



HAL
open science

Rational solutions to the KPI equation of order 7 depending on 12 parameters

Pierre Gaillard

► **To cite this version:**

Pierre Gaillard. Rational solutions to the KPI equation of order 7 depending on 12 parameters. 2018.
hal-01736228

HAL Id: hal-01736228

<https://hal.science/hal-01736228v1>

Preprint submitted on 16 Mar 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Rational solutions to the KPI equation of order 7 depending on 12 parameters

Pierre Gaillard
Université de Bourgogne
Institut de mathématiques de Bourgogne
9 avenue Alain Savary BP 47870
21078 Dijon Cedex France
E-mail: Pierre.Gaillard@u-bourgogne.fr

January 27, 2018

Abstract

We construct in this paper, rational solutions as a quotient of two determinants of order $2N = 14$ and we obtain what we call solutions of order $N = 7$ to the Kadomtsev-Petviashvili equation (KPI) as a quotient of 2 polynomials of degree 112 in x , y and t depending on 12 parameters. The maximum of modulus of these solutions at order 7 is equal to $2(2N + 1)^2 = 450$. We make the study of the patterns of their modulus in the plane (x, y) and their evolution according to time and parameters $a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5, b_6$. When all these parameters grow, triangle and ring structures are obtained.

Keywords: KPI equation; Fredholm determinants; Wronskians; rogue waves; lumps.

PACS numbers :
33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd

1 Introduction

We consider the Kadomtsev-Petviashvili equation (KPI), first proposed in 1970 [1] in the following normalization

$$(4u_t - 6uu_x + u_{xxx})_x - 3u_{yy} = 0. \quad (1)$$

As usual, subscripts x , y and t denote partial derivatives.

The first rational solutions were constructed in 1977 by Manakov, Zakharov, Bordag and Matveev [2]. Other more general rational solutions of the KPI

equation were found by Krichever in 1978 [3, 4], Satsuma and Ablowitz in 1979 [5], Matveev in 1979 [6], in particular among many works on this subject. We construct rational solutions of order N depending on $2N - 2$ parameters which can be written as a ratio of two polynomials in x , y and t of degree $2N(N + 1)$.

The maximum of the modulus of these solutions at order N is equal to $2(2N + 1)^2$. Here we construct the explicit rational solutions of order 7, depending on 12 real parameters, and the representations of their modulus in the plane of the coordinates (x, y) according to the real parameters $a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, a_5, b_5, a_6, b_6$ and time t . When the parameters grow, we obtain $\frac{N(N+1)}{2}$ peaks in particular structures, such as triangles, rings, or concentric rings.

2 Rational solutions to KPI equation of order N depending on $2N - 2$ parameters

The rational solutions to the KPI equation are given by the following result [38, 40]:

Theorem 2.1 *The function v defined by*

$$v(x, y, t) = -2 \frac{|\det((n_{jk})_{j,k \in [1, 2N]})|^2}{\det((d_{jk})_{j,k \in [1, 2N]})^2} \quad (2)$$

is a rational solution of the KPI equation (1), where

$$\begin{aligned} n_{j1} &= \varphi_{j,1}(x, y, t, 0), \quad 1 \leq j \leq 2N & n_{jk} &= \frac{\partial^{2k-2} \varphi_{j,1}}{\partial \epsilon^{2k-2}}(x, y, t, 0), \\ n_{jN+1} &= \varphi_{j,N+1}(x, y, t, 0), \quad 1 \leq j \leq 2N & n_{jN+k} &= \frac{\partial^{2k-2} \varphi_{j,N+1}}{\partial \epsilon^{2k-2}}(x, y, t, 0), \\ d_{j1} &= \psi_{j,1}(x, y, t, 0), \quad 1 \leq j \leq 2N & d_{jk} &= \frac{\partial^{2k-2} \psi_{j,1}}{\partial \epsilon^{2k-2}}(x, y, t, 0), \\ d_{jN+1} &= \psi_{j,N+1}(x, y, t, 0), \quad 1 \leq j \leq 2N & d_{jN+k} &= \frac{\partial^{2k-2} \psi_{j,N+1}}{\partial \epsilon^{2k-2}}(x, y, t, 0), \\ & & & 2 \leq k \leq N, \quad 1 \leq j \leq 2N \end{aligned} \quad (3)$$

The functions φ and ψ are defined in (4), (5), (6), (7).

