

Global tracking-stabilization control of mobile robots with parametric uncertainty

Mohamed Maghenem, Antonio Loria, Elena Panteley

▶ **To cite this version:**

Mohamed Maghenem, Antonio Loria, Elena Panteley. Global tracking-stabilization control of mobile robots with parametric uncertainty. IFAC 2017 - 20th World Congress of the International Federation of Automatic Control, Jul 2017, Toulouse, France. pp.4114 - 4119, 10.1016/j.ifacol.2017.08.797 . hal-01744931

HAL Id: hal-01744931
https://hal.science/hal-01744931

Submitted on 5 Mar 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Global tracking-stabilization control of mobile robots with parametric uncertainty ⋆

Mohamed Maghenem * **Antonio Loria** ** **Elena Panteley** ***

* *Université Paris Saclay, 91190, Gif sur-Yvette, France (e-mail: maghenem@l2s.centralesupelec.fr).*
** *CNRS, 91190, Gif sur-Yvette, France (e-mail: loria@lss.supelec.fr)*
*** *CNRS, 91190, Gif sur-Yvette, France; ITMO University, Saint Petersburg, Russia (e-mail: panteley@lss.supelec.fr)*

Abstract: We present a simple time-varying controller for tracking problem of mobile robots. We consider the full model of autonomous vehicles, including both the kinematics and the Lagrangian dynamics equations. Our control approach relies on designing a controller at the kinematics level, under any integrable virtual leader velocities, which is robust to any controller, at the torc level, that guarantees that the velocity errors are square integrable. In addition, we assume that the inertia is unknown hence, we use a passivity-based adaptive controller that guarantees the convergence of the velocity tracking errors.

Keywords: Tracking control, adaptive control, stabilization, nonholonomic systems

1. INTRODUCTION

Tracking control of mobile robots has been thoroughly studied since the early 1990s –see *e.g.*, [Kanayama et al., 1990] and [Kanayama et al., 1991] where the authors introduced a follow-the-leader control approach. It consists in defining a virtual robot that generates a reference trajectory which is supposed to be followed by the controlled robot. Depending on the velocities of the virtual robot, we distinguish the tracking problem, in which the leader velocities are generic time varying functions, and the stabilization problem, in which the leader velocities are null. In the tracking-stabilization problem the leader velocities are functions of time and asymptotically converge to zero; such problem may be regarded as a robust stabilization problem with respect to the leader velocities.

The stabilization problem of nonholonomic mobile robots has been widely treated in the literature since the seminal work of [Brockett, 1983], which establishes the non existence of any autonomous smooth feedback that globally stabilizes the origin of the closed loop. Some articles provide discontinuous controllers [Astolfi, 1999, Pourboghrat, 2002], time-varying continuous controllers [Morin and Samson, 1997], and a smooth time-varying controllers [Samson, 1995, Loría et al., 1999, 2002].

In [Samson, 1995] the translational velocity input is used to bound the trajectories of the closed-loop system to allow the convergence of one of the two translational coordinates $(\dot{x}, \dot{y})$ to zero. Then, the rotational velocity input is designed so as to remain excited until the resting translational coordinates converge to zero. This idea inspired [Loría et al., 1999, 2002] where the so-called δ-persistently exciting controllers are introduced and uniform global asymptotic stability for the closed-loop system is established. Many other articles propose similarly structured controllers, albeit using different methods [Jiang et al., 2001, Lee et al., 2001, Wang et al., 2015, Do et al., 2004].

Even if some of these works solve more elaborated problems such as simultaneous stabilization and tracking, they do not establish uniformity with respect to time since the main convergence argument remains on the use of Barbalăt's Lemma. In this paper we provide a simple procedure to analyze the stability of mobile robots under such a class of smooth time varying-controllers using properties of persistently exciting signals, as in [Loría et al., 2002]. Uniform global asymptotic stability of the kinematics closed-loop system is established for any integrable leader velocities. Then, robustness of the kinematics controller with respect to any torque level controller, allowing the convergence of the error velocities with finite $\mathcal{L}_2$ norm is concluded.

The rest of the paper is organized as follows. In next section we describe the problem statement and its solution. Our main result is presented in Section 3 and our stability proofs are presented in Section 4. Some simulation results are presented in Section 5 before concluding with some remarks in Section 6.

