility function is the one used e.g, in [28] [33] [34] and the references therein:

$$w^{\mathrm{EE}}(a_0, a_1, ..., a_K) = \sum_{i=1}^{K} \frac{R_0 \times \psi(\mathrm{SINR}_i)}{a_i + P_0} \tag{26}$$

where R_0 is the raw data rate (in bit/s) and ψ is a function which represents the net data rate. The function ψ might represent the packet success rate (see e.g., [29] where $\psi(x) = (1 - e^{-x})^M$, $M \geqslant 1$ being the packet length), the complementary of the outage probability (see e.g., [30] where $\psi(x) = e^{-\frac{c}{x}}$, $c > 0$ being a constant related to spectral efficiency), or the Shannon spectral efficiency (see e.g., [14] [31] [32] where $\psi(x) = \log(1 + x)$). The raw data rate is the same as [14], i.e., $R_0 = 1$ Mbit/s. At last, the constant P_0 represents the power consumed by the transmitter when the radiated power is zero. For instance, in [33] it may represent the computation power, the circuit power, or the base station power consumption as in [34] [35].

To implement Algorithm 1, quantized channel gains are used. To obtain the channel gain alphabet $\mathcal{G}$ in which each channel gain $g_{ij}^b = |h_{ij}^b|$ lies, we apply a maximum entropy quantizer [36] to the modulus of h_{ij}^b, the real and imaginary parts of h_{ij}^b being Rayleigh distributed. Also we will assume that $V = const$ and $\gamma_m = 0$. At the end of this section, however, we shall provide numerical results to get a bit more insights about the choice of the auxiliary variable V, knowing that the proper design of the coordination key is an interesting issue which is left as an extension of this paper.

A. Influence of the channel estimation quality of the individual CSI on the shape of decision functions

In this subsection, the cardinality of $\mathcal{G}$ is set to 15: $|\mathcal{G}| = 15$. For the ease of exposition, we shall choose the reference scenario given by the following choices for the model parameters: $K = 2$, $B = 1$, $\sigma^2 = 10$ mW, $P_{\max} = 100$ mW, $\mathbb{E}(g_{ii}) = 1$, for $j \neq i$, $10\log_{10}\left(\frac{\mathbb{E}(g_{ii})}{\mathbb{E}(g_{ji})}\right) = 5$ dB, $\mathcal{A}_i = \left\{0, \frac{P_{\max}}{|\mathcal{P}|-1}, \frac{2P_{\max}}{|\mathcal{P}|-1}, ..., P_{\max}\right\}$ for the power level alphabet with $|\mathcal{P}| = 75$, $\gamma_m = 0$ and $R_0 = 1$ Mbit/s (or 1 MHz). We assume that Transmitter i, $i \in \{1, ..., K\}$ has some imperfect knowledge about the individual CSI i.e., $s_i = \widehat{g}_{ii}$. To obtain the channel gain estimate $\widehat{g}_{ii}$ we consider a noisy version of the actual (continuous) channel gain with $\widetilde{g}_{ii} = g_{ii} + z_i$ (z_i being an AWGN) and apply the aforementioned quantization operation to obtain $\widehat{g}_{ii}$. This defines a certain estimation SNR (ESNR) which is given by:

$$\text{ESNR}_i = \frac{\mathbb{E}[g_{ii}^2]}{\mathbb{E}[(\widehat{g}_{ii} - g_{ii})^2]}. \tag{27}$$