$$\begin{aligned} \varphi_{4j+1,k} &= \gamma_k^{4j-1} \sin X_k, & \varphi_{4j+2,k} &= \gamma_k^{4j} \cos X_k, \\ \varphi_{4j+3,k} &= -\gamma_k^{4j+1} \sin X_k, & \varphi_{4j+4,k} &= -\gamma_k^{4j+2} \cos X_k, \end{aligned} \quad (4)$$

for $1 \leq k \leq N$, and

$$\begin{aligned} \varphi_{4j+1,N+k} &= \gamma_k^{2N-4j-2} \cos X_{N+k}, & \varphi_{4j+2,N+k} &= -\gamma_k^{2N-4j-3} \sin X_{N+k}, \\ \varphi_{4j+3,N+k} &= -\gamma_k^{2N-4j-4} \cos X_{N+k}, & \varphi_{4j+4,N+k} &= \gamma_k^{2N-4j-5} \sin X_{N+k}, \end{aligned} \quad (5)$$

for $1 \leq k \leq N$.

The functions $\psi_{j,k}$ for $1 \leq j \leq 2N$, $1 \leq k \leq 2N$ are defined in the same way, the term X_k is only replaced by Y_k .

$$X_\nu = \frac{\kappa_\nu x}{2} + i\delta_\nu y - i\frac{x_{3,\nu}}{2} - i\frac{\tau_\nu}{2} t - i\frac{e_\nu}{2},$$

$$Y_\nu = \frac{\kappa_\nu x}{2} + i\delta_\nu y - i\frac{x_{1,\nu}}{2} - i\frac{\tau_\nu}{2}t - i\frac{e_\nu}{2},$$

for $1 \leq \nu \leq 2N$, with κ_ν , δ_ν , $x_{r,\nu}$ defined in (9) and parameters e_ν defined by (10).

$$\begin{aligned} \psi_{4j+1,k} &= \gamma_k^{4j-1} \sin Y_k, & \psi_{4j+2,k} &= \gamma_k^{4j} \cos Y_k, \\ \psi_{4j+3,k} &= -\gamma_k^{4j+1} \sin Y_k, & \psi_{4j+4,k} &= -\gamma_k^{4j+2} \cos Y_k, \end{aligned} \quad (6)$$

for $1 \leq k \leq N$, and

$$\begin{aligned} \psi_{4j+1,N+k} &= \gamma_k^{2N-4j-2} \cos Y_{N+k}, & \psi_{4j+2,N+k} &= -\gamma_k^{2N-4j-3} \sin Y_{N+k}, \\ \psi_{4j+3,N+k} &= -\gamma_k^{2N-4j-4} \cos Y_{N+k}, & \psi_{4j+4,N+k} &= \gamma_k^{2N-4j-5} \sin Y_{N+k}. \end{aligned} \quad (7)$$

Real numbers λ_j are such that $-1 < \lambda_\nu < 1$, $\nu = 1, \dots, 2N$ depending on a parameter ϵ which will be intended to tend towards 0; they can be written as

$$\lambda_j = 1 - 2\epsilon^2 j^2, \quad \lambda_{N+j} = -\lambda_j, \quad 1 \leq j \leq N. \quad (8)$$

The terms κ_ν , δ_ν , γ_ν , τ_ν and $x_{r,\nu}$ are functions of λ_ν , $1 \leq \nu \leq 2N$; they are defined by the formulas :

$$\begin{aligned} \kappa_j &= 2\sqrt{1-\lambda_j^2}, & \delta_j &= \kappa_j \lambda_j, & \gamma_j &= \sqrt{\frac{1-\lambda_j}{1+\lambda_j}}, \\ x_{r,j} &= (r-1) \ln \frac{\gamma_j - i}{\gamma_j + i}, & r &= 1, 3, & \tau_j &= -12i\lambda_j^2 \sqrt{1-\lambda_j^2} - 4i(1-\lambda_j^2) \sqrt{1-\lambda_j^2}, \\ \kappa_{N+j} &= \kappa_j, & \delta_{N+j} &= -\delta_j, & \gamma_{N+j} &= \gamma_j^{-1}, \\ x_{r,N+j} &= -x_{r,j}, & \tau_{N+j} &= \tau_j & j &= 1, \dots, N. \end{aligned} \quad (9)$$

e_ν , $1 \leq \nu \leq 2N$ are defined in the following way :

$$\begin{aligned} e_j &= 2i \left(\sum_{k=1}^{1/2 M-1} a_k (je)^{2k-1} - i \sum_{k=1}^{1/2 M-1} b_k (je)^{2k-1} \right), \\ e_{N+j} &= 2i \left(\sum_{k=1}^{1/2 M-1} a_k (je)^{2k-1} + i \sum_{k=1}^{1/2 M-1} b_k (je)^{2k-1} \right), \quad 1 \leq j \leq N, \\ a_k, b_k &\in \mathbf{R}, \quad 1 \leq k \leq N-1. \end{aligned} \quad (10)$$

ϵ_ν , $1 \leq \nu \leq 2N$ are real numbers defined by :

$$\epsilon_j = 1, \quad \epsilon_{N+j} = 0 \quad 1 \leq j \leq N. \quad (11)$$

3 Explicit expression of rational solutions of order 7 depending on 12 parameters

In the the following, we explicitly construct rational solutions to the KPI equation of order 7 depending on 12 parameters.