2. PROBLEM FORMULATION AND SOLUTION

Let us consider the Lagrangian dynamic model of a wheeled mobile robot as given, for instance, in [Do, 2007],

$$\dot{z} = J(z)\nu \tag{1a}$$

⋆ The work of the third author is supported by Government of Russian Federation (grant 074-U01).

$$M\dot{\nu} + C(\dot{z})\nu = \tau. \tag{1b}$$

The vector $z := [x,\ y,\ \theta]$ contains the Cartesian coordinates and orientation of the robot. More precisely, (x, y) corresponds to the middle point in between the axis connecting the two wheels. Then, $\tau \in \mathbb{R}^2$ is the torque control input; $\nu := [\nu_1\ \nu_2]$ stands for the angular velocities corresponding to the two robot's wheels, M is the inertia matrix and C is the matrix of Coriolis forces. The former is constant, symmetric and positive definite while the latter is skew-symmetric. We assume that the inertia parameters and the constants contained in C are unknown.

In addition, we use the coordinate transformation matrix

$$J(z) = \frac{r}{2}\begin{bmatrix}\cos(\theta) & \cos(\theta)\\ \sin(\theta) & \sin(\theta)\\ 1/b & -1/b\end{bmatrix}$$

where r is the radius of either wheel and b is the distance from the center of either wheel to the Cartesian point (x, y) hence, r and b are considered to be known. The relation between the wheels' velocities, ν, and the robot's velocities in the fixed frame, $\dot{z}$, is given by

$$\begin{bmatrix}v\\ \omega\end{bmatrix} = \frac{r}{2b}\begin{bmatrix}b & b\\ 1 & -1\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix} \quad \Leftrightarrow \quad \begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix} = \frac{1}{r}\begin{bmatrix}1 & b\\ 1 & -b\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix} \tag{2}$$

which may be used in (1a) to obtain the familiar model

$$\begin{cases}\dot{x} &= v\cos\theta\\ \dot{y} &= v\sin\theta\\ \dot{\theta} &= \omega.\end{cases} \tag{3}$$

The tracking-stabilization control problem for (1) or, equivalently, for (1b) and (3), consists in designing a control law τ such that the robot follows the trajectory generated by a fictitious robot with kinematics

$$\begin{cases}\dot{x}_r &= v_r\cos\theta_r\\ \dot{y}_r &= v_r\sin\theta_r\\ \dot{\theta}_r &= \omega_r.\end{cases} \tag{4}$$

which is assumed to describe a trajectory that vanishes into a set-point that is, we have

$$\lim_{t\to\infty} |v_r(t)| + |\omega_r(t)| = 0, \tag{5}$$

so the robot is required to stabilize and stop at a point.

We solve this problem by separating the stabilization tasks at the kinematics and the force levels. For Equation (3) we design virtual control laws v^* and ω^* to stabilize the origin for the velocity kinematics errors –see Section 3.2. For the Lagrangian dynamics (1b) we design an adaptive controller that guarantees that $v \to v^*$ and $\omega \to \omega^*$ in spite of the parametric uncertainty regarding M and C.

3. CONTROL DESIGN

3.1 Control of the force-dynamics equation

Let v^* and ω^* be given virtual control laws. For the time being we only assume that they are once continuously differentiable functions of t and the tracking errors, yet to be defined. Then, we can compute $\nu^* := [\nu_1^*\ \nu_2^*]^\top$ using (2) and, for the time being, consider torque control input

$$\tau = M\dot{\nu}^* + C(\dot{z})\nu^* - k_d\tilde{\nu}, \quad k_d > 0 \tag{6}$$

where $\tilde{\nu} := \nu - \nu^*$. That is, we temporarily assume that the parameters are known. Using the expression (6) in (1b), we obtain

$$M\dot{\tilde{\nu}} + \big[C(\dot{z}(t)) + k_d I\big]\tilde{\nu} = 0, \tag{7}$$

in which we have replaced $\dot{z}$ with the trajectories $\dot{z}(t)$ to regard this system as (linear) time-varying, with state $\tilde{\nu}$. Now, due to the skew-symmetry of $C(\cdot)$ the total derivative of

$$V(\tilde{\nu}) = \frac{1}{2}\tilde{\nu}^\top M\tilde{\nu}, \tag{8}$$

along the trajectories of (7) yields

$$\dot{V}(\tilde{\nu}) \le -k_d|\tilde{\nu}|^2. \tag{9}$$

Strictly speaking, this inequality holds only in the interval of existence of $\dot{z}(t)$ but, since it holds uniformly in the latter, we shall assume, for the sake of argument, that the solutions are forward complete.

