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# 1 **MFD-based simulation: Spillbacks in multi-reservoir** 2 **networks**

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1 **ABSTRACT**

2 The wide majority of large-scale traffic flow models based on the Macroscopic Fundamental Dia-  
3 gram (MFD) are still grounded on what has been called the “accumulation-based” model by some  
4 authors. Recent studies have highlighted the limitations of such an approach in fast-varying condi-  
5 tions, and have focused on a better and more flexible representation of the reservoir inner dynamics,  
6 which has been referred as the “trip-based” model. However, when connecting several reservoirs  
7 together, the management of inflows and outflows to propagate congestion properly through the  
8 reservoirs is still a challenge in both modeling approaches. In particular, in saturated traffic condi-  
9 tions, handling inflow merge at a reservoir entry or outflow diverge at exit is of crucial importance  
10 as it governs the global system state evolution. Unfortunately, this has not been deeply investigated  
11 in the literature.

12 In this paper, we propose a thorough analysis of the way congestion is usually handled  
13 in the accumulation-based framework. This serves as a basis to implement a proper congestion  
14 propagation model in the trip-based approach. Theoretical and simulation studies show that in  
15 case of several trip lengths in a zone, there exists only one form of inflow limitation at the reservoir  
16 entry that complies with the global constraints on flow and production. Moreover, the outflows  
17 from all trips exiting the same zone have to be all inter-dependent to satisfy the main hypothesis  
18 of the reservoir model, i.e. that all users are traveling at the same mean speed. This has strong  
19 implications when several reservoirs are connected together. We notably point out that the system  
20 could converge to global gridlock if a safe outflow management based on the most constrained exit  
21 of each reservoir is not adopted. Simulation examples are given.

22 *Keywords:* Macroscopic Fundamental Diagram, reservoir systems, congestion propagation, trip  
23 length, accumulation-based model, trip-based model

## 1 INTRODUCTION

2 Over the past decade, the Macroscopic Fundamental Diagram (MFD) has appeared to be a power-  
3 ful tool to describe traffic states at the network level with few implementation and computational  
4 efforts. Many studies have notably used MFD-based traffic simulators for several promising ap-  
5 plications, like traffic state estimation (Knoop and Hoogendoorn, Yildirimoglu and Geroliminis,  
6 1, 2), perimeter control (Haddad, Haddad and Mirkin, Ampountolas et al., 3, 4, 5), route guid-  
7 ance at large scale (Hajiahmadi et al., Ding et al., 6, 7), or analyzing cruising-for-parking issues  
8 (Leclercq et al., Cao and Menendez, Zheng and Geroliminis, 8, 9, 10). Their modeling approaches  
9 take advantage of the multi-reservoir representation of a city, where the dynamics of each urban  
10 region (also called “reservoir”) are described by the single reservoir model of Daganzo (11). This  
11 framework, also referred as the “accumulation-based” model, assumes that the reservoir outflow  
12 is proportional to the total circulating flow inside the zone if one consider a constant average trip  
13 length shared by all travelers. Some authors have extended this approach to account for multi-  
14 ple trip lengths in a reservoir, either to develop new applications like modeling search-for-parking  
15 (Geroliminis, 12, 13) and macroscopic routing (Yildirimoglu et al., Ramezani et al., 14, 15); or to  
16 highlight inaccuracies in MFD-based models due the constant trip length hypothesis (Yildirimoglu  
17 and Geroliminis, Leclercq et al., 2, 16).

18 More recently, a “trip-based” formulation of the single reservoir model has gained a new  
19 interest in the community. Based on a idea of Arnott (17), this approach has been exploited in Da-  
20 ganzo and Lehe (18) and then Leclercq et al., Lamotte and Geroliminis (8, 19). The principle is that  
21 all users travel at the same space-mean speed (given by the MFD) at a given time, and exit the zone  
22 once they have completed their individually assigned trip length. As shown in a thorough compari-  
23 son with the accumulation-based model by Mariotte et al. (20), the trip-based approach gives more  
24 accurate results during transient phases. Nevertheless, some authors like Haddad and Mirkin (21)  
25 suggest that the inaccuracies of the accumulation-based model can be taken into account directly  
26 by implementing delays in the control-oriented formulation of the conservation equation. While  
27 being interesting for control applications, we find this approach still limited in a multi-reservoir  
28 context as it relies on a quasi-static approximation for calculating delays.

29 However, from the modeler’s perspective, despite all these recent advances in MFD-based  
30 simulation, congestion propagation in a multi-reservoir framework is not fully understood yet.  
31 Notably, the questions of if and how boundary flows should be limited when a reservoir is over-  
32 saturated, and how to distribute inflows and outflows are rarely addressed. In details, as the wide  
33 majority of MFD-based simulators are still developed for control applications, most authors argue,  
34 with reason, that the controllers will not allow the reservoir to reach highly congested states, so  
35 that the aforementioned concerns may be eclipsed (see e.g. Kouvelas et al., 22). However, other  
36 applications of MFD-based models should not ignore them. Actually some interesting works al-  
37 ready propose incomplete but viable solutions to deal with congestion propagation. Hajiahmadi  
38 et al., Lentzakis et al. (6, 23), whose simulator is based on the Network Transmission Model (NTM)  
39 of Knoop and Hoogendoorn (1), consider exogenous boundary capacities between reservoirs and  
40 a global entry supply function per reservoir. Their approach ensures a perfect protection of the  
41 reservoirs from global gridlock, nevertheless this one can hardly be extended to heterogeneous trip  
42 length situations. Yildirimoglu and Geroliminis (2) certainly developed the more advanced tool  
43 in MFD-based simulation, as they account for different trip lengths and manage flow exchanges  
44 with a Dynamic Traffic Assignment (DTA) procedure on macro-routes (succession of reservoirs).  
45 However, they handle each boundary between two adjacent reservoirs separately with a pro-rata

1 inflow merge, and do not provide any further information on how the global protection of each  
2 reservoir is ensured.

3 In this paper, we investigate the constraints and the requirements to design a proper con-  
4 gestion propagation model, applicable in any situation in an MFD-based multi-reservoir system.  
5 For the accumulation-based model, we will refer to the framework of [Yildirimoglu and Gerolim-  
6 minis, Geroliminis \(2, 13\)](#). For the trip-based model, we will pursue the effort of [Mariotte et al.  
7 \(20\)](#) who provide a first attempt to handle spillbacks in this formulation. For both modeling ap-  
8 proaches, it appears that elementary constraints on flow and production lead to a unique definition  
9 of the entry supply function for each accumulation or trip length category in one reservoir. More-  
10 over, applying the same constraints for exit flows allow us to show that each partial outflow cannot  
11 be treated independently from the other categories in a reservoir. It follows that if one wants to  
12 preserve the consistency of an MFD-based model (i.e. that the users are traveling at the same mean  
13 speed at each time), one must let the inflow merge and outflow diverge allocation be endogenously  
14 defined by the reservoir state. Consequently, only a few degrees of freedom are left to the mod-  
15 eler to control flow exchanges in simulation. We then propose two approaches of outflow share  
16 complying with all these constraints.

17 This paper is organized as follows: section 2 first introduces spillbacks modeling in a  
18 single reservoir with one trip length, then section 3 extends this framework to different trip length  
19 categories in one reservoir.

## 20 **SPILLBACKS IN A SINGLE RESERVOIR WITH UNIQUE TRIP LENGTH**

### 21 **Accumulation-based formulation**

22 In this study, we focus on how a reservoir should interact with its neighbors in the context of a  
23 multi-reservoir representation of a city.

