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# Anharmonicity and Disorder in the Black Phases of Cesium Lead Iodide used for Stable Inorganic Perovskite Solar Cells

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## Abstract

Hybrid organic-inorganic perovskites emerged as a new generation of absorber materials for high-efficiency low-cost solar cells in 2009. Very recently, fully inorganic perovskite quantum dots also led to promising efficiencies, making them a potentially stable and efficient alternative to their hybrid cousins. Currently, the record efficiency is obtained with  $CsPbI_3$  whose crystallographical characterization is still limited. Here we show through high resolution in-situ synchrotron XRD measurements that  $CsPbI_3$  can be undercooled below its transition temperature and temporarily maintained in its perovskite structure down to room temperature, stabilizing a metastable perovskite polytype (black  $\gamma$ -phase) crucial for photovoltaic applications. Our analysis of the structural phase transitions reveals a highly anisotropic evolution of the individual lattice parameters *versus* temperature. Structural, vibrational and electronic properties of all the experimentally observed black phases are further inspected based on several theoretical approaches. While the black  $\gamma$ -phase is shown to behave harmonically around equilibrium, for the tetragonal phase density functional theory reveals the same anharmonic behavior, with a Brillouin zone-centered double-well instability, as for the cubic phase. Using total energy and vibrational entropy calculations, we highlight the competition between all the low-temperature phases of  $CsPbI_3$  ( $\gamma$ ,  $\delta$ ,  $\beta$ ) and show that avoiding the order-disorder entropy term arising from double-well instabilities is key in order to prevent the formation of the yellow perovskitoid phase. A symmetry-based tight-binding model, validated by self-consistent GW calculations including spin-orbit coupling, affords further insight into their electronic properties, with evidence of Rashba effect for both cubic and tetragonal phases when using the symmetry breaking structures obtained through frozen phonon calculations.

## Keywords

inorganic perovskite solar cells, anharmonicity, cesium, phonons, DFT, SXRD, Rashba

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3 The interest of the photovoltaics community for hybrid perovskite solar cells (PSCs) has  
4 been growing rapidly since the first demonstration in 2009,<sup>1</sup> mainly because PSCs combine  
5 ease and low-cost fabrication of organic electronics with efficiencies which compete with  
6 those of traditional sectors, considering the highest certified efficiency of 22.7%.<sup>2</sup> In the  
7 race to commercialization, the methylammonium-based perovskite  $CH_3NH_3PbI_3$  (MAPbI<sub>3</sub>  
8 or "MAPI") will probably be outpaced by more stable perovskite structures because of  
9 its poor stability.<sup>3</sup> Several strategies are currently being explored, such as mixed cation  
10 recipes,<sup>4</sup> layered-perovskites,<sup>5</sup> and nano-structures such as quantum dots.<sup>6</sup> In fact, after  
11 Eperon *et al.* found in 2015 a new experimental method in order to maintain  $CsPbI_3$  stable  
12 in its black phase at room temperature and realized the first working cesium lead iodide  
13 solar cell,<sup>7</sup> a cell exceeding 10% efficiency was reported in 2016.<sup>8</sup> It was also shown<sup>9</sup> that  
14 adding a small quantity of  $EDAPbI_4$  (EDA = ethylenediamine) prevents the formation of the  
15 nonperovskite yellow phase of  $CsPbI_3$  and leads to a reproducible efficiency of 11.8%. The  
16 recent report of a 13.4% efficient cesium lead iodide perovskite quantum dot solar cell<sup>6</sup> has  
17 demonstrated that purely inorganic perovskite solar cells have definitively become a stable  
18 and efficient alternative to their hybrid cousins. Besides, a four-terminal tandem cell using  
19 the formamidinium organic cation along with a small fraction of cesium as well as halogen  
20 alloying, in order to be able to tune the band gap, has reached more than 25% efficiency.<sup>10</sup>

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It is currently well-known that the crystal structure has a direct impact on device per-  
formance and it is thus of crucial importance to have high-quality crystallographic data so  
as to obtain valuable references for structural characterization of thin films or quantum dots  
used in those devices. So far, the structure of  $CsPbI_3$  has mainly been studied through  
its similarities with the lead-free perovskite  $CsSnI_3$ . In fact, the structure of the black or-  
thorhombic  $\gamma$ -phase of  $CsSnI_3$  was experimentally measured in 1991,<sup>11</sup> and twenty years  
later all its crystallographic phases have reached a comprehensive understanding (experi-  
mental and theoretical).<sup>1,12-16</sup> For  $CsMI_3$  (M=Pb, Sn), four phases are expected:<sup>12,17</sup> cubic  
( $\alpha$ ), tetragonal ( $\beta$ ), and two orthorhombic phases (a black  $\gamma$  and a non-perovskite yellow

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3  $\delta$ -phase), thus including transitions between perovskite phases and non-perovskite polytypes  
4 (perovskitoids)<sup>18</sup> at low temperature. All the phases of  $CsSnI_3$  have been thoroughly char-  
5 acterized,<sup>12,19</sup> as well as the room temperature  $\delta$ -phase of  $CsPbI_3$ .<sup>17,19,20</sup> More recently, some  
6 of us investigated the temperature dependence of  $CsPbI_3$ 's band gap and how reversible the  
7 non-perovskite to perovskite structural transformations at high temperature<sup>21</sup> can be. It  
8 was shown that, after heating the samples above 360 °C, the room temperature  $\delta$ -phase  
9 (yellow) converts to the black perovskite  $\alpha$ -phase. At variance, during the cooling step, the  
10 perovskite structure converts at 260 °C to the  $\beta$ -phase and at 175 °C to the  $\gamma$ -phase; both  
11 these phases are black. Only after a few days the yellow  $\delta$ -phase is obtained again. This  
12 work was the first report on the black orthorhombic  $\gamma$ -phase that is crucial in the context of  
13 photovoltaics applications.

