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## A multiphase approach to the construction of pod-rom for flows induced by rotating solids

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## INTRODUCTION

- **Objective:** Construct reduced order models (ROM) for the simulation of turbomachinery with *imposed rotation velocity* by proper orthogonal decomposition (POD).
- **Difficulty:** The POD yields a *spatial* basis from temporal correlations (here of the velocity).
- **Approach:**
  1. Extend the Navier-Stokes equations to the solid (rotor) domain by the multiphase approach. The body velocity is enforced via distributed Lagrange multipliers.
  2. Build a single POD basis for the multiphase velocity and project the governing equations.

## REFERENCES

- [1] Glowinski, R., Pan, T. W., Hesla, T. I., & Joseph, D. D. (1999). A distributed Lagrange multiplier/fictitious domain method for particulate flows. *International Journal of Multiphase Flow*, 25(5), 755-794.
- [2] Liberge, E., & Hamdouni, A. (2010). Reduced order modelling method via proper orthogonal decomposition (POD) for flow around an oscillating cylinder. *Journal of fluids and structures*, 26(2), 292-311.
- [3] Mosquera, R., Hamdouni, A., El Hamidi, A., Allery, C. (2018). POD Basis Interpolation via Inverse Distance Weighting on Grassmann Manifolds. Manuscript submitted to AIMS Journals.

## 1. MULTIPHASE APPROACH

**Level-set** Signed distance to the fluid/solid interface  $\Gamma_I(t)$ :

$$\chi(\mathbf{x}, t) = \begin{cases} +d(\mathbf{x}, \Gamma_I(t)) & \text{if } \mathbf{x} \in \Omega_S(t) \cup \Gamma_I(t), \\ -d(\mathbf{x}, \Gamma_I(t)) & \text{if } \mathbf{x} \in \Omega_F(t) \end{cases}$$

**Smoothed Heaviside** (immersion depth  $\epsilon$ )

$$h_\epsilon(\bullet) = \frac{1}{2} \left( 1 + \tanh \left( \frac{\pi \bullet}{\epsilon} \right) \right).$$

**Membership function for  $\Omega_S(t)$**

$$\mathbf{1}_{\Omega_S(t)}(\mathbf{x}) = h_\epsilon(\chi(\mathbf{x}, t)).$$

**Multiphase quantities**

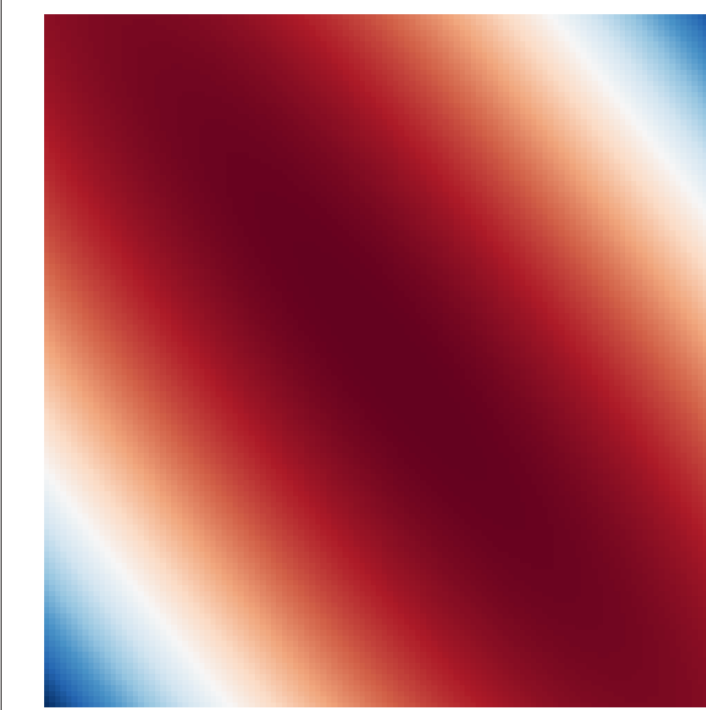
Velocity field over  $\Omega = \Omega_S(t) \cup \Omega_F(t)$ :

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{1}_{\Omega_S(t)}(\mathbf{x}) \mathbf{u}_S(\mathbf{x}, t) + (\mathbf{I} - \mathbf{1}_{\Omega_S(t)}(\mathbf{x})) \mathbf{u}_F(\mathbf{x}, t).$$