Let us now assume that the constant parameters in M and C are unknown and let us denote by $\hat{M}$ and $\hat{C}(\dot{z})$ the estimates of the inertia and the coriolis matrices respectively. Then, we introduce the certainty-equivalence control law

$$\tau = \hat{M}\dot{\nu}^* + \hat{C}(\dot{z})\nu^* - k_d\tilde{\nu}, \quad k_d > 0, \tag{10}$$

which, together with (1b) leads to the closed-loop equation

$$M\dot{\tilde{\nu}} + \big[C(\dot{z}(t)) + k_d I\big]\tilde{\nu} = [\hat{M} - M]\dot{\nu}^* + [\hat{C}(\dot{z}) - C(\dot{z})]\nu^*.$$

Then, since C is linear in the unknown parameters, as it is customary in adaptive control of linearly-parameterized systems, we collect all the lumped constant parameters in the vector $\Theta \in \mathbb{R}^m$ and, respectively, its estimates in $\hat{\Theta}$ so that the previous expression may be written in the compact form

$$M\dot{\tilde{\nu}} + \big[C(\dot{z}(t)) + k_d I\big]\tilde{\nu} = \Phi(\dot{z}, \nu^*, \dot{\nu}^*)^\top\tilde{\Theta} \tag{11}$$

where $\tilde{\Theta} = \hat{\Theta} - \Theta$.

Thus, if we define the update law as

$$\dot{\hat{\Theta}} = -\gamma\Phi(\dot{z}, \nu^*, \dot{\nu}^*)\tilde{\nu}, \quad \gamma > 0, \tag{12}$$

the total derivative of

$$V(\tilde{\nu}, \tilde{\Theta}) := \frac{1}{2}\big[\,|\tilde{\nu}|^2 + \frac{1}{\gamma}|\tilde{\Theta}|^2\,\big] \tag{13}$$

along the trajectories of (11) satisfies (9). Integrating the latter and using the fact that V is positive definite and radially unbounded, we conclude that $\tilde{\nu}$, $\tilde{\Theta} \in \mathcal{L}_\infty$ and, moreover, $\tilde{\nu} \in \mathcal{L}_2$. From the closed-loop equation (7) we obtain that $\dot{\tilde{\nu}} \in \mathcal{L}_\infty$ as well. Thus, $\tilde{\nu} \to 0$ asymptotically. In view of (2), we also obtain that

$$\tilde{v},\ \tilde{\omega} \in \mathcal{L}_2 \tag{14}$$

$$\lim_{t\to\infty} |\tilde{v}| + |\tilde{\omega}| = 0. \tag{15}$$

The control design and the analysis carried out so far is fairly standard. Yet, its simplicity should not jeopardize the importance of the main result. We shall prove that a fairly simple controller achieves the tracking-stabilization control goal under weak conditions. Our next step is the design of the virtual control laws v^* and ω^* to stabilize the kinematics equations (3) around the trajectory generated by the virtual leader robot (4). We design a controller that

is robust to vanishing "disturbances" $\tilde{v}$ and $\tilde{\omega}$ under the weak property (14). In other words, our kinematics controller is compatible with *any* force controller guaranteeing (14), (15).

3.2 Control of the kinematics errors

As usual, we introduce the position and orientation errors

$$\begin{aligned} p_\theta &= \theta_r - \theta \\ p_x &= x_r - x - d_x \\ p_y &= y_r - y - d_y, \end{aligned}$$

and we perform a convenient rotation to represent the errors on the fixed frame. That is, let

$$\begin{bmatrix} e_\theta \\ e_x \\ e_y \end{bmatrix} := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_\theta \\ p_x \\ p_y \end{bmatrix}. \tag{16}$$

Then, differentiating the latter we recover the well-known model for the error velocity-kinematics,

$$\dot{e}_\theta = \omega_r - \omega \tag{17a}$$
$$\dot{e}_x = \omega e_y - v + v_r \cos(e_\theta) \tag{17b}$$
$$\dot{e}_y = -\omega e_x + v_r \sin(e_\theta) \tag{17c}$$

The tracking control problem boils down to stabilizing the origin for the system (17). in particular, it is required to steer the error $e := [e_\theta,\ e_x,\ e_y]^\top$ to zero. We propose to do so with the δ-persistently exciting controller (see [Loría et al., 1999]) given by

$$v^* := k_x e_x + v_r(t)\cos(e_\theta), \quad k_x > 0 \tag{18a}$$
$$\omega^* := \omega_r(t) + k_\theta e_\theta + k_y p(t) e_y, \quad k_y,\ k_\theta > 0 \tag{18b}$$

where $p(t)$, $v_r(t)$, and $\omega_r(t)$ are smooth bounded functions.