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25 In addition, vibrational properties of cesium halide perovskites have proven to be a key  
26 factor in determining the stability of phases with temperature. In  $CsPbCl_3$  the role of  
27 phonons in phase transitions was pointed out already in the seventies.<sup>22</sup> Recently, first prin-  
28 ciples calculations on  $CsSnI_3$  revealed soft phonon modes and strong anharmonicity.<sup>23,24</sup>  
29 Conversely, in the case of  $CsPbI_3$  (and  $RbPbI_3$ ), the role of vibrational properties in de-  
30 termining the phase stability is still under-studied, although large values of Born effective  
31 charges hinted towards possible structural instabilities.<sup>25</sup> Very recently, some of us evidenced  
32 unexpected anharmonic features in the form of Brillouin zone-centered double-well instabil-  
33 ity for both the cubic  $\alpha$ -phase and the yellow orthorhombic  $\delta$ -phase of  $CsPbI_3$ ,<sup>26</sup> but due  
34 to lack of experimental data on the other phases, our understanding remained incomplete.  
35 As for hybrid perovskites, several groups<sup>27,28</sup> revealed small lifetimes for the phonons, a  
36 consequence of double-well potential energy profiles at M and R points in the BZ of cubic  
37  $CH_3NH_3PbI_3$ . These structural effects in hybrid organic-inorganic perovskites, complicated  
38 by the rotational motion of the cation, have an influence on the optoelectronic properties.  
39 For instance, octahedral tilting has been shown to have a direct impact on the continuum  
40 band gap.<sup>29,30</sup> This was further investigated by Yang *et al.*<sup>31</sup> who considered the influence of  
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3 temperature on the electronic band structure as well as on the optical absorption threshold  
4 of cubic  $MAPbI_3$ . However, from this and other works<sup>27,32</sup> it seems that the influence of  
5 soft phonon modes on the absorption threshold in  $MAPbI_3$  can be neglected.  
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9 In this paper, we report the detailed experimental structures of the crucial black  $\gamma$ -phase  
10 of  $CsPbI_3$  at 325K, along with the structures of the  $\alpha$ - (645K) and  $\beta$ - (510K) phases, using  
11 synchrotron X-ray powder diffraction (SXRD). The temperature-distortion of these three  
12 phases is studied. Using DFPT (Density Functional Perturbation Theory) methods similar  
13 to our previous work,<sup>26</sup> we investigate the vibrational properties and the energy landscape  
14 of both the low-temperature orthorhombic  $\gamma$ -phase and the tetragonal  $\beta$ -phase of  $CsPbI_3$ .  
15 For the tetragonal phase, the instabilities at the  $\Gamma$  point are further studied performing  
16 total energy calculations on a path following the soft phonon eigenvectors (frozen phonons  
17 method). The energy competition between the low-temperature phases of  $CsPbI_3$  is studied  
18 using total energy and vibrational entropy calculations. Finally, we used symmetry-based  
19 tight-binding modeling and the self-consistent many-body (scGW) approximation to derive  
20 the electronic band structure, using our experimental data on the different phases of  $CsPbI_3$   
21 and a previously developed tight binding model for hybrid perovskite  $MAPbI_3$ .<sup>33</sup> We explore  
22 the Rashba effect for both cubic and tetragonal phases using the new symmetry-breaking  
23 structures revealed by our frozen phonon calculations.  
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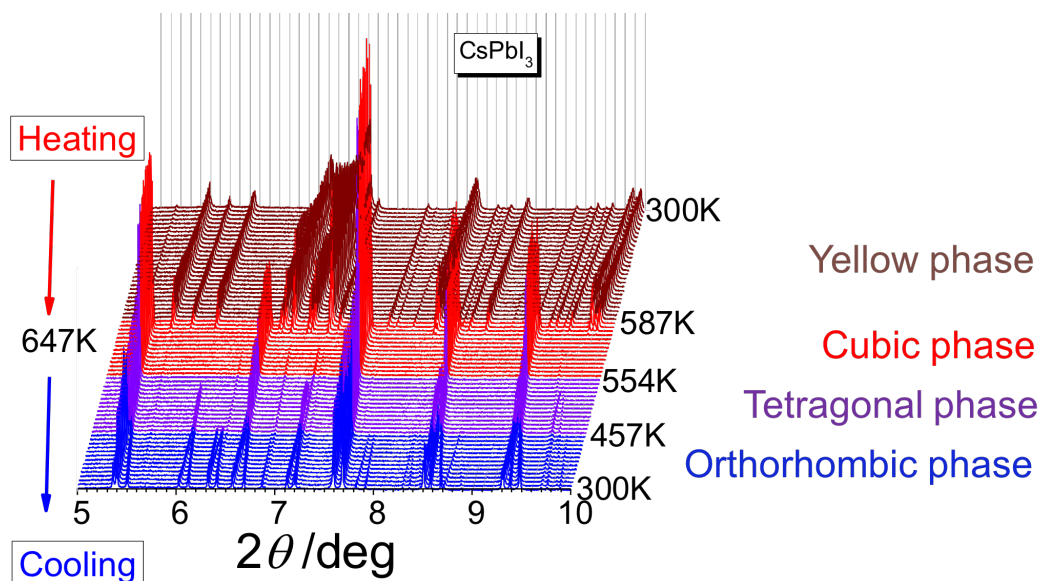
## 41 Results and Discussion

### 42 Structural distortion of the black phases of $CsPbI_3$

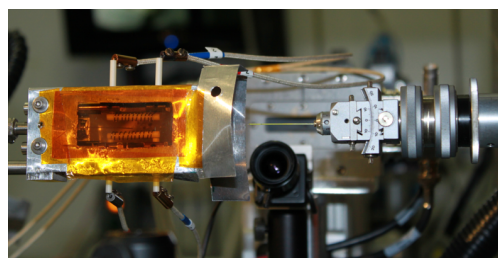
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45  $CsPbI_3$  is one of the earliest known ternary plumbahalides, originally discovered by Møller,<sup>20</sup>  
46 having the distinctive characteristic that it adopts a different, "perovskitoid",<sup>18</sup> structure  
47 compared to its well-known congeners  $CsPbBr_3$  and  $CsPbCl_3$  that adopt a regular per-  
48 ovskite structure.<sup>34</sup> Later studies have in fact shown that  $CsPbI_3$  can become a perovskite  
49 at elevated temperatures (around 310°C),<sup>17</sup> but its structural reconfiguration as it cools  
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3 back to room temperature is unknown. We show here that  $CsPbI_3$  can be undercooled be-  
4 low its transition temperature and temporarily retain its perovskite structure down to room  
5 temperature as shown by high resolution synchrotron X-ray diffraction (figure 1a), using  
6 an in-situ experimental setup (figure 1b,1c). The structural analysis confirms the change  
7 of the yellow phase ( $\delta$ -phase) to its black polytype ( $\alpha$ -phase) on heating but upon cooling  
8 the perovskite does not return to its original yellow phase immediately. Instead,  $CsPbI_3$   
9 adopts the metastable perovskite polytype on cooling back to room temperature, revealing a  
10 phase transition pathway reminiscent of  $CsPbBr_3$ ,<sup>35,36</sup>  $CsSnBr_3$ <sup>37,38</sup> and  $CsSnI_3$ <sup>12,39</sup> where  
11 the high symmetry cubic phase ( $\alpha$ -phase) distorts initially to a tetragonal phase ( $\beta$ -phase)  
12 followed by a further change to the orthorhombic ( $\gamma$ -phase) which persists down to room  
13 temperature (figure 2).  
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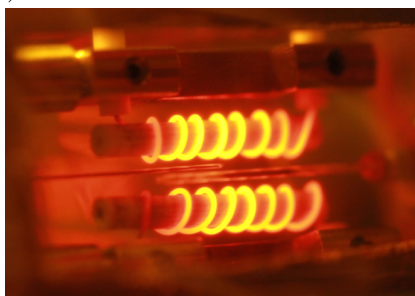
25 The conversion of the  $\delta$ - $CsPbI_3$  to the  $\alpha$ - $CsPbI_3$  involves a dramatic change in the unit  
26 cell volume (figures 3a, 3b) which is compensated by a large increase in the thermal pa-  
27 rameters of the iodide ions, signaling a very large increase in the dynamic motion of the  
28 corner-connected  $[PbI_{6/2}]^-$  octahedra. The dynamic motion, which is corroborated by the  
29 emergence of stereochemical activity at higher temperature, known as "emphanisis",<sup>40,41</sup>  
30 progressively relaxes to less dynamic states ( $\beta$ - $CsPbI_3$ ,  $\gamma$ - $CsPbI_3$ ) as the perovskite cools.  
31 Quite remarkably, whereas the overall thermal expansion coefficient is positive, the evolution  
32 of the individual lattice parameters is highly anisotropic involving a combination of nega-  
33 tive and positive thermal expansion coefficients (figure 3c). Initially the crystallographic  
34 c-axis expands on cooling in the tetragonal phase regime, followed by a large expansion of  
35 the crystallographic b-axis in the orthorhombic phase which is largely compensated by the  
36 enormous decrease in the crystallographic a-axis. The competing thermal expansion leads  
37 to a crossing of the increasing b-axis and the decreasing c-axis which become equivalent  
38 slightly above room temperature. The complex phase transitions in  $CsPbI_3$  have also been  
39 confirmed by Differential Thermal Analysis (DTA) which supports all the phase changes  
40 observed experimentally by XRD.  
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(a)



