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Abstract

Identifiability is the property that a mathematical model must satisfy to guarantee an unambiguous mapping between its parameters and the output trajectories. It is of prime importance when parameters must be estimated from experimental data representing input-output behavior and clearly when parameter estimation is used for fault detection and identification. Definitions of identifiability and methods for checking this property for linear and nonlinear systems are now well established and, interestingly, some scarce works ([8, 16]) have provided identifiability definitions and numerical tests in a bounded-error context. This paper resumes and better formalizes the two complementary definitions of set-membership identifiability and μ-set-membership identifiability of [16] and presents a method applicable to nonlinear systems for checking them. This method is based on differential algebra and makes use of relations linking the observations, the inputs and the unknown parameters of the system. Using these results, a method for fault detection and identification is proposed. The relations mentioned above are used to estimate the uncertain parameters of the model. By building the parameter estimation scheme on the analysis of identifiability, the solution set is guaranteed to reduce to one connected set, avoiding this way the pessimism of classical set-membership estimation methods. Fault detection and identification are performed at once by checking the estimated values against the parameter nominal ranges. The method is illustrated with an example describing the capacity of a macrophage mannose receptor to endocytose a specific soluble macromolecule.

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1. Introduction

Fault detection and identification via parameter estimation relies on the principle that possible faults in the monitored system can be associated with specific parameters of the mathematical model of the system given in the form of an input-output relation $y(t) = g(u(t), e(t), \theta, x(t))$, where $y(t)$ represents the output vector, $u(t)$ the input vector, and $x(t)$ the state variables which are partially measurable. θ represents the non measurable parameters which are likely to change on the occurrence of a fault, and $e(t)$ the modeling error and/or noise terms affecting the process.

Identifiability is the property that the mathematical model must satisfy to guarantee an unambiguous mapping between its parameters θ and the output trajectories $y(t)$. It is of prime importance when parameters are to be estimated from experimental data representing input-output behavior and clearly when parameter estimation is used for fault detection and identification. If the model is not identifiable, it may as well be that faulty parameters and non faulty parameters are not discriminable through the estimation scheme.

Parameter estimation methods are generally cast in a stochastic framework in which uncertainty is taken into account through appropriate assumptions about noise and model error probability distributions. However, some sources of uncertainty are not well-suited to the stochastic uncertainty assumption and are better modeled as bounded uncertainty. This is typically the case for modeling tolerances on the parameter values, for which the manufacturer provides low and high bounds corresponding to the inherent variability of technological processes. Set-membership (SM) models naturally cope with this type of uncertainties and with model errors and noises assumed to be bounded but otherwise unknown.

SM methods have been the focus of a growing interest in the last decade and they have been applied to many tasks ([3, 18, 20]). The literature on this topic shows interesting progress, for example, [34]. It confirms that SM estimation is an interesting alternative to stochastic estimation methods. Advantageously, SM parameter estimation methods are able to squarely reject models that are not consistent with data and error bounds. SM estimation can be based on interval analysis that was introduced by [30] and several algorithms have been proposed, like in [17, 18, 21, 35]. Other approaches dedicated to linear models include ellipsoid shaped methods [15, 24, 29], parallelotopes and zonotopes [2].

Surprisingly, the interest for SM estimation methods has not been underpinned by investigations about identifiability and only two works can be mentioned. The pioneering paper by [8] outlines that interval based methods and interval constraint propagation can be used to test for a new definition of global identifiability. In contrast to structural global identifiability [32], the new property no longer allows for the existence of atypical regions in the domain of interest. This is actually a byproduct of using interval methods for testing it. But in this work, what is really an interpretation of identifiability in the SM context is only presented as a practical condition. Indeed, instead of imposing parameters corresponding to a given input-output trajectory to be strictly different, they are allowed to be distant by a given ε, which provides a stopping condition to the numerical method. It is only recently – ten years later – that [16] formalized both the above property and test by introducing two complementary definitions for the identifiability of error-bounded uncertain models, namely *set-membership identifiability* (*SM identifiability*) and *μ-set-membership identifiability* (*μ-SM identifiability*). The first one is conceptual whereas an instance of the second, called *ε-SM identifiability*, can be put in correspondence with interval based parameter estimation methods and the specified stopping condition precision threshold ε.

One of the benefits of SM identifiability is that it bypasses standard identifiability and allows one to give (set) estimates of parameters that are unidentifiable in the classical sense (see [40] for a good survey of classical definitions). SM identifiability indeed guarantees that there exists a mapping of the parameter space into connected subsets so that every subset can be associated with a distinguishable output behavior.

In this paper, a more formal characterization than the one in [16] is proposed for μ-SM identifiability, based on the topological concept of contraction mapping [31]. This definition nicely captures the intuition that a parameter set that is associated with distinguishable behavior may be reduced to some extent, hence the contraction, and still retain the same property. If the set can be contracted as small as desired, μ-SM identifiability meets classical identifiability. Otherwise comes the concept of ε-SM identifiability.