Proposition 1. Consider the system (17); let $\tilde{v} := v - v^*$ and $\tilde{\omega} := \omega - \omega^*$ where v^* and ω^* are defined in (18). Assume that (5) holds,

$$\lim_{t\to\infty} |\tilde{v}(t)| + |\tilde{\omega}(t)| = 0, \tag{19}$$

$$v_r, \omega_r \in \mathcal{L}_1, \quad \tilde{v} \in \mathcal{L}_2, \tag{20}$$

and, in addition, that there exist μ and $T > 0$ such that

$$\int_t^{t+T} \dot{p}(s)^2 ds \geq \mu \quad \forall\, t \geq 0. \tag{21}$$

Then,

$$\lim_{t\to\infty} |e(t)| = 0. \tag{22}$$

Moreover, when $\tilde{v}(t) = \tilde{\omega}(t) = 0$, the origin, for the system (17), is UGAS. □

Remark 1. The condition (19), as well as $\tilde{v} \in \mathcal{L}_2$, are established for the adaptive controller (10), (12) in the previous section. The condition that $(v_r, \omega_r) \in \mathcal{L}_1$, however, imposes a mild restriction on the convergence speed of (v_r, ω_r); for instance, $v_r(t)$ and $\omega_r(t)$ has to converge faster than $1/t$. Finally, the condition on p, (21), is met by design; for instance, any periodic function fits.

For clarity of exposition, the proof of Proposition 1 is presented in the following section. We wrap up this section with our main statement, whose proof is direct from Proposition 1 and the developments in Section 3.1.

Proposition 2. (Main result). Consider the system (1) in closed loop with (10), (12) and

$$\nu^* = \frac{1}{r}\begin{bmatrix} 1 & b \\ 1 & -b \end{bmatrix} \begin{bmatrix} v^* \\ \omega^* \end{bmatrix}, \tag{23}$$

where v^* and ω^* are defined in (18). If, in addition, (19)–(21) hold, so does (22).

4. PROOF OF PROPOSITION 1

The proof follows three main logical steps:

- to compute the closed-loop equations,
- to establish that the closed-loop solutions are bounded,
- to show that the nominal error system (with $\tilde{\nu} = \nu - \nu^* = 0$) is uniformly globally asymptotically stable.

Generally speaking, we regard the closed-loop equations as a stable nominal system with state e and a vanishing perturbation –see (19). The second item is established using standard arguments as on Lyapunov, as well as input-output stability. Then, to establish the last item we exploit some fundamental structural properties of the closed-loop system and we rely on different analysis tools for nonlinear systems, including output-injection and cascaded-systems arguments.

4.1 The closed-loop equations

First, we replace $v = \tilde{v} + v^*$ and we use (18a) in (17b) and, correspondingly, we replace $\omega = \tilde{\omega} + \omega^*$ and we use (18b) in (17a) to obtain

$$\dot{e}_\theta = -k_\theta e_\theta - k_y e_y p(t) - \tilde{\omega} \tag{24a}$$
$$\dot{e}_x = -k_x e_x + \omega e_y + \tilde{v} \tag{24b}$$
$$\dot{e}_y = -\omega e_x + v_r \sin(e_\theta). \tag{24c}$$

Equivalently, for the first two equations we have

$$\begin{aligned} \dot{e}_x &= -k_x e_x + \big[\tilde{\omega} + \omega_r(t) + k_\theta e_\theta + k_y p(t) e_y\big] e_y + \tilde{v} \\ \dot{e}_y &= -\big[\tilde{\omega} + \omega_r(t) + k_\theta e_\theta + k_y p(t) e_y\big] e_x + v_r \sin(e_\theta) \end{aligned}$$

Hence, using the error coordinates $e := [e_\theta\, e_x\, e_y]^\top$ and defining $u := [\tilde{v}\, \tilde{\omega}]^\top$, the closed-loop system becomes

$$\dot{e} = f_e(t, e) + g(t, e) u \tag{25}$$

where the the input-gain matrix g is given by

$$g(t, e) := \begin{bmatrix} 0 & -1 \\ 1 & -e_y \\ 0 & e_x \end{bmatrix}$$

and the drift of the nominal system is

$$f_e(t, e) := \begin{bmatrix} -k_\theta e_\theta - k_y e_y p \\ -k_x e_x + (k_\theta e_\theta + k_y p e_y)\, e_y \\ -(k_\theta e_\theta + k_y p e_y)\, e_x \end{bmatrix} +$$

$$\begin{bmatrix} 0 \\ \omega_r e_y \\ -\omega_r e_x + v_r \sin e_\theta \end{bmatrix}$$

Following the proof-lines of [Panteley and Loría, 2001, Lemma 1] we conclude that, for the system (25), $e \to 0$ if:

(A) the solutions are uniformly globally bounded,
(B) the origin of $\dot{e} = f_e(t, e)$ is uniformly globally asymptotically stable,
(C) the perturbation $u \to 0$ asymptotically.

The last condition, item (C) holds by assumption. We proceed to prove the first two.

