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ANALYTICAL APPROACH TO STUDY NOISE AND VIBRATION OF A SYNCHRONOUS PERMANENT MAGNET MACHINE

A. Ait-Hammouda, M. Hecquet, M. Goueygou*, P. Brochet, A.Randria**.

L2EP - Ecole Centrale de LILLE

* IEMN D.O.A.E. UMR 8520 CNRS, ** Alstom- ORNANS.

Ecole Centrale de Lille, Cité scientifique, B.P. 48, 59651 Villeneuve D'Ascq Cedex, France.

Ait-hammouda.amine@ec-lille.fr; michel.hecquet@ec-lille.fr, marc.goueygou@ec-lille.fr

Abstract

Electrical machines operating at variable speed generate vibrations, which can be harmful for the machine itself and for its environment. Thus, it is necessary for the manufacturer to take into account noise and vibration at the design stage.

There are many tools and methods of analysis, which enable to study coupled phenomena : a classical method is the finite element method (F.E.M) in magneto-dynamics including coupling with circuit and load. However, in the case of strong coupling, these electromagnetic models, vibroacoustic, or thermics, would take a considerable computing time, making structure optimization practically impossible. In order to solve this problem, an analytical multiphysical model is developed for a synchronous machine.

I-Introduction

The vibration analysis of electrical machines is a rather old problem. During the 40s and 50s, the problem was deeply studied by various researchers [1], [2]. Vibrations of electromechanical system are due to excitation of forces, some of which are of magnetic origin. Other vibrations such due to aerodynamics or bearings, aren't considered in this study.

The aim of the study is to predict the acoustic noise of a synchronous permanent magnet machine. Effectively two ways can be adopted to reduce noise and vibrations: by controlling the excitation or by modifying the system structure. In this work, only the second solution is explored. Three models are presented : electromagnetic, mechanical of vibration and acoustic. For each part, comparisons with F.E.M. and experiments have been made.

II. Electromagnetic model

Its assumed that forces in the air gap of the machine are the main mechanical excitation. To characterize induction in the air-gap, the proposed method is based on the calculation of the air-gap permeance (P_e) and the magnetomotive force (mmf) [3]. To establish the analytical expression of the permeance, a few assumptions are made :

- the magnetic circuit has a high permeability and a linear characteristic,
- the tangential component of the air-gap flux density is negligible relative to the radial component.

In order to facilitate the comprehension of formula used, a complete nomenclature of notation used is given on §VI.

1. Analytical expression of the air gap permeance

The permeance is inversely proportional to the thickness of the air-gap. This thickness is a function of dimensions of the stator and rotor structure. The air-gap permeance is given by [3] for P.M.M :

$$p(\alpha^s) = p_0 + \sum_{k_s=-\infty}^{+\infty} P_{k_s} \cos(k_s N_s \alpha^s) \quad (1)$$

2. Analytical expression of the magnetomotive force and of the flux density created by the magnet of the rotor

Considering the B(H) curve of the magnet given, the induction created by the magnet at the operating point is [4] :

$$B_{aim} = \frac{H_c \cdot h_{aim} \cdot \mu_0}{\left(e + \frac{h_{aim}}{\mu_r}\right)} \quad (2)$$

Given the expression of the magnetomotive force as a function of the fields and the height of magnet:

$$mmf = H_c \cdot h_{aim} \quad (3)$$

The air-gap permeance of ' p_e ' is equal to:

$$P_e = \frac{B_{aim}}{mmf} = \frac{\mu_0}{\left(e + \frac{h_{aim}}{\mu_r}\right)} \quad (4)$$

The shape of the magnetomotive force created by a magnet rotor is assumed to be as fig.1.

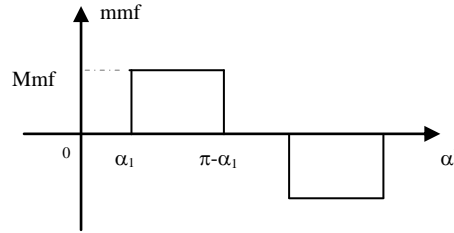


Fig.1 : Shape of the magneto motiveforce created by the magnet rotor.