24 The concept of the single reservoir model has been first presented in [Daganzo, Geroliminis  
25 and Daganzo \(11, 24\)](#). It corresponds to a given zone in a urban network where the traffic states  
26 are characterized by a well-defined production-MFD  $P(n)$  (in [veh.m/s]), or equivalently, a speed-  
27 MFD  $V(n) = P(n)/n$  (in [m/s]), where  $n$  (in [veh]) is the accumulation (number of circulating  
28 vehicles in the reservoir) . The reservoir entry (also conceptually called “upstream boundary”)  
29 is the aggregation of all individual entry nodes of the network; similarly the reservoir exit (or  
30 “downstream boundary”) aggregates all the exit nodes. Through the entry is defined the total  
31 effective inflow  $q_{in}(t)$ , and through the exit the total effective outflow  $q_{out}(t)$  (in [veh/s]). In a first  
32 approach, we do not consider internal trips (trips that start or end into the reservoir), and assume  
33 that traffic states result from “external” trips only (trips from the aggregated entry to the aggregated  
34 exit). The question of internal trips will come up in the discussion (section 4). In this framework,  
35 the reservoir dynamics are governed by the following vehicle conservation equation ([Daganzo,  
36 11](#)):

$$\frac{dn}{dt} = q_{in}(t) - q_{out}(t) \quad (\text{reservoir dynamics}) \quad (1)$$

37 The accumulation and trip-based models differ on the definition of  $q_{in}(t)$  and  $q_{out}(t)$ . We present  
38 here the accumulation-based model first. Most of the previous studies dealing with MFD-based  
39 aggregated dynamics actually do the distinction between internal and external trips, however their  
40 treatments are often mixed in the same modeling approach, so that a proper definition of inflow  
41 and outflow is sometimes missing. In control-oriented works, some authors like [Ampountolas  
42 et al., Kouvelas et al., Aboudolas and Geroliminis \(5, 22, 25\)](#) split the inflow into the receiving

1 flow from adjacent reservoirs (for which the controllers apply), and the “uncontrolled demand”  
 2 which may be internal or external. Because we focus on a reservoir interacting with its neighbors  
 3 here, we propose a definition of flow exchange at boundaries inspired by the Cell Transmission  
 4 Model (CTM) of [Daganzo \(26\)](#). Thus, at the reservoir entry, the effective inflow results from the  
 5 competition between a given demand  $\lambda(t)$  from some sending reservoirs and an entry supply func-  
 6 tion  $I(n)$  depending on the reservoir state and restraining the inflow when the reservoir becomes  
 7 congested:

$$q_{\text{in}}(t) = \min[\lambda(t); I(n)] \quad (\text{effective inflow}) \quad (2)$$

8 At the reservoir exit, the effective outflow balances a given supply  $\mu(t)$  (inflow restriction into  
 9 some destination reservoirs in case of congestion), and an outflow demand function  $O(n)$  also  
 10 depending on the reservoir state:

$$q_{\text{out}}(t) = \min[\mu(t); O(n)] \quad (\text{effective outflow}) \quad (3)$$

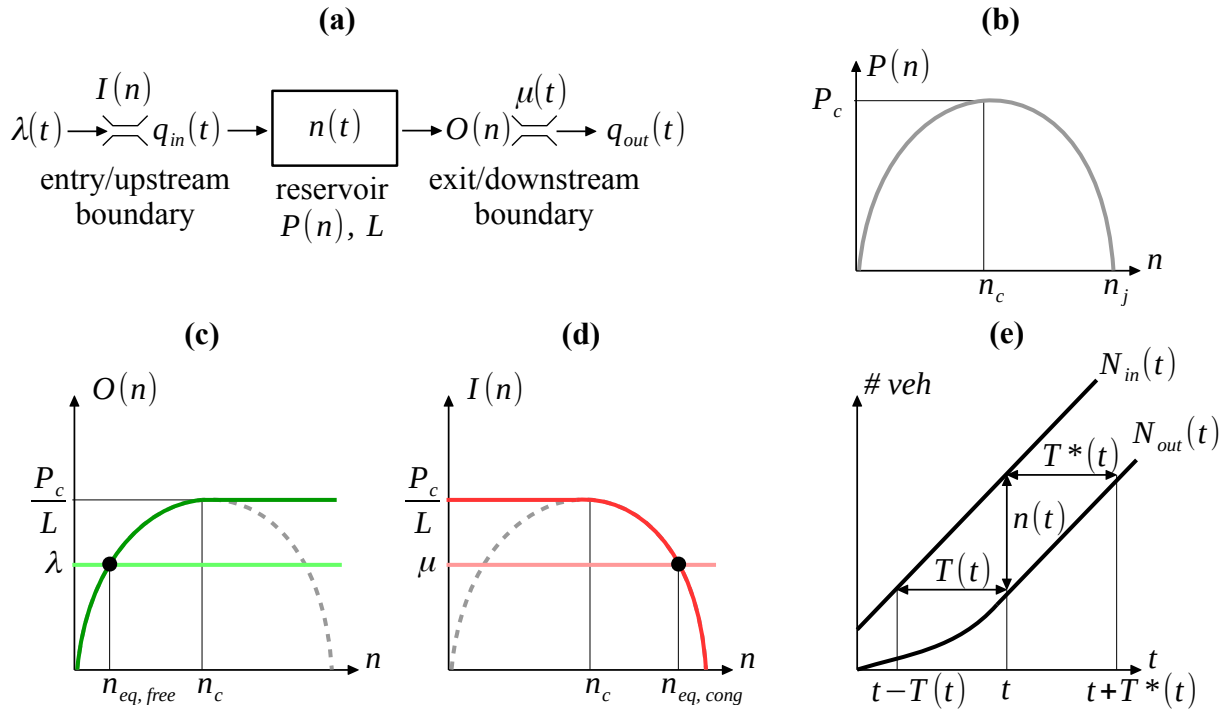
11 The single reservoir model with its boundary conditions is represented in figure 1(a). As there is no  
 12 spatial extension in the reservoir, the simplest version of the accumulation-based model assumes  
 13 an average trip length  $L$  for all travelers and applies the queuing formula of [Little \(27\)](#) to define  
 14 the “trip completion rate”  $G(n) = n/L \cdot V(n) = P(n)/L$  ([Daganzo, 11](#)). This quasi-static approach  
 15 has several limitations as detailed in [Mariotte et al. \(20\)](#). To our best knowledge, all the studies  
 16 from the literature consider that the system outflow always equals  $G(n)$ . In our opinion however,  
 17 we believe that this is only true to model internal congestion for the outflow of inner trips, but  
 18 that a distinction between  $O(n)$  and  $G(n)$  should be made for external trips. Otherwise, we may  
 19 encounter unrealistic situations where the reservoir state is stucked to an equilibrium point on the  
 20 congested part of the MFD during a temporary supply reduction at exit. In such a case, the system  
 21 would be unable to retrieve a free-flow situation after the end of the supply reduction. The reader  
 22 can refer to [Mariotte et al. \(20\)](#) for more details, but this situation will be also illustrated in the next  
 23 section. This can be avoided if we adopt the following definition of outflow demand:

$$O(n) = \begin{cases} \frac{n}{L}V(n) = G(n) & \text{if } n < n_c \\ \frac{P_c}{L} & \text{otherwise} \end{cases} \quad (\text{outflow demand function}) \quad (4)$$

24 At the reservoir entry, the general shape of the entry supply function  $I(n)$  is first introduced in [Da-](#)  
 25 [ganzo \(11\)](#), but the authors do not mention any explicit formulation. Like in [Knoop and Hoogen-](#)  
 26 [doorn, Hajiahmadi et al., Lentzakis et al. \(1, 6, 23\)](#), we define  $I(n)$  in accordance with the basic  
 27 principles of traffic flow theory:

$$I(n) = \begin{cases} \frac{P_c}{L} & \text{if } n < n_c \\ \frac{n}{L}V(n) = G(n) & \text{otherwise} \end{cases} \quad (\text{entry supply function}) \quad (5)$$

28 These two functions are represented in figures 1(c) and (d), for a typical shape of the production-  
 29 MFD in figure 1(b). Note that  $O(n)$  and  $I(n)$  are similar to the entry and exit functions of a cell  
 30 in the CTM ([Daganzo, 26](#)). When dealing with external trips only, such a formulation for the  
 31 accumulation-based model is fully consistent to handle both free-flow and congested situations in  
 32 the reservoir. Figures 1(c) and (d) show examples of equilibrium states reached in free-flow and  
 33 congestion respectively with given boundary conditions.