Remark 2. It is worth remarking that, in the context of this paper, our main statement, *i.e.*, Proposition 2, establishes (only) the convergence of the tracking errors. However, from our proof the much stronger property of uniform global asymptotic stability may be concluded if item (C) is reinforced to requiring that the origin be uniformly globally asymptotically stable, for the dynamics of the "input" u. In particular, the convergence of e is uniform if so is that of u, however, this property cannot be established in the adaptive case-scenario broached here since the convergence of the reference trajectories v_r, ω_r prevents the regressor Φ in (12) from being persistently exciting and guaranteeing the convergence of the parametric estimation errors to zero.

4.2 Item (A): boundedness of the solutions

Since p is bounded and $k_\theta > 0$, it follows that the solutions of (25) are bounded. On the other hand, the compact expression of the closed-loop dynamics, (24), is particularly convenient to establish that the closed-loop solutions are globally bounded. To show this, let

$$W(e) := \ln(1 + V_1(e)) \tag{26}$$
$$V_1(e) := \frac{1}{2}\big[\, e_x^2 + e_y^2 \,\big]. \tag{27}$$

The total derivative of V_1 along the trajectories of (24b), (24c) yields

$$\dot{V}_1(e) \le -k_x e_x^2 + |e_x||\tilde{v}| + |v_r||\sin(e_\theta)||e_y| \tag{28}$$

hence,

$$\dot{W}(e) \le \frac{1}{1+V_1}\Big[-\frac{k_x}{2} e_x^2 + |v_r||e_y| + \frac{\tilde{v}^2}{2k_x}\Big] \tag{29}$$
$$\le \frac{|e_y|}{1+V_1}|v_r| + \frac{1}{2k_x[1+V_1]}\tilde{v}^2. \tag{30}$$

Integrating on both sides of the latter from 0 to t and using (20) we see that $W(e(t))$ is bounded for all $t \ge 0$. Boundedness of $e_x(t)$ and $e_y(t)$ follows since W is positive definite and radially unbounded in (e_x, e_y).

Next, consider the third closed-loop equation, (24a). It corresponds to an exponentially stable system with bounded input $u(t) = -k_y e_y(t) p(t) - \tilde{\omega}(t)$ hence, we also have $e_\theta \in \mathcal{L}_\infty$.

4.3 Item (B): UGAS of the nominal system

This corresponds to the proof of the second claim of proposition 1. Let $\tilde{v} = \tilde{\omega} = 0$ and, for further development, let us split the drift of the nominal system $\dot{e} = f_e(t, e)$ into the output injection form:

$$f_e(t, e) = F(t, e) + K(t, e) \tag{31}$$

where

$$K(t, e) := \begin{bmatrix} 0 \\ \omega_r e_y \\ -\omega_r e_x + v_r \sin e_\theta \end{bmatrix} \tag{32}$$

and

$$F(t, e) := \begin{bmatrix} -k_\theta & 0 & -k_y p \\ 0 & -k_x & (k_\theta e_\theta + k_y p e_y) \\ 0 & -(k_\theta e_\theta + k_y p e_y) & 0 \end{bmatrix} \begin{bmatrix} e_\theta \\ e_x \\ e_y \end{bmatrix}$$

To establish UGAS for the origin of $\dot{e} = f_e(t, e)$ we invoke the output-injection statement [Panteley et al., 2001, Proposition 3]. According to the latter, UGAS follows if:

(i) the origin of $\dot{e} = f_e(t, e)$ is uniformly globally stable;
(ii) the origin of $\dot{e} = F(t, e)$ is UGAS;
(iii) there exist an "output" y, non decreasing functions k_1, k_2, β: $\mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$, and class $\mathcal{K}_\infty$ function k, as well as a positive definite function γ such that

$$|K(t, e)| \le k_1(|e|)k(|y|) \tag{33}$$
$$|y(t, e)| \le k_2(|e|) \tag{34}$$
$$\int_0^\infty \gamma\big(|y(t)|\big) \le \beta(|e(0)|). \tag{35}$$

Condition (i): uniform global stability –see [Hahn, 1967], is tantamount to uniform stability and uniform global boundedness of the solutions. The latter was established already for the closed-loop system under the action of the "perturbation" u hence, it holds all the more in this case, where $u = 0$. Then, uniform stability follows from uniform global stability of the linearization of $\dot{e} = f_e(t, e)$ about the origin, *i.e.*,

$$\dot{e} = \begin{bmatrix} -k_\theta & 0 & -k_y p(t) \\ 0 & -k_x & 0 \\ 0 & 0 & 0 \end{bmatrix} e + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \omega_r(t) \\ v_r(t) & -\omega_r(t) & 0 \end{bmatrix} e.$$