(b)



(c)

Figure 1: **Temperature dependent synchrotron XRD (SXR)**. (a) SXR spectra for  $CsPbI_3$ . The original yellow phase, converts to the black phase upon heating, but does not return to the original structure upon cooling. (b), (c) The experimental setup used for the *in situ* SXR measurement of  $CsPbI_3$  before and during the experiments. Photos (b) and (c) are courtesy of 11BM of Advanced Photon Source at Argonne National Laboratory.



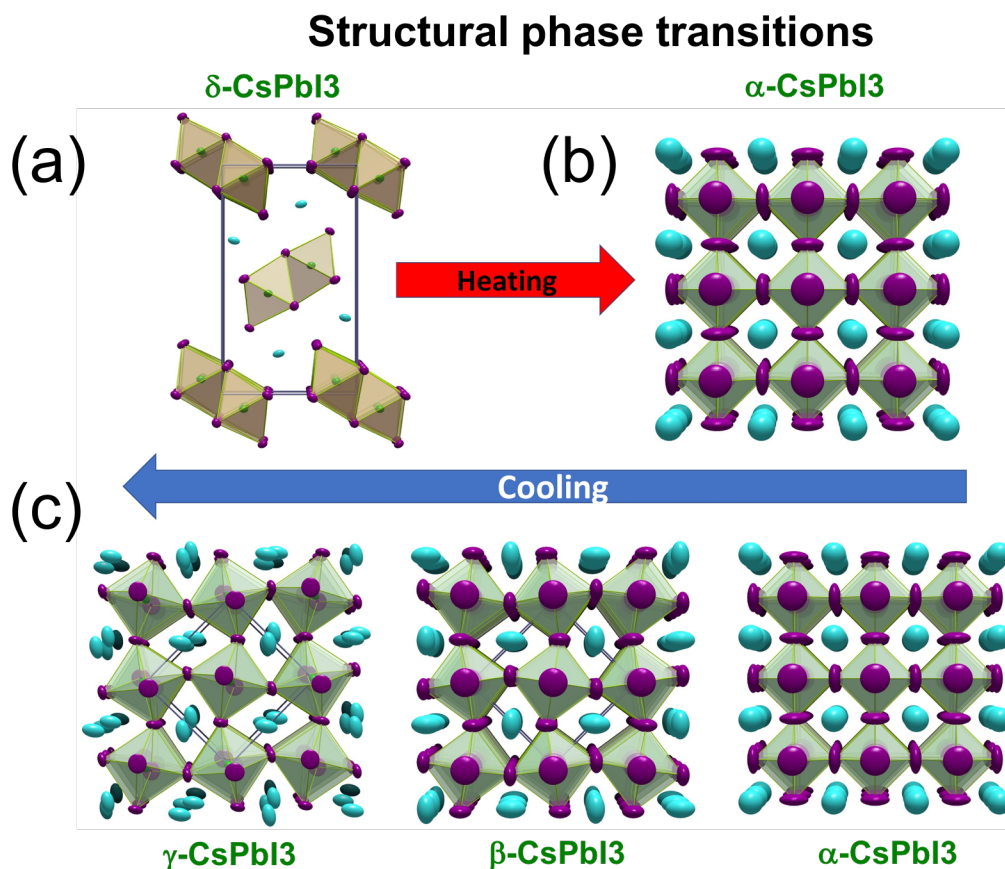


Figure 2: **Structural phase transitions in  $CsPbI_3$  versus temperature.** (a) The initial yellow perovskitoid phase ( $\delta$ - $CsPbI_3$ ,  $NH_4CdCl_3$ -type) converts to (b) the black perovskite phase ( $\alpha$ - $CsPbI_3$ ,  $CaTiO_3$ -type) as the temperature exceeds the transition temperature. (c) Upon cooling, the black perovskite phase is retained and can be undercooled below the transition temperature, where the typical perovskite distortions (tetragonal  $\beta$ - $CsPbI_3$ , orthorhombic  $\gamma$ - $CsPbI_3$ ) can be observed all the way to room temperature. These phases are metastable and transform to the thermodynamic  $\delta$ - $CsPbI_3$  upon standing.

## Vibrational properties and phase competition

In order to get further insight into these black-perovskite phases, we used DFT for structural optimization, which is mandatory to allow inspection of vibrational properties (phonons). We start from the above experimental crystal structures (Figure S4) for the  $\alpha$  (645K),  $\beta$  (510K) and  $\gamma$  (325K) phases having, respectively,  $Pm\bar{3}m$ ,  $P4/mbm$  and  $Pbnm$  space groups. Relaxing the structure from these experimental data sets, we obtain a good agreement on the lattice constants between our XRD experiments (table 1) and our DFT calculations