Besides, a differential algebra based method for checking SM identifiability and μ-SM identifiability is proposed. This method makes use of relations linking the observations, the inputs and the unknown parameters of the system. Building on this method, fault detection and identification are achieved via parameter estimation. The relations mentioned above are used to estimate the parameters of the model in a set-membership framework, through an analytic solution. The estimated value sets are then checked against the parameter nominal ranges. The identification of the fault(s) comes as a byproduct of this detection test.

The paper is organized as follows. Section 2 provides the formal description of the systems that are analyzed and of the problem. Section 3 presents the two definitions of SM identifiability. The differential algebra based method to analyze SM identifiability is given in Section 4. It is taken as a basis for parameter estimation in Section 5, and the approach is illustrated with an example describing the capacity of a macrophage mannose receptor to endocytose a specific soluble macromolecule. Finally, some conclusions are outlined in Section 6.

2. Problem formulation

The problem considered in this paper is the one of detecting and identifying faults that may occur on a (controlled or uncontrolled) system with bounded uncertain parameters represented by a SM model of the following form:

$$\Gamma_1^P = \begin{cases} \dot{x}(t,p) = f(x(t,p),u(t),p), \\ y(t,p) = h(x(t,p),p), \\ x(t_0,p) = x_0 \in X_0, \\ p \in P \subset \mathcal{U}_{\mathcal{P}}, \;\; t_0 \leq t \leq T, \end{cases} \tag{1}$$

where:

- $x(t,p) \in \mathbb{R}^n$ and $y(t,p) \in \mathbb{R}^m$ denote the state variables and the outputs at time t respectively,
- $u(t) \in \mathbb{R}^r$ is the input vector at time t,
- the initial conditions x_0, if any, are supposed to belong to a bounded set X_0,
- the functions f and h are real and analytic on M (in particular, they are infinitely differentiable), where

M is an open set of $\mathbb{R}^n$ such that $x(t, p) \in M$ for every $t \in [t_0,\, T]$ and $p \in P$. T is a finite or infinite time bound,

- the vector of parameters p belongs to a connected set of parameters P. P is supposed to belong to $\mathcal{U}_{\mathcal{P}}$ where $\mathcal{U}_{\mathcal{P}}$ is an a priori known set of admissible parameters, $\mathcal{U}_{\mathcal{P}}$ is either included in $\mathbb{R}^p$ or equal to $\mathbb{R}^p$.

In this paper, fault detection is approached via parameter estimation and relies on the identifiability assessment of the system as defined in section 3. It uses some relations obtained during the identifiability analysis process presented in section 4.2.

3. Set-membership identifiability

This section proposes a formulation of the SM identifiability problem for the class of systems formalized by (1). It resumes the definitions proposed in [16] and proposes a more formal characterization than the one in [16] for μ-SM identifiability.

3.1. Definitions

Two definitions of global SM identifiability are provided, as well as their local counterpart. The first one is a conceptual definition, whereas the second one, relying on the definition of a contraction mapping μ, can be put in correspondence with operational set-membership estimation methods. In these definitions, $Y(P, u)$ (respectively $Y(P)$) denotes the set of output trajectories, solution of Γ_1^P with the input u (resp. when $u = 0$). The two following definitions apply to controlled systems, but they can be stated similarly for uncontrolled systems.

Definition 3.1. *The nonempty bounded connected set $P^* \subseteq \mathcal{U}_{\mathcal{P}}$ is globally SM identifiable if there exists an input u such that $Y(P^*, u) \neq \emptyset$ and*
$Y(P^*, u) \cap Y(\bar{P}, u) \neq \emptyset,\ \bar{P} \subseteq \mathcal{U}_{\mathcal{P}} \Longrightarrow P^* \cap \bar{P} \neq \emptyset.$

Under the conditions of definition (3.1), we may equivalently say that the model Γ_1^P given by (1) is globally SM identifiable with respect to P^*.

Let us now consider a nonempty bounded connected set Π of $\mathbb{R}^p$ and d a classical metric on $\mathbb{R}^p$ [7], [31]. On the metric space (Π, d), let μ be a continuous map from Π to Π. μ is a contraction if there is a nonnegative number $k < 1$ such that for all $\pi_1,\ \pi_2$ in Π, $d(\mu(\pi_1), \mu(\pi_2)) < kd(\pi_1, \pi_2)$ [31]. Let us also define the diameter of Π by the least upper bound of $\{d(\pi_1,\ \pi_2),\ \pi_1, \pi_2 \in \Pi\}$.

In the following definition, the set P^* is supposed to be a bounded connected set and μ is a contraction from P^* to P^*.

Definition 3.2. *The nonempty bounded connected set $P^* \subseteq \mathcal{U}_{\mathcal{P}}$ is globally μ-SM identifiable if, for all contractions μ from P^* to P^*, $\mu(P^*)$ is globally SM identifiable.*

Under the conditions of definition (3.2), we may equivalently say that the model Γ_1^P given by (1) is globally μ-SM identifiable with respect to P^*.

Definition 3.2 differs from definition 3.1 in the sense that the set P^* may be reduced as small as desired by the contraction μ while still retaining the property of SM identifiability. This is true by Banach fixed-point theorem, which implies that the diameter of $\mu(P^*)$ tends to zero [31]. In this case, μ-SM identifiability meets classical identifiability and, interestingly, it means that classical identifiability holds for any $p \in P^*$. In the opposite case, i.e. if the diameter of $\mu(P^*)$ cannot be lower than ε without eventually loosing SM identifiability, we refer to *ε-SM identifiability*. This definition will be shown to have practical importance in section 3.3.