Fig. 1 represents the decision function $\overline{f}_i(s_i)$ provided by Algorithm 1 for various values of ESNR while maximizing the sum-energy-efficiency, with equal weights for individual utilities, i.e. $\forall i \in \{1, ..., K\}, \lambda_i = \frac{1}{K}$. For this figure we assume that $\psi(x) = e^{-\frac{c}{x}}$, $c = 1$, and $P_0 = 0$. At least two very interesting practical insights can be extracted from the figure. First, when perfect individual CSI is available (i.e., when $\text{ESNR} \rightarrow \infty$), the optimal decision function naturally exhibits a threshold below which the transmitter should not transmit. This is very interesting since, to our knowledge, no paper on energy-efficient power control (at least in the sense as defined as in the present paper) has exhibited the need for a threshold, and this, in the absence of QoS constraints. This also allows one to make an interesting connection with [9] where the sum-rate maximization is obtained with a thresholding technique and by merely using binary power control; our results shows that this is more general and applies to other utility functions. Second, we see that, when only noisy estimation are available for the direct channel gain g_{ii}, the optimal decision functions comprises some piecewise constant parts. This shows, in particular, that not all available transmit power levels are exploited to maximize the average sum-energy-efficiency. Indeed, here 75 levels are available but it is seen e.g., that when $\text{ESNR} = 6$ dB only some of them are exploited on the plot represented here. This situation may be referred to as a *"cooling effect"* since less and less power levels are exploited as the ESNR decreases (in connection with the literature of learning when the chosen action consists of a tradeoff between exploration and exploitation -see e.g., [7]). Indeed, when the estimation noise is stronger ($\text{ESNR} = 0$ dB) this cooling effect on the decision function is completely apparent since only one transmit power level is exploited. In this respect, Fig. 2 represents the decision functions provided Algorithm 1 when $\psi(x) = \log_2(1 + x)$ and $P_0 = 10$ mW (as chosen in [14]). Interestingly, the cooling effect appears again and even in the absence of estimation noise, showing it is strongly related to the utility function form. Of the key messages our study conveys is that the best global performance may still be obtained even after reducing the possible choices in terms of transmit power levels. Having reduced action spaces may be very attractive in terms of computational complexity but also for measuring or sensing accurately the activity of the transmitters of interest.

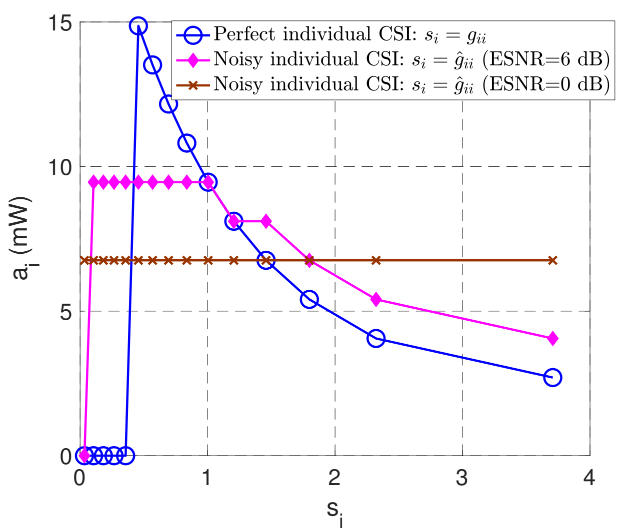

Fig. 1: When the estimation noise level increases only some of the available transmit power levels are exploited to maximize sum-energy-efficiency (namely, by using Algorithm 1) here with $\psi(x) = e^{-\frac{c}{x}}$ and $P_0 = 0$. In connection with the literature [7] on learning we refer to this effect as a cooling effect.

B. Influence of the available CSI on the achievable long-term utility region

Another type of precious information which is currently not available in the literature is the utility region for the problem of power control under partial information. Indeed, the knowledge of the long-term utility region is instrumental since it allows the best performance of the system to be fully characterized. In particular, any proposed power control scheme can be represented on the utility region and therefore, assessed in terms of efficiency. Again, for ease of exposition, we assume two transmitter-receiver pairs ($K = 2$), which means that the utility region can be represented in a plane