We cannot give the complete analytic expressions of the solutions to the KPI equation of order 7 with twelve parameters because of their lengths.

The rational solutions to the KPI equation can be written as $v(x, y, t) = -2 \frac{|d_3(x, y, t)|^2}{d_1(x, y, t)^2}$,

with d_3 and d_1 polynomials of degree 112 in x , y and t . The number of terms of the polynomials of the numerator d_3 and denominator d_1 of the solutions are shown in the table below (Table 1) when only one of the parameters a_i and b_i are set non equal to 0.

| a_i | d_3 | d_1 | b_i | d_3 | d_1 |
|-------|--------|--------|-------|--------|--------|
| 1 | 86 927 | 46 383 | 1 | 86 926 | 45 036 |
| 2 | 55 509 | 29 584 | 2 | 55 509 | 28 790 |
| 3 | 42 219 | 22 489 | 3 | 42 309 | 21 962 |
| 4 | 34 968 | 18 608 | 4 | 34 968 | 18 167 |
| 5 | 30 342 | 16 132 | 5 | 30 342 | 15 778 |
| 6 | 26 595 | 14 099 | 6 | 26 595 | 13 846 |

Table 1: Number of terms for the polynomials d_3 and d_1 of the solutions to the KPI equation in the case $N = 7$.

We give patterns of the modulus of the solutions in the plane (x, y) of coordinates in functions of parameters $a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5, b_6$ and time t .

The maximum of modulus of these solutions is checked equal in this case $N = 7$ to $2(2N + 1)^2 = 2 \times 15^2 = 450$.

When all the parameters are equal to 0, we obtain the lump L_7 with a highest amplitude of the modulus equal to 450.

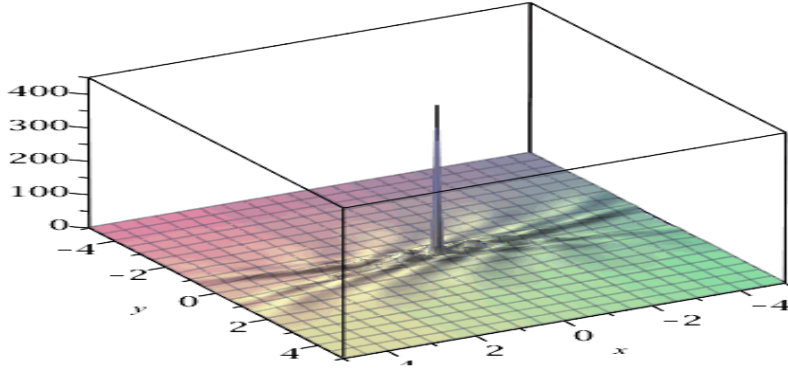


Figure 0. Solution of order 7 to KPI, for $t = 0$ when all parameters equal to 0.

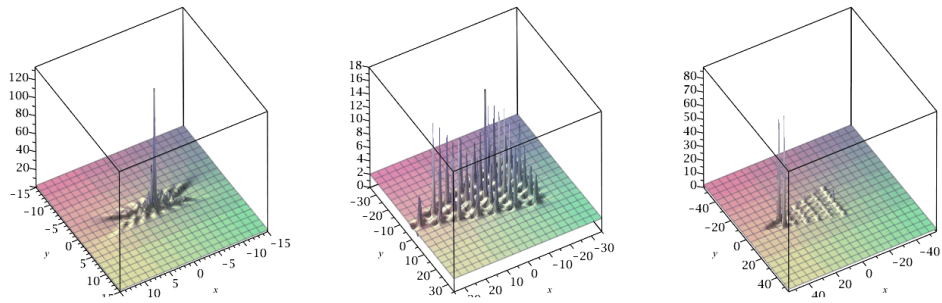


Figure 1. Solution of order 7 to KPI, for $a_1 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $a_1 = 1$; in the center for $t = 0$ and $a_1 = 10^3$; on the right for $t = 1$ and $a_1 = 10^5$.

