Emission Noise in an Interacting Quantum Dot: Role of Inelastic Scattering and Asymmetric Coupling to the Reservoirs – Supplemental Material

A. Crépieux¹, S. Sahoo^{2,3}, T.Q. Duong¹, R. Zamoum⁴, and M. Lavagna^{2,5}

A. Crepteux, S. Sanoo ¹, T.G. Duong, R. Zanoun, and M. Euragina
 ¹ Aix Marseille Univ, Université de Toulon, CNRS, CPT UMR 7332, 13288 Marseille, France
 ² Univ. Grenoble Alpes, CEA, INAC-Pheliqs, 38000 Grenoble, France
 ³ Physics Department and Research Center OPTIMAS,

University of Kaiserslautern, 67663 Kaiserslautern, Germany

Université de Bouira, rue Drissi Yahia, Bouira 10000, Algeria and

⁵ Centre National de la Recherche Scientifique - CNRS - 38042 Grenoble, France

In this Supplemental Material (SM), we first present the detailed calculation of the finite-frequency noise for an interacting quantum dot (QD) with asymmetric couplings to the reservoirs (Section A), secondly we prove the relation $M_{LL}^{LL}(\varepsilon,\nu) = |t_1 + t_2 + t_3|^2$ (Section B), where the second formulation appears when we take the coherent superposition of the processes P_1 , P_2 and P_3 . Thirdly, we give the relation between the various transmission amplitudes and coefficients in the case of a non-interacting QD, or when only elastic scattering is present, for which the optical theorem holds, and we derive the noise matrix elements in that case (Section C). Fourthly we give the expression of the self-consistent equations of motion used to determine numerically the retarded Green function in the case of an interacting QD (Section D), and end up by discussing the results obtained for the differential conductance in the case of symmetric and asymmetric couplings (Section E).

A – CALCULATION OF THE CURRENT NOISE FOR AN INTERACTING QD WITH ARBITRARY COUPLING SYMMETRY

We start from the expression for the current noise in a QD given by Eqs. (A11-A15) in Ref. 1 obtained in the flat wideband limit (FWBL) for the conduction band. We get $S_{\alpha\beta}(\nu) = \sum_{i=1}^{5} C_{\alpha\beta}^{(i)}(\nu)$ with

$$\mathcal{C}_{\alpha\beta}^{(1)}(\nu) = \frac{e^2}{h} \delta_{\alpha\beta} \int_{-\infty}^{\infty} d\varepsilon \left[G_{\sigma}^{<}(\varepsilon) \Sigma_{\alpha,\sigma}^{>}(\varepsilon - h\nu) + \Sigma_{\alpha,\sigma}^{<}(\varepsilon) G_{\sigma}^{>}(\varepsilon - h\nu) \right], \tag{S1}$$

$$\mathcal{C}_{\alpha\beta}^{(2)}(\nu) = -\frac{e^2}{h} \int_{-\infty}^{\infty} d\varepsilon \Big[G_{\sigma}^r(\varepsilon) \Sigma_{\alpha,\sigma}^<(\varepsilon) + G_{\sigma}^<(\varepsilon) \Sigma_{\alpha,\sigma}^a(\varepsilon) \Big] \Big[G_{\sigma}^r(\varepsilon - h\nu) \Sigma_{\beta,\sigma}^>(\varepsilon - h\nu) + G_{\sigma}^>(\varepsilon - h\nu) \Sigma_{\beta,\sigma}^a(\varepsilon - h\nu) \Big] , (S2)$$

$$\mathcal{C}^{(3)}_{\alpha\beta}(\nu) = \frac{e^2}{h} \int_{-\infty}^{\infty} d\varepsilon \Big[\Sigma^{<}_{\alpha,\sigma}(\varepsilon) G^r_{\sigma}(\varepsilon) \Sigma^r_{\beta,\sigma}(\varepsilon) + \Sigma^a_{\alpha,\sigma}(\varepsilon) G^{<}_{\sigma}(\varepsilon) \Sigma^r_{\beta,\sigma}(\varepsilon) + \Sigma^a_{\alpha,\sigma}(\varepsilon) G^a_{\sigma}(\varepsilon) \Sigma^{<}_{\beta,\sigma}(\varepsilon) \Big] G^{>}_{\sigma}(\varepsilon - h\nu) , \qquad (S3)$$