The mmf created by the rotor according to the angle α^r can be expressed as a Fourier series in the rotor reference frame:

$$mmf(\alpha^r) = \sum_{h_r=1}^{+\infty} 4 \cdot \frac{Mmf}{h_r \pi} \cos(h_r \cdot \alpha) \sin(h_r \cdot p \cdot \alpha^r) \quad (\text{Where } P \text{ is the number of pole pair}) \quad (5)$$

The flux density created by the rotor according to the angle α^s in the stator reference frame is:

$$b(\alpha^s, t) = P(\alpha^s, t) mmf = \sum_{h_r=1}^{+\infty} \sum_{k_s=-\infty}^{+\infty} 4 \cdot P k_s \frac{Mmf}{h_r \pi} \cdot \cos(h_r \cdot p \cdot \alpha) \sin([h_r + k_s N_s] \cdot p \cdot \alpha^s + h_r \cdot p \cdot \omega_r \cdot t) \quad (6)$$

$$b(\alpha^s, t) = \sum_{v=1}^{+\infty} \sum_{u=1}^{+\infty} B_{ph} \sin(v \cdot \alpha^s + u \cdot \omega_r \cdot t) \quad (7)$$

with :

$$\alpha^s = \alpha^r - \theta \quad (\text{speed rotation of the rotor } \theta = p \cdot \omega_r \cdot t)$$

Starting from flux density in the air-gap, the force applied to the stator is determined by :

$$f(\alpha^s, t) = \frac{[\sum b(\alpha^s, t)]^2}{2\mu_0} \quad (8)$$

$$f(\alpha^s, t) = \frac{1}{4\mu_0} \sum_{u1} \sum_{u2} \sum_{v1} \sum_{v2} B_{p1h1} B_{p2h2} \{ \cos [(u1+u2)\omega t - (v1+v2)\alpha^s] + \cos [(u1-u2)\omega t - (v1-v2)\alpha^s] \} \quad (9)$$

A component of force will be written in the form (fig.2), figure 3 presents the radial force applied to the stator versus space and time :

$$f(\alpha^s, t) = \sum_{m=1}^{+\infty} \sum_{\omega=1}^{+\infty} \hat{F} \cos(m \cdot \alpha^s + \omega t) \quad (10)$$

Where : $\omega = (u1 \pm u2)\omega_r$ is the pulsation of the force.
 $m = (v1 \pm v2)$ order of the mode.

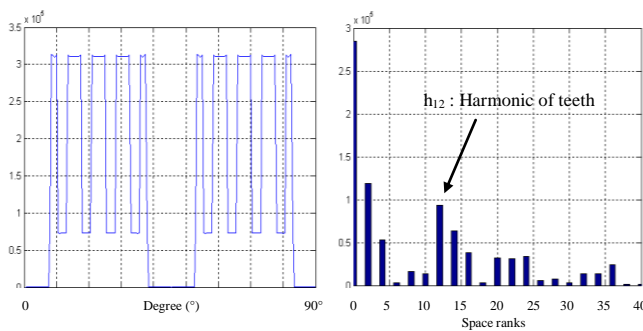


Fig.2 : Representation of the force and its FFT in space

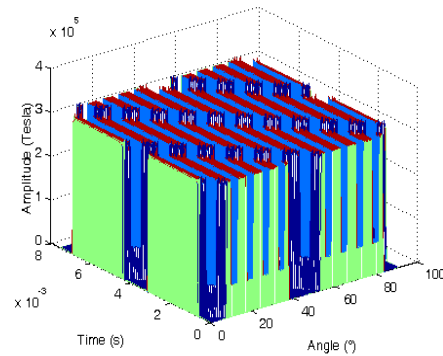


Fig.3 : The force in space and the time created by the rotor of the synchronous machine

3. Finite element validation

Using the finite element software OPERA-2D [5], the air-gap induction created by the magnet rotor as a function of space and time has been calculated. In figure.4, only a comparison between induction wave shapes versus the angle is presented. The results of comparison are satisfactory, the form of induction and harmonics determined analytically is validated numerically [6].