**FIGURE 1** (a) Single reservoir with its boundaries, (b) production-MFD, (c) outflow demand function  $O(n)$  with the equilibrium point for a given inflow demand  $\lambda$  in free-flow, (d) inflow supply function  $I(n)$  with the equilibrium point for a given exit restriction  $\mu$  in congestion, and (e) cumulative count curves with the accumulation  $n(t)$ , the experienced travel time  $T(t)$  and the exact predictive travel time  $T^*(t)$

1 **Trip-based formulation**

2 The theoretical background of the trip-based model has been settled by [Arnott \(17\)](#). Let consider  
 3 a single reservoir with a unique trip length  $L$ . It is assumed that at each time  $t$ , all the vehicles are  
 4 traveling at the same speed  $V(n(t))$ . A user exiting the reservoir at  $t$  has traveled during a period of  
 5  $T(t)$  by definition,  $T(t)$  being the user experienced travel time. This user thus entered the reservoir  
 6 at  $t - T(t)$ , and his/her trip distance was  $L$ . The trip-based model considers that the accumulation  
 7 and therefore the mean speed may change during the user’s trip, which is mathematically expressed  
 8 as:

$$L = \int_{t-T(t)}^t V(n(s)) ds \tag{6}$$

9 By using basic relationships based on entering and exiting count curves, it can be shown that the  
 10 derivative of equation 6 leads to (see e.g. [Arnott, Mariotte et al., 17, 20](#)):

$$q_{out}(t) = q_{in}(t - T(t)) \cdot \frac{V(n(t))}{V(n(t - T(t)))} \tag{7}$$

11 Using equation 7 to solve the conservation equation 1 leads to a differential equation with endoge-  
 12 nous delay. Despite being mathematically intractable, this formulation of the outflow can allow the  
 13 development of efficient numerical resolution schemes ([Mariotte et al., 20](#)). These methods work

1 in free-flow only, where  $q_{in}(t)$  is the input, equal to the inflow demand  $\lambda(t)$ , and where  $q_{out}(t)$  is  
 2 the consequence of the system evolution. In congestion however, the role of inflow and outflow  
 3 are switched, as  $q_{out}(t)$  becomes the input, equal to the outflow supply  $\mu(t)$ , and  $q_{in}(t)$  has now to  
 4 adapt to the system evolution due to the restriction at exit. It can be shown that equation 7 can be  
 5 reversed to express  $q_{in}(t)$  as a function of  $q_{out}(t)$ :

$$q_{in}(t) = q_{out}(t + T^*(t)) \cdot \frac{V(n(t))}{V(n(t + T^*(t)))} \quad (8)$$

6 where  $T^*(t)$  is the exact predictive travel time, i.e. the time during which the user entering at  $t$  will  
 7 travel, see also figure 1(e). By construction we have:  $T(t) = T^*(t - T(t))$ . But equation 8 suggests  
 8 that (i) it is not possible to deduce the inflow when downstream supply restriction should apply  
 9 (this would require the knowledge of the system future evolution), and (ii) if this was possible we  
 10 have no clue on how to made the switch.

11 Thus in practice, this model needs to be coupled with another model for reproducing con-  
 12 gestion propagation. A first attempt has been made by Mariotte et al. (20). They assume a free-  
 13 flow evolution of the system and then apply the outflow reduction and the minimum principle of  
 14 Newell (28) on the inflow. This method with off-line calculations is sufficient for the analysis of  
 15 a single reservoir, but not suitable in a multi-reservoir context where reservoir states may be all  
 16 inter-dependent with time. A simple way to perform in-line computations of inflow limitations  
 17 is to switch to the accumulation-based framework in congestion, by using the same entry supply  
 18 function  $I(n)$ .

### 19 Trip-based numerical implementation

20 In the following of this paper, we will use the event-based scheme from Mariotte et al. (20) to  
 21 solve numerically the trip-based model. In congestion, the reservoir exit flow is limited to  $\mu(t)$   
 22 at each time by retaining the vehicles inside the zone until the next exit time, even if they have  
 23 already completed their trip length. At entry, the inflow limitation is ensured with the definition of  
 24 a minimum or supply time for entering the reservoir:

$$t_{\text{entry supply}}^{N^{\text{in}}} = t_{\text{entry}}^{N^{\text{in}}-1} + \frac{1}{I(n)} \quad (\text{entry supply time}) \quad (9)$$

25 where  $t_{\text{entry supply}}^{N^{\text{in}}}$  is the supply time for the  $N^{\text{in}}$ th vehicle to enter the reservoir,  $t_{\text{entry}}^{N^{\text{in}}-1}$  is the entering  
 26 time of the previous vehicle, and  $I(n)$  is the entry supply function of the accumulation-based model,  
 27 see equation 5.

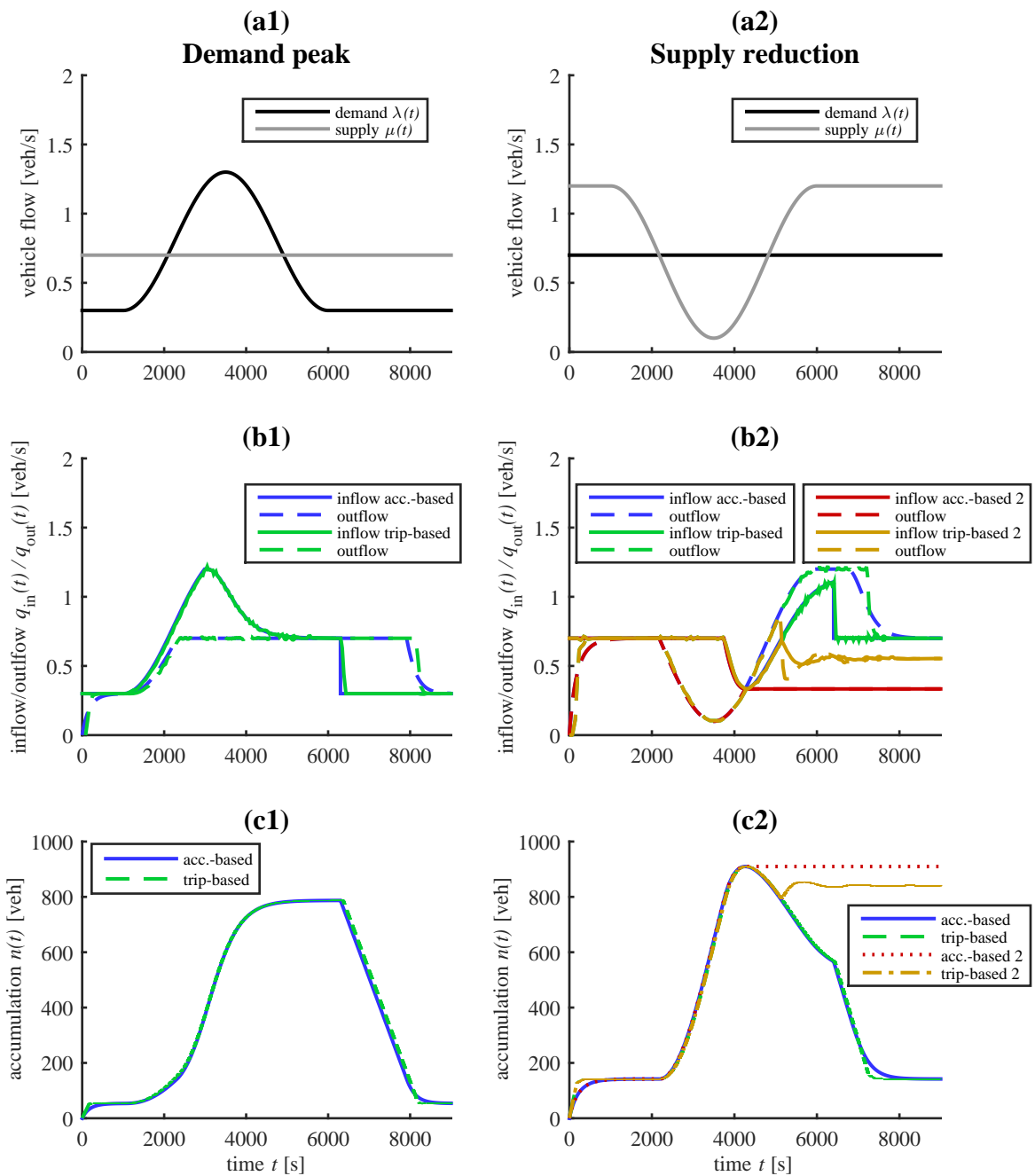
28 The application of this method is illustrated with two test cases. The first one is about a de-  
 29 mand peak temporarily exceeding the exit supply, and the second one concerns a supply reduction  
 30 at exit below the demand level at entry. These numerical examples consider a single reservoir with  
 31 maximum accumulation  $n_j = 1000$  veh, average trip length  $L = 2.5$  km, free-flow speed  $u = 15$  m/s,  
 32 and characterized by a parabolic production-MFD with maximum production  $P_c = 3000$  veh.m/s  
 33 and critical accumulation  $n_c = 400$  veh.