The definition of μ-SM identifiability hence subsumes classical identifiability, while leading to the concept of ε-SM identifiability, which is an extension to the SM framework.

To account for possible singularities in $\mathcal{U}_{\mathcal{P}}$, μ-SM identifiability can be generically extended into *structural μ-SM identifiability*, which means that the model set Γ_1^P is μ-SM identifiable with respect to $P^* \subset \mathcal{U}_{\mathcal{P}}$ except for a subset of points of zero measure in $\mathcal{U}_{\mathcal{P}}$. Let us notice that defining the structural counterpart of SM identifiability, as given by definition 3.1, is not relevant because in this definition, P^* cannot be of zero measure. The same is true for ε-SM identifiability as explained in [8].

Proposition 3.1. *Global μ-SM identifiability with respect to P^* implies global SM identifiability with respect to P^* but the inverse is not true.*

Proof – The implication is obvious. The fact that the inverse implication is not true is proved with a counterexample. Let us consider the following uncertain model in which ω is an unknown parameter allowed to take an interval value:

$$\left\{ \begin{array}{l} \dot{x} = x + t\cos(\omega), \\ x(0) = x_0 \in X_0. \end{array} \right. \tag{2}$$

Its solution is $x(t) = x_0 e^t + (-1 - t + e^t)\cos(\omega)$. An admissible set for ω is taken as $\mathcal{U}_{\mathcal{P}} = [0, 2\pi]$. If

$\omega \in P^* = \left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$, the model is globally SM identifiable but not μ-SM identifiable. Indeed, no trajectory arising from the systems whose parameter ω is in P^* is identical to a trajectory arising from the complementary set of P^* in $\mathcal{U}_\mathcal{P}$, hence SM identifiability of P^*. However, if the diameter of P^* is smaller, there may exist two disjoint subsets of P^*, namely P_1^* and P_2^*, which result in common trajectories. This is the case, for example, of $P_1^* = [\pi, \dfrac{3\pi}{2}]$ and $P_2^* = [\dfrac{\pi}{2}, \dfrac{3\pi}{4}]$. Consequently, P^* cannot be contracted as small as desired, and is not hence μ-SM identifiable. □

Local definitions of (μ-)SM identifiability can be given by considering an open neighborhood W of P^* in which Γ_1^P is globally (μ-)SM identifiable with respect to P^* with $\mathcal{U}_\mathcal{P}$ restricted to W.

If the model (1) is neither globally (μ-)SM identifiable nor locally (μ-)SM identifiable, it is said *non (μ-)SM identifiable*.

In the case of uncontrolled models, similar definitions can be considered without input.

3.2. μ-SM-identifiability and parameter estimation

Parameter estimation algorithms consist in most of the cases in comparing the observed signal with the system output obtained with candidate (interval) parameter vectors. The observed signal is generally uncertain and can be defined within bounds distant from a given λ. The parameters whose trajectories fall within these bounds are admissible, whereas the others are not. μ-SM identifiability with respect to P^* implies that the trajectories arising from the two sets, namely P^* and its complementary set, are not identical. But this is not enough to guaranty that, if the trajectories arising from P^* fall within the bounds, none of the trajectories of the complementary set does.

Using Gronwall lemma and assuming that the functions f and h defining model (1) are Lipschitz, it has been proved in [39] that $y(t,p)$ is M-Lipschitz with respect to the second variable, i.e. the parameter vector. The constant M has been defined explicitly so that the conditions under which the output trajectories $y(t,p)$ are distant by a given λ can be determined.

This condition together with μ-SM identifiability with respect to P^* not only imply that the trajectories arising from a set P^* and its complementary set are different but also that there exist at least a time point for which they are distant by more than λ.

In other words, one can determine the minimal distance between two parameter vectors for which it is possible to distinguish numerically the trajectories by λ. Moreover, once the parameter vector precision is given, the time point for which the distance between two trajectories, in P^* and its complementary set, is above λ can be determined.

3.3. Correspondence with operational interval based parameter estimation

This section first provides some concepts related to the manipulation of sets, then discusses interval based set inversion, exemplified by the algorithm SIVIA (Set Inversion Via Interval Analysis), as a framework in which the parameter estimation problem can be cast [18]. An interpretation of ε-SM identifiability is then exhibited and shown to be a formalization of the interval based test proposed in [8] in the framework of Interval Constraint Propagation (ICP). ε-SM identifiability is then shown to generalize classical identifiability to sets whose dimension can be controlled.

3.3.1. Interval set inversion

When manipulating sets of values, it is important to be able to check whether one set is included in another set or not. Given two subsets $\mathbb{S}_1$ and $\mathbb{S}_2$ of $\mathbb{R}^n$, one wants to test whether $\mathbb{S}_1$ is included in $\mathbb{S}_2$ or not. This test, known as the *inclusion* test is used to prove that all points in a given set satisfy a given property or to prove that none of them does.