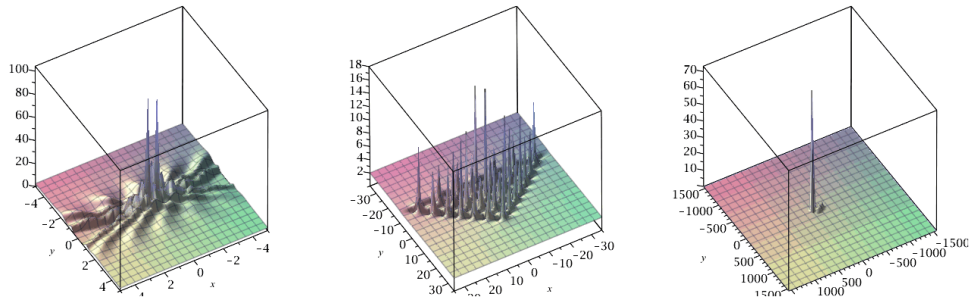


Figure 2. Solution of order 7 to KPI, for $b_1 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $b_1 = 1$; in the center for $t = 0$ and $b_1 = 10^3$; on the right for $t = 10$ and $b_1 = 10^6$.

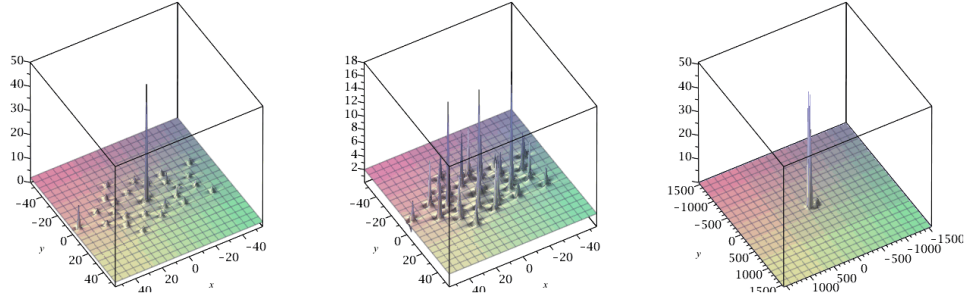


Figure 3. Solution of order 7 to KPI, for $a_2 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $a_2 = 10^6$; in the center for $t = 0.1$ and $a_2 = 10^6$; on the right for $t = 10$ and $a_2 = 10^3$.

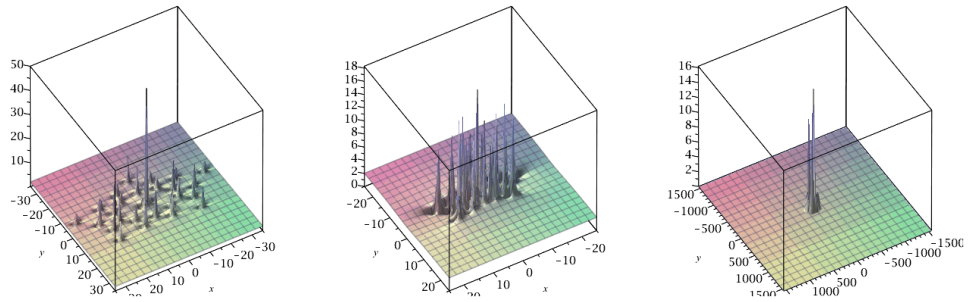


Figure 4. Solution of order 7 to KPI, for $b_2 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $b_2 = 10^5$; in the center for $t = 0.1$ and $b_2 = 10^3$; on the right for $t = 10$ and $b_2 = 10^5$.

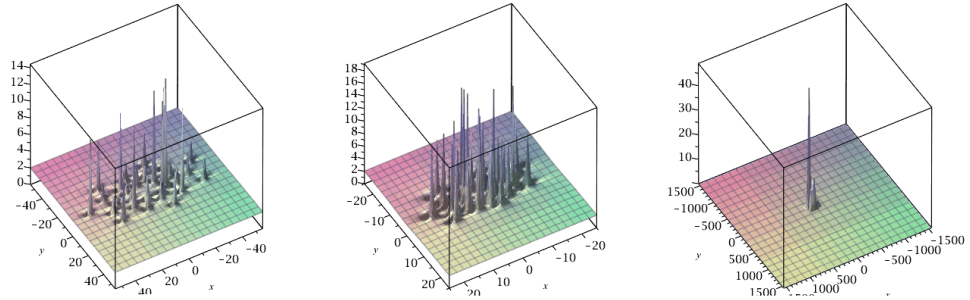


Figure 5. Solution of order 7 to KPI, for $a_3 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $a_3 = 10^8$; in the center for $t = 0.1$ and $a_3 = 10^5$; on the right for $t = 10$ and $a_3 = 10^3$.