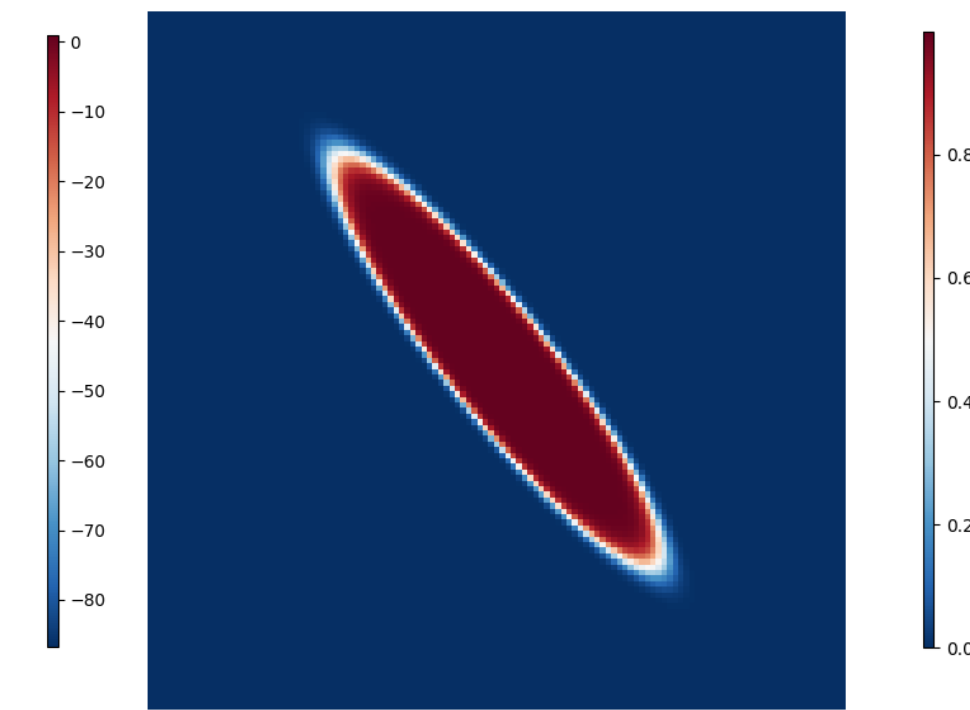
Material properties (density and viscosity):

$$\begin{aligned} \rho(\mathbf{x}, t) &= \mathbf{1}_{\Omega_S(t)}(\mathbf{x}) \rho_S + (\mathbf{I} - \mathbf{1}_{\Omega_S(t)}(\mathbf{x})) \rho_F \\ \nu(\mathbf{x}, t) &= \mathbf{1}_{\Omega_S(t)}(\mathbf{x}) \nu_S + (\mathbf{I} - \mathbf{1}_{\Omega_S(t)}(\mathbf{x})) \nu_F. \end{aligned}$$