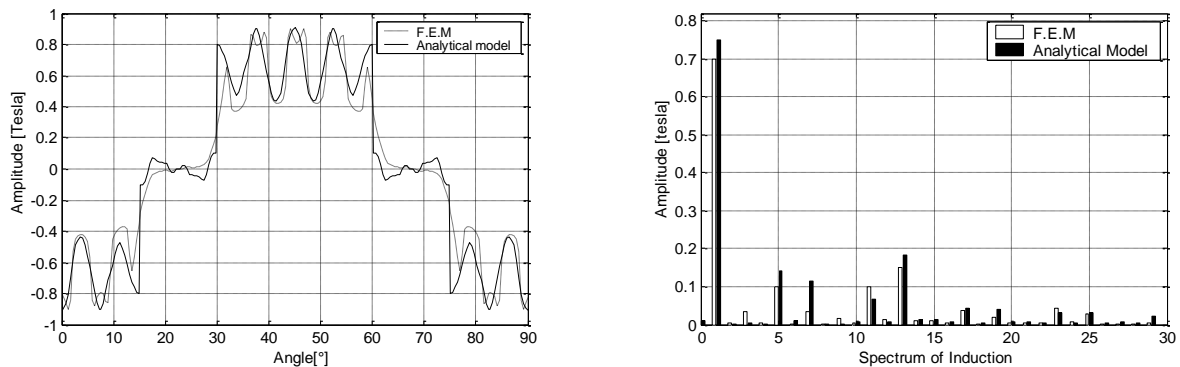


Fig.4 : Comparison of the form induction and FFT

III. Development of a vibratory model

In this study, vibrations are the consequence of excitation of a mechanical system by electromagnetic forces. Once the forces applied to the stator have been determined, we can now study the vibrations which correspond to deformations whose amplitudes will have to be calculated. For that purpose some

parameters have to be determined : damping, mode shapes and frequencies of resonance for each mode.

The studied analytical model takes into account the yoke, the frame, the teeth and winding.

1. Natural frame mode

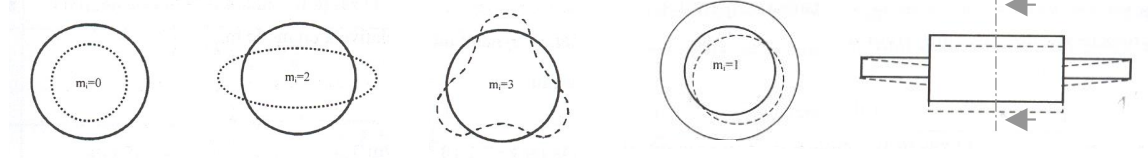


Fig.5 : Example of deformations of the stator for various modes.

According to the order of the mode (fig.5), the deformation arises in various forms. For example [7], the wave of force responsible for the oval deformation corresponds to mode 2, triangular with mode 3. Our study, as those which were done before, showed that the most significant modes 'm' lie between 1et 4, and especially mode 2 which can be dangerous if it is excited.

2. frequencies of resonance

Each mode shape is associated with a mechanical frequency of resonance. The rather simple formulas given by JORDAN [7] easily make it possible to find the frequency of resonance of each mode of a stator (yoke + teeth). The model was improved by VERMA [8] to take winding into account.

For the $m=0$ mode, attractive effort of the cylinder head occurs, the amplitude of the static deformation of the stator is given by :

$$F_0 = \frac{1}{2\pi} \sqrt{\frac{E}{\rho \cdot R^2 \cdot \Delta}} \quad (11)$$

where :

$\Delta = 1 + \frac{M_{te}}{M_{yo}}$; ρ : density of stator material ; E : modulus of elasticity.

The stator teeth and frame are taken into account directly in the coefficient Δ . In the case where the stator is wound, the coefficient Δ is modified by the integration of the weight of winding :

$$\Delta = 1 + \frac{M_{te} + M_{wi} + M_{ca}}{M_{yo}}$$

For the $m>1$ modes, the yoke is subjected to flexion and one obtains the following equation:

$$F_m = F_0 \frac{h \cdot m \cdot (m^2 - 1)}{2\sqrt{3} \cdot R_c \cdot \sqrt{m^2 + 1}} \quad (12)$$

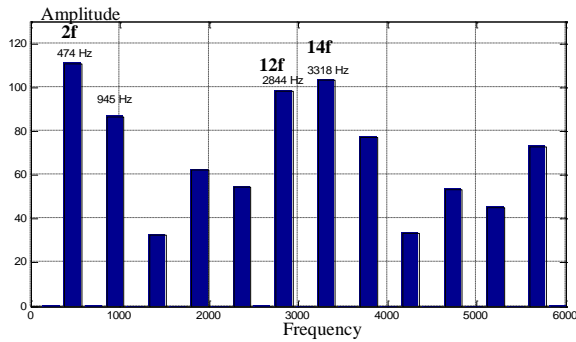
f_r : frequency of vibration ; F_m : mechanical frequency of resonance ; ξ_a : damping coefficient

Maximum radial vibrations of the stator occur when excitation frequencies of forces harmonics are equal or close to one of the above frequencies F_m , and that is possible when we have a coincidences of the modes.