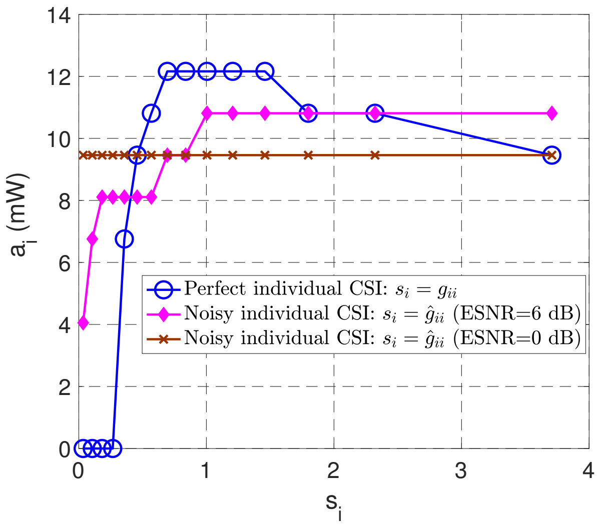

Fig. 2: The cooling effect observed for the previous figure is confirmed when considering other definitions for the energy-efficiency-based utility function (here with $\psi(x) = \log_2(1+x)$ and $P_0 = 10$ mW). In fact, it may also occur even in the absence of estimation noise.

but the proposed framework is valid for any value of $K \geqslant 2$. Additionally, we choose $10\log_{10}\left(\frac{\mathbb{E}(g_{ii})}{\mathbb{E}(g_{ji})}\right) = 0$ dB ($j \neq i$) to better illustrate the influence of the different channels. The other system configurations are the same as Sec. V-A.

Fig. 3 represents the Pareto-frontier of the long-term utility region (thus in the (U_1, U_2) plane) for the same scenarios as in the previous subsection. This allows one to see the impact of the individual channel gain quality in terms of achievable utility. The outer curve is obtained by assuming perfect individual CSI at the transmitters ($s_i = g_{ii}$) and using exhaustive search, which means that the curve represents exactly the best performance achievable under the considered partial information; interestingly, almost the same curve has been obtained by using Algorithm 1, which indicates that the corresponding optimality loss is negligible here. The other curves are obtained with the different values of ESNR considered for Fig. 1 (namely, ESNR $\in \{0, 6, +\infty\}$ dB) and using Algorithm 1. This result brings new insights w.r.t. existing works since it allows one to quantify the impact of the channel estimation quality on the final performance of the resource allocation policy i.e., when measured in terms of energy-efficiency. In the scenario, the cost of imperfect knowledge in terms of individual CSI is seen to be approximately 25% when the estimates are very noisy (namely ESNR $= 0$ dB).

From here on, we assume no estimation noise and rather assess the influence of partial information in terms of what channel gains is known. We define four information scenarios: 1. Perfect global CSI: $s_i = (g_{11}, g_{12}, g_{21}, g_{22})$; 2. Perfect direct CSI $s_i = (g_{11}, g_{22})$; 3. Perfect local CSI $s_i = (g_{ii}, g_{ji}), j \neq i$; 4. Perfect individual CSI: $s_i = g_{ii}$. Fig. 4 represents the Pareto-frontier of the long-term utility region (always in the (U_1, U_2) plane) for these four scenarios. Several useful observations can be made. First, moving from individual to local CSI does only bring a very marginal improvement in terms of energy-efficiency. On the other hand, knowing all the direct channels is definitely very useful for reaching good global performance. Third, the loss induced by not having global CSI is clearly assessed here and might be found to be acceptable. However, note that for the sake of representation, we assume two transmitter-receiver pair here. For more users, these conclusions would need to be refined. Further, we provide simulations for more typical number of users and assess the performance under these conditions.

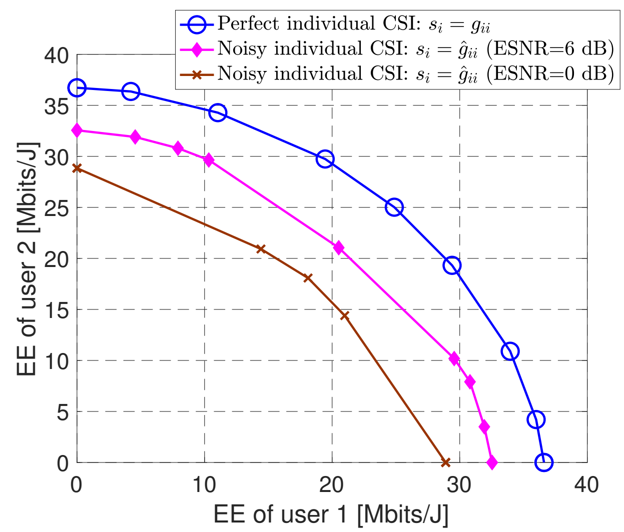

Fig. 3: Interestingly, the loss induced by having noisy individual channel gain estimates instead of perfect estimates is seen to be reasonable even when the estimates are very noisy.