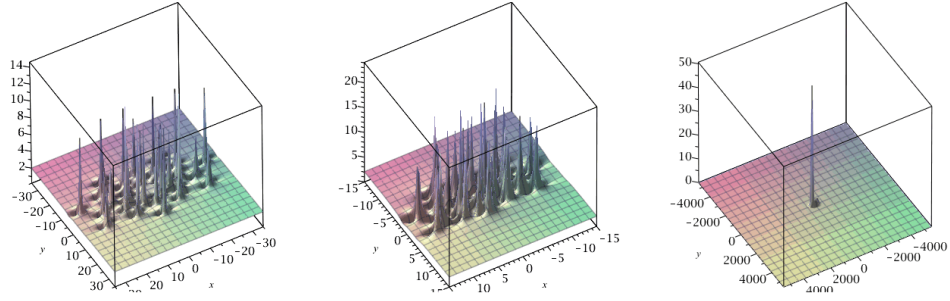


Figure 6. Solution of order 7 to KPI, for $b_3 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $b_3 = 10^7$; in the center for $t = 0.1$ and $b_3 = 10^4$; on the right for $t = 10$ and $b_3 = 10^3$.

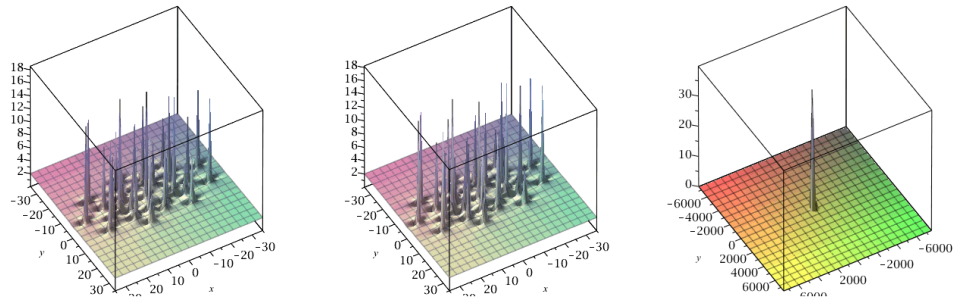


Figure 7. Solution of order 7 to KPI, for $a_4 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $a_4 = 10^9$; in the center for $t = 0.1$ and $a_4 = 10^9$; on the right for $t = 10$ and $a_4 = 10^3$.

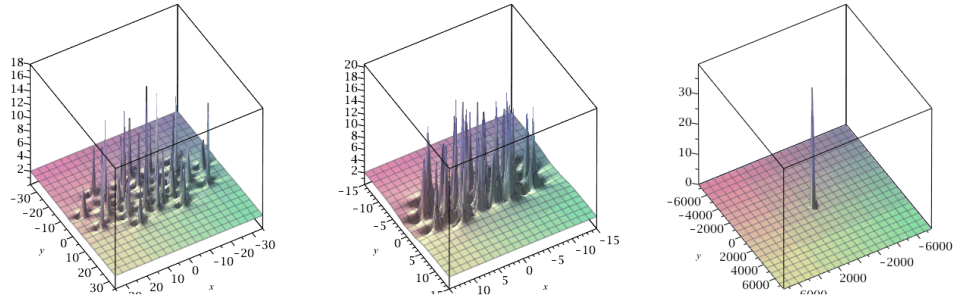


Figure 8. Solution of order 7 to KPI, for $b_4 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $b_4 = 10^9$; in the center for $t = 0.1$ and $b_4 = 10^5$; on the right for $t = 10$ and $b_4 = 10^3$.

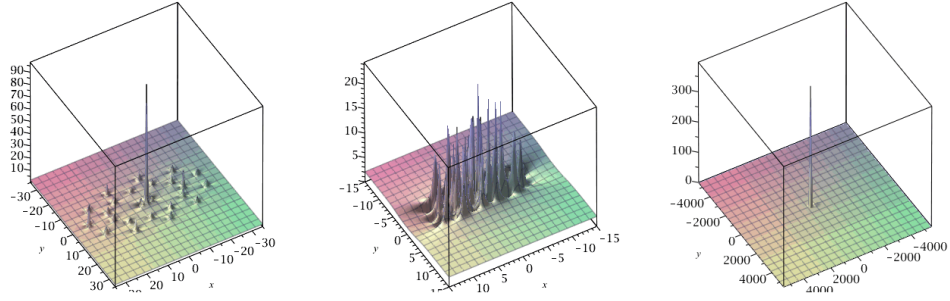


Figure 9. Solution of order 7 to KPI, for $a_5 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $a_5 = 10^{11}$; in the center for $t = 0.1$ and $a_5 = 10^5$; on the right for $t = 20$ and $a_5 = 10^{11}$.