Conversely, if two sets intersect, their intersection inherits the properties of the two sets. It is hence often desirable to reduce a set to its intersection with respect to another set, which is obtained through *contraction*. The *contraction*[1] of $\mathbb{S}_1$ with respect to $\mathbb{S}_2$ is a smaller set s such that $\mathbb{S}_1 \cap \mathbb{S}_2 = s \cap \mathbb{S}_2$. If $\mathbb{S}_2$ is the feasibility set of a problem and s turns out to be empty, then the set $\mathbb{S}_1$ does not contain the solution.

Interval analysis makes use of specific sets, also known as *boxes*. A real interval is a closed and connected subset of $\mathbb{R}$ denoted $[\underline{x}, \overline{x}] = \{x \in \mathbb{R}, \mid \underline{x} \leq x \leq \overline{x}\}$. A box is an interval vector $[\underline{X}, \overline{X}]$, that is a vector with interval components.

In SIVIA, inclusion and contraction are used to test if a box can or cannot be removed from the solution set. When no conclusion can be drawn, the box is bisected and each of the sub-boxes can be tested in turn in a *branch-and-bound* manner. The same principles are used in ICP.

[1] Although the intuition behind *contraction* as used here and *contraction* as used in section 3.1 is similar, the two concepts are not the same. We use the same term because it is used as so by the interval analysis community.

Consider the problem of determining the solution set for the unknown quantity u defined by:

$$\begin{aligned} S &= \{u \in U \mid \Phi(u) \in [y]\}, \\ &= \Phi^{-1}([y]) \cap U, \end{aligned} \tag{3}$$

where $[y]$ is known a priori, U is an a priori search set for u and Φ a nonlinear function not necessarily invertible in the classical sense. (3) involves computing the reciprocal image of Φ. This can be solved using the algorithm *SIVIA*, which recursively explores all the search space without loosing any solution. SIVIA delivers a guaranteed enclosure of the solution set S as follows:

$$\underline{S} \subseteq S \subseteq \overline{S}. \tag{4}$$

The inner enclosure $\underline{S}$ is composed of the boxes that have been proved feasible. To prove that a box $[u]$ is feasible it is sufficient to prove that $\Phi([u]) \subseteq [y]$. Reversely, if it can be proved that $\Phi([u]) \cap [y] = \emptyset$, then the box $[u]$ is unfeasible. Otherwise, no conclusion can be reached and the box $[u]$ is said undetermined. The latter is then bisected in two sub-boxes that are tested until their size reaches a user-specified precision threshold $\varepsilon > 0$. Such a termination criterion ensures that *SIVIA* terminates after a finite number of iterations.

In the above solving schema, the number of bisections to be performed is generally prohibitive. Hence, recent algorithms take advantage of constraint propagation techniques to reduce the width of the boxes to be tested [12], [13], [42]. In this context, the inclusion relations and the model equations can be interpreted as constraints. A Constraint Satisfaction Problem (CSP) can be formulated as below.

A Constraint Satisfaction Problem $H = (\mathcal{X}, \mathcal{D}, \mathcal{C})$ is defined by:

- a set of variables $\mathcal{X} = \{x_1, ..., x_n\}$,
- a set of nonempty domains $\mathcal{D} = \{D_1, ..., D_n\}$ where D_i is the domain associated to the variable x_i,
- a set of constraints $\mathcal{C} = \{C_1, ..., C_m\}$, so that the associated variables to each constraint are defined in $\mathcal{X}$.

Given a CSP and an initial box $\mathbb{S}_1$ (which may result from a bisection operation as explained above), different types of so-called *contractors* can be used [17][9] to contract $\mathbb{S}_1$ into a set s that satisfies the constraints. Among the most well-known is the forward-backward contractor [5] which is based on constraint propagation and consists in contracting the domain of the CSP by taking into account iteratively each constraint of the set $\{C_1, ..., C_m\}$.

3.3.2. Interpretation of ε-SM identifiability

SM identifiability does not provide the means to control the set of interest P^*, i.e. the parameter solution set. This points out the practical interest of μ-SM identifiability which is defined through a contraction mapping $\mu(.)$ that allows one to control the diameter of P^*. In particular, ε-SM identifiability means that the diameter of P^* cannot be lower than ε without eventually loosing SM identifiability. The diameter of P^* can hence be put in correspondance with the user-specified precision threshold of SIVIA. Consequently, ε-SM identifiability provides the means to guarantee that the estimate provided by the set inversion algorithm when the precision threshold is taken equal to ε consists of a connected set.

ε-SM identifiability is actually a formalization of the ICP numerical test proposed by [8] to check global identifiability in a domain. Instead of imposing parameters corresponding to a given input-output trajectory to be strictly different, [8] allows them to be distant by a given ε, which provides a stopping condition to the ICP numerical method.

Ultimately, μ-SM identifiability subsumes classical identifiability and SM identifiability as it provides the means to control the set P^* thanks to the contraction mapping $\mu(.)$. When the diameter of P^* tends to 0, μ-SM identifiability comes back to classical identifiability. Interestingly, μ-SM identifiability with respect to P^* means that classical identifiability holds for any $p \in P^*$, which provides a way to test classical identifiability for a set of parameters as a whole as shown in section 4. When this diameter is necessarily higher or equal to ε, it results in ε-SM identifiability.