34 Figure 2(a1) shows the demand  $\lambda(t)$  and supply  $\mu(t)$  profiles for the demand peak case.  
 35 The simulation scenario has been designed to let the congestion reach the entry before the de-  
 36 mand decreases. The reservoir state evolution is presented in figures 2(b1) and (c1) with the in-  
 37 flow/outflow and accumulation. The blue curves correspond to the accumulation-based model, the



1 green ones to the trip-based model. All graphs show similar results for both modeling approaches.  
2 This was actually expected, since the modeling of spillbacks is handled in the same manner in  
3 both models. This also proves that the switch to the accumulation-based model works well in the  
4 trip-based framework with few modifications in the event-based resolution scheme.

5 Figure 2(a2) shows the demand  $\lambda(t)$  and supply  $\mu(t)$  profiles for the supply reduction case.  
6 Similarly, the simulation scenario has been designed to let the congestion reach the entry before the  
7 exit supply increases again. In figures 2(b2) and (c2), the red and yellow curves corresponds to the  
8 evolution of inflow/outflow and accumulation when  $O(n)$  always equals  $G(n)$  as it is traditionally  
9 assumed in the literature. In the accumulation-based model (in red), we observe that the system  
10 reaches an equilibrium point once inflow equals outflow shortly after 4000 s. Then, the reservoir  
11 state does not evolve anymore because after this point the outflow corresponds to the exit demand  
12  $O(n)$ , and thus  $q_{\text{out}}(t)$  is not impacted by an increase of  $\mu(t)$ , see equation 7. In the trip-based  
13 approach (in yellow), the users travel at a low mean speed after 4000 s to adapt the exit supply  
14 reduction. But when this limitation disappears, the vehicle exit rate is still the same because the  
15 mean speed remains low, and consequently the system cannot recover from congestion in this  
16 framework too. We can fix this problem if we keep the outflow demand  $O(n)$  maximum during  
17 severe congestion periods. This can be modeled in the trip-based framework only if we force  
18 the travelers to complete their trip length when  $n \geq n_c$ . Theoretically, it implies that the users  
19 concerned will have a speed different from  $V(n)$  during congested situations. This formulation  
20 happens to be equivalent to the outflow demand definition of equation 4. This is illustrated by the  
21 blue and green curves in figures 2(b2) and (c2), which also show similar results for both models.



**FIGURE 2** (a1) Demand peak at the reservoir entry, demand  $\lambda(t)$  and supply  $\mu(t)$  profiles, (b1) inflow  $q_{in}(t)$  and outflow  $q_{out}(t)$  and (c1) accumulation  $n(t)$  for the accumulation and trip-based models. (a2) Supply reduction at the reservoir exit, demand and supply profiles, (b2) inflow and outflow and (c2) accumulation for the accumulation and trip-based models, where model “2” corresponds to the assumption that  $O(n)$  always equals  $G(n)$

1 **CONGESTION PROPAGATION WITH SEVERAL TRIP CATEGORIES**

2 **Accumulation-based framework**

3 The aim of our study is to propose a robust modeling framework for congestion propagation in  
 4 a multi-reservoir environment. Like in the approach of [Yildirimoglu and Geroliminis \(2\)](#), we  
 5 consider that users are assigned to a set of given “macro-routes”, i.e. successions of reservoirs, as  
 6 illustrated in figure 3(a), and that the system state can be described at the level of a macro-route,  
 7 later simply referred as “route”. As a reservoir can be crossed by different routes with different  
 8 trip lengths, the thorough understanding of flow dynamics in one reservoir with heterogeneous trip  
 9 lengths is essential to build a proper multi-reservoir simulation tool.

10 The extension of the single reservoir model with one trip length to several trip lengths has  
 11 been first established in ([Geroliminis, 12, 13](#)). It is inspired by the two-bin model of [Daganzo](#)  
 12 ([11](#)). This theoretical framework has then been used in various studies with more complex multi-  
 13 reservoir settings (e.g. [Yildirimoglu and Geroliminis, Ramezani et al., 2, 15](#)). Let consider a single  
 14 reservoir with  $N$  trip length categories  $L_i$ , or  $N$  routes with length  $L_i$ , as presented in figure 3(b).  
 15 All accumulations  $n_i$  in each route  $i$  should satisfy the following system ([Geroliminis, 13](#)):

$$\forall i \in \{1, \dots, N\}, \quad \frac{dn_i}{dt} = q_{in,i}(t) - q_{out,i}(t) \quad (10)$$

16 where  $q_{in,i}(t)$  and  $q_{out,i}(t)$  are respectively the effective inflow and outflow for route  $i$ . Con-  
 17 ceptually, the reservoir is split into “sub-reservoirs” governed by the accumulation  $n_i$ . These  
 18 sub-reservoirs are coupled together by the mean speed  $V(n)$  or the total production  $P(n)$ , where  
 19  $n = \sum_{i=1}^N n_i$ . It is assumed that in slow-varying conditions, the trip completion rate  $G_i$  of each route  
 20  $i$  satisfies the queuing formula of [Little \(27\)](#):

$$G_i(n_1, \dots, n_N) = G_i(n_i, n) = \frac{n_i}{L_i} V(n) = \frac{n_i}{n} \frac{P(n)}{L_i} \quad (\text{trip completion rate}) \quad (11)$$

21 It follows that the definition of the outflow demand  $O_i$  for each category  $i$  should be:

$$O_i(n_i, n) = \begin{cases} \frac{n_i}{n} \frac{P(n)}{L_i} = G_i(n_i, n) & \text{if } n < n_c \\ \frac{n_i}{n} \frac{P_c}{L_i} & \text{otherwise} \end{cases} \quad (\text{outflow demand}) \quad (12)$$

with by definition:

$$n = \sum_{i=1}^N n_i \quad (\text{total accumulation}) \quad (13)$$

$$G(n) = \sum_{i=1}^N G_i(n_i, n) \quad (\text{total trip completion rate}) \quad (14)$$

$$O(n) = \sum_{i=1}^N O_i(n_i, n) \quad (\text{total outflow demand}) \quad (15)$$

22 Similarly to the single trip length case, we suppose that the outflow demand is maximum in over-  
 23 saturated situations ( $n > n_c$ ).

1 Note also that a dynamic average trip length  $L(t)$  can be defined in applying Little's formula  
 2 at the reservoir scale:  $G(n) = n/L \cdot V(n)$ . It comes (Geroliminis, 12):

$$L(t) = \frac{n(t)}{\sum_{i=1}^N \frac{n_i(t)}{L_i}} = \frac{\sum_{i=1}^N G_i(n_i(t), n(t)) L_i}{G(n(t))} \quad (\text{average trip length}) \quad (16)$$

3 The major difference with the unique trip length model is that here, each outflow demand  $O_i$  de-  
 4 pends not only on  $n_i$ , but also on the other accumulations  $n_1, \dots, n_N$  through the total accumulation  
 5  $n$ .