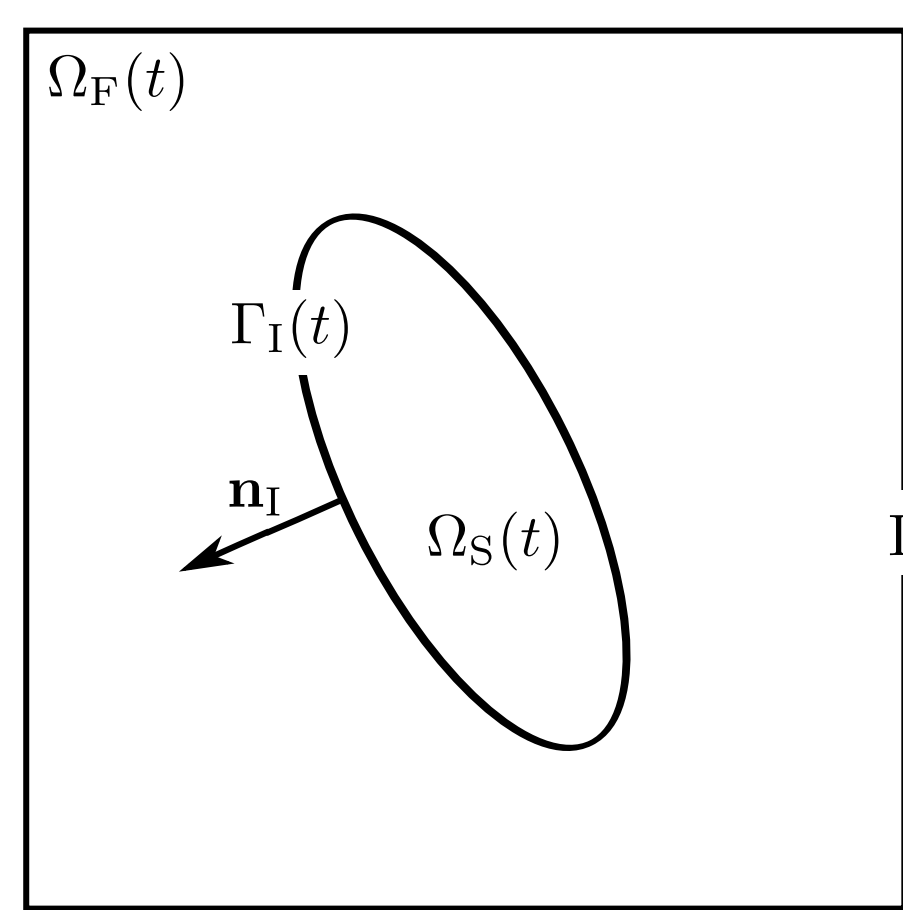
$\chi(\mathbf{x}, t)$



$\mathbf{1}_{\Omega_S(t)}(\mathbf{x})$



## 2. FULL ORDER MODEL [1]



Denoting  $\mathbf{u}_\omega$  the rotation velocity,  $\lambda$  the Lagrange multiplier and  $\mu$  the test function associated with the rotation constraint, the weak form of the coupled problem is

$$0 = \int_{\Omega} \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \nabla \mathbf{u} \cdot \mathbf{u} \right) \cdot \mathbf{v} \, d\mathbf{x} - \int_{\Omega} \mathbf{f} \cdot \mathbf{v} \, d\mathbf{x} + \int_{\Omega} 2\nu \operatorname{Tr}(\mathbf{D}(\mathbf{u}) \cdot \mathbf{D}(\mathbf{v})) \, d\mathbf{x} - \int_{\Omega} p \nabla \cdot \mathbf{v} \, d\mathbf{x} + \int_{\Omega_S(t)} \lambda \cdot \mathbf{v} \, d\mathbf{x} + \int_{\Omega} q \nabla \cdot \mathbf{u} \, d\mathbf{x} + \int_{\Omega_S(t)} \mu \cdot (\mathbf{u} - \mathbf{u}_\omega) \, d\mathbf{x},$$

with an appropriate standard functional setting.

## 4. PROPOSED POD-ROM

⇒ **POD of the membership function**

**POD basis**  $\Lambda = (\Lambda_i(\mathbf{x}))_{1 \leq i \leq N_\Lambda}$ ,  $N_\Lambda \ll N_X$ .

**Decomposition**

$$\mathbf{1}_{\Omega_S(t)}(\mathbf{x}) \simeq \sum_{i=1}^{N_\Lambda} \Lambda_i(\mathbf{x}) \gamma_i(t)$$

**Periodicity**

Coefficients  $\gamma_i(t) \rightarrow \hat{\gamma}_i(\theta)$  determined *a priori*.

**Insertion in the standard POD-ROM**

$$\hat{\mathbf{A}}(t) \cdot \frac{d\alpha(t)}{dt} = \hat{\mathbf{B}}(t) \cdot \alpha(t) + \hat{\mathbf{C}}(t) : \alpha(t) \otimes \alpha(t) + \hat{\mathbf{F}}(t),$$

⇒ **Matrices evaluation at each timestep (cost  $\sim N_\Lambda$ ).**

$$\hat{A}_{ij}(\theta) = \bar{a}_{i,j} + \sum_{k=1}^{N_\Lambda} \bar{a}_{i,j,k} \gamma_k(\theta),$$

$$\hat{B}_{ij}(\theta) = \bar{b}_{i,j} + \sum_{k=1}^{N_\Lambda} \bar{b}_{i,j,k} \gamma_k(\theta),$$

$$\hat{C}_{ijk}(\theta) = \bar{c}_{i,j,k} + \sum_{l=1}^{N_\Lambda} \bar{c}_{i,j,k,l} \gamma_l(\theta),$$

$$\hat{F}_i(\theta) = \bar{f}_i + \sum_{j=1}^{N_\Lambda} \bar{f}_{i,j} \gamma_j(\theta) + \lambda_i.$$