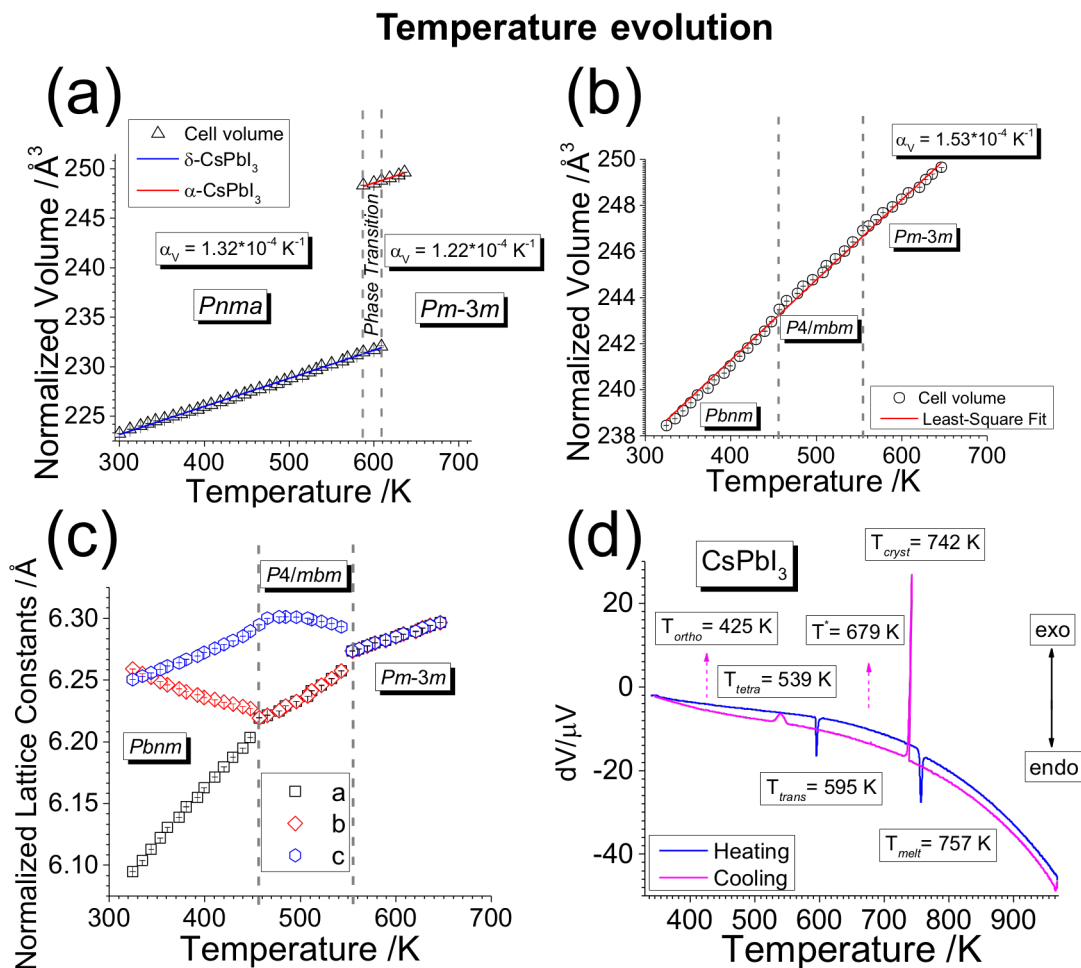


Figure 3: **Temperature evolution of  $CsPbI_3$ .** (a)  $\delta$ - $CsPbI_3$  ( $Pnma$ ) expands linearly on heating up to the transition temperature at around 600K where the phase transition to  $\alpha$ - $CsPbI_3$  ( $Pm\bar{3}m$ ) produces a large increase in the cell volume. (b) On cooling,  $\alpha$ - $CsPbI_3$  undergoes successive phase transitions to  $\beta$ - $CsPbI_3$  ( $P4/mbm$ ) and  $\gamma$ - $CsPbI_3$  ( $Pbnm$ ) without converting back to the original  $\delta$ - $CsPbI_3$  phase. (c) The anisotropic temperature evolution of the  $CsPbI_3$  perovskite revealing competitive negative and positive thermal expansion trends among the individual lattice parameters of the low temperature phases. (d) Differential thermal analysis (DTA) is in good agreement with XRD experiments. Legend:  $T_{melt}$  = melting point,  $T_{cryst}$  = crystallization point,  $T_{trans}$  =  $\delta$ - to  $\alpha$ - $CsPbI_3$  transition,  $T_{tetra}$  =  $\alpha$ - to  $\beta$ - $CsPbI_3$  transition,  $T_{ortho}$  =  $\beta$ - to  $\gamma$ - $CsPbI_3$  transition,  $T^*$  =  $Cs_4PbI_6$  formation.

(table 2), the latter corresponding to a temperature of 0 K. For instance, the normalized volume at 0 K can be extrapolated from figure 3b and we found  $227 \text{ \AA}^3$  which is in good agreement with the DFT value obtained for the  $\delta$ -phase ( $215 \text{ \AA}^3$ ). We also report in tables 1 and 2 the octahedral tilting with angles  $\beta$  (in-plane) and  $\delta$  (out-of-plane) calculated as

in Ref. 42, as increased octahedral tilting can be directly traced back to an increase of the continuum band gap in addition to the direct impact of the expansion/contraction of the unit cell parameters. Note that for the orthorhombic phase in the DFT calculations (and later in the phonon calculations) we use the *Pnam* convention, which is equivalent to *Pbnm* by a permutation of axes.

Table 1: Normalized lattice constants ( $\text{\AA}$ ) and octahedral tilting angles<sup>42</sup> of the 3 black phases of *CsPbI<sub>3</sub>* from SXRD.

	<i>Pbnm</i> exp. (324.6 K)	<i>P4/mbm</i> exp. (511.7 K)	<i>Pm<math>\bar{3}m</math></i> exp. (646.5 K)
a	6.095	6.241	6.297
b	6.259	6.241	6.297
c	6.250	6.299	6.297
angle $\beta$	11°5	8°6	0
angle $\delta$	9°9	0	0

Table 2: Normalized lattice constants ( $\text{\AA}$ ) and octahedral tilting angles<sup>42</sup> of the 3 black phases of *CsPbI<sub>3</sub>* as derived from DFT calculations (at 0 K). We give both the symmetric structures ( $\eta = 0$  in figure 4b) and the new equilibrium structures ( $\eta \neq 0$  in figure 4b) of the double well found for the *P4/mbm* and *Pm $\bar{3}m$*  phases.

	<i>Pnam</i>	<i>P4/mbm</i> ( $\eta = 0$ )	<i>P4/mbm</i> ( $\eta \neq 0$ )	<i>Pm<math>\bar{3}m</math></i> ( $\eta = 0$ )	<i>Pm<math>\bar{3}m</math></i> ( $\eta \neq 0$ )
a	6.335	5.973	5.717	6.149 <sup>26</sup>	6.183 <sup>26</sup>
b	5.606	5.973	6.222	6.149 <sup>26</sup>	6.183 <sup>26</sup>
c	6.112	6.273	6.248	6.149 <sup>26</sup>	6.183 <sup>26</sup>
angle $\beta$	0	16°5	10°3 - 22°8	0	2°9 - 3°3
angle $\delta$	0	0	3°7	0	3°6

The orthorhombic, low temperature,  $\gamma$ -phase of *CsPbI<sub>3</sub>* matches the corresponding phase studied experimentally for similar perovskite *CsSnI<sub>3</sub>* in 1991 by Yamada *et al.*<sup>11</sup> who identified it with the *Pbnm* space group symmetry. This is a totally different phase compared to the *Pnma* orthorhombic structure (yellow  $\delta$ -phase) obtained by Trots<sup>17</sup> and whose phonon

spectrum was already investigated using first-principles calculations.<sup>26</sup> The  $\gamma$ -phase's structure corresponds to an antiferro-distortion of the  $Pm\bar{3}m$  perovskite structure as shown previously.<sup>19</sup> Contrary to our previous findings for the  $\alpha$ - and  $\delta$ -phases,<sup>26</sup> the phonon dispersion of the orthorhombic  $\gamma$ -phase of  $CsPbI_3$ , calculated with DFPT, does not present any imaginary modes at the gamma point (see figure 4d). Furthermore, there are no unstable phonons at the other high symmetry points. This was expected since at low temperature this phase is the most stable perovskite phase.

The phonon dispersions of  $CsPbI_3$  in the tetragonal phase (space group  $P4/mbm$ ), with the structure (denoted as " $\eta = 0$ " in table 2) optimized starting from our experimental data are shown in figure 4a. The imaginary modes that can be found at different high-symmetry points in the Brillouin zone arise from the instabilities of the tetragonal phase in this zero Kelvin DFT calculation. These modes should be stabilized when including temperature effects *via* vibrational entropy.