Now, let $f : \mathbb{R}_{\ge 0} \times \mathbb{R}_{\ge 0}$ be defined as

$$\dot{f} = -k_\theta f + k_y p, \tag{36}$$

and let

$$e_z := e_\theta + f e_y.$$

The dynamics of the linearized model in the new error coordinates (e_z, e_x, e_y) is given by

$$\begin{bmatrix} \dot{e}_z \\ \dot{e}_x \\ \dot{e}_y \end{bmatrix} = \begin{bmatrix} -k_\theta & 0 & 0 \\ 0 & -k_x & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_z \\ e_x \\ e_y \end{bmatrix} + \begin{bmatrix} f(e_z - f e_y) & -f e_x \\ 0 & e_y \\ e_z - f e_y & -e_x \end{bmatrix} \begin{bmatrix} v_r(t) \\ \omega_r(t) \end{bmatrix}. \tag{37}$$

Uniform global stability of the origin of (37), under the integrability of (v_r, ω_r), can be proved using the Lyapunov function

$$V(t, e) := \frac{G(t)}{2}\left[e_z^2 + e_x^2 + e_y^2 \right], \tag{38}$$

where $G(t)$ is defined as:

$$G(t) := e^{-2\gamma \int_0^t (|v_r(s)|+|\omega_r(s)|)ds},$$
$$\gamma := \bar{f}^2 + 3\bar{f} + 1, \quad \bar{f} := \sup_t |f(t)|. \tag{39}$$

Remark 3. Note that the existence of a positive constant $\bar{f}$ follows from the boundedness of $p(t)$ and (36).

The function $V(t,e)$ is a Lyapunov function candidate that satisfies

$$e^{-2\gamma\beta}|e|^2 \le V(t,e) \le |e|^2,$$
$$\beta \ge \int_0^\infty (|v_r(s)| + |\omega_r(s)|)\, ds. \tag{40}$$

Moreover, the derivative of V along trajectories of (37) verifies

$$\dot{V}(t,e) = G(t)Q(t,e) + \frac{\dot{G}(t)}{2}\left[e_z^2 + e_x^2 + e_y^2\right] + G(t)\left[-k_\theta e_z^2 - k_x e_x^2\right], \tag{41}$$

where

$$\begin{aligned} Q(t,e) :=& f e_z^2 v_r - f\omega_r e_x e_z + (1-f^2)e_y e_z v_r - f v_r e_y^2 \\ \le& \bar{f} e_z^2 |v_r| + \bar{f}|\omega_r||e_x e_z| + (1+\bar{f}^2)|e_y e_z||v_r| \\ &+ \bar{f}|v_r| e_y^2 \\ \le& \gamma(|v_r| + |\omega_r|)\left[e_z^2 + e_x^2 + e_y^2\right]. \end{aligned} \tag{42}$$

Hence, using (39) and (42), we obtain

$$\begin{aligned} \dot{V}(t,e) \le& \left(\dot{G}(t)/2 + G(t)(|v_r| + |\omega_r|)\gamma\right)\left[e_z^2 + e_x^2 + e_y^2\right] \\ &- G(t)\left[k_\theta e_z^2 + k_x e_x^2\right] \\ \le& - G(t)\left[k_\theta e_z^2 + k_x e_x^2\right], \end{aligned} \tag{43}$$

Condition (ii): The nominal system has the convenient cascaded form

$$\dot{e}_\theta = -k_\theta e_\theta - k_y p(t) e_y \tag{44a}$$
$$\begin{bmatrix}\dot{e}_x \\ \dot{e}_y\end{bmatrix} = \underbrace{\begin{bmatrix} -k_x & \tilde{\psi}(t,e_y) \\ -\tilde{\psi}(t,e_y) & 0\end{bmatrix}\begin{bmatrix} e_x \\ e_y\end{bmatrix}}_{F_{xy}(t,e_x,e_y)} \tag{44b}$$

where

$$\tilde{\psi}(t,e_y) = k_\theta e_\theta(t) + k_y p(t) e_y. \tag{45}$$

Note that we have replaced e_θ with $e_\theta(t)$ to regard the system as a cascade Loría [2008]. Uniform global asymptotic stability follows from the following facts: 1) the system (44a) is input-to-state stable; 2) the interconnection term $k_y p(t)$ is bounded, and 3) the origin for (44b) is uniformly globally asymptotically stable.