3. Calculation of the deformations

The electromagnetic forces are responsible for vibrations of the stator. They are characterized by the deformations Y_{md} whose amplitudes has to be calculated. After having established the frequencies of resonance of the structure, we define below the dynamic deformations for modes $m>1$.

$$Y_{md} = \frac{12 \cdot R_a \cdot R_c^3 \cdot \hat{F}}{E \cdot h^3 \cdot (m^2 - 1)^2 \sqrt{\left(1 - \left(\frac{f_r}{F_m}\right)^2\right)^2 + \left(2 \cdot \xi_a \cdot \frac{f_r}{F_m}\right)^2}} \quad (13)$$



Speed	3555 rpm
Frequency of the supply f_r	237
Rotational frequency f_{rot}	237/p
Frequencies of components of forces (multiple of 2f)	$h \cdot 237$ ($h=2,4,6 \dots$)

Fig.6 : Total analytical vibratory spectrum with 3555 rpm.

Let us note that damping coefficient ξ_a cannot be given theoretically, however JORDAN [7] considers that for a synchronous machine it one is between 0,01 and 0,04. The total vibratory spectrum obtained by our analytical model is presented above (fig.6).

The results of simulations agree well with the theory. In addition, the proximity of the frequency of excitation mode 0 with the frequency of resonant mode 0 (at 2844 Hz) explains the vibration peak around 2900Hz. However let us point out that precautions must be taken when analyzing the results.

The model giving induction is not perfect (saturation is neglected) and the formulas of TIMAR [2] giving the vibrations are also approximated. What is of interests is to determine the frequency of the main peaks and to be able to range their amplitudes.

4. Calculation of the resonance frequencies and F.E validation

The analytical vibration model was validated numerically by ANSYS: calculation of the vibration modes of the stator structure, including : yoke, yoke&teeth, yoke&teeth&frame and yoke&teeth&frame&winding. A part of the obtained results of simulations are presented in Table.1.

Range of the mode	Analytical method (Hz)	F.E.M (Hz)
0	3063	3151
2	243	268
3	688	732
4	1319	1349
5	2134	2078

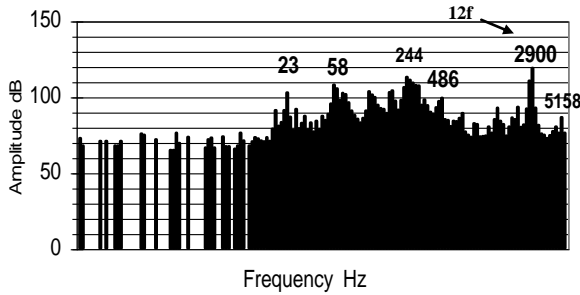
Table.1 : Comparative table of the resonance frequencies for each mode (teeth&yoke)

5. Experimental validation of the model

The resonant modes of P.M.M were studied experimentally with various configurations : without flask and rotor, without rotor, complete machine. The machine is suspended to approach the free-free mode. The measurement of the frequencies of resonance is carried out in the following way :

Using a spectrum analyzer, we determine the transfer function : acceleration (measured by an accelerometer) divided by force (applied by a hammer). Maxima of this function correspond to the resonance frequencies of the structure. The obtained results are only compared with the analytical model (Table.2), because F.E.M, does not take into the windings.