C. Comparison between Algorithm 1 and the state-of-the-art

Here (see Fig. 5), we consider a more general and generic wireless scenario namely, a small cell network (see e.g. [36] [37]) for which the interaction between $K = 9$ neighboring cells is studied. The communication scenario is the same as the one considered in [36]. In this model, the path loss effects for the link ij are denoted by $\mathbb{E}(g_{ij}^s)$ and is inversely proportional to the distance between Transmitter i and Receiver j. More precisely, $\mathbb{E}(g_{ij}^s) = \left(\frac{d_0}{d_{ij}}\right)^2$ where d_{ij} denotes the aforementioned distance and $d_0 = 5$ m is a normalization factor. All small base stations are considered to be in the center of their own cells, whereas the mobile stations $\mathrm{MS}_1, \ldots, \mathrm{MS}_9$ have been chosen to have the following normalized coordinates : $(3.8, 3.2)$, $(7.9, 1.4)$, $(10.2, 0.7)$, $(2.3, 5.9)$, $(6.6, 5.9)$, $(14.1, 9.3)$, $(1.8, 10.6)$, $(7.1, 14.6)$, $(12.5, 10.7)$. One can obtain the real coordinates for the mobile stations by scaling these coordinates by a multiplying factor of $\frac{\mathrm{ISD}}{d_0}$, where ISD denotes the inter-site distance. The choices made for the parameter values are almost the same as in [14] except for the SNR which is set here to a value that is more suitable for small cell networks (namely, SNR $= 30$ dB). More precisely:

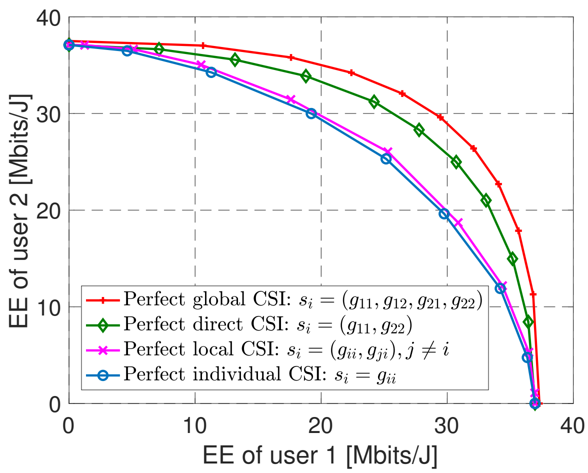

Fig. 4: Here, the knowledge is assumed to be partial but perfect. While knowing the cross channel gains is seen to bring very marginal improvement, the knowledge of direct channels allows one to bridge a quite good fraction of the gap between the individual CSI and global CSI scenarios.

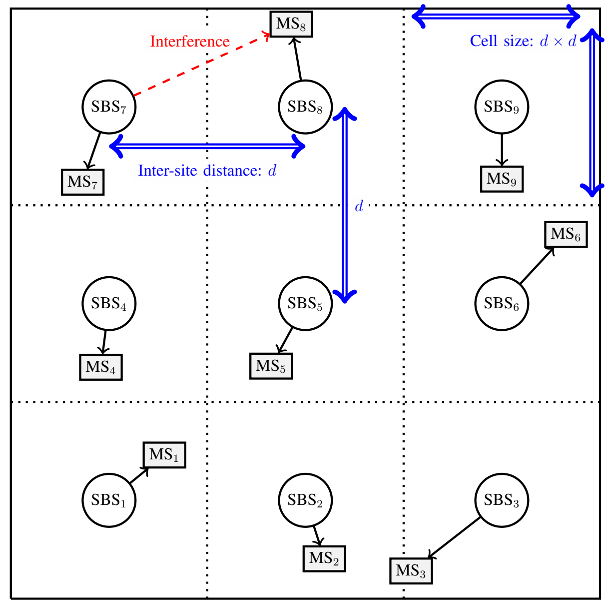

Fig. 5: Assumed small cell scenario for Sec. V-C.