$$\mathcal{C}_{\alpha\beta}^{(4)}(\nu) = \frac{e^2}{h} \int_{-\infty}^{\infty} d\varepsilon G_{\sigma}^{<}(\varepsilon) \Big[\Sigma_{\alpha,\sigma}^r(\varepsilon - h\nu) G_{\sigma}^r(\varepsilon - h\nu) \Sigma_{\beta,\sigma}^{>}(\varepsilon - h\nu) + \Sigma_{\alpha,\sigma}^{>}(\varepsilon - h\nu) G_{\sigma}^a(\varepsilon - h\nu) G_{\sigma}^a(\varepsilon - h\nu) \Big], \qquad (S4)$$

$$\mathcal{C}_{\alpha\beta}^{(5)}(\nu) = -\frac{e^2}{h} \int_{-\infty}^{\infty} d\varepsilon \Big[G_{\sigma}^{<}(\varepsilon) \Sigma_{\beta,\sigma}^{r}(\varepsilon) + G_{\sigma}^{a}(\varepsilon) \Sigma_{\beta,\sigma}^{<}(\varepsilon) \Big] \Big[G_{\sigma}^{>}(\varepsilon - h\nu) \Sigma_{\alpha,\sigma}^{r}(\varepsilon - h\nu) + G_{\sigma}^{a}(\varepsilon - h\nu) \Sigma_{\alpha,\sigma}^{>}(\varepsilon - h\nu) \Big] \Big] ,$$
(S5)

where $\sum_{\alpha,\sigma}^{r,a,\gtrless}(\varepsilon) = \sum_{k\in\alpha} |V_{\alpha}|^2 g_{k,\alpha,\sigma}^{r,a,\gtrless}(\varepsilon)$ is the contribution to the self-energy brought by the tunneling between the α reservoir and the QD. $G_{\sigma}^{r,a,\gtrless}(\varepsilon)$ and $g_{k,\alpha,\sigma}^{r,a,\gtrless}(\varepsilon)$ are the retarded, advanced, Keldysh greater and lesser Green functions in the QD and in the disconnected α reservoir respectively. The approximation made to get Eqs. (S1-S5) amounts to having factorized the two-particle Green functions in the QD into a product of two single-particle Green functions. Provided that this approximation is made together with the FWBL assumption, the latter expression for the noise is general and valid for any value of the Coulomb interactions in the QD and any symmetry of the tunneling couplings of the dot to the two reservoirs. We remark that even if the spin index σ appears in the Green function and self-energy notations, it does not appear as an index in the noise notation since the system we consider in this Letter is spin unpolarized. When the system is in a steady state, $G_{\sigma}^{\gtrless}(\varepsilon)$ is simply given by [2, 3]

$$G^{\gtrless}_{\sigma}(\varepsilon) = G^r_{\sigma}(\varepsilon) \Sigma^{\gtrless}_{\text{tot},\sigma}(\varepsilon) G^a_{\sigma}(\varepsilon) .$$
(S6)

⁴ Faculté des sciences et des sciences appliquées,

In the presence of interactions, the total self-energy can be put in the form [2, 4]

$$\Sigma_{\text{tot},\sigma}^{r,a,\gtrless}(\varepsilon) = \Sigma_{L,\sigma}^{r,a,\gtrless}(\varepsilon) + \Sigma_{R,\sigma}^{r,a,\gtrless}(\varepsilon) + \Sigma_{\text{int},\sigma}^{r,a,\gtrless}(\varepsilon) , \qquad (S7)$$

where $\Sigma_{\text{int},\sigma}^{r,a,\gtrless}(\varepsilon)$ is the additional contribution brought by the interactions residing in the central region. In the FWBL, $\Sigma_{\alpha,\sigma}^{r}(\varepsilon) = -i\Gamma_{\alpha}/2, \Sigma_{\alpha,\sigma}^{a}(\varepsilon) = i\Gamma_{\alpha}/2, \Sigma_{\alpha,\sigma}^{<}(\varepsilon) = i\Gamma_{\alpha}f_{\alpha}^{e}(\varepsilon), \text{ and } \Sigma_{\alpha,\sigma}^{>}(\varepsilon) = -i\Gamma_{\alpha}f_{\alpha}^{h}(\varepsilon), \text{ where } \Gamma_{\alpha} = 2\pi\rho_{\alpha}|V_{\alpha}|^{2}, \rho_{\alpha} \text{ being the density of states of the } \alpha \text{ reservoir and } V_{\alpha} \text{ the electron hopping amplitude between the QD and the } \alpha \text{ reservoir.}$