The first two of the latter statements are obvious. To see the third, we invoke [Panteley et al., 2001, Theorem 1], by observing that $\tilde{\psi}$ is uniformly δ-persistently exciting with respect to e_y, which means that for each $\delta > 0$, there exists two numbers μ and $T > 0$, such that:

$$|e_y| \ge \delta \implies \int_t^{t+T} |\tilde{\psi}(s, e_y(s))|^2 ds \ge \mu. \tag{46}$$

Indeed, this is the case, under the condition that $\dot{p}$ is persistently exciting. To see this, we observe that

$$\dot{\tilde{\psi}} = -\left[k_\theta + k_y p(t) e_x\right]\tilde{\psi} + k_y e_y \dot{p}(t) \tag{47}$$

(for which computation we used that, here, $v_r = \omega_r = 0$), the term $k_y e_y \dot{p}(t)$ verifies (46) which means that it is uniformly δ-persistently exciting with respect to e_y, and the trajectories of (44) are globally bounded. Hence, by the filtering lemma [Panteley et al., 2001, Lemma 1], we conclude that $\tilde{\psi}$ also is uniformly δ-persistently exciting with respect to e_y.

Condition (iii): Using (32), a direct computation shows that there exists $c > 0$ such that

$$|K(t,e)| \le c\left|\left[e_\theta\, e_x\, e_y\right]\right|\left|\left[v_r\, \omega_r\right]\right|$$

hence, (33) holds with $k_1(s) = cs$, $k(s) = s$, and $y := [v_r\, \omega_r]$. Moreover, (34) and (35) hold with $\gamma(s) = s$, since $[v_r\, \omega_r] \in \mathcal{L}_1$, for a constant functions β and k_2 independent of the initial state.

Remark 4. Even if $y(t)$ in [Panteley et al., 2001, Proposition 3] is considered as an output of the system, the statement remains valid when $y(t)$ is an external signal such that (33) and (35) hold uniformly with respect to time.

This concludes the proof of UGAS for the nominal system $\dot{e} = f_e(t,e)$ hence, condition (B) is verified and, actually, it also concludes the proof of Proposition 1. ■

5. SIMULATION RESULTS

We present some brief simulation results that illustrate the performance of the δ-PE controller (10), (12), (18), and (23). The physical parameters are taken from [Fukao et al., 2000]:

$$M = \begin{bmatrix} m_1 & m_2 \\ m_2 & m_1\end{bmatrix}, \quad C(\dot{z}) = \begin{bmatrix} 0 & c\omega \\ -c\omega & 0\end{bmatrix},$$

with $m_1 = 0.6227$, $m_2 = -0.2577$, $c = 0.2025$, $r = 0.15$, and $b = 0.5$. The parameters $(\gamma, k_x, k_y, k_\theta, k_d)$ are taken equal to $(10^{-5}, 1, 1, 1, 20)$ respectively and the time varying function $p(t) = 12\sin(0.5t)+13$. The initial condition is set to: $z(0) = (1,1,1)$, $\nu(0) = (0,0)$, and $\hat{\Theta}(0) = (\hat{m}_1, \hat{m}_2, \hat{c}) = (0,0,0)$. The reference trajectory consists in an interrupted straight path, *i.e.*, $\omega_r \equiv 0$, $v_r = 5e^{-t}$.

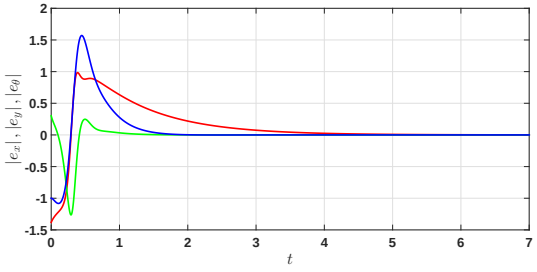


Fig. 1. Simulation of the tracking errors

6. CONCLUSION

This paper provides a simple way to analyze stability of nonholonomic mobile robots in closed loop with a class

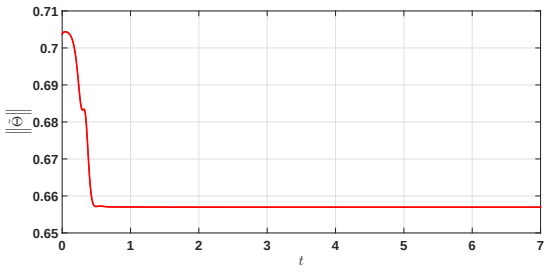


Fig. 2. Simulation of the parametric estimation erros

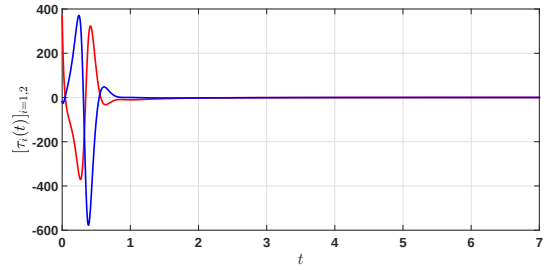


Fig. 3. Control input torque

of smooth time varying δ-persistently exciting controllers. These controllers work, in particular, for the case of straight-path vanishing trajectories. An original contribution is to establish uniform global asymptotic stability for the system at the kinematics level. Further research is being carried out to design δ-persistently exciting controllers for the control of swarms of vehicles. Also, the extension of our results to the case of output-feedback control is under investigation.