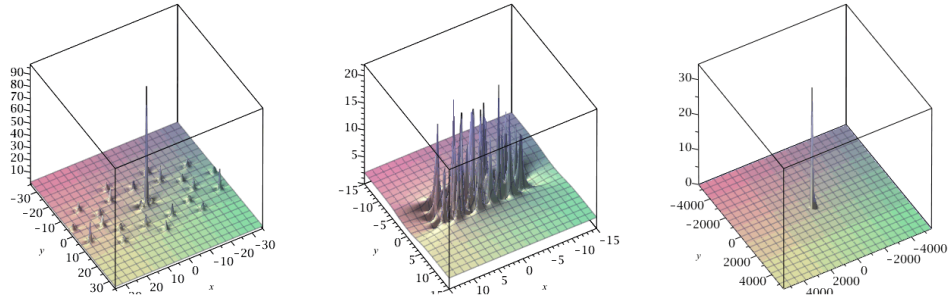


Figure 10. Solution of order 7 to KPI, for $b_5 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $b_5 = 10^{12}$; in the center for $t = 0.1$ and $b_5 = 10^5$; on the right for $t = 50$ and $b_5 = 10^{11}$.

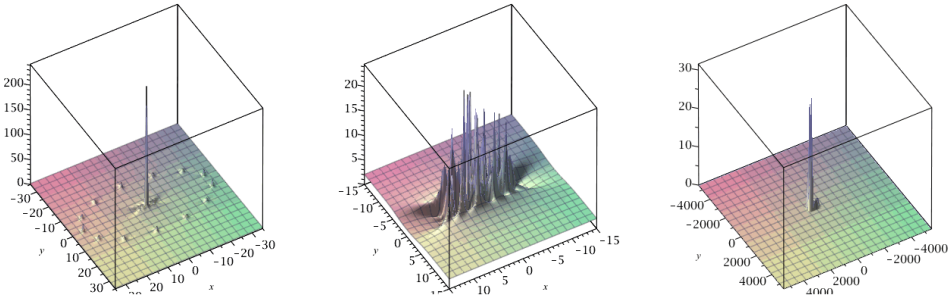


Figure 11. Solution of order 7 to KPI, for $a_6 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $a_6 = 10^{14}$; in the center for $t = 0.1$ and $a_6 = 10^5$; on the right for $t = 20$ and $a_6 = 10^{11}$.

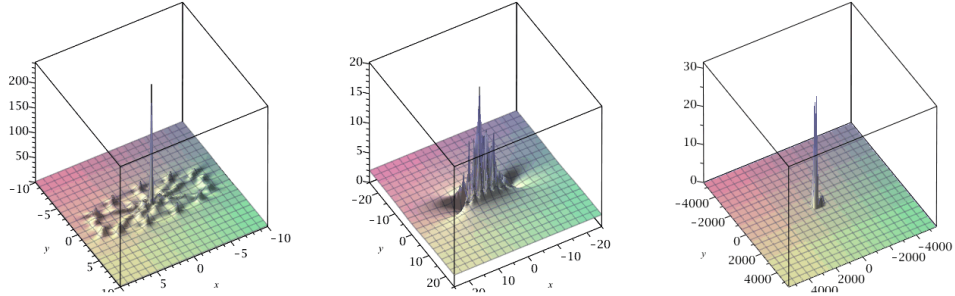


Figure 12. Solution of order 7 to KPI, for $b_6 \neq 0$ and all other parameters equal to 0; on the left for $t = 0$ and $b_6 = 10^6$; in the center for $t = 0.1$ and $b_6 = 10^3$; on the right for $t = 20$ and $b_6 = 10^{11}$.

4 Conclusion

We construct 7-th order rational solutions to the KPI equation depending on 12 real parameters. These solutions can be expressed in terms of a ratio of two polynomials of degree $2N(N + 1) = 112$ in x , y and t . The maximum of the modulus of these solutions is equal to $2(2N + 1)^2 = 450$; this solution which can be called lump L_7 is obtained when all parameters are equal to 0 at the instant $t = 0$. Here we have given a complete description of rational solutions of order 7 with 12 parameters by constructing explicit expressions of polynomials of these solutions.