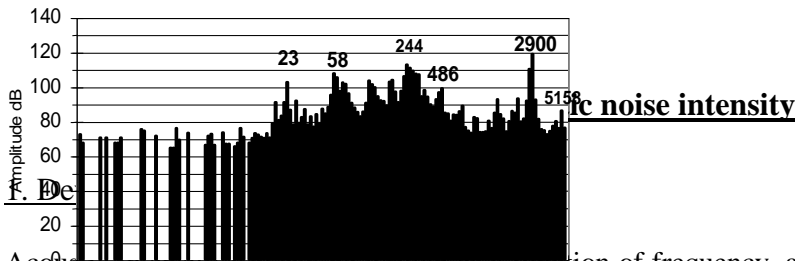
In order to study the vibrations generated by the operating P.M.M. an accelerometer is positioned on the frame of the machine and measures the deformations of the frame. The vibratory spectrum gives lines identical to those obtained by the noise measurement, it displays a dominant line at 2900 Hz, that corresponds to theoretical excitation mode 0 predicted at 2844 Hz (Fig.7).



Range of the mode	Analytical Method (Hz)	Experimental method (Hz)
0	2736	2855
2	308	376
3	871	1004
4	1670	1720
5	2702	2757

fig.7: Vibratory spectrum measured with 3555 rpm with 1/12 of octave.

Table.2 : Comparative table of the resonance frequencies for each mode (yoke&teeth&winding&frame)



Acoustic intensity $I(x)$ can be written as a function of frequency, amplitude of vibrations, mode order and stator surface [9]:

$$I(x) = \frac{\sigma 8200 f_r^2 Y_{md}^2 S_e}{4 \pi x^2 (2m+1)} \quad (14)$$

The coefficient σ is called factor of radiation, it represents the capacity of a machine to be a good sound generator and can be calculated in two different ways according to whether one assumes the machine to be a sphere or a cylinder. σ is a factor which varies with λ (wavelength) and the diameter of the machine. It also depends on the mode shape [2]:

$$\sigma = f(\pi \frac{D}{\lambda}), \quad \lambda = \frac{c}{f_r}$$

c : traveling speed of sound (344m/s); f_r : Frequency of the vibration.

It appears that $I(x)$ is inversely proportional to the order of the mode, in addition the acoustic intensity is proportional to the square of the vibration amplitude. In general, we define I , and W in decibels we thus define the levels of acoustic pressure, acoustic intensity and sound power:

$$L_p = 20 \log(\frac{P}{P_0}), \quad L_i = 10 \log(\frac{I}{I_0}), \quad L_w = 10 \log(\frac{W}{W_0})$$

With :

$$P_0 = 20 \mu\text{Pa}, \quad I_0 = 10^{-12} \text{ W/m}^2, \quad W_0 = 10^{-12} \text{ W}$$

The spectrum of the total noise obtained by our analytical model is presented below (fig.8).

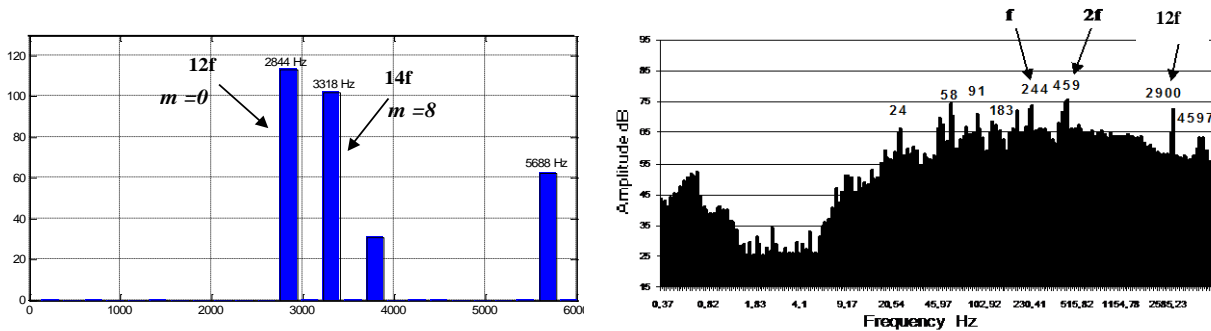


Fig.8 : Spectrum of the total analytical noise with 3555 rpm.

Fig.9 : Spectrum of the noise measured with 3555 rpm.

The lines below 2844Hz do not appear because the effect of σ is relatively significant on the low frequencies ($\sigma=0$).

2. Experimental validation of the acoustic model

Figure.9 presents the measured acoustic noise spectrum at 3555 rpm, lines are located at the same frequencies as in the vibration spectrum. The first line determined by measurements is located at 2900 Hz (12f). In theory, the harmonic of teeth (12f) is located at 2844 Hz.