## ACKNOWLEDGMENT

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## 3. STANDARD POD-ROM [2]

**Mean field**  $\bar{\mathbf{u}}(\mathbf{x}) = \frac{1}{N_T} \sum_{n=1}^{N_T} \mathbf{u}(\mathbf{x}, t_n)$

**Fluctuating filed**  $\tilde{\mathbf{u}}(\mathbf{x}, t) = \mathbf{u}(\mathbf{x}, t) - \bar{\mathbf{u}}(\mathbf{x})$

**Data matrix**  $U_{mn} \equiv \tilde{\mathbf{u}}(\mathbf{x}_m, t_n)$

**POD basis:** left singular vectors of  $\mathbf{U}$

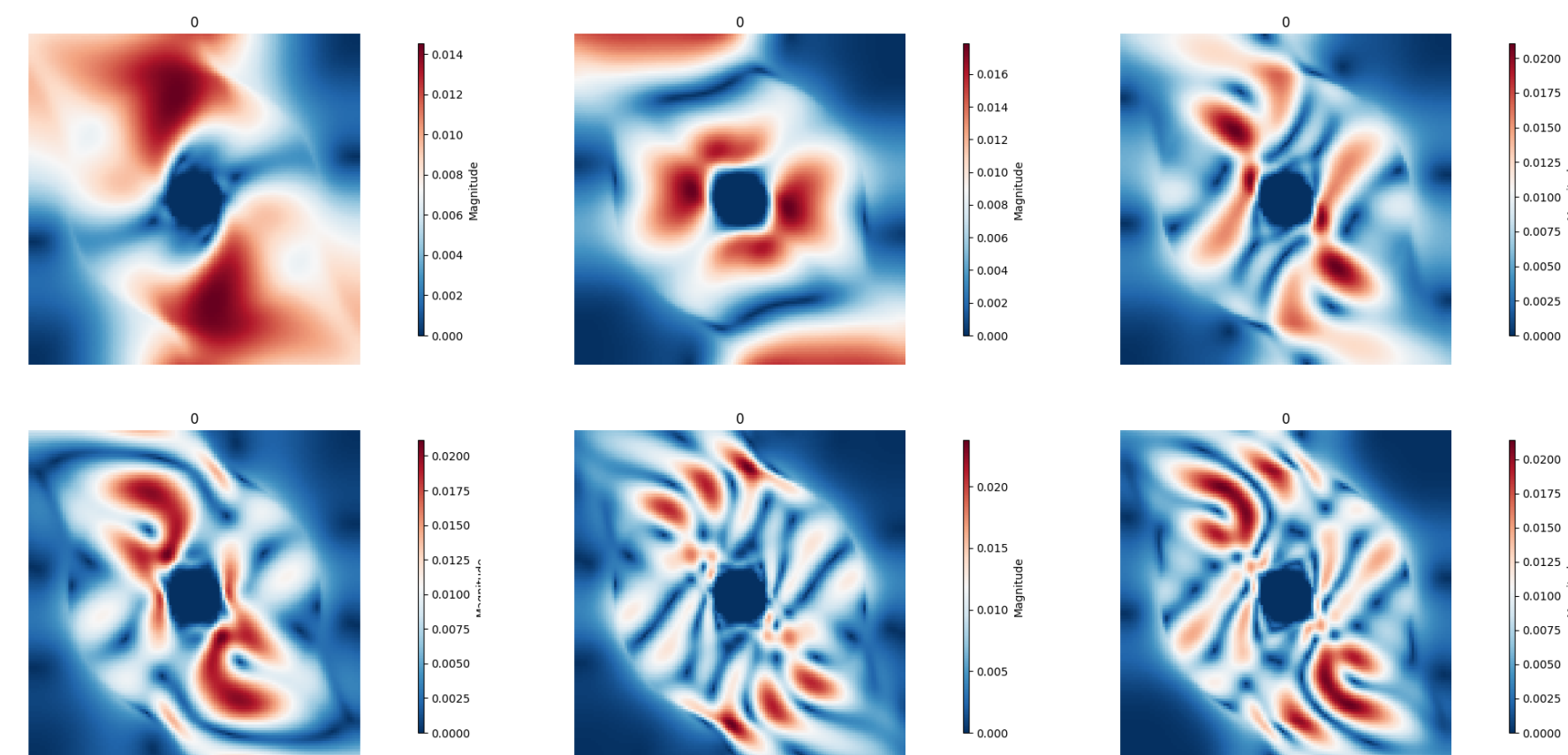
$$\Phi = (\phi_i(\mathbf{x}))_{1 \leq i \leq N_\Phi}, \quad N_\Phi \ll N_T \ll N_X$$