We deduce the construction of the modulus of solutions in the (x, y) plane of coordinates; different structures appear. For a given t close to 0, when one parameter grows and the other ones are equal to 0 we obtain triangles, rings or concentric rings. There are six types of patterns. In the cases $a_1 \neq 0$ or $b_1 \neq 0$ we obtain triangles with a maximum of 28 peaks (figures 1 and 2); for $a_2 \neq 0$ or $b_2 \neq 0$, we have 3 concentric rings with two of them with 10 peaks and an another with 5 peaks (figures 3 and 4). For $a_3 \neq 0$ or $b_3 \neq 0$, we obtain 4 concentric rings without central peak with 7 peaks on each of them (figures 5 and 6). For $a_4 \neq 0$ or $b_4 \neq 0$, we have 3 concentric rings with 9 peaks, with a peak in the center (figures 7 and 8). For $a_5 \neq 0$ or $b_5 \neq 0$, we obtain 2 concentric rings without central peak with 11 peaks on each of them (figures 9 and 10). For $a_6 \neq 0$ or $b_6 \neq 0$, we have only one ring with 13 peaks (figures 11 and 12). But, when t grows, all the structures disappear very quickly and the heights of the peaks decrease even more quickly.

References

- [1] B.B. Kadomtsev, V.I. Petviashvili, On the stability of solitary waves in weakly dispersing media, Sov. Phys. Dokl., V. 15, N. 6, 539-541, 1970

- [2] S.V. Manakov, V.E. Zakharov, L.A. Bordag, V.B. Matveev, Two-dimensional solitons of the Kadomtsev-Petviashvili equation and their interaction, *Phys. Letters*, V. 63A, N. 3, 205-206, 1977
- [3] I. Krichever, Rational solutions of the Kadomtcev-Petviashvili equation and integrable systems of n particles on a line, *Funct. Anal. and Appl.*, V. 12, N. 1 : 76-78, 1978
- [4] I. Krichever, S.P. Novikov Holomorphic bundles over Riemann surfaces and the KPI equation, *Funkt. Ana. E Pril.*, V. 12, 41-52, 1979
- [5] J. Satsuma, M.J. Ablowitz, Two-dimensional lumps in nonlinear dispersive systems, *J. Math. Phys.*, V. 20, 1496-1503, 1979
- [6] V.B. Matveev, Darboux transformation and explicit solutions of the Kadomtcev-Petviashvili equation depending on functional parameters, *Letters in Mathematical Physics*, V. 3, 213-216, 1979
- [7] P. Gaillard, V.B. Matveev, Wronskian addition formula and its applications, *Max-Planck-Institut für Mathematik*, MPI 02-31, V. **161**, 2002
- [8] P. Gaillard, A new family of deformations of Darboux-Pöschl-Teller potentials, *Lett. Math. Phys.*, V. **68**, 77-90, 2004
- [9] P. Gaillard, V.B. Matveev, New formulas for the eigenfunctions of the two-particle Calogero-Moser system, *Lett. Math. Phys.*, V. **89**, 1-12, 2009
- [10] P. Gaillard, V.B. Matveev, Wronskian and Casorai determinant representations for Darboux-Pöschl-Teller potentials and their difference extensions, *RIMS Kyoto*, N. 1653, 1-19, 2009
- [11] P. Gaillard, V.B. Matveev, Wronskian and Casorai determinant representations for Darboux-Pöschl-Teller potentials and their difference extensions, *J. Phys A : Math. Theor.*, V. **42**, 1-16, 2009
- [12] P. Gaillard, From finite-gap solutions of KdV in terms of theta functions to solitons and positons, *halshs-00466159*, 2010
- [13] P. Gaillard, Families of quasi-rational solutions of the NLS equation and multi-rogue waves, *J. Phys. A : Meth. Theor.*, V. **44**, 1-15, 2011
- [14] P. Gaillard, Wronskian representation of solutions of the NLS equation and higher Peregrine breathers, *J. Math. Sciences : Adv. Appl.*, V. **13**, N. 2, 71-153, 2012
- [15] P. Gaillard, Degenerate determinant representation of solution of the NLS equation, higher Peregrine breathers and multi-rogue waves, *J. Math. Phys.*, V. **54**, 013504-1-32, 2013
- [16] P. Gaillard, Wronskian representation of solutions of NLS equation and seventh order rogue waves, *J. Mod. Phys.*, V. **4**, N. 4, 246-266, 2013