The presence of soft modes is less expected at the  $\Gamma$ -point (1 doubly degenerated phonon at  $-23 \text{ cm}^{-1}$ , circled in red), where only acoustic modes should be found. Consistently with earlier findings on the  $\alpha$  and  $\delta$  phases,<sup>26</sup> these instabilities correspond to previous neutron scattering experiments showing very low acoustic phonon density of states at  $80^\circ\text{C}$ .<sup>43</sup> Similar results were obtained more recently for  $CH_3NH_3PbI_3$ <sup>28</sup> and  $CH_3NH_3PbBr_3$ .<sup>44</sup> These instabilities, especially the one at  $\Gamma$ , were previously reported and analyzed by some of us<sup>26</sup> for both the  $\delta$ -phase and the  $\alpha$ -phase in inorganic  $CsPbI_3$ . In order to solve this anomaly and to further explore the energy landscape around equilibrium, we use here a similar method of frozen phonons which we do not detail further in this paper.

Using Landau theory<sup>45</sup> and the adiabatic approximation, the anharmonic energy<sup>22,46</sup> as a function of displacement is given by:

$$U(\eta) = U_0 + \frac{\eta^2}{4} \hbar \omega_{\nu\Gamma}(\mathbf{q}_\Gamma) + \frac{\eta^4}{24} \Phi_{4,\nu\Gamma} + O(\eta^6) \quad (1)$$

This model fits well with the results, shown in figure 4. This means that the previous

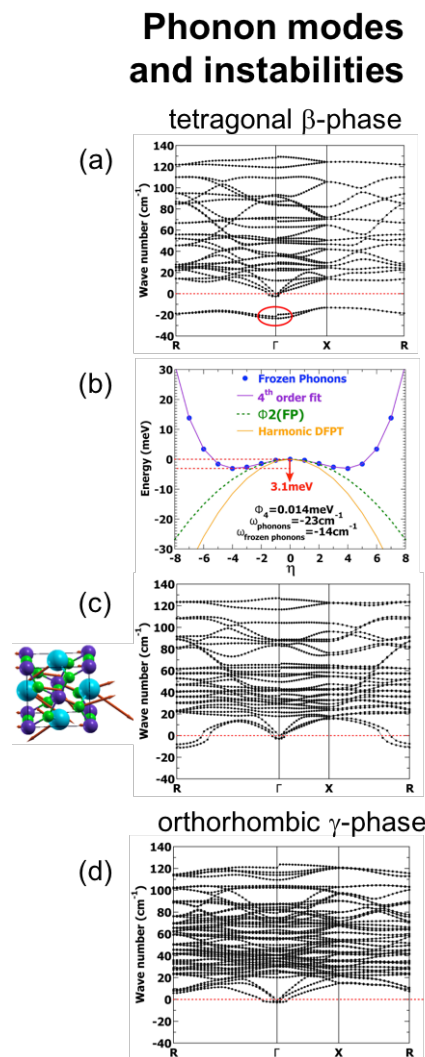


Figure 4: **Phonon modes in  $CsPbI_3$ .** (a) Phonon modes of the tetragonal  $\beta$ -phase of  $CsPbI_3$  in the structure optimized from our experimental data (denoted as " $\eta = 0$ " in table 2). Given that the tetragonal structure relaxes to a slightly orthorhombic one when breaking the symmetry (figure c), we use for this figure the same orthorhombic convention for the high-symmetry points of the Brillouin zone as the one used in (c) and (d). (b) Potential energy surface for tetragonal  $CsPbI_3$  on a path following the most unstable optical phonon's eigenvector at  $\Gamma$  versus displacement parameter  $\eta$ . A  $0.38\text{\AA}$  3N dimensional displacement is required to attain the new minimum.  $\omega_{\text{frozen phonons}}$  and  $\Phi_4$  are obtained by fitting equation 1. (c) Phonon modes of tetragonal  $CsPbI_3$  at the true equilibrium structure (slightly orthorhombic) obtained through geometry optimization of the symmetry-broken structure (denoted as " $\eta \neq 0$ " in table 2). The imaginary modes at  $\Gamma$  were eliminated thanks to tight convergence thresholds and the frozen phonon method. (d) Phonon modes of the orthorhombic  $\gamma$ -phase of  $CsPbI_3$  in the structure optimized from our experimental data (see table 2).

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3 structure was not the proper equilibrium state and that the tetragonal phase of  $CsPbI_3$   
4 behaves anharmonically around equilibrium. The structure will thus oscillate between the  
5 two minima and through this very weak energy barrier (3.1 meV). In order to estimate the  
6 frequency of these oscillations, one can write:  
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$$\tau = \tau_0 e^{\frac{E}{k_B T}} \quad (2)$$

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16 for an energy barrier  $E$  (and the Boltzmann constant  $k_B$ ). Assuming for  $\tau_0$  the lowest  
17 optical phonon frequency ( $\omega \approx 23 \text{ cm}^{-1}$ ), one finds a typical oscillation time of  $0.4 \cdot 10^{-12} \text{ s}$   
18 at room temperature. After relaxation from the minimum structure obtained by fitting the  
19 4th order model, the energy barrier increases to 8.1 meV. This leads to the new equilibrium  
20 structure denoted as " $\eta \neq 0$ ".  
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26 After relaxing from the new symmetry breaking equilibrium positions ( $\eta \neq 0$ ), we could  
27 totally eliminate the soft modes at  $\Gamma$  (see figure 4c). To conclude, for the tetragonal phase  
28 of  $CsPbI_3$  the imaginary modes at  $\Gamma$  were eliminated thanks to both tight convergence  
29 thresholds and the frozen phonon method. As a result, this anharmonic phonon at  $\Gamma$  will  
30 not condense at low temperatures and will not yield one of the orthorhombic phases whereas  
31 the remaining instabilities at the R point correspond to phonon modes which condense at  
32 lower temperatures to transition to the orthorhombic black  $\gamma$ -phase.<sup>21,43</sup>  
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40 To summarize our comprehensive vibrational study, we have shown here and in Ref. 26  
41 that  $CsPbI_3$  presents this anharmonic feature in three out of four phases, at variance with  
42  $MAPbI_3$  for which instabilities at the center of the BZ were not found.  
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46 Next, we briefly investigate the phase competition and vibrational entropy of the low  
47 temperature phases. Table 3 summarizes the total energies of the two different orthorhombic  
48 phases of  $CsPbI_3$  along with that computed for the tetragonal  $\beta$ -phase. For each phase,  
49 we report here the lowest energy obtained by DFT optimization, which means using the  
50 symmetry breaking structures for the  $\delta$ - and  $\beta$ -phases ( $\eta \neq 0$ ).  
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Table 3: Total energies (U) and free energies (F) of  $CsPbI_3$  phases, given per formula unit of 5 atoms.

Phase	Experimental Temperature (K)	U (meV)	F at 400 K (meV)
$\delta$	325	34 <sup>26</sup>	88.5
$\gamma$	450	0	0
$\beta$	510	54	

We find that the  $\gamma$ -phase has a lower total energy than the  $\delta$ -phase, contrary to what one would expect given their respective temperature of stability. We carefully checked that this conclusion is not changed when using PBE or even HSE exchange-correlation instead of LDA, or when spin orbit coupling is taken into account (see Supplementary Information). Given the structural instabilities of these structures, especially for the  $\delta$ -phase,<sup>26</sup> one can anticipate relatively high values for the vibrational entropies, potentially reversing this order for the free energy  $F = U - TS$ . We calculated the free energy F by including the  $-TS_{vib}$  term (quasi-harmonic approximation<sup>47</sup>). This method, using the phonon density of states, cannot be straightforwardly applied<sup>23</sup> when unstable phonon modes are present, and can thus be used only for low-temperature orthorhombic phases, once their unstable modes at  $\Gamma$  have been removed (see computational methods for technical details).