$K = 9$, $S = 1$, $P_{\max} = 10$ dBW, $\sigma^2 = 10$ dBm, $P_0 = 10$ dBm, $|\mathcal{P}| = 2000$ with uniform power increment, $|\mathcal{G}| = 6$.

For Fig. 6, it is assumed that $\psi(x) = 1 - e^{-x}$ whereas, for Fig. 7 $\psi(x) = \log_2(1 + x)$ is assumed. Both figures represent the sum-energy-efficiency against the inter-site distance namely, the distance between two small base stations. Three curves are represented. The top curve corresponds to the team power control provided by Algorithm 1. The curve in the middle depicts the performance for the cooperative solution derived by Zappone et al on [14]. At last, the bottom curve corresponds to the non-cooperative solution which is described e.g., in [14]. For both choices of ψ, the gain brought by using Algorithm 1 is seen to be very significant. Note that here we assume perfect individual CSI, which makes the comparison with the closest state-of-the-art technique possible. However, note that the power control schemes in [14] have not been designed to deal with noisy estimates or an arbitrary partial knowledge about the global CSI, as opposed to Algorithm 1 that can always be used even in these complex scenarios.

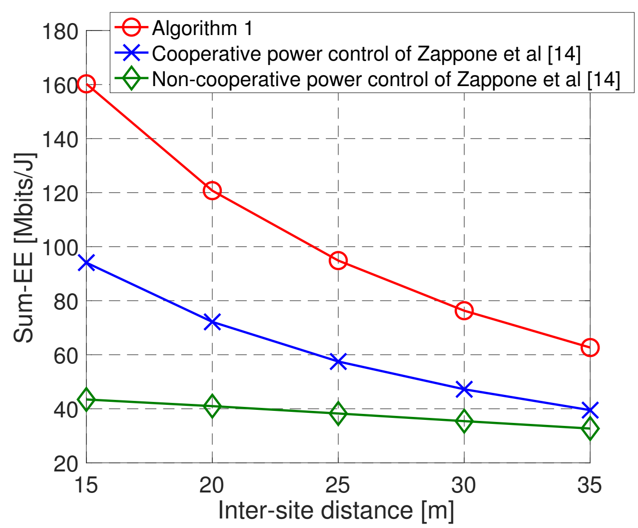

Fig. 6: Comparison of Algorithm 1 with state-of-the-art power control schemes for $\psi(x) = 1 - e^{-x}$ for the scenario of Fig. 5 (namely, with $K = 9$ users.)

D. Additional simulations

To conclude the simulation section, we first want to know more about the importance of the choice of auxiliary variable, which allows one to generate suitable coordination keys. To this effect, we consider a simple scenario ($K = 2$) and the sum-rate as the performance criterion. More precisely, we assume a multiple access channel (MAC) scenario, which is a special case of the interference channel scenario. The channel between Transmitter i and Receiver i is denoted by g_i and the sum-rate is defined as

$$u_{\text{MAC}}^{\text{SR}}(a_1, \ldots, a_K; g_1, \ldots, g_K) = \sum_{i=1}^{K} \log_2 \left(1 + \frac{g_i a_i}{\sigma^2 + \sum_{j \neq k} g_j a_j} \right). \tag{28}$$

All the alphabets are considered to be binary with $P_i \in \{0, P_{max}\}$, $g_i \in \{0.3, 1\}$ and $V \in \{V_1, V_2\}$. The probability for each channel realization is half, namely, for $i \in \{1, 2\}$, $\Pr(g_i = 0.3) = \Pr(g_i = 1) = 50\%$. For the QoS constraints, we considered the asymmetric case where $u_1 \geqslant 0.45 \times \sum_{k=1}^{K} \log_2(1 + \text{SNR})$ and $u_2 \geqslant 0.15 \times \sum_{k=1}^{K} \log_2(1 + \text{SNR})$ with