In the absence of interactions, $\Sigma_{\text{int},\sigma}(\varepsilon) = 0$ and the self-energy contains only the tunneling contributions. As a result, the following relation holds: $G_{\sigma}^{r}(\varepsilon) - G_{\sigma}^{a}(\varepsilon) = -iG_{\sigma}^{r}(\varepsilon)(\Gamma_{L} + \Gamma_{R})G_{\sigma}^{a}(\varepsilon)$, ensuring the optical theorem to be satisfied; the expression for the noise in this case is given in the Section C of this SM.

In the general case, when interactions are present, the latter relation no longer holds due to the contribution of the interaction self-energy. However the calculation of the noise can still be done according to the procedure presented below provided that the system is in a steady state. By incorporating Eqs. (S6) and (S7) into the expression of the noise given in Eqs. (S1-S5), one gets

$$\begin{aligned} \mathcal{S}_{\alpha\beta}(\nu) &= \frac{e^2}{h} \Gamma_{\alpha} \delta_{\alpha\beta} \int_{-\infty}^{\infty} d\varepsilon \bigg[f^h_{\alpha}(\varepsilon - h\nu) G^r_{\sigma}(\varepsilon) \sum_{\gamma} \Gamma_{\gamma} f^e_{\gamma}(\varepsilon) G^a_{\sigma}(\varepsilon) + f^e_{\alpha}(\varepsilon) G^r_{\sigma}(\varepsilon - h\nu) \sum_{\gamma} \Gamma_{\gamma} f^h_{\gamma}(\varepsilon - h\nu) G^a_{\sigma}(\varepsilon - h\nu) \bigg] \\ &+ \frac{e^2}{h} \Gamma_{\alpha} \Gamma_{\beta} \int_{-\infty}^{\infty} d\varepsilon \bigg[G^r_{\sigma}(\varepsilon) \sum_{\gamma} \Gamma_{\gamma} f^e_{\gamma}(\varepsilon) G^a_{\sigma}(\varepsilon) G^r_{\sigma}(\varepsilon - h\nu) \sum_{\delta} \Gamma_{\delta} f^h_{\delta}(\varepsilon - h\nu) G^a_{\sigma}(\varepsilon - h\nu) \\ &- f^e_{\alpha}(\varepsilon) f^h_{\beta}(\varepsilon - h\nu) G^r_{\sigma}(\varepsilon) G^r_{\sigma}(\varepsilon - h\nu) - f^e_{\beta}(\varepsilon) f^h_{\alpha}(\varepsilon - h\nu) G^a_{\sigma}(\varepsilon) G^a_{\sigma}(\varepsilon - h\nu) \\ &- i \big[f^e_{\alpha}(\varepsilon) G^r_{\sigma}(\varepsilon) - f^e_{\beta}(\varepsilon) G^a_{\sigma}(\varepsilon) \big] G^r_{\sigma}(\varepsilon - h\nu) \sum_{\gamma} \Gamma_{\gamma} f^h_{\gamma}(\varepsilon - h\nu) G^a_{\sigma}(\varepsilon - h\nu) \\ &+ i \big[f^h_{\alpha}(\varepsilon - h\nu) G^a_{\sigma}(\varepsilon - h\nu) - f^h_{\beta}(\varepsilon - h\nu) G^r_{\sigma}(\varepsilon - h\nu) \big] G^r_{\sigma}(\varepsilon) \sum_{\gamma} \Gamma_{\gamma} f^e_{\gamma}(\varepsilon) G^a_{\sigma}(\varepsilon) \bigg] \,. \end{aligned}$$

The r.h.s. of Eq. (S8) results from the contribution proportional either to $\Sigma_{\gamma,\sigma}^{<}(\varepsilon)\Sigma_{\delta,\sigma}^{>}(\varepsilon-h\nu)$ or to $\Sigma_{\gamma,\sigma}^{>}(\varepsilon)\Sigma_{\delta,\sigma}^{<}(\varepsilon-h\nu)$ in Eqs. (S1-S5), once we have inserted Eqs. (S6) and (S7). We have checked that the remaining contributions coming from the linear and quadratic terms in $\Sigma_{int,\sigma}^{\geq}(\varepsilon)$, $\Sigma_{int,\sigma}^{\geq}(\varepsilon-h\nu)$ cancel in the steady state.