**Decomposition**

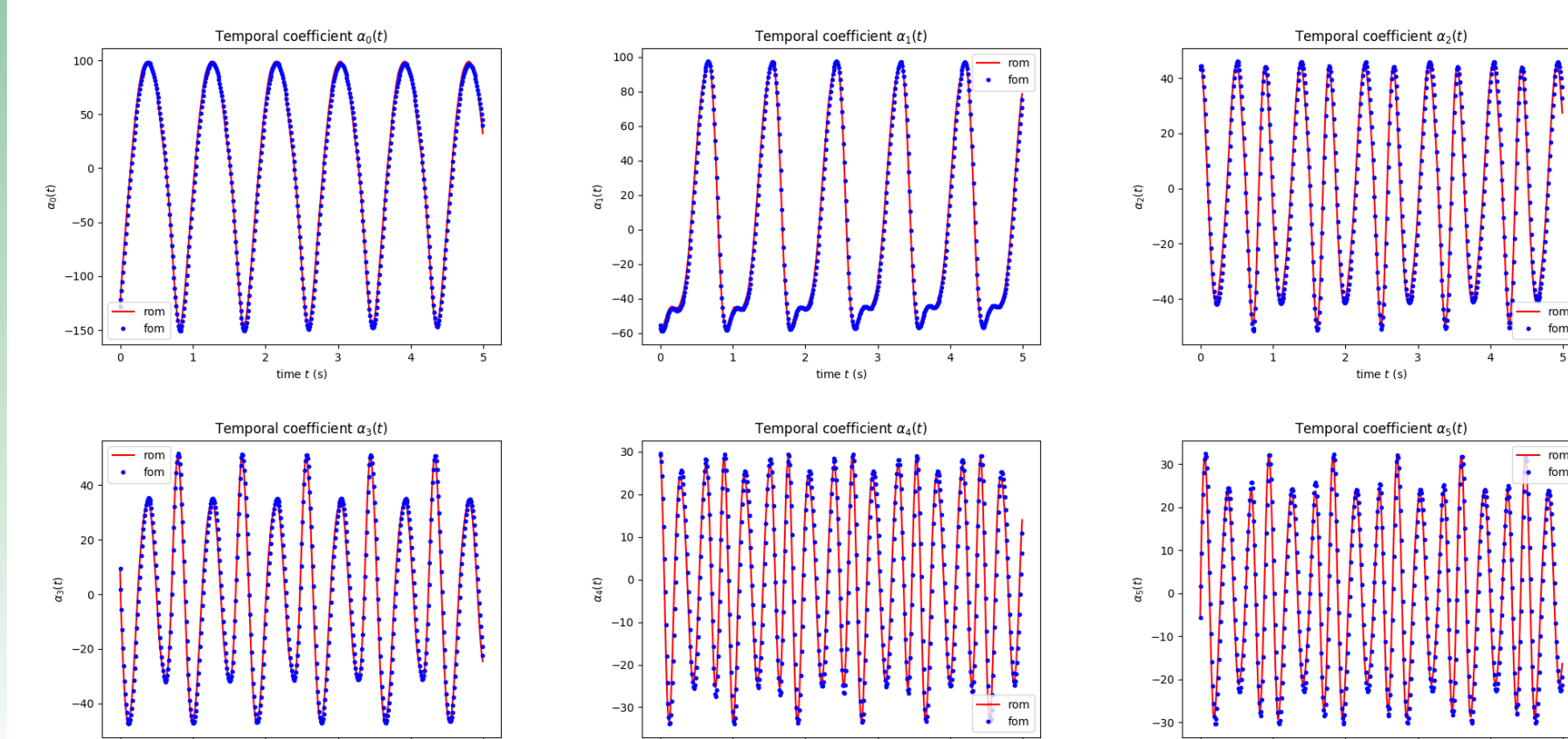
$$\mathbf{u}(\mathbf{x}, t) \simeq \bar{\mathbf{u}}(\mathbf{x}) + \sum_{i=1}^{N_\Phi} \phi_i(\mathbf{x}) \alpha_i(t)$$