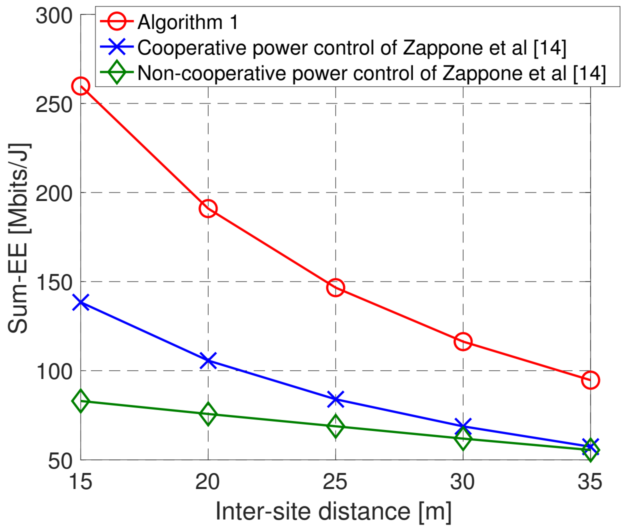

Fig. 7: Comparison of Algorithm 1 with state-of-the-art power control schemes for $\psi(x) = \log_2(1+x)$ for the scenario of Fig. 5 (namely, with $K = 9$ users.)

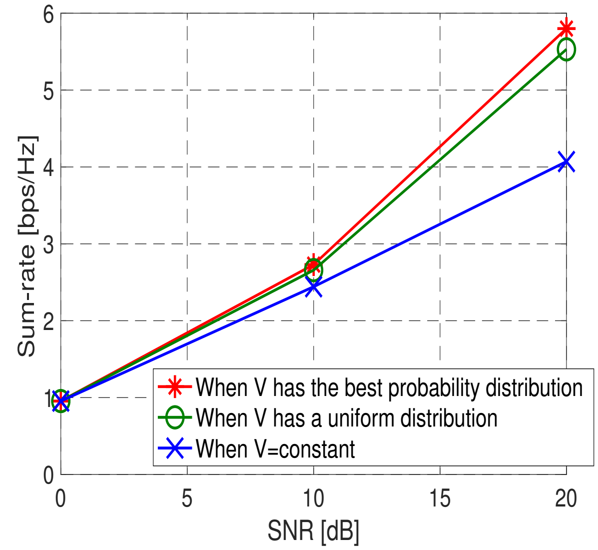

Fig. 8: Considering the sum-rate utility for $K = 2$ with quality of service constraints, it is seen that exchanging a coordination key offline (and built from a lottery given by the random auxiliary variable V) brings a non-negligible improvement, especially at high SNR and regarding the underlying ease of implementation.

$\text{SNR} = \frac{P_{\max}}{\sigma^2}$. For maximizing the utility sum-rate for $K = 2$ transmitters, Fig. 8 represents the sum-rate against the SNR for three different cases: without auxiliary variable (bottom curve), with a uniformly distributed auxiliary variable (middle curve), and with the optimally distributed auxiliary variable (top curve). The figure shows that having a coordination key can effectively improve the performance; the improvement is especially apparent at high SNR regime where, unsurprisingly, coordination plays a more important role. Obtained gains are appreciable since they can be obtained quite easily. Indeed, exchanging offline a sequence of binary realizations of a Bernouilli variable is perfectly doable when designing a real wireless system.

Another interesting case to consider is the multi-band MAC scenario, which has been treated in [29]. We consider the sum-energy-efficiency utility and compare it with the scheme derived in [29]. For this, we assume $K = 3$ users, 2 possible operating bands, and $\psi(x) = (1 - e^{-x})^{100}$. The other parameters are: $P_{\max} = 10$ dBW, $\sigma^2 = 10$ dBm, $P_0 = 0$, $|\mathcal{P}| = 2000$ with uniform power increment, $|\mathcal{G}| = 6$. Fig. 9 shows the sum-rate performance against the average link quality $\mathbb{E}(g_i)$ for the scheme obtained from Algorithm 1 and the scheme proposed in [29]. The performance gain is appreciable even for a quite small ratio $\frac{K}{B}$; indeed, the sum-utility performance is improved by more than three times. Moreover, it is has been observed for other simulations that the gain is even more significant when the load per band increases, e.g., in the scenario $K = 9$, $B = 2$.