4. Analysis of set-membership identifiability

In the literature, different approaches have been proposed for studying global identifiability of nonlinear systems, for example, the revisited Taylor Series approach of [33], those based on the local state isomorphism theorem, [10], [11], [19], [41], or those based on differential algebra, [4], [28], [36], [38]. Most of them can be adapted in order to test global (μ-)SM identifiability. In [39], the authors propose two methods for testing the global (μ-)SM identifiability: the first one is based on the Taylor Series approach, the second one on differential algebra. The latter is used in this paper because, as it will be seen in section 5, it permits to deduce a numerical procedure for SM fault detection. The notion of partial injectivity of [23] is needed and is recalled in the first subsection.

4.1. Partial injectivity

Definition 4.1. *Consider a function $f : \mathcal{A} \rightarrow \mathcal{B}$ and any set $\mathcal{A}_1 \subseteq \mathcal{A}$. The function f is said to be a partial injection of $\mathcal{A}_1$ over $\mathcal{A}$, noted $(\mathcal{A}_1, \mathcal{A})$-injective, if $\forall a_1 \in \mathcal{A}_1$, $\forall a \in \mathcal{A}$,*

$$a_1 \neq a \Rightarrow f(a_1) \neq f(a).$$

f is said to be $\mathcal{A}$-injective if it is $(\mathcal{A}, \mathcal{A})$-injective.

In [23], an algorithm based on interval analysis for testing the injectivity of a given differentiable function is presented and a solver called ITVIA (Injectivity Test Via Interval Analysis) implemented in C++ is mentioned.

4.2. Testing global (μ-)SM identifiability by Differential Algebra

The proposed approach is an extension of [16] and is based on [38]. [14] has proved that, choosing the appropriate elimination order $\{p\} < \{y, u\} < \{x\}$ (which consists in eliminating unobservable state variables), the differential algebra approach [22] allows one to obtain relations between inputs, outputs and parameters. These relations can be expressed as:

$$\begin{aligned} R_i(y,u,p) = \quad & \theta_0^i(y,u) \\ & + \sum_{k=1}^{n_i} \theta_k^i(p) m_k^i(y,u),\ i = 1,\dots,m, \end{aligned} \tag{5}$$

where $(\theta_k^i)_{1\leq k\leq n_i}$ are rational in p, $\theta_u^i \neq \theta_v^i$ $(u \neq v)$, $(m_k^i(y,u))_{1\leq k \leq n_i}$ are differential polynomials with respect to y and u and $\theta_0^i \neq 0$.

The size of the system is the number of observations. For simplicity, we assume that $i = 1$, i.e. there is only one output variable and $n = n_1$, $m_k(y,u) = m_k^1(y,u)$, $\theta_k = \theta_k^1$. The higher order derivative of y in (5) is noted l.

The following theorem provides necessary and sufficient conditions for global SM identifiability or μ-SM identifiability.

Theorem 1. *Assume that the functional determinant $\triangle R(y,u) = det(m_k(y,u), k = 1,\dots,n)$, is not in the ideal $\mathcal{I}_p^0$ obtained after eliminating state variables. Consider P^* a subset of $\mathcal{U}_\mathcal{P}$ for which the function ϕ defined by*

$$\begin{aligned} \phi : p = (p_1,\dots,p_p) \mapsto \quad & (\theta_1(p),\dots,\theta_n(p), y(t_0^+, p), \\ \dots, \quad & y^{(l-1)}(t_0^+, p)) \end{aligned}$$

verifies:

$$\forall p^* \in P^*, \forall \bar{p} \notin P^*,\ \phi(p^*) \neq \phi(\bar{p}). \tag{6}$$

Then the model Γ_1^P is globally SM identifiable with respect to P^*.
If the model Γ_1^P is globally SM identifiable with respect to P^* and ϕ is $(P^*,\mathcal{U}_\mathcal{P})$-injective then the model is μ-SM identifiable with respect to P^*.
In the two cases, if the coefficient of $y^{(l)}$ in (5) is not equal to 0 at t_0, then the reciprocal is valid.

Proof – Sufficiency Consider P^* verifying the hypothesis of the theorem. Suppose there exists an input u^* such that $Y(P^*, u^*) \neq \emptyset$ and $y^* \in Y(P^*, u^*) \cap Y(\bar{P}, u^*)$ for a cartesian product of intervals $\bar{P} \in \mathcal{U}_\mathcal{P}$. Thus, there exists $p^* \in P^*$, $\bar{p} \in \bar{P}$ such that $y^* = y(., p^*) = y(., \bar{p})$ and $R(y^*, u^*, p^*) = R(y^*, u^*, \bar{p})$.
Denote $Q(y^*, u^*) = R(y^*, u^*, p^*) - R(y^*, u^*, \bar{p})$. Since
$det(Q)(y^*, u^*) = det(m_k(y^*, u^*),\ k = 0,\dots,n) = \triangle R(y^*, u^*)$ is not equal to zero, $\theta_k(p^*) = \theta_k(\bar{p})$ for $k = 1,\dots,n$. Furthermore, recall that $y(., p^*) = y(., \bar{p})$ in particular $y^{(j)}(t_0, p^*) = y^{(j)}(t_0, \bar{p})$ for $j = 0,\dots,l-1$. Since the function ϕ is supposed to verify condition (6), one gets $\bar{p} \in P^*$ that is $P^* \cap \bar{P} \neq \emptyset$.
If ϕ is $(P^*,\mathcal{U}_\mathcal{P})$-injective then, ϕ is in particular injective on P^* and we always have $p^* = \bar{p}$, that is $P^* \cap \bar{P} \neq \emptyset$.