- [17] P. Gaillard, V.B. Matveev, Wronskian addition formula and Darboux-Pöschl-Teller potentials, *J. Math.*, V. **2013**, ID 645752, 1-10, 2013
- [18] P. Gaillard, Two parameters deformations of ninth Peregrine breather solution of the NLS equation and multi rogue waves, *J. Math.*, V. **2013**, 1-111, 2013
- [19] P. Gaillard, Two-parameters determinant representation of seventh order rogue waves solutions of the NLS equation, *J. Theor. Appl. Phys.*, V. **7**, N. **45**, 1-6, 2013
- [20] P. Gaillard, Six-parameters deformations of fourth order Peregrine breather solutions of the NLS equation, *J. Math. Phys.*, V. **54**, 073519-1-22, 2013
- [21] P. Gaillard, Deformations of third order Peregrine breather solutions of the NLS equation with four parameters, *Phys. Rev. E*, V. **88**, 042903-1-9, 2013
- [22] P. Gaillard, Ten parameters deformations of the sixth order Peregrine breather solutions of the NLS equation, *Phys. Scripta*, V. **89**, 015004-1-7, 2014
- [23] P. Gaillard, The fifth order Peregrine breather and its eight-parameters deformations solutions of the NLS equation, *Commun. Theor. Phys.*, V. **61**, 365-369, 2014
- [24] P. Gaillard, Higher order Peregrine breathers, their deformations and multi-rogue waves, *J. Of Phys. : Conf. Ser.*, V. **482**, 012016-1-7, 2014
- [25] P. Gaillard, M. Gastineau, Eighteen parameter deformations of the Peregrine breather of order ten solutions of the NLS equation, *Int. J. Mod. Phys. C*, V. **26**, N. 2, 1550016-1-14, 2014
- [26] P. Gaillard, Two parameters wronskian representation of solutions of nonlinear Schrödinger equation, eight Peregrine breather and multi-rogue waves, *J. Math. Phys.*, V. **5**, 093506-1-12, 2014
- [27] P. Gaillard, Hierarchy of solutions to the NLS equation and multi-rogue waves, *J. Phys. : Conf. Ser.*, V. **574**, 012031-1-5, 2015
- [28] P. Gaillard, Tenth Peregrine breather solution of the NLS, *Ann. Phys.*, V. **355**, 293-298, 2015
- [29] P. Gaillard, M. Gastineau, The Peregrine breather of order nine and its deformations with sixteen parameters solutions of the NLS equation, *Phys. Lett. A.*, V. **379**, 13091313, 2015
- [30] P. Gaillard, Other $2N-2$ parameters solutions to the NLS equation and $2N+1$ highest amplitude of the modulus of the N -th order AP breather, *J. Phys. A: Math. Theor.*, V. **48**, 145203-1-23, 2015

- [31] P. Gaillard, Multi-parametric deformations of the Peregrine breather of order N solutions to the NLS equation and multi-rogue waves, *Adv. Res.*, V. **4**, 346-364, 2015
- [32] P. Gaillard, Higher order Peregrine breathers solutions to the NLS equation, *Jour. Phys. : Conf. Ser.*, V. **633**, 012106-1-6, 2016
- [33] P. Gaillard, M. Gastineau Patterns of deformations of Peregrine breather of order 3 and 4, solutions to the NLS equation with multi-parameters, *Journal of Theoretical and Applied Physics*, V. **10**,1-7, 2016
- [34] P. Gaillard, M. Gastineau Twenty parameters families of solutions to the NLS equation and the eleventh Peregrine breather, *Commun. Theor. Phys*, V. **65**, 136-144, 2016
- [35] P. Gaillard, Rational solutions to the KPI equation and multi rogue waves, *Annals Of Physics*, V. **367**, 1-5, 2016
- [36] P. Gaillard, M. Gastineau Twenty two parameters deformations of the twelfth Peregrine breather solutions to the NLS equation, *Adv. Res.*, V. , 1-11, 2016
- [37] P. Gaillard, Towards a classification of the quasi rational solutions to the NLS equation, *Theor. And Math. Phys.*, V. **189**, 1440-1449, 2016
- [38] P. Gaillard, Fredholm and Wronskian representations of solutions to the KPI equation and multi-rogue waves, *Jour. of Math. Phys.*, V. **57**, 063505-1-13, doi: 10.1063/1.4953383, 2016
- [39] P. Gaillard, M. Gastineau Families of deformations of the thirteenth Peregrine breather solutions to the NLS equation depending on twenty four parameters, *Jour. Of Bas. And Appl. Res. Int.*, V. **21**, N. 3, 130-139, 2017
- [40] P. Gaillard, From Fredholm and Wronskian representations to rational solutions to the KPI equation depending on $2N^2$ parameters, *Int. Jour. of Appl. Sci. And Math.*, V. **4**, N. 3, 60-70, 2017
- [41] P. Gaillard, Families of Rational Solutions of Order 5 to the KPI Equation Depending on 8 Parameters, *New Hor. in Math. Phys.*, V. 1, N. 1, 26-31, 2017
- [42] P. Gaillard, 6-th order rational solutions to the KPI Equation depending on 10 parameters, *Jour. Of Bas. And Appl. Res. Int.*, V. **21**, N. 2, 92-98, 2017