Expression for the auto-correlators $S_{LL}(\nu)$ and $S_{RR}(\nu)$

From Eq. (S8), taking $\alpha = \beta = L$ and rearranging the various terms, we get the expression of the auto-correlator noise associated with the L reservoir, that is

$$S_{LL}(\nu) = \frac{e^2}{h} \int_{-\infty}^{\infty} d\varepsilon \bigg[\sum_{\gamma} \Gamma_L \Gamma_{\gamma} G_{\sigma}^r(\varepsilon) G_{\sigma}^a(\varepsilon) f_{\gamma}^e(\varepsilon) f_L^h(\varepsilon - h\nu) + \sum_{\gamma} \Gamma_L \Gamma_{\gamma} G_{\sigma}^r(\varepsilon - h\nu) G_{\sigma}^a(\varepsilon - h\nu) f_L^e(\varepsilon) f_{\gamma}^h(\varepsilon - h\nu) + \sum_{\gamma,\delta} \Gamma_L^2 \Gamma_{\gamma} \Gamma_{\delta} G_{\sigma}^r(\varepsilon) G_{\sigma}^a(\varepsilon) G_{\sigma}^r(\varepsilon - h\nu) G_{\sigma}^a(\varepsilon - h\nu) f_{\gamma}^e(\varepsilon) f_{\delta}^h(\varepsilon - h\nu) - \Gamma_L^2 \big[G_{\sigma}^r(\varepsilon) G_{\sigma}^r(\varepsilon - h\nu) + G_{\sigma}^a(\varepsilon) G_{\sigma}^a(\varepsilon - h\nu) \big] f_L^e(\varepsilon) f_L^h(\varepsilon - h\nu) - i \sum_{\gamma} \Gamma_L^2 \Gamma_{\gamma} \big[G_{\sigma}^r(\varepsilon) - G_{\sigma}^a(\varepsilon) \big] G_{\sigma}^r(\varepsilon - h\nu) G_{\sigma}^a(\varepsilon - h\nu) f_L^e(\varepsilon) f_{\gamma}^h(\varepsilon - h\nu) + i \sum_{\gamma} \Gamma_L^2 \Gamma_{\gamma} \big[G_{\sigma}^a(\varepsilon - h\nu) - G_{\sigma}^r(\varepsilon - h\nu) \big] G_{\sigma}^r(\varepsilon) G_{\sigma}^a(\varepsilon) f_{\gamma}^e(\varepsilon) f_L^h(\varepsilon - h\nu) \bigg] .$$
(S9)

Introducing the transmission amplitude $t_{\alpha\beta}(\varepsilon) = i\sqrt{\Gamma_{\alpha}\Gamma_{\beta}}G_{\sigma}^{r}(\varepsilon)$ and the transmission coefficient $\mathcal{T}_{\alpha\beta}(\varepsilon) = |t_{\alpha\beta}(\varepsilon)|^{2} = \Gamma_{\alpha}\Gamma_{\beta}G_{\sigma}^{r}(\varepsilon)G_{\sigma}^{a}(\varepsilon)$ in this expression, and performing the sum over γ and δ , we get