Necessity Let's prove the contrapositive. Suppose there exists $\bar{P}$, $P^* \cap \bar{P} = \emptyset$ containing $\bar{p}$ such that $\phi(p^*) = \phi(\bar{p})$ for a certain $p^* \in P^*$. Since the coefficient of $y^{(l)}$ in (5) is not equal to 0 at t_0 and the differential polynomials $(m_k)_{k=1,\dots,n}$ have a degree 1 in $y^{(l)}$ ([14]), any time derivative $y^{(r)}(t_0^+, p^*)$, $r \geq l$ can be rewritten as a function of $y^{(l-1)}(t_0^+, p^*), \dots, y(t_0^+, p^*), \theta_1(p^*), \dots, \theta_n(p^*)$. According to the hypothesis, the coefficients of $y(t, p^*)$ in the Taylor expansion are the same as those of $y(t, \bar{p})$. Thus, $y^* := y(t, p^*) = y(t, \bar{p})$ and $y^* \in Y(P^*, u) \cap Y(\bar{P}, u)$. Consequently, the model is not globally SM identifiable for P^*. $\square$

Example: Consider the uncertain model:

$$\begin{cases} \dot{x}_1 = x_1^2 + (1-p_2)x_2, \\ \dot{x}_2 = \sin(p_1)x_1, \\ x_1(0) = (p_1 + 2(1-p_2)\cos(p_1)), \\ x_2(0) = 0, \\ y = x_1, \end{cases} \tag{7}$$

where $(p_1, p_2) \in P^* = [-1, 4] \times [0, 1/10]$.
We want to know whether the model is globally μ-SM identifiable with respect to P^*. By setting $c_1 = \sin(p_1)$

and with the elimination order $\{c_1, p_2\} < \{y\} < \{x_1, x_2\}$, the package diffalg of Maple gives the following input-output polynomial:

$$R(y, u) = \ddot{y} - 2\dot{y}y - (1 - p_2)c_1 y.$$

In that case, the functional determinant is reduced to $\triangle R(y, u) = y$ and in using the function $belong\_to$ of the package diffalg of Maple [6], we verify that it is not in $\mathcal{I}_p^0$.
In order to consider the initial condition, the function $\phi : (p_1, p_2) \to ((1 - p_2)\sin(p_1), (p_1 + 2(1 - p_2)\cos(p_1)))$ has to be studied. Notice that the coefficient of $\ddot{y}$ in (5) is not equal to 0 at 0. The solver ITVIA ([23]) allows one to obtain the partition of the box P^* on which the function Φ is partially injective and Φ can be proved to be not injective over P^*. Thus the model is not μ-SM identifiable with respect to P^*.

5. Fault detection and identification

5.1. Problem setting

SM fault detection may use state estimation or parameters estimation. When the model is nonlinear like (1), the sets to be characterized (state or parameter values) may be nonconvex and may even consist of several disconnected sets. In the latter case, interval analysis encloses such sets in the convex hull and the usual drawback is to obtain very conservative interval solution vectors, which result in missing alarms. Recent methods such as [1] or the one proposed in this paper should provide significant improvements in this direction.

In this work, the same analysis is used for identifiability checking and for estimating the parameters. Advantageously, after analyzing identifiability, we can guarantee that the solution set for the system (1) reduces to one connected set.

In this section, a numerical method deduced from section 4.2 is proposed to estimate the unknown constant parameters of a non linear system like (1).

The output y is supposed to be disturbed by a bounded additive noise η, $\eta(t) \in [\eta(t)]$ and the parameter vector p belongs to P where P is an interval vector. The polynom (5) can be used to estimate the interval vector P. Consider Θ_k the associated expression of θ_k defined in the polynom (5), where p is substituted by P. $\Theta_k(P)$ is a connected set for all connected P since it involves sum, difference and product of connected sets. Suppose that the observations are done at discrete times t_j, $0 \le j \le M$ and they are noted $y_j = y(t_j)$.

Then, the following system whose interval vector $(\Theta_k(P))_{1\le k\le n}$ is unknown can be deduced:

$$\begin{array}{l} \forall j = 0, \ldots, M, \\ 0 \in m_0(y_j, u_j) + \sum_{k=1}^{n} \Theta_k(P) m_k(y_j, u_j). \end{array} \tag{8}$$

Notice that (8) is linear with respect to $\{\Theta_1(P), \ldots, \Theta_n(P)\}$. Solving the previous system comes back to solving $0 \in [A][x] - [b]$ or $[A][x] = [b]$ where $[A]_j = ([m_k(y_j, u_j)])_{k=1,\ldots,n}$ is the j^{th} line of the interval matrix $[A]$ and $[b]_j = -[m_0(y_j, u_j)]$ is the j^{th} line of the interval vector $[b]$.