### 6 Calculating perimeter inflows

7 All the effective inflows  $q_{in,i}(t)$  for each route  $i$  may be treated as independent variables. They are  
 8 the result of the competition between the corresponding demand  $\lambda_i(t)$  and an entry supply function  
 9  $I_i(n_1, \dots, n_N)$ :

$$\forall i \in \{1, \dots, N\}, \quad q_{in,i}(t) = \min[\lambda_i(t); I_i(n_1(t), \dots, n_N(t))] \quad (\text{effective inflow route } i) \quad (17)$$

10 There is no clear consensus on the definition of each entry supply function  $I_i(\cdot)$  in the literature.  
 11 Knoop and Hoogendoorn (1) consider a global supply function  $I(n)$  at the reservoir entry that  
 12 applies for all inflows, while Yildirimoglu and Geroliminis, Geroliminis, Ramezani et al. (2, 12, 15)  
 13 treat each boundary flow separately (exchange with adjacent reservoirs). Knoop and Hoogendoorn  
 14 (1) use the same definition as in equation 5, but they also define exogenous boundary capacities  
 15 between adjacent reservoirs. Ramezani et al. (15) simplify the evolution of  $I(n)$  for  $n > n_c$  with  
 16 a linear curve, however they give no further details on the maximum flow capacity they use for  
 17  $n < n_c$ . The general idea is to allocate portion of flows regarding a global inflow limitation for the  
 18 whole reservoir to prevent it from gridlock (our point is that gridlock may happen due to internal  
 19 demand but not due to external loading at the perimeter). Main issues are the definitions of: (i) the  
 20 allocation scheme, (ii) the maximum allowed flow for each route (capacity at entry), and (iii) the  
 21 distinction between under- and oversaturated states.

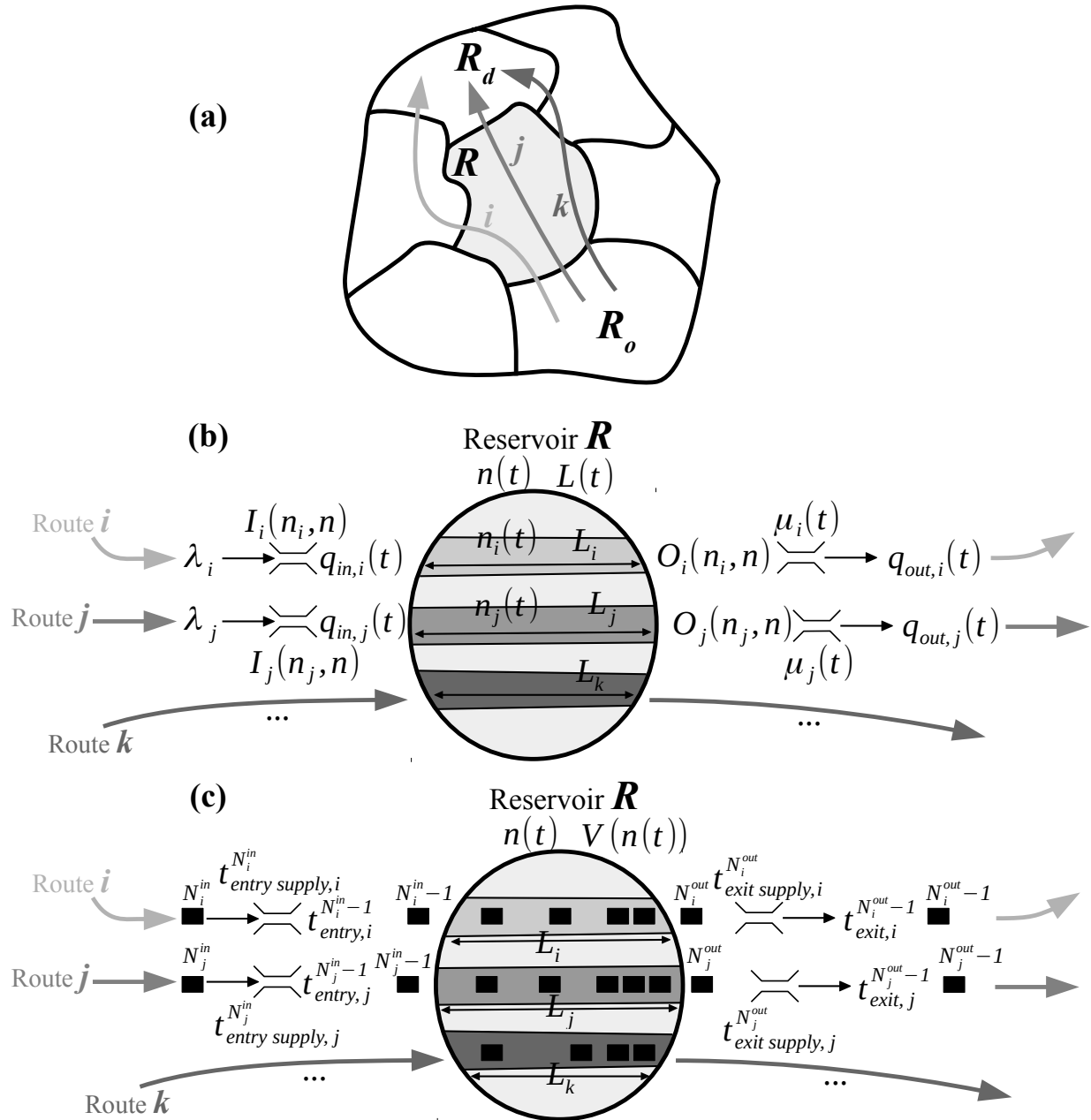
The functions  $I_i(\cdot)$  must be designed to ensure that the total effective inflow and total entry  
 production (i) never exceed the reservoir capacity and (ii) adapt to the reservoir state in oversat-  
 ured regime. Similarly to the case of one trip length, we suppose that under- and oversaturated  
 states are distinguished by the critical accumulation  $n_c$ . Therefore the entry supply functions must  
 comply with the two following global constraints at any time, on flow and production respectively  
 (the  $n_i$  are omitted):

$$\sum_{i=1}^N I_i = \begin{cases} \frac{P_c}{L} & \text{if } n < n_c \\ \frac{P(n)}{L} & \text{otherwise} \end{cases} \quad (\text{flow constraint}) \quad (18)$$

$$\sum_{i=1}^N L_i I_i = \begin{cases} P_c & \text{if } n < n_c \\ P(n) & \text{otherwise} \end{cases} \quad (\text{production constraint}) \quad (19)$$

22 where  $L$  is the average trip length defined in equation 16. We will demonstrate that under these  
 23 two constraints, the unique possible definition of the  $I_i(\cdot)$  is:

$$\forall i \in \{1, \dots, N\}, \quad I_i(n_i, n) = \begin{cases} \frac{n_i P_c}{n L_i} & \text{if } n < n_c \\ \frac{n_i P(n)}{n L_i} = \frac{n_i}{L_i} V(n) & \text{otherwise} \end{cases} \quad (\text{entry supply function}) \quad (20)$$



**FIGURE 3** (a) Examples of three routes ( $i, j, k$ ) for a macro-OD ( $R_o, R_d$ ) in a multi-reservoir system, (b) representation of the reservoir  $R$  crossed by the routes in the accumulation-based and (c) trip-based frameworks

1 *Proof.* Let assume without loss of generality that each function  $I_i$  can be regarded as a portion of a  
 2 global entry supply  $I(n)$ :  $\forall i \in \{1, \dots, N\}, I_i = \alpha_i I(n)$  where the  $\alpha_i(\cdot)$  are functions of  $(n_1, \dots, n_N)$ .  
 3 As we have  $\sum_{i=1}^N I_i = I(n)$ , the functions  $\alpha_i(\cdot)$  should verify  $\sum_{i=1}^N \alpha_i = 1$ . We first consider that  
 4  $n \geq n_c$ . The application of the flow constraint (equation 18) forces  $I(n) = P(n)/L$ , whereas the  
 5 application of the production constraint (equation 19) leads to  $I(n) \sum_{i=1}^N L_i \alpha_i = P(n)$ , and thus  
 6  $\sum_{i=1}^N L_i \alpha_i = L$ . This suggests the following expression of the  $\alpha_i(\cdot)$  functions:  $\forall i \in \{1, \dots, N\}, \alpha_i =$