The results show that taking into account the vibrational entropy does not change the conclusion: the  $\gamma$ -phase is calculated to have a lower energy than the  $\delta$ -phase (table 3). This might be due to the fact the  $\delta$ -phase could actually be further stabilized through an additional order-disorder  $\Delta S$  stochastic entropy term<sup>48</sup> associated to the structural instabilities (and related to the fourth-order term in equation 1 that we reported for this phase). This term naturally does not apply to the more stable  $\gamma$ -phase.

We think that avoiding the disorder of the non-perovskite phase is key in order to keep  $CsPbI_3$  in its stable black phases. In a very recent paper,<sup>9</sup> Zhang *et al.* noticed that the stabilization of  $\alpha$ - $CsPbI_3$  is accompanied by a significant reduction in grain size. In light of our findings on the ferroelectric disorder of the  $\delta$ -phase,<sup>26</sup> the reduced grain size could

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3 prevent the perovskite from forming very small ferroelectric domains and allow it to remain  
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5 in its ordered black phases.  
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## 8 **Electronic properties of the black perovskite phases**

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10 We further use the aforementioned XRD and DFT-optimized structures for the different  
11 perovskite phases of  $CsPbI_3$  to investigate their electronic properties. To this end we employ  
12 both semi-empirical and state-of-the-art many-body DFT techniques (see computational  
13 section for details). To model non-cubic phases of  $CsPbI_3$ , we customized the  $sp^3$  tight-  
14 binding (TB) model recently developed for the cubic phase of  $MAPbI_3$ ,<sup>33</sup> considering a  
15 simple  $d^{-2}$  Harrison law<sup>49</sup> to handle bond length variations, along with the matrix elements  
16 as given by Slater and Koster.<sup>50</sup> Given the very limited experimental data available for the  
17 black phases of  $CsPbI_3$ , we used the TB parameters reported previously and optimized for  
18  $MAPbI_3$ .<sup>33</sup>  
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29 The electronic band structures computed with the TB model using the experimental  
30 crystallographic data recorded for the  $\alpha$  (645K),  $\beta$  (510K) and  $\gamma$  (325K) phases of  $CsPbI_3$   
31 are shown in figure 5a, 5b and 5c, respectively. All electronic band gaps are direct and  
32 corresponding values are summarized in table 4. For the cubic  $\alpha$ -phase, the computed band  
33 gap (1.6 eV) agrees nicely with the experimental one (1.7 eV).<sup>7,51</sup> As expected, because of  
34 band folding<sup>52</sup> the band gap of  $CsPbI_3$  shifts from the R point of the Brillouin zone in the  
35 cubic  $\alpha$ -phase to Z and  $\Gamma$  for the  $\beta$  and  $\gamma$ -phases, respectively. The electronic band gap  
36 undergoes a progressive increase when going from the cubic phase to the more distorted  $\beta$ -  
37 and  $\gamma$ - phases. This increase is a direct consequence of the lead-iodide octahedra rotations  
38 that stabilizes the top of the valence band (VBM) and destabilizes the bottom of the con-  
39 duction band (CBM) due to, respectively, anti-bonding and bonding character of the orbital  
40 overlaps.<sup>52</sup>  
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53 Given the limited experimental data, we also performed calculations at a relevant level of  
54 theory required to assess bandgaps, namely using the quasiparticle self-consistent GW theory  
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(scGW) along with spin-orbit coupling (SOC).<sup>53,54</sup> The agreement with the TB model is fairly good (table 4 and figure 5a/b/c), particularly because the parameters of our TB model have not been refined to match any experimental data nor those computed at the scGW+SOC level. The latter confirm the progressive decrease of the bandgap from the  $\gamma$ -phase to the  $\alpha$ -phase. These predictions clearly prompt further detailed experimental investigation of all three perovskite black phases.

Table 4: Comparison of electronic band gaps obtained within the TB model and from scGW calculations including SOC for the experimental determined crystal structures (figure 2c).

Phase	$E_G$ (eV)	$E_G$ (eV)	$E_G$ (eV)
	TB	scGW+SOC	exp.
$\gamma$	2.25	2.53	
$\beta$	1.83	1.89	
$\alpha$	1.61	1.48	1.73 <sup>7,51</sup>

Next, we further use our TB model to investigate the electronic properties of the DFT optimized crystal structures of  $CsPbI_3$  (table 2). Corresponding electronic band structures are shown in figure 5d and 5e and bandgaps are summarized in table 5. Overall, for the high symmetry structures ( $\eta = 0$ ), the computed band structures reveal the very same features as those obtained for the experimental crystal structures (figure 5). The bandgap of the cubic  $Pm\bar{3}m$  structure is significantly reduced as a result of the smaller unit cell parameter (table 5). For the low-symmetry structures ( $\eta \neq 0$ ) band gaps of both the cubic and tetragonal structures undergo a significant increase consistent with octahedral distortions (vide supra). More, the energy minima are shifted away from the high symmetry points of the Brillouin zone (R and Z for the  $\alpha$  and  $\beta$  phases, respectively) due to the Rashba effect that lifts the spin degeneracy.<sup>53</sup>