$$S_{LL}(\nu) = \frac{e^2}{h} \int_{-\infty}^{\infty} d\varepsilon \Big[\mathcal{T}_{LR}(\varepsilon) \mathcal{T}_{LR}(\varepsilon - h\nu) f_R^e(\varepsilon) f_R^h(\varepsilon - h\nu) \\ + \Big[\mathcal{T}_{LL}(\varepsilon) + \mathcal{T}_{LL}(\varepsilon - h\nu) + \mathcal{T}_{LL}(\varepsilon) \mathcal{T}_{LL}(\varepsilon - h\nu) + t_{LL}(\varepsilon) t_{LL}(\varepsilon - h\nu) + t_{LL}^*(\varepsilon) t_{LL}^*(\varepsilon - h\nu) \\ - 2 \operatorname{Re} \{ t_{LL}(\varepsilon) \} \mathcal{T}_{LL}(\varepsilon - h\nu) - 2 \operatorname{Re} \{ t_{LL}(\varepsilon - h\nu) \} \mathcal{T}_{LL}(\varepsilon) \Big] f_L^e(\varepsilon) f_L^h(\varepsilon - h\nu) \\ + \Big[1 - \Big[2 \operatorname{Re} \{ t_{LL}(\varepsilon) \} - \mathcal{T}_{LL}(\varepsilon) \Big] \Big] \mathcal{T}_{LR}(\varepsilon - h\nu) f_L^e(\varepsilon) f_R^h(\varepsilon - h\nu) \\ + \Big[1 - \Big[2 \operatorname{Re} \{ t_{LL}(\varepsilon - h\nu) \} - \mathcal{T}_{LL}(\varepsilon - h\nu) \Big] \Big] \mathcal{T}_{LR}(\varepsilon) f_R^e(\varepsilon) f_L^h(\varepsilon - h\nu) \Big] .$$
(S10)

We have used this result to write the expression of the matrix elements $M_{LL}^{\gamma\delta}(\varepsilon,\nu)$ along the first row of Table I introducing an effective transmission coefficient defined as: $\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon) = 2\text{Re}\{t_{LL}(\varepsilon)\} - \mathcal{T}_{LL}(\varepsilon)$. The expression for the auto-correlator noise in the *R* reservoir, $\mathcal{S}_{RR}(\nu)$ given along the second row of Table I, is obtained from $\mathcal{S}_{LL}(\nu)$ by interchanging the indices *L* and *R*.

Expression for the cross-correlators $S_{LR}(\nu)$ and $S_{RL}(\nu)$

From Eq. (S8), taking $\alpha = L$ and $\beta = R$ and rearranging the various terms, we get the expression for the cross-correlator noise, that is

$$S_{LR}(\nu) = \frac{e^2}{h} \Gamma_L \Gamma_R \int_{-\infty}^{\infty} d\varepsilon \bigg[\sum_{\gamma,\delta} \Gamma_{\gamma} \Gamma_{\delta} G_{\sigma}^r(\varepsilon) G_{\sigma}^a(\varepsilon) G_{\sigma}^r(\varepsilon - h\nu) G_{\sigma}^a(\varepsilon - h\nu) f_{\gamma}^e(\varepsilon) f_{\delta}^h(\varepsilon - h\nu) - G_{\sigma}^r(\varepsilon) G_{\sigma}^r(\varepsilon - h\nu) f_L^e(\varepsilon) f_R^h(\varepsilon - h\nu) - G_{\sigma}^a(\varepsilon) G_{\sigma}^a(\varepsilon - h\nu) f_R^e(\varepsilon) f_L^h(\varepsilon - h\nu) - i \sum_{\gamma} \Gamma_{\gamma} \big[f_L^e(\varepsilon) G_{\sigma}^r(\varepsilon) - f_R^e(\varepsilon) G_{\sigma}^a(\varepsilon) \big] G_{\sigma}^r(\varepsilon - h\nu) G_{\sigma}^a(\varepsilon - h\nu) f_{\gamma}^h(\varepsilon - h\nu) + i \sum_{\gamma} \Gamma_{\gamma} \big[f_L^h(\varepsilon - h\nu) G_{\sigma}^a(\varepsilon - h\nu) - f_R^h(\varepsilon - h\nu) G_{\sigma}^r(\varepsilon - h\nu) \big] G_{\sigma}^r(\varepsilon) G_{\sigma}^a(\varepsilon) f_{\gamma}^e(\varepsilon) \bigg] .$$
(S11)