The lines at low frequencies (between 24 Hz and 459 Hz) are not found in theory, because they are related in majority part to the background noise, they are not generated by the synchronous machine, but by the driving motor and ventilator (Fig.10).

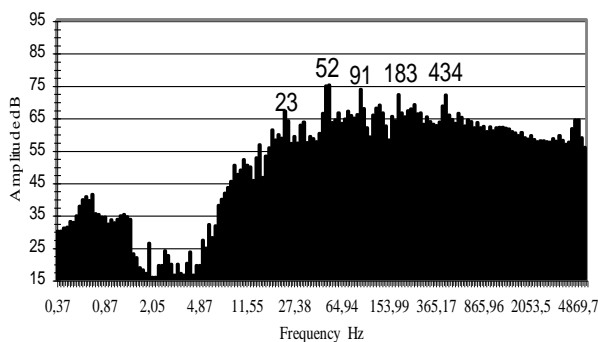


Fig.10 : Spectrum of the noise of driving motor & ventilator measured with 3555 rpm (1/12 octave)

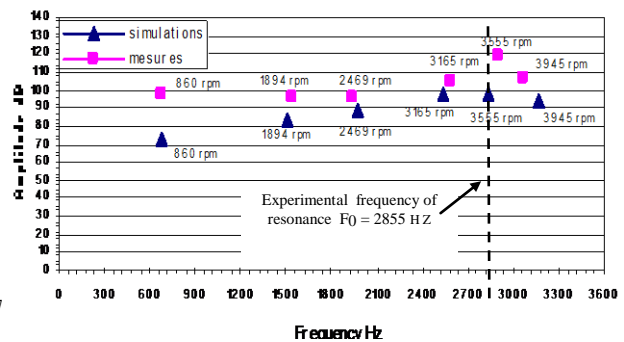


Fig.11 : Level of the 12th harmonic vs. rotation speed
Vibratory comparisons

The machine was tested at various speeds, that allowed us to highlight a particularly dangerous speed. Moreover, some results are over-estimated but the quality of those is respected (Fig.11). In spite of the inaccuracies, major lines appear, which is of primary importance in view of noise reduction.

To know which lines to reduce does not require to know its amplitude precisely. Its frequency, on the other hand, must be well given. Lastly, taking into account the complexity of the studied phenomena and the many steps of calculations making it possible to lead to the results, the latter seem very satisfactory.

V. CONCLUSION

In this paper, an analytical model allowing the prediction of the vibrations and the noise of a synchronous permanent magnet machine is proposed.

The model takes account of various parameters such as for example the height of magnet, the number of slots stator and the height of cylinder head.

Tests have been carried out by measuring vibrations and noise radiated by a synchronous permanent magnet machine at various speed. The vibration and noise measured enable to highlight the most

significant lines which are multiple of f_r (frequency of supply) and among which one is dissociated : the harmonic of teeth at frequency with $12f$ ($N_s=12$). At frequencies close to the frequencies of resonance 2736Hz for mode $m=0$, corresponds of the high vibratory lines and noise.

Electromagnetic, vibratory and acoustic model was validated numerically and in experimentally. Nevertheless, some improvements will be necessary such as for example the determination of the damping coefficient.

VI. Nomenclature

b : Instantaneous flux density,	B_{aim} : flux density in the magnet,
c : traveling speed of sound (344m/s),	D : outside diameter of the stator,
e : thickness of the air-gap,	h : height of the stator yoke,
h_{aim} : thickness of the magnet,	H_c : coercive field of the magnet,
H_{aim} : fields in the magnet,	M_{te} : total mass of the stator teeth,
M_{wi} : mass of the stator windings,	M_{ca} : mass of the frame,
M_{yo} : mass of the stator yoke,	m : mode number,
mmf : magneto motive force,	N_s : number of stator slot per pair of pole,
$p = 4$: number of pole-pairs,	P_e : permeance,
R_a : interior radius stator,	R_c : average radius of stator,
S_e : outside surface of the stator,	ω_r : angular rotor frequency,
ω_f : pulsation of the force,	ξ_a : damping coefficient,
α' : angle, compared to the reference axis rotor,	α' : angle, compared to the reference axis stator,
μ_0 : permeability of free space,	μ_r : relative recoil permeability of the magnet.

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