1  $\beta_i L/L_i$ , where  $\beta_i(\cdot)$  are functions of  $(n_1, \dots, n_N)$  and satisfy  $\sum_{i=1}^N \beta_i = 1$ . Knowing that  $\sum_{i=1}^N \alpha_i = 1$ ,  
 2 it results that  $\sum_{i=1}^N \beta_i = 1/L = \sum_{i=1}^N n_i/(nL_i)$ . Again, this suggests the following expression of the  
 3  $\beta_i(\cdot)$  functions:  $\forall i \in \{1, \dots, N\}, \beta_i = \gamma_i n_i/n$ , where  $\gamma_i(\cdot)$  are functions of  $(n_1, \dots, n_N)$  and must  
 4 verify  $\sum_{i=1}^N \beta_i = \sum_{i=1}^N \gamma_i n_i/n = 1$ . Because the last equality is true whatever the accumulations  
 5  $(n_1, \dots, n_N)$ , it follows that  $\forall i \in \{1, \dots, N\}, \gamma_i = 1$ , and therefore  $\forall i \in \{1, \dots, N\}, \alpha_i = n_i/n \cdot L/L_i$ .  
 6 Finally we have shown that  $\forall i \in \{1, \dots, N\}, I_i = n_i/n \cdot P(n)/L_i$  when  $n \geq n_c$ , and the same demon-  
 7 stration applies similarly for the case  $n < n_c$ .  $\square$

### 8 Calculating perimeter outflows

9 Because they are the result of the reservoir inner dynamics, the situation may be different for the  
 10 outflows  $q_{out,i}(t)$  of each route  $i$ . Without loss of generality we can consider that each outflow is a  
 11 fraction of the total outflow  $q_{out}$  (time  $t$  is omitted):  $\forall i \in \{1, \dots, N\}, q_{out,i} = \alpha_i q_{out}$  where the  $\alpha_i(\cdot)$   
 12 may eventually be function of  $(n_1, \dots, n_N)$ . Because by definition  $\sum_{i=1}^N q_{out,i} = q_{out}$ , the coefficients  
 13  $\alpha_i(\cdot)$  verify  $\sum_{i=1}^N \alpha_i = 1$ . If we assume that the expression of the average trip length  $L(t)$  is always  
 14 defined by equation 16 at any time (this is equivalent to assuming that Little's formula applies at  
 15 any time at the reservoir scale), then the total exit production should verify:

$$\sum_{i=1}^N L_i q_{out,i}(t) = L(t) q_{out}(t) \quad (\text{exit production}) \quad (21)$$

16 Replacing  $q_{out,i}$  by  $\alpha_i q_{out}$  in equation 21 leads to:  $\sum_{i=1}^N L_i \alpha_i = L$ . Thus our previous demonstration  
 17 about inflow perimeter can also apply here. Knowing that  $\sum_{i=1}^N \alpha_i = 1$ , such a relationship results  
 18 in:  $\forall i \in \{1, \dots, N\}, \alpha_i = n_i/n \cdot L/L_i$ .

19 Therefore it appears that, to be consistent with the reservoir inner dynamics (characterized  
 20 by the average trip length), the definition of the exit production imposes the exit flows to be all inter-  
 21 dependent. Because in our study the supplies  $\mu_i(t)$  represent the connection to the “downstream”  
 22 reservoirs, they may take any values, which in general have few chances to comply with these  
 23 inter-dependence relationships. Actually, our analysis shows that there exist only one degree of  
 24 freedom to restrain the outflow of all routes at the reservoir exit. This is similar to what is observed  
 25 for the diverge model of Newell (29) where the FIFO conditions apply to all vehicles whatever  
 26 their destinations.

In a first approach, we may use the total supply  $\mu(t) = \sum_{i=1}^N \mu_i(t)$  to limit the total outflow  
 demand. We call this the “global supply” restriction approach, and it gives the following effective  
 outflows:

$$q_{out}(t) = \min[\mu(t); O(n(t))] \quad (\text{total effective outflow}) \quad (22)$$

$$\forall i \in \{1, \dots, N\}, \quad q_{out,i}(t) = \frac{n_i(t)}{n(t)} \frac{L(t)}{L_i} q_{out}(t) \quad (\text{effective outflow route } i) \quad (23)$$

27 Note that in free-flow, the expressions of  $q_{out,i}(t)$  simplify to  $G_i(n_i, n)$ . During congestion, this  
 28 choice ensures that the reservoir exits the maximum flow possible allowed by all downstream  
 29 reservoirs. However, there is a chance that one or more routes exceed the local exit supply, i.e. there  
 30 may exist a given  $i$  for which  $q_{out,i}(t) > \mu_i(t)$  because no constraint is applied locally. This would  
 31 be critical for the corresponding downstream reservoir (next reservoir in route  $i$ ), as this would

1 mean that an excess of inflow could enter ( $\mu_i(t)$  represents the inflow limitation of this downstream  
2 reservoir).

For this reason, we propose a second approach called the “most constrained” supply method. To avoid that one route might send excess of flow to the next reservoir, we have to ensure that all exit restrictions are respected:  $\forall i \in \{1, \dots, N\}, q_{\text{out},i}(t) \leq \mu_i(t)$ . This is possible if we define all the outflows thanks to the most restricted exit  $k$ :

$$q_{\text{out},k}(t) = \mu_k(t) \quad (\text{most constrained outflow}) \quad (24)$$

where:  $O_k(n_k, n) = \max_{O_i(n_i, n) > \mu_i(t)} O_i(n_i, n)$

$$\forall i \in \{1, \dots, N\}, i \neq k \quad q_{\text{out},i}(t) = \frac{n_i(t)}{n_k(t)} \frac{L_k}{L_i} q_{\text{out},k}(t) \quad (\text{effective outflow route } i) \quad (25)$$

3 Note that in free-flow, no exit is constrained so that  $q_{\text{out},k}(t) = O_k(n_k, n)$ . In congestion, with this  
4 formulation, the system will adapt to the limitation  $\mu_k(t)$  for route  $k$ , so that at equilibrium we have  
5  $G_k(n_k, n) = n_k(t)/L_k V(n(t)) = \mu_k(t)$  (assuming that  $\mu_k(t)$  is constant after a given time). Knowing  
6 that outflow  $k$  is chosen as the maximum of the constrained outflows (equation 24), all effective  
7 outflows  $i$  in equation 25 will be automatically lower than their respective limitations  $\mu_i(t)$ . But in  
8 such a case, each route downstream capacity may be underused.

9 Note that if users travel the same distance, i.e.  $\forall i \in \{1, \dots, N\}, L_i = L(t) = L_0$ , all our con-  
10 clusions still apply with few simplifications when considering different accumulation categories  
11 (users are distinguished by their route or destination).

## 12 Implementation in the trip-based model

13 The management of both inflows and outflows can be easily implemented in the trip-based frame-  
14 work. Inflow restrictions are described following the same principle as in the single reservoir  
15 model, we switch to the accumulation-based for each route in congestion. As for the inter-  
16 dependence between the outflows, this is even more simple here: keeping the global order of  
17 the vehicles by their arrival times ensures that the reservoir inner dynamics are preserved on each  
18 route. In practice, there is a waiting list of users which may have different trip lengths in the reser-  
19 voir, and because there are all traveling at the same speed, they can be simply ordered by their  
20 remaining travel distance.

21 At the reservoir entry, each route  $i$  may restrain its inflow by a supply time  $t_{\text{entry supply},i}^{N_i^{\text{in}}}$  for  
22 the  $N_i^{\text{in}}$ th vehicle willing to enter:

$$\forall i \in \{1, \dots, N\} \quad t_{\text{entry supply},i}^{N_i^{\text{in}}} = t_{\text{entry},i}^{N_i^{\text{in}}-1} + \frac{1}{I_i(n_i, n)} \quad (\text{entry supply time}) \quad (26)$$

23 where  $t_{\text{entry},i}^{N_i^{\text{in}}-1}$  is the entry time of the previous vehicle in route  $i$ , see also figure 3(c).