Finding a solution for (8) requires an evaluation of a finite number of measurement derivatives. In this work, these derivatives are estimated by using Higher Order Sliding Mode (HOSM) differentiators developped in [25], [27].

5.2. Derivative estimation

In the works [25], [26], [27] concerning HOSM differentiators, the input signal $y(t)$ to be derivated is considered as a function defined on $[0, +\infty[$. The signal $y(t)$ is supposed to consist of a bounded Lebesgue-measurable noise (bounded by a positive constant α) with unknown features and an unknown base signal $y_0(t)$ with the m^{th} derivative having a known Lipschitz constant $C > 0$. The successive derivatives of a signal $y(t)$ are estimated by $z_0(t), z_1(t), ..., z_m(t)$ as described below:

$$\left\{\begin{array}{rl} \dot{z_0} = & v_0, \\ v_0 = & -\lambda_0 \, |z_0 - y|^{\frac{m}{(m+1)}} \operatorname{sign}(z_0 - y) + z_1, \\ \dot{z_1} = & v_1, \\ v_1 = & -\lambda_1 \, |z_1 - v_0|^{\frac{(m-1)}{m}} \operatorname{sign}(z_1 - v_0) + z_2, \\ \vdots & \\ \dot{z_{m-1}} = & v_{m-1}, \\ v_{m-1} = & -\lambda_{m-1} \, |z_{m-1} - v_{m-2}|^{\frac{1}{2}} \operatorname{sign}(z_{m-1} - v_{m-2}) \\ & +z_m, \\ \dot{z_m} = & -\lambda_m \operatorname{sign}(z_m - v_{m-1}), \end{array}\right.$$

where $\lambda_j \in \mathbb{R}$, $j = 0, \ldots, m$ represent the differentiator parameters. Generally, these parameters are chosen experimentally (for more details, see [25], [26]).
It has been proved in [26] that the best estimate accuracy of the k^{th} derivative is proportional to:

$$acc_k = \mu_k C^{\frac{k}{m+1}} \alpha^{\frac{m+1-k}{m+1}}, \;\; k = 0, 1, 2, ...m$$

where $\mu_k \ge 1$ depends only on λ_j ($j = 0, \ldots, m$). In this paper, we compute an interval containing the exact k^{th} derivative of y. We note $y_p^{(k)}$ the estimate of the

k^{th} exact derivative of y, then this computed interval is given by:

$$[y_p^{(k)} - acc_k,\ y_p^{(k)} + acc_k],$$

with $y_p^{(k)} = z_k$ [37].

5.3. Case study

The following example taken from [38] is considered. In [38], the proposed model is cast in a stochastic framework in which uncertainty is taken into account through appropriate assumptions about noise and model error probability distributions.

This model allows one to explore the capacity of the macrophage mannose receptor to endocytose soluble macromolecule and to quantify the different aspects of such a process:

$$\begin{cases} \dot{x}_1 = \alpha_1(x_2 - x_1) - \frac{V_m x_1}{1+x_1}, \\ \dot{x}_2 = \alpha_2(x_1 - x_2), \\ x_1(0) \in [0.62,\ 0.63],\ x_2(0) = 0, \\ y = x_1, \end{cases} \tag{9}$$

where x_1 (resp. x_2) is the enzyme concentration outside (resp. inside) the macrophage and $p = (\alpha_1, V_m, \alpha_2)$ are the unknown parameters which have to be identified. The parameter α_1 is the rate constant of the transfer from Compartment 1 (or the central compartment), practically plasma, to Compartment 2 (or the peripheral compartment), which represents the part of the extravascular extracellular fluid accessible. Furthermore, α_2 is the rate constant of the transfer from Compartment 2 to Compartment 1.

This model can be easily proved to be globally μ-SM identifiable with respect to $(\mathbb{R}^+)^3$. The numerical study has been conducted in simulation in Matlab using Intlab. The simulated outputs are disturbed by a truncated gaussian noise η such that $\eta(t) \in [-0.001,\ 0.001]$. Thus, $y(t) = \bar{y}(t) + \eta(t)$ where $\bar{y}$ is the exact output corresponding to the exact value of parameters: $\alpha_1 = 0.011$, $\alpha_2 = 0.02$ and $V_m = 0.1$. The observations are supposed to be done at discrete times $(t_j)_{j=1,\ldots,N}$ on the interval $[0, 60]$ with a sampling period equal to 1. The polynom $R(y, u)$ is given by:

$$R(y,u) = \ddot{y}(1+y)^2 + \gamma_1\dot{y}(1+y)^2 + \gamma_2 y(1+y) + \gamma_3\dot{y},$$

with $\gamma_1 = \alpha_1 + \alpha_2$, $\gamma_2 = \alpha_2.V_m$ and $\gamma_3 = V_m$.