Introducing the transmission amplitude and coefficients, and performing the sum over γ and δ , we get

$$S_{LR}(\nu) = \frac{e^2}{h} \int_{-\infty}^{\infty} d\varepsilon \bigg[\bigg[\mathcal{T}_{LR}(\varepsilon) \mathcal{T}_{LL}(\varepsilon - h\nu) - t_{LL}(\varepsilon) \mathcal{T}_{LR}(\varepsilon - h\nu) - \mathcal{T}_{LR}(\varepsilon) t_{LL}^*(\varepsilon - h\nu) \bigg] f_L^e(\varepsilon) f_L^h(\varepsilon - h\nu) + \bigg[\mathcal{T}_{LR}(\varepsilon) \mathcal{T}_{RR}(\varepsilon - h\nu) - t_{RR}^*(\varepsilon) \mathcal{T}_{LR}(\varepsilon - h\nu) - \mathcal{T}_{LR}(\varepsilon) t_{RR}(\varepsilon - h\nu) \bigg] f_R^e(\varepsilon) f_R^h(\varepsilon - h\nu) + \bigg[\mathcal{T}_{LR}(\varepsilon) \mathcal{T}_{LR}(\varepsilon - h\nu) + t_{LR}(\varepsilon) t_{LR}(\varepsilon - h\nu) - t_{RR}(\varepsilon) \mathcal{T}_{LR}(\varepsilon - h\nu) - \mathcal{T}_{LR}(\varepsilon) t_{LL}(\varepsilon - h\nu) \bigg] f_L^e(\varepsilon) f_R^h(\varepsilon - h\nu) + \bigg[\mathcal{T}_{LR}(\varepsilon) \mathcal{T}_{LR}(\varepsilon - h\nu) + t_{LR}^*(\varepsilon) t_{LR}^*(\varepsilon - h\nu) - t_{LL}^*(\varepsilon) \mathcal{T}_{LR}(\varepsilon - h\nu) - \mathcal{T}_{LR}(\varepsilon) t_{RR}^*(\varepsilon - h\nu) \bigg] f_R^e(\varepsilon) f_R^h(\varepsilon - h\nu) \bigg] .$$
(S12)

We have used this result to write the expression of the matrix elements $M_{LR}^{\gamma\delta}(\varepsilon,\nu)$ along the third row of Table I. The cross-correlator $S_{RL}(\nu)$, given along the fourth row of Table I, is obtained from the expression of $S_{LR}(\nu)$ by interchanging the indices L and R.

B – **PROOF OF THE RELATION:** $M_{LL}^{LL}(\varepsilon, \nu) = |t_1 + t_2 + t_3|^2$

From Table I, we have

$$M_{LL}^{LL}(\varepsilon,\nu) = |t_{LL}(\varepsilon) - t_{LL}(\varepsilon - h\nu)|^2 + \mathcal{T}_{LR}^{\text{eff},L}(\varepsilon)\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon - h\nu) , \qquad (S13)$$

with $\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon) = 2\text{Re}\{t_{LL}(\varepsilon)\} - \mathcal{T}_{LL}(\varepsilon)$, thus

$$M_{LL}^{LL}(\varepsilon,\nu) = |t_{LL}(\varepsilon) - t_{LL}(\varepsilon - h\nu)|^2 + \left[2\operatorname{Re}\{t_{LL}(\varepsilon)\} - \mathcal{T}_{LL}(\varepsilon)\right] \left[2\operatorname{Re}\{t_{LL}(\varepsilon - h\nu)\} - \mathcal{T}_{LL}(\varepsilon - h\nu)\right]$$

$$= |t_{LL}(\varepsilon) - t_{LL}(\varepsilon - h\nu)|^2 + \left[t_{LL}(\varepsilon) + t_{LL}^*(\varepsilon)\right] \left[t_{LL}(\varepsilon - h\nu) + t_{LL}^*(\varepsilon - h\nu)\right]$$

$$+ \mathcal{T}_{LL}(\varepsilon)\mathcal{T}_{LL}(\varepsilon - h\nu) - 2\operatorname{Re}\{t_{LL}(\varepsilon)\}\mathcal{T}_{LL}(\varepsilon - h\nu) - 2\operatorname{Re}\{t_{LL}(\varepsilon - h\nu)\}\mathcal{T}_{LL}(\varepsilon).$$
(S14)