24 At the reservoir exit, the vehicles are kept in order inside the reservoir until the next exit  
25 time. The two outflow management methods differ on the definition of the exit supply time of  
26 the first vehicle  $N_j^{\text{out}}$  of the waiting list. Here, the  $\{N_i^{\text{out}}\}_{1 \leq i \leq N}$  represent the numbers of the next  
27 vehicles to exit in each route  $i$ . For the global supply approach, the calculation of the total flow  
28 supply  $\mu(t)$  as the sum of all  $\mu_i(t)$  is translated in a mean time headway estimation:

$$t_{\text{exit supply}}^{N_j^{\text{out}}} = t_{\text{exit},k}^{N_k^{\text{out}}-1} + \frac{1}{\sum_{i=1}^N 1 / (t_{\text{exit supply},i}^{N_i^{\text{out}}} - t_{\text{exit},i}^{N_i^{\text{out}}-1})} \quad (\text{global supply time}) \quad (27)$$

1 where  $t_{\text{exit supply}}^{N_j^{\text{out}}}$  is the global supply time for the next vehicle to exit (to a given route  $j$ ),  $t_{\text{exit},k}^{N_k^{\text{out}-1}$  is  
 2 the exit time of the previous vehicle (to another route  $k$ ), and  $\left(t_{\text{exit supply},i}^{N_i^{\text{out}}} - t_{\text{exit},i}^{N_i^{\text{out}-1}\right)$  represents the  
 3 admissible headway for entering the next reservoir in route  $i$ .

4 For the most constrained exit approach, respecting the downstream inflow limitations is  
 5 ensured by defining the exit supply time as the supply time of the actual route of vehicle  $N_j^{\text{out}}$ :

$$t_{\text{exit supply}}^{N_j^{\text{out}}} = t_{\text{exit supply},j}^{N_j^{\text{out}}} \quad (\text{local supply time}) \quad (28)$$

6 where  $t_{\text{exit supply},j}^{N_j^{\text{out}}}$  is the local supply time of route  $j$ , defined by the next reservoir entry limitation.

### 7 **Example for two routes**

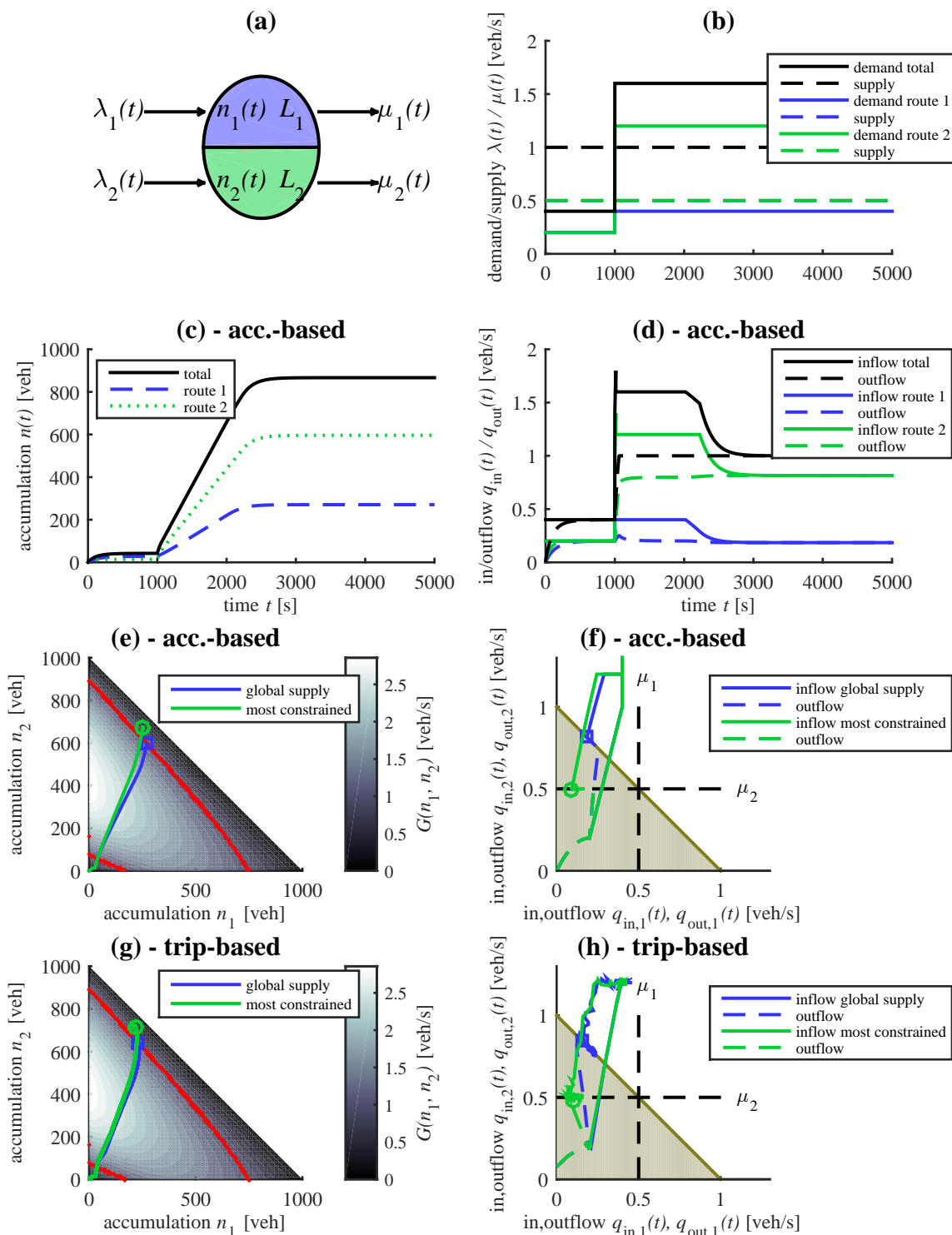
8 We present here the above-mentioned results with a simple case of a sudden demand increase in  
 9 two routes. The network and MFD characteristics are the same as in section 2. The reservoir  
 10 configuration is presented in figure 4(a), route 1 has a length of  $L_1 = 2000$  m, route 2 of  $L_2 =$   
 11  $1000$  m. Figure 4(b) presents a scenario of a demand gap on route 2 while the demand for route  
 12 1 is constant. After 1000 s we have  $\lambda_1(t) = 0.4$  veh/s and  $\lambda_2(t) = 1.2$  veh/s, whereas the exit  
 13 supplies are  $\mu_1(t) = \mu_2(t) = 0.5$  veh/s. Figures 4(c) and (d) show the evolution of accumulations  
 14 and inflows/outflows respectively for the global supply approach in the accumulation-based model.  
 15 In figure 4(d), we notice that the total inflow or outflow corresponds to  $\mu_1(t) + \mu_2(t) = 1$  veh/s  
 16 after 3000 s, however it appears clearly that the flow on route 2 exceeds its limitation ( $q_{\text{out},2}(t) =$   
 17  $0.8$  veh/s  $> \mu_2(t)$  at equilibrium). Consequently, the next reservoir in route 2 may encounter  
 18 gridlock if such a situation lasts a significant period. As explained previously, this approach is  
 19 unacceptable in a multi-reservoir context, which explains why the second method is the most  
 20 relevant.

21 In the second approach, outflows are defined by the most constrained exit. In practice,  
 22 while this is easy to implement in the trip-based framework as detailed previously, it is much  
 23 harder to determine this exit in the accumulation-based model, because we need to know the out-  
 24 flow share in steady state. To this end, one can solve the system equation 10 under congested  
 25 conditions with the first approach. Using equations 20 and 23 leads to:  $\forall i \in \{1, \dots, N\}, q_{\text{in},i}(t) =$   
 26  $n_i(t)/L_i V(n(t)) = q_{\text{out},i}(t) = \frac{n_i(t)}{n(t)} \frac{L(t)}{L_i} \mu(t)$ , thus  $n(t)/L(t)V(n(t)) = \mu(t)$ , but this is insufficient to  
 27 get the final solution. Actually, solving analytically this system seems intractable as the equilib-  
 28 rium state in congestion also depends on the demands  $\lambda_i(t)$ . Consequently, an iterative search on  
 29 all exits is implemented in the accumulation-based model to determine the most limiting outflow  
 30 at each time. Its principle is the following. (i) For all exits where demand flow exceeds supply,  
 31 the exit with the highest supply is assumed to be the most restrictive one. (ii) We calculate other  
 32 outflows through equation 25 and check that they are all below their respective supplies. (iii) If  
 33 one exit does not respect its local constraint, then it is assumed to be the most restrictive one. We  
 34 go back to (ii) and loop until all constraints are verified.