If we denote $y_p^{(1)}(t_j)$ (resp. $y_p^{(2)}(t_j)$) the estimate of $\dot{y}(t_j)$ (resp. $\ddot{y}(t_j)$), the obtained system which has to be solved is $[A][x] = [b]$ where $[A]_j = ([y_p^{(1)}(t_j)(1 + y(t_j))^2],\ \ [y(t_j)(1 + y(t_j))],\ \ [y_p^{(1)}(t_j)])$ and $[b]_j = [-y_p^{(2)}(t_j)(1 + y(t_j))^2]$.

$y_p^{(1)}(t_j)$ and $y_p^{(2)}(t_j)$ are obtained by using HOSM differentiators presented in subsection 5.2. The parameters of the HOSM differentiators are given by $\lambda_0 = 3$, $\lambda_1 = 0.2$ and $\lambda_2 = 0.1$.

Solving this system can be cast into the set inversion framework for which we used the SIVIA algorithm complented by the forward-backward propagation to contract the initial parameter box. The problem solved here is to find $[x]$ such that $0 \in [A][x] - [b]$, given initial intervals for γ_1, γ_2 and γ_3 obtained from prior knowledge.

Case of nominal behaviour: By using initial intervals given by $\gamma_1 = [0,\ 0.04]$, $\gamma_2 = [0,\ 0.003]$, $\gamma_3 = [0,\ 0.2]$ and the bisection precision $\varepsilon = 0.001$, we obtain in 14.18 seconds : $\alpha_1 = [0,\ 0.0401]$, $\alpha_2 = [0,\ 0.0437]$ and $V_m = [0.06875,\ 0.13203]$,

by using the equations: $V_m = \gamma_3$, $\alpha_2 = \gamma_2/V_m$, $\alpha_1 = \gamma_1 - \alpha_2$. All these intervals contain the normal values, confirming normal behavior.

Then, by using $\gamma_1 = [0,\ 0.04]$, $\gamma_2 = [0,\ 0.003]$, $\gamma_3 = [0,\ 0.2]$ and the bisection precision $\varepsilon = 0.0001$, we obtain in 177.55 seconds : $\alpha_1 = [0,\ 0.0329]$, $\alpha_2 = [0.0071,\ 0.0317]$ and $V_m = [0.094824,\ 0.10527]$.

All these intervals contain the normal values.

Case of a fault on parameter α_2: In this simulation, we assume a fault on $\alpha_2 = 2$, which means that the rate of the transfer from Compartment 2 to Compartment 1 is high.

After 25.15 minutes, by using $\gamma_1 = [0,\ 3]$, $\gamma_2 = [0,\ 1]$, $\gamma_3 = [0,\ 0.2]$, we obtain: $\alpha_1 = [0.0000,\ 0.5050]$, $\alpha_2 = [1.1200,\ 10.4435]$ and $V_m = [0.0242,\ 0.1790]$.

The real faulty value of α_2 is contained in the estimated interval for α_2, which allows us to detect and localize the fault. Moreover, there is no intersection between the estimated interval for α_2 and the one obtained for normal behaviour.

Case of a fault on parameter α_2 at $t = 15$s: Consider now the case of an abrupt change in the value of α_2 during the test and let us assume a fault $\alpha_2 = 1$ at time $t = 15$s. This fault is detected in $t = 0.05$s after its occurence.

After the detection of this fault, by using $\gamma_1 = [0,\ 1.05]$, $\gamma_2 = [0,\ 0.12]$, $\gamma_3 = [0,\ 0.12]$, we obtain in 22.6 seconds $\alpha_1 = [0,\ 0.0238]$, $\alpha_2 = [0.0738,\ 1.0500]$ and $V_m = [0,\ 0.1200]$. These intervals on α_1 and V_m

contain the normal values whereas the one obtained for α_2 contains the faulty value.

Case of a fault on parameter V_m: In this simulation, we assume a fault on $V_m = 0.2$ at $t = 27s$. This fault is detected in $t = 0.02$s after its occurence. Once the fault is detected, the estimation algorithm is initialized with $\gamma_1 = [0,\ 0.04]$, $\gamma_2 = [0,\ 0.007]$, $\gamma_3 = [0,\ 0.3]$, and we obtain in 33.43s the intervals $\alpha_1 = [0,\ 0.0150]$, $\alpha_2 = [0,\ 0.0248]$ and $V_m = [0.0242,\ 0.3]$.
The intervals on α_1 and α_2 contain the normal values whereas the one for V_m contains the faulty value, hence confirming the fault.

6. Conclusion

This paper proposes a fault detection and identification method for bounded uncertainty nonlinear models relying on an original parameter identifiability scheme. It takes benefit of a differential algebra based method for checking SM identifiability and its operational counterpart μ-SM identifiability. These notions provide a way to study different aspects of identifiability for uncertain bounded-error systems, in particular systems that represent an infinite family of nonlinear systems. By building the parameter estimation scheme on the analysis of identifiability, we guarantee that the solution set reduces to one connected set, avoiding this way the pessimism of SM methods. Identifiability is closely related to diagnosability as it provides the guaranty that two situations corresponding to different parameterized settings are distinguishable. The proposed method has been applied to an example describing the capacity of a macrophage mannose receptor to endocytose a specific soluble macromolecule. Different normal and faulty scenarios have been considered. For every scenario, the parameters have been estimated correctly with reasonable precision.

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