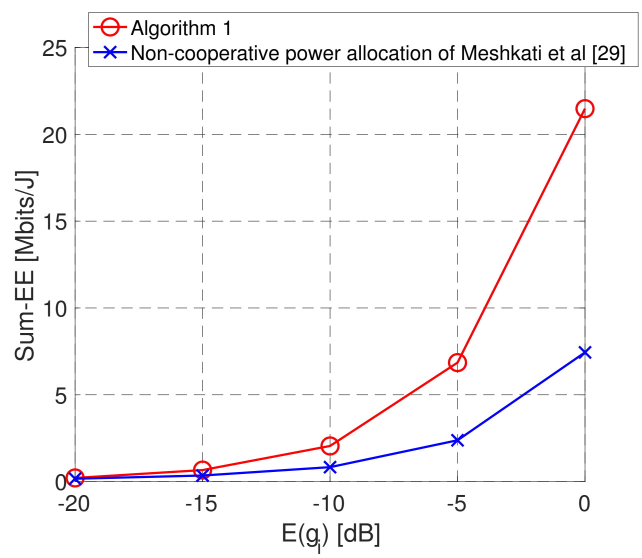

Fig. 9: For multi-band MAC and typical values for the channel gain mean ($\mathbb{E}(g_i) \geqslant 0.1$), the proposed power control scheme is shown to provide a significant performance gain over the technique proposed in [29].

VI. Conclusion

To summarize in a concise way what the proposed approach brings w.r.t. the state-of-the-art and what its limitations are, we propose to describe its strengths and weaknesses under a list form.

Strong features of our approach

▸ In contrast with the state-of-the-art, by making a fruitful connection between power control and information theory, our approach allows one to characterize the best performance a set of transmitters can achieve in terms of power control under partial information. In particular, this allows one to measure the efficiency of any proposed power control scheme.

▸ Both the limiting performance analysis and the proposed algorithm work for a broad class of utility functions and not only for a specific utility function as often assumed in the literature.

▸ Both the limiting performance analysis and the proposed algorithm work for a broad class of partial observation structures and not only for a very specific observation structure as often assumed in the literature. For instance, the vast majority of power control and radio resource allocation schemes (see e.g., [9] [14] [19] [20] [38] [39]) makes information assumptions such as perfect individual or global CSI but does not allow one to deal with noisy estimation or other arbitrary partial and perfect information.

Limitations of our approach

▸ Although assuming the power control actions and network state to be discrete does not constitute a limitation for the limiting performance analysis since the continuous case follows by specialization (namely, as a limiting case of the discrete case as done for classical coding theorems), it typically involves some complexity limitations for the proposed algorithm. The proposed algorithm corresponds to one possible numerical solution to determine good power control functions, however finding low complexity numerical routines constitutes a very relevant issue to be explored.

▸ Both the limiting performance analysis and the proposed algorithm assume utility functions under the form $u_i(a_0, a_1, ..., a_K)$ when a_0 corresponds to the realizations of an i.i.d. random process $(A_{0,t})_{t \geqslant 1}$ and the partial information available to Transmitter i (namely, s_i) is the output of discrete memoryless channel. In this paper, the channel state is assumed to be i.i.d. which is a common and very well accepted assumption. If the state or the observation structure happens to be with memory, the derived results would not hold anymore. They would need to be generalized. This would be necessary for instance, for a Markovian state.

▸ As many related papers, the proposed algorithm provides power control functions under a numerical form but not in an analytical form. However, the obtained numerical results may be used as a source of inspiration to propose relevant classes of functions which are suited to the considered setup. Thresholding, saturation, steps, scaling are examples of operations which may be exhibited and used.

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