35 The differences between the two outflow management approaches are presented in fig-  
 36 ures 4(e) and (f) for the accumulation-based model, and 4(g) and (h) for the trip-based model.  
 37 Like the case of one trip length in section 2, the accumulation- and trip-based models give similar  
 38 results in congestion for both approaches. Figures 4(e) and (g) show the evolution of  $(n_1(t), n_2(t))$   
 39 in the accumulation plane, where the total trip completion rate function  $G(n_1, n_2)$  is also plot-  
 40 ted. The red line corresponds to all the equilibrium points verifying  $G(n_1, n_2) = \mu(t) = 1$  veh/s.



1 Although the evolution of accumulations is quite similar in both outflow managements, the flow  
2 equilibrium is completely different. Figures 4(f) and (h) show the evolution of  $(q_{in,1}(t), q_{in,2}(t))$   
3 and  $(q_{out,1}(t), q_{out,2}(t))$  in the flow plane. Each route exit limitations  $\mu_1, \mu_2$  are indicated, and the  
4 brown area shows where the total flow is lower than  $\mu_1 + \mu_2$ . One clearly notices that the only point  
5 allowing a maximum total outflow and complying with each local constraint is  $\mu_1 = \mu_2 = 0.5$  veh/s.  
6 This would be possible if  $L_1 = L_2$  and  $\lambda_1 = \lambda_2$ . But here the second route sends the higher outflow,  
7 this is the critical exit in this scenario. With the first approach, its outflow exceeds the downstream  
8 limitation, leading the next reservoir to gridlock. Whereas with the second approach, its outflow is  
9 set to the right limitation, reducing automatically the outflow of route 1 (the longest) in the same  
10 time.



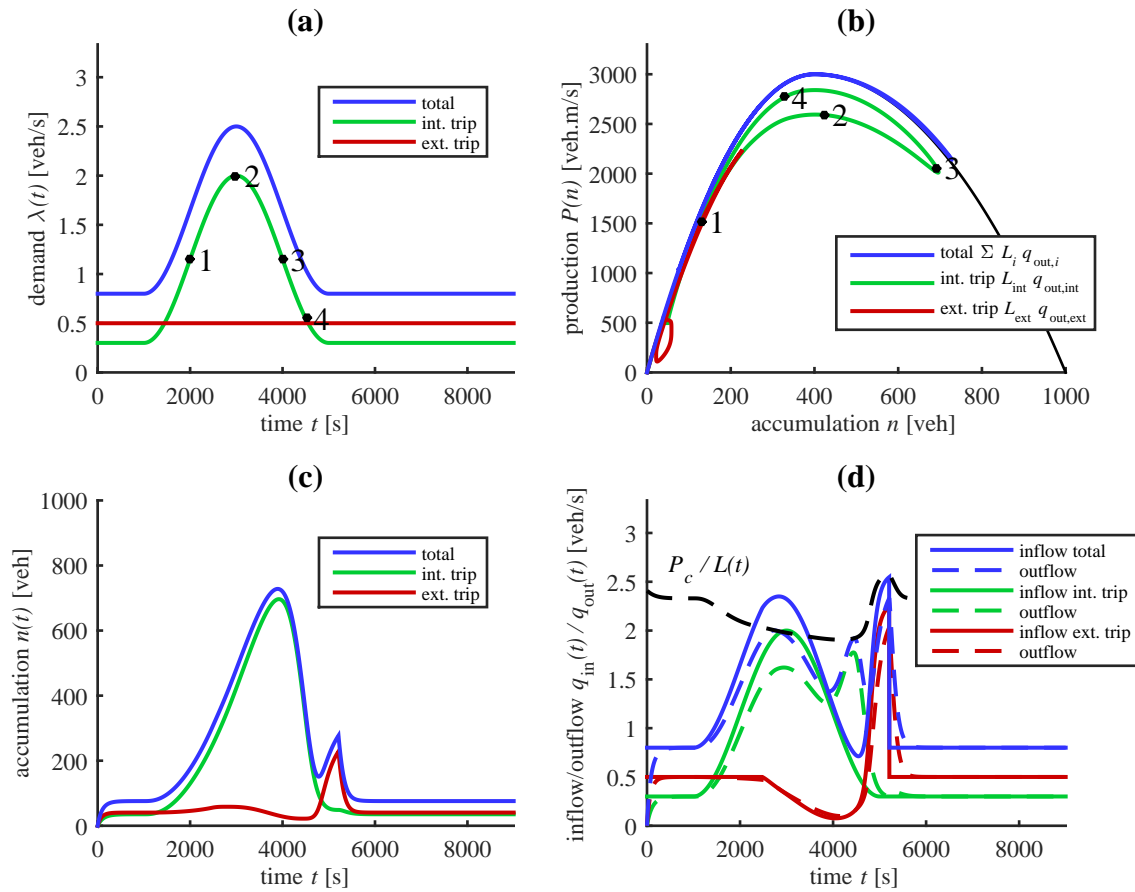
**FIGURE 4** (a) Two routes in a reservoir, (b) demand gap with demand and supply profiles, (c) accumulation and (d) inflow/outflow for each route. (e) Accumulation-based model with the global supply and most constrained supply approaches, evolution of  $(n_1(t), n_2(t))$  in the accumulation plane, (f) evolution of  $(q_{in,1}(t), q_{in,2}(t))$  and  $(q_{out,1}(t), q_{out,2}(t))$  in the flow plane. (g) Trip-based model with the global supply and most constrained supply approaches, evolution of accumulation and (h) inflow/outflow

## 1 DISCUSSION AND CONCLUSION

2 This study proposed a general framework to handle spillbacks in multi-reservoir systems for the  
3 accumulation and trip-based models. We focused on a single reservoir crossed by multiple macro-  
4 routes (or trip categories) because it corresponds to the building block of any multi-reservoir sim-  
5 ulator. A major contribution of this paper is the proper treatment of flow exchanges at a reservoir  
6 perimeter when congestion spillbacks. The framework we developed in such a case (entry flow  
7 functions and allocation scheme for downstream capacities) resorts to the accumulation-based for-  
8 mulation, but we demonstrated that it can be easily implemented in the trip-based formulation too  
9 while smoothly handling the transition with the free-flow case and the regular trip-based function-  
10 ing. Not surprisingly, the results are similar for both modeling approaches when congestion is  
11 propagating.

12 The last ingredient we should include in our framework is internal trips (starting or ending  
13 in the reservoir). Inflows and outflows for such trips should have special treatments as they can  
14 start or end anywhere within the reservoir and should not be restricted by perimeter constraints.  
15 As such, we assume that internal inflow is unrestricted, and that internal outflow (rate at which the  
16 users reach their destination) decreases proportionally to the production-MFD in the accumulation-  
17 based model, or that the vehicles go on driving at the speed-MFD during congestion until they reach  
18 their destination for the trip-based model. The resulting simulation is presented in figure 5(a) for a  
19 simple demand peak scenario with two trips, internal and external. The system evolution is shown  
20 in figures 5(b), (c) and (d). It clearly appears that even with the inflow boundary at the perimeter the  
21 reservoir can reach oversaturated regimes due to the presence of inner trips. Figure 5(d) shows that  
22 the total inflow temporarily exceeds the reservoir capacity  $P_c/L(t)$ . We can also notice a counter-  
23 clockwise hysteresis loop for the exit production of internal trips. Note that inner trips may easily  
24 lead the reservoir to gridlock when the internal inflow is high, see [Mahmassani et al. \(30\)](#) for more  
25 information on urban gridlock. Although this simple test case certainly needs more investigations,  
26 it proves that the integration of internal trips is straightforward in our framework.

27 Further developments of our MFD-based simulator will include a macro-route choice set  
28 generator and a DTA module for several applications like routing strategies, search-for-parking or  
29 perimeter control.



**FIGURE 5 Demand peak for internal trips in a single reservoir. (a) demand profile  $\lambda(t)$  for internal trips and external trips, (b) evolution of the system in the (accumulation  $n_i$ , production  $P_i = L_i q_{out,i}$ ) plane for each trip category  $i$  (“internal” or “external”), (c) evolution of accumulation and (d) inflow/outflow for each trip category**

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4 MAGnUM project).

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