REFERENCES

A. Astolfi. Exponential stabilization of a wheeled mobile robot via discontinuous control. *ASME J. Dyn. Syst. Meas. Contr.*, 121(1):121–126, 1999.

R. Brockett. Asymptotic stability and feedback stabilization. In R. S. Millman R. W. Brocket and H. J. Sussmann, editors, *Differential geometric control theory*, pages 181–191. Birkhäuser, 1983.

K. D. Do. Formation tracking control of unicycle-type mobile robots. In *Proceedings 2007 IEEE International Conference on Robotics and Automation*, pages 2391–2396, April 2007. doi: 10.1109/ROBOT.2007.363677.

K. D. Do, Z-P. Jiang, and J. Pan. Simultaneous tracking and stabilization of mobile robots: an adaptive approach. *IEEE Trans. on Automat. Contr.*, 49(7):1147–1152, 2004.

T. Fukao, H. Nakagawa, and N. Adachi. Adaptive tracking control of a nonholonomic mobile robot. *IEEE Trans. on Robotics Automat.*, 16(5):609–615, 2000.

W. Hahn. *Stability of motion*. Springer-Verlag, New York, 1967.

Z-P. Jiang, E. Lefeber, and H. Nijmeijer. Saturated stabilization and tracking of a nonholonomic mobile robot. *Syst. & Contr. Letters*, 42(5):327–332, 2001.

Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Naguchi. A stable traking control scheme for an autonomous vehicle. In *Proc. IEEE Conf. Robotics Automat.*, pages 384–389, 1990.

Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Noguchi. A stable tracking control method for a non-holonomic mobile robot. In *Proc. IROS IEEE/RSJ International Workshop on Intelligent Robots and Systems. Intelligence for Mechanical Systems*, pages 1236–1241, 1991.

T-C. Lee, K-T. Song, C-H. Lee, and C-C. Teng. Tracking control of unicycle-modeled mobile robots using a saturation feedback controller. *IEEE Trans. Contr. Syst. Technol.*, 9(2):305–318, 2001.

A. Loría. From feedback to cascade-interconnected systems: Breaking the loop. In *Proc. 47th. IEEE Conf. Decision Contr.*, pages 4109–4114, Cancun, Mex., 2008.

A. Loría, E. Panteley, and A. Teel. A new persistency-of-excitation condition for UGAS of NLTV systems: Application to stabilization of nonholonomic systems. In *Proc. 5th. European Contr. Conf.*, pages 1363–1368, Karlsrühe, Germany, 1999.

A. Loría, E. Panteley, and K. Melhem. UGAS of skew-symmetric time-varying systems: application to stabilization of chained form systems. *European J. of Contr.*, 8(1):33–43, 2002.

A. Loría E. Panteley, D. Popovic, and A. Teel. δ-persistency of excitation: a necessary and sufficient condition for uniform attractivity. In *Proc. 41th. IEEE Conf. Decision Contr.*, pages 3506–3511, Las Vegas, CA, USA, 2002. Paper no. REG0623 .

P. Morin and C. Samson. Application of backstepping techniques to the time-varying exponential stabilisation of chained form systems. *European J. of Contr.*, 3(1): 15–36, 1997.

E. Panteley and A. Loría. Growth rate conditions for stability of cascaded time-varying systems. *Automatica*, 37(3):453–460, 2001.

E. Panteley, A. Loría, and A. Teel. Relaxed persistency of excitation for uniform asymptotic stability. *IEEE Trans. on Automat. Contr.*, 46(12):1874–1886, 2001.

F. Pourboghrat. Exponential stabilization of nonholonomic mobile robots. *J. Comput. Elec. Engin.*, 28(5): 349–359, 2002.

C. Samson. Control of chained system: Application to path following and time-varying point stabilization of mobile robots. *IEEE Trans. on Automat. Contr.*, 40(1):64–77, 1995.

Y. Wang, Z. Miao, H. Zhong, and Q. Pan. Simultaneous stabilization and tracking of nonholonomic mobile robots: A lyapunov-based approach. *IEEE Transactions on Control Systems Technology*, 23(4): 1440–1450, July 2015. ISSN 1063-6536. doi: 10.1109/TCST.2014.2375812.