## Electronic band structures

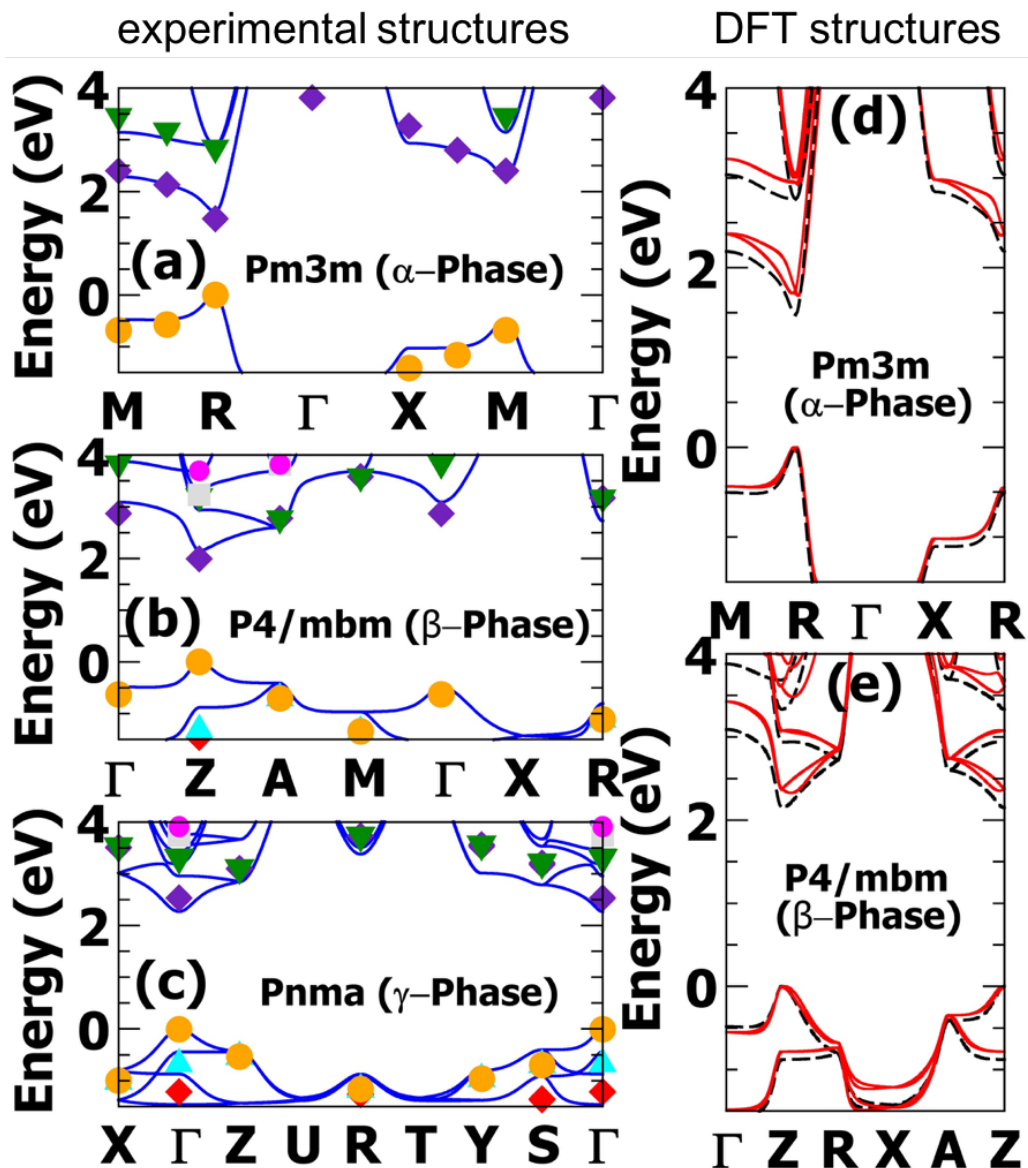


Figure 5: **Electronic band structures.** (a), (b) and (c) Computed electronic band structures using the TB model (full lines) and at the scGW+SOC level of theory (symbols) for the experimentally determined crystal structures (figure 2c). (d) and (e) Electronic band structures calculated from the TB method for the DFT optimized structures (d) cubic  $\alpha$ -phase and (e) tetragonal  $\beta$ -phase. Dashed lines correspond to the symmetric structures ( $\eta = 0$ ) whereas straight lines correspond to the new equilibrium structures ( $\eta \neq 0$ ).

Table 5: Data relevant to the Rashba effect: band gap energies for the DFT optimized structures of the cubic  $\alpha$ - and tetragonal  $\beta$ -phases.  $\eta = 0$  and  $\eta \neq 0$  (figure 4b) correspond to the symmetric and new equilibrium structures, respectively.  $k_{minCB}$  indicates the position in reciprocal space of the conduction band minimum.  $\Delta E_C$  is the energy shift between R-point for the  $\alpha$ -phase (Z point for the  $\beta$ -phase) and the new conduction band minimum.

Phase	$E_G$ (eV) $\eta = 0$	$E_G$ (eV) $\eta \neq 0$	$k_{minCB} (\frac{\pi}{a}, \frac{\pi}{b}, \frac{\pi}{c})$	$\Delta E_C$ (eV)
$\beta$	2.15	2.38	(0.166,0,1)	0.05
$\alpha$	1.47	1.75	0.963(1, 1, 1)	0.06

Thus, these results strongly support the hypothesis of dynamical Rashba effect in  $CsPbI_3$ . They also demonstrate that it is an intrinsic property of the inorganic lattice, even though its strength might be influenced by the nature of the organic cations such as MA or FA.

## Concluding Remarks

Using high resolution synchrotron XRD measurements, we showed that  $CsPbI_3$  can be undercooled below its transition temperature and temporarily retain its perovskite structure down to room temperature, stabilizing a metastable perovskite polytype (black  $\gamma$ -phase). Our detailed study of the temperature distortion revealed complex phase transitions with a highly anisotropic evolution of the individual lattice parameters and even a crossing of the b-axis and c-axis in the  $Pbnm$  phase. Our ab-initio vibrational calculations have shown that out of the three black phases of  $CsPbI_3$ , the only phase that presents a harmonic energy landscape around equilibrium is the orthorhombic  $\gamma$ -phase. Avoiding the anharmonic order-disorder entropy term of the non perovskite  $\delta$ -phase is key in order to keep  $CsPbI_3$  in its black phase at room temperature. As for the new tetragonal  $\beta$ -phase, we evidenced unexpected anharmonic effects in the form of a Brillouin zone-centered double-well. This instability could affect in particular the static dielectric constant through the coupling between phonons and the electric field in these materials.<sup>55</sup> In addition, the perovskite oscillations through the double well could be at the origin of the dynamical Rashba effect and the ferroelectricity foreseen in halide perovskites.<sup>53</sup> In fact, we evidenced this Rashba effect for the cubic and

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3 tetragonal phase and studied the electronic structure of all three black phases of  $CsPbI_3$   
4 combining tight-binding modeling and self-consistent GW calculations in the framework of  
5 many body perturbation theory.  
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## 11 Methods

### 15 Synthesis

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18 PbO (100 mmol, 22.32 g) was dissolved in 100 ml of 57% aqueous HI.  $Cs_2CO_3$  (50 mmol,  
19 16.29 g) was dissolved in 100 ml of 57% aqueous HI. A yellow solid is obtained from the  
20 mixture of CsI and PbO solutions, leaving an essentially colorless supernatant liquid (usu-  
21 ally pale yellow because of the presence of dissolved  $I_2$ ; colorless starting HI should give a  
22 practically colorless supernatant). The addition was done slowly, under constant stirring at  
23 the initial steps to avoid local concentration of CsI. The mixture was left to cool down to  
24 ambient temperature and immediately filtered through a fritted-dish funnel under vacuum.  
25 The solid was washed with 5% aqueous HI (5 parts of water/one part of concentrated HI)  
26 followed by methanol and left for overnight drying. The solid was further dried in an oven  
27 at about 70°C in air. The yield is 72g (around 100%).  
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38 An amount of 36 mg (0.05 mmol) of  $CsPbI_3$  were mixed with 57 mg (0.95 mmol) of fused  
39  $SiO_2$  and transferred into a mortar which were thoroughly ground into a homogeneous pale  
40 yellow powder. The resulting powder was transferred in a 0.5 mm OD fused  $SiO_2$  capillary,  
41 filling it to around 2/3 of its length. Neat fused  $SiO_2$  was added on top to facilitate sealing.  
42 The capillary was transferred into a vacuum line and flame sealed through the neat  $SiO_2$  at  
43  $10^{-4}$  mbar. For measuring, the capillary was placed into a brass pin sample holder base, held  
44 in place by fast drying epoxy, mounted on the goniometer and measured under the following  
45 conditions.  
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## High-resolution temperature-dependent X-ray diffraction

High resolution synchrotron powder diffraction experiments were performed at the beamline 11-BM of the Advanced Photon Source (APS), Argonne Nat. Lab.. An average wavelength of  $\lambda = 0.413906\text{\AA}$  was used.<sup>56</sup>