## 5. RESULTS

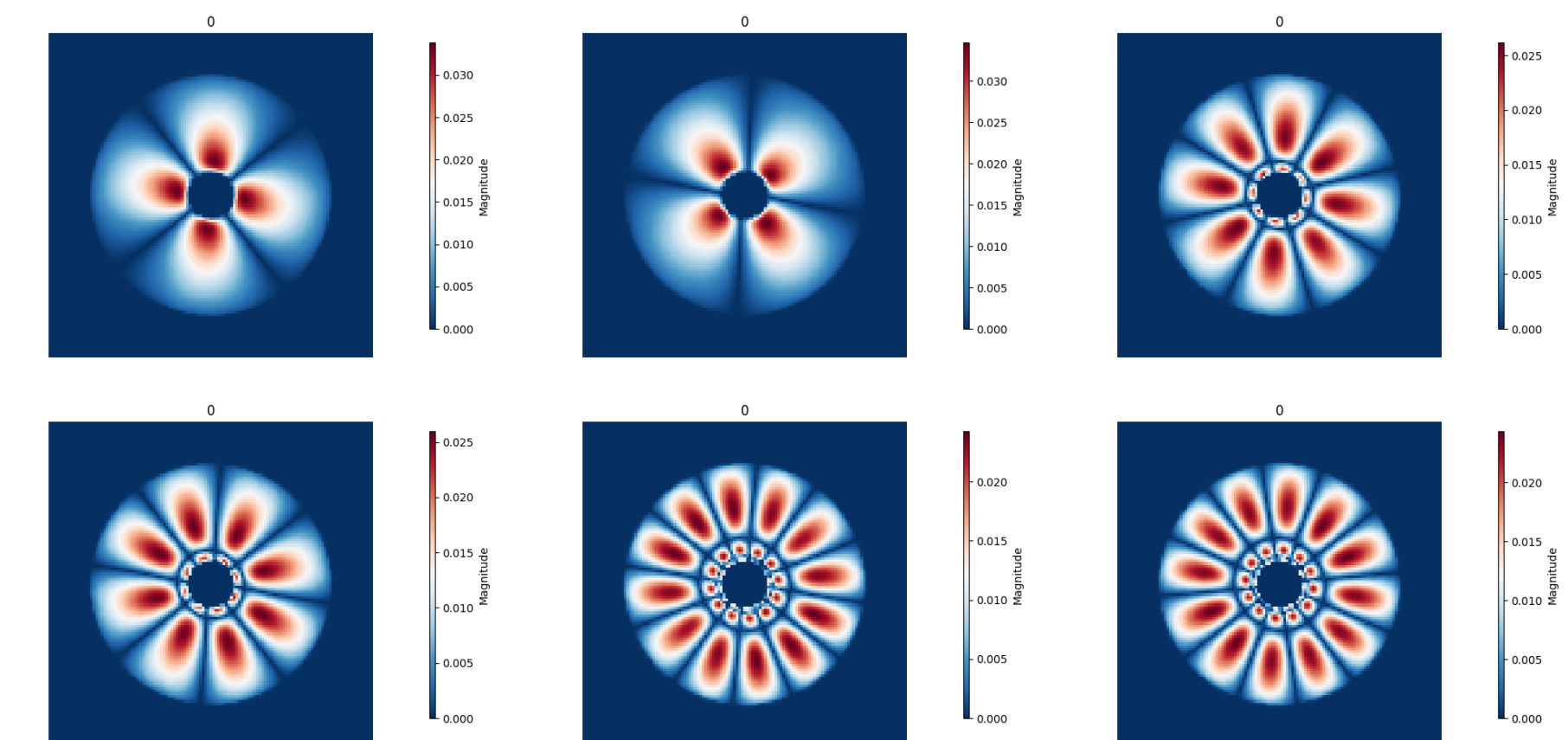
$\Phi$ : POD modes for the velocity



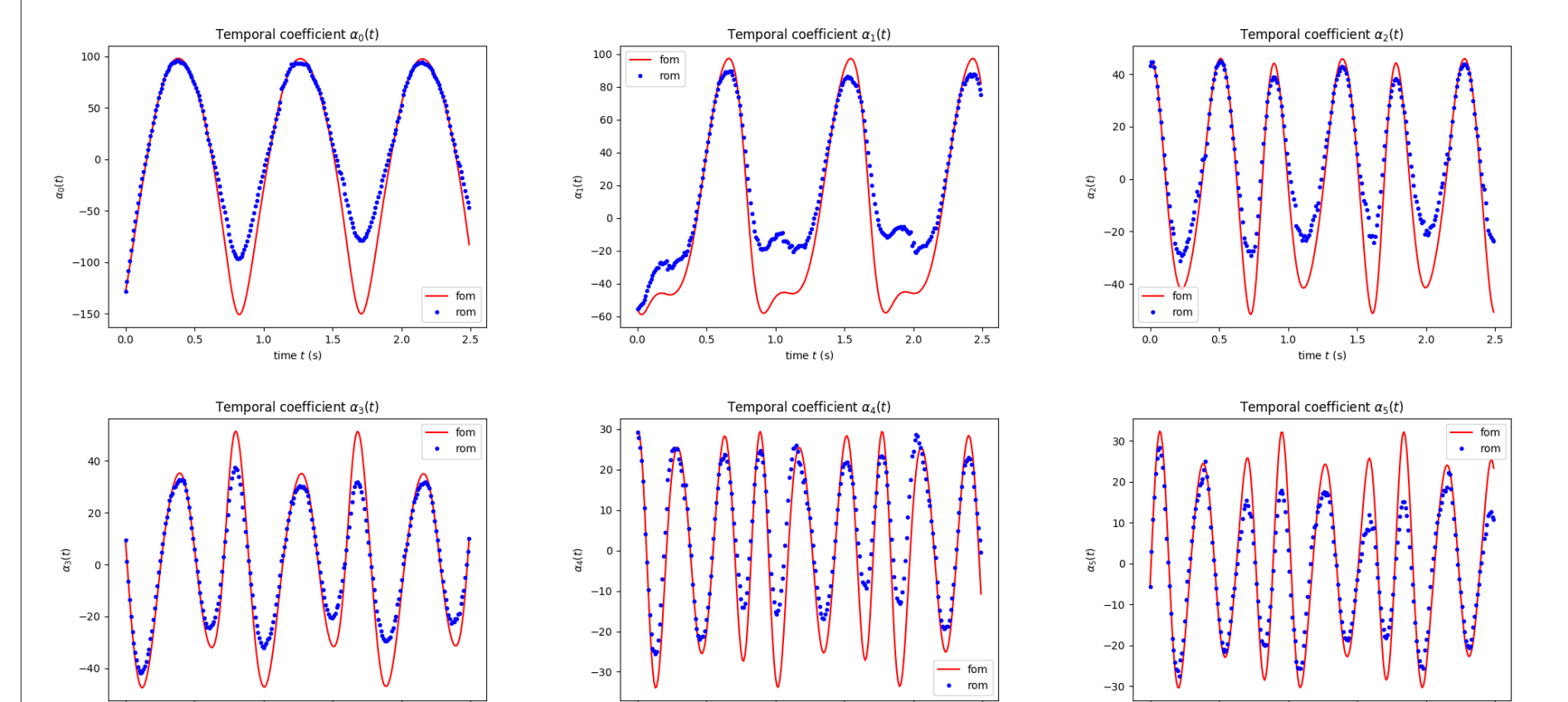
Coeff  $\alpha_i(t)$ : **full order** Vs. **standard POD-ROM**.



$\Lambda$ : POD modes for the membership function



Coeff  $\alpha_i(t)$ : **full order** Vs. **proposed POD-ROM**.



## CONCLUSION

**Contributions**

- Efficient reconstruction of the velocity in both the fluid and solid domains, while substantially reducing the computational cost.
- Very general: Any simulation code for the incompressible Navier-Stokes eq. can be used to generate the data  $(\mathbf{u}(\mathbf{x}, t_n))_{1 \leq n \leq N_T}$ .

**Perspectives**

- Cope with the reconstruction of the velocity in the solid domain at each iteration by rewriting the governing equations for a rotating subdomain.
- Interpolate between the POD-ROMs over the Grassmann manifold (see *e.g.* [3]).