A home-made resistive heater device was used to control the temperature of the samples over the range 300-650 K (see figure 1b). The temperature was ramped at a rate of  $2.5^\circ\text{K}/\text{min}$  and diffraction patterns were recorded every 4 min covering a temperature window of around  $10^\circ\text{K}/\text{pattern}$ . A thermocouple, protected by a fused  $\text{SiO}_2$  sheath, was used to measure the temperature close to the bottom of the sample capillary, from where the data were collected (see figure 1c). The diffractometer was controlled using the EPICS code.<sup>57</sup>

## Powder pattern refinements

The powdered patterns were refined using the cyclic refinement function of Jana2006.<sup>58</sup> The initial room temperature pattern was refined using a pseudo-Voigt peak shape model with 20 Legendre polynomial terms used to model the background. The atomic coordinates of the known structure of  $\delta\text{-CsPbI}_3$  were used to initiate the Rietveld refinement with all atoms refining anisotropically. The higher temperature patterns were refined in an automated manner with the initial guesses for each pattern being provided by the preceding one. Upon each phase transition, where the fitting quality decreased significantly, a new initial structural refinement was made manually and the cyclic refinement was repeated until the next phase transition. The initial guesses for the space groups and the atomic positions of  $\alpha\text{-CsPbI}_3$ ,  $\beta\text{-CsPbI}_3$  and  $\gamma\text{-CsPbI}_3$ , were found based on screening through the most common perovskite tilting modes.<sup>59</sup> The detailed refinement data is given in the Supplementary Information file.

## Density functional theory for vibrational properties study

Electronic-structure calculations were performed within the Density-Functional Theory (DFT)<sup>60 61</sup> framework, as implemented in the Quantum Espresso code.<sup>62</sup> The vibrational properties were studied using the Local Density Approximation (LDA) and plane waves with a cutoff of 70 Ry. The details of the pseudopotentials that we used here, both the scalar relativistic ones and the fully relativistic used to check the influence of spin-orbit coupling, have been described in Ref. 26. Additional calculations to test the effect of the exchange correlation functionals, using PBE and HSE, are described in the Supporting Information. The following Monkhorst-Pack meshes<sup>63</sup> (centered on  $\Gamma$ ) were used to describe the various phases:  $\delta$ -*CsPbI<sub>3</sub>*:  $10 \times 5 \times 3$ ,  $\alpha$ -*CsPbI<sub>3</sub>*:  $8 \times 8 \times 8$ ,  $\beta$ -*CsPbI<sub>3</sub>*:  $7 \times 5 \times 5$  and  $\gamma$ -*CsPbI<sub>3</sub>*:  $6 \times 6 \times 4$ .

We used Density-Functional Perturbation Theory<sup>47</sup> (DFPT) to compute the phonon spectra, as implemented in the Quantum Espresso code.<sup>62</sup> Born effective charges and high-frequency dielectric tensors were computed with linear response theory and gave us access to the long range contributions to the dynamical matrices. A resolution of 5  $\mathbf{q}$ -points/ $\text{\AA}^{-1}$  was chosen for the phonon spectra. To remove spurious phonon modes, we used strict convergence thresholds of  $10^{-4} \text{ Ry/bohr}$  (forces) and  $10^{-14}$  (phonon self-consistency).

The vibrational entropy was estimated for the  $\delta$  and  $\gamma$ -phases by adding a temperature dependent term to the free energy (quasi-harmonic approximation<sup>47</sup>). The phonon density of states was calculated from the dynamical matrices obtained by DFPT. We neglect thermal expansion, *i.e.*, the phonon frequencies used for the calculation of the vibrational entropy are computed once for all at the zero temperature ground state. The densities of states are shown in figure S6, the corresponding vibrational entropies used for the free energies for the two phases are shown in figure S7. The free energy of  $\gamma$ -phase is always lower than that of the  $\delta$ -phase.

## Many-body perturbation theory for electronic properties (scGW)

DFT calculations, as a starting point for perturbation theory, are performed using the plane-wave projector augmented wave (PAW) implementation available in the VASP software.<sup>64–66</sup> The generalized gradient approximation (GGA-PBE) is used for the exchange-correlation functional.<sup>67,68</sup> The spin-orbit coupling (SOC), mandatory for achieving relevant electronic structure calculations on lead-based perovskites,<sup>52–54</sup> is used as implemented in the VASP code and detailed in Ref. 69. To avoid the underestimation of the electronic band gaps of semi-local DFT with SOC,<sup>52–54</sup> we used quasiparticle self-consistent GW theory<sup>70,71</sup> (scGW) as implemented in the VASP code.<sup>72–74</sup> We used the GGA-PBE relativistic pseudopotentials available in VASP with Cs [ $5s^25p^66s^1$ ], Pb [ $6s^26p^25d^{10}$ ] and I [ $5s^25p^5$ ]. A plane-wave basis set with an energy cut-off of 500 eV is used to expand the electronic wave-functions. The reciprocal space integration is performed over a  $2\times 2\times 2$ ,  $2\times 2\times 2$  and  $4\times 4\times 4$  Monkhorst-Pack grid<sup>75</sup> for the  $\gamma$ ,  $\beta$  and  $\alpha$  phases respectively. To reach energy convergence we imposed a tolerance factor of  $5.10^{-7}$  eV on the residual potential.

## Tight binding model

The  $sp^3$  TB model presented here has been adapted from the one recently developed for the hybrid organic perovskites  $MAPbI_3$  in its cubic phase.<sup>33</sup> It is based on a set of 16 basis functions, when spin orbit coupling (SOC) is disregarded, and 32 basis functions when SOC is included. For a cubic  $AMX_3$  halide perovskite, it consists in one  $s$  and three  $p$  orbitals for the M atom, and the same holds for X atoms. No basis function is needed for the A cation and the model includes nearest neighbor interactions between M and X atoms. The nine parameters to consider (without SOC) are: (i) four matrix elements related to M and X atoms:  $E_{sM}$ ,  $E_{pM}$ ,  $E_{sX}$  and  $E_{pX}$  and (ii) five matrix elements referring to the atomic overlap integrals:  $V_{ss}$ ,  $V_{sMpX}$ ,  $V_{pMsX}$ ,  $V_{pp\sigma}$ ,  $V_{pp\pi}$ . With SOC, the degenerate p states split into  $J = 1/2$  and  $3/2$  bands and this splitting is taken into account both for the metal ( $\Delta_{soM}$ ) and halogens ( $\Delta_{soX}$ ). To model the perovskite black phases of  $CsPbI_3$ , we used

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3 the parameters determined for  $MAPbI_3$ <sup>33</sup> without further refinement. To address the non-  
4 cubic structures having different Metal-Halogen bond lengths, we adapted this TB model  
5 considering the expression for the matrix elements reported in the seminal work by Slater  
6 and Koster<sup>50</sup> and an additionnal  $d^{-2}$  Harrison law,<sup>49</sup> without the need of any additional  
7 parameter (for instance, no adjustable prefactor).  
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## 13 14 15 **Associated Content**

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18 The authors declare no competing financial interest.  
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## 50 51 **Supporting Information Available**

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53 We provide in the supporting information:  
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- Additional information about the temperature-dependent XRD measurements (refined patterns of mixed phases)
- Detailed structures and cif files
- A study of the influence of SOC on the energy landscape
- Results on the phase competition when considering alternative exchange-correlation functionals (PBE, HSE) and SOC
- Details on the vibrational entropies

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## Graphical TOC Entry