Knowing that $|t_{LL}(\varepsilon) - t_{LL}(\varepsilon - h\nu)|^2 = \mathcal{T}_{LL}(\varepsilon) + \mathcal{T}_{LL}(\varepsilon - h\nu) - t^*_{LL}(\varepsilon)t_{LL}(\varepsilon - h\nu) - t_{LL}(\varepsilon)t^*_{LL}(\varepsilon - h\nu)$, this leads to

$$M_{LL}^{LL}(\varepsilon,\nu) = \mathcal{T}_{LL}(\varepsilon) + \mathcal{T}_{LL}(\varepsilon - h\nu) + t_{LL}(\varepsilon)t_{LL}(\varepsilon - h\nu) + t_{LL}^*(\varepsilon)t_{LL}^*(\varepsilon - h\nu) + \mathcal{T}_{LL}(\varepsilon)\mathcal{T}_{LL}(\varepsilon - h\nu) - 2\operatorname{Re}\{t_{LL}(\varepsilon)\}\mathcal{T}_{LL}(\varepsilon - h\nu) - 2\operatorname{Re}\{t_{LL}(\varepsilon - h\nu)\}\mathcal{T}_{LL}(\varepsilon) , \qquad (S15)$$

which can be factorized under the form

$$M_{LL}^{LL}(\varepsilon,\nu) = \mathcal{T}_{LL}(\varepsilon) + [1 - t_{LL}(\varepsilon)][1 - t_{LL}^*(\varepsilon)]\mathcal{T}_{LL}(\varepsilon - h\nu) + t_{LL}(\varepsilon)[1 - t_{LL}^*(\varepsilon)]t_{LL}(\varepsilon - h\nu) + t_{LL}^*(\varepsilon)[1 - t_{LL}(\varepsilon)]t_{LL}^*(\varepsilon - h\nu) = \mathcal{T}_{LL}(\varepsilon) + \mathcal{R}_{LL}(\varepsilon)\mathcal{T}_{LL}(\varepsilon - h\nu) + t_{LL}(\varepsilon)r_{LL}^*(\varepsilon)t_{LL}(\varepsilon - h\nu) + t_{LL}^*(\varepsilon)r_{LL}(\varepsilon)t_{LL}^*(\varepsilon - h\nu) , \quad (S16)$$

where we have used the definitions of the reflection amplitude: $r_{LL}(\varepsilon) = 1 - t_{LL}(\varepsilon)$, and the reflection coefficient: $\mathcal{R}_{LL}(\varepsilon) = r_{LL}(\varepsilon)r_{LL}^*(\varepsilon)$. Note that we have also the relation: $\mathcal{R}_{LL}(\varepsilon) = 1 - \mathcal{T}_{LR}^{\text{eff},L}(\varepsilon)$.

The terms appearing in Eq. (S16) correspond precisely to the terms appearing in $|t_1 + t_2 + t_3|^2$, with $t_1 = t_{LL}(\varepsilon)t_{LL}^*(\varepsilon - h\nu)$, $t_2 = t_{LL}(\varepsilon)r_{LL}^*(\varepsilon - h\nu)$ and $t_3 = r_{LL}(\varepsilon)t_{LL}^*(\varepsilon - h\nu)$, so that we finally obtain

$$M_{LL}^{LL}(\varepsilon,\nu) = |t_1 + t_2 + t_3|^2 .$$
(S17)

C – NON-INTERACTING LIMIT OR ELASTIC SCATTERING LIMIT

In this Section, we show that when we neglect the interactions in the QD, or when we have only elastic scattering, the Table I of Ref. 1 can be derived from the Table I given in the main text of this Letter. Indeed, in that case there are specific relations between the transmission and reflection coefficients which can be obtained from the optical theorem as summarized below.

1 – Optical theorem

We define the S-matrix of a QD connected to a L reservoir and a R reservoir as $\mathbf{S} = \mathbb{1} + i\mathbf{T}$, where the T-matrix is given by

$$\mathbf{T} = \begin{pmatrix} \tau_{LL}(\varepsilon) & \tau_{LR}(\varepsilon) \\ \tau_{RL}(\varepsilon) & \tau_{RR}(\varepsilon) \end{pmatrix} , \qquad (S18)$$

in the $\{L, R\}$ basis, where $\tau_{\alpha\beta}(\varepsilon) = it_{\alpha\beta}(\varepsilon)$ with $t_{\alpha\beta}(\varepsilon)$ the transmission amplitude from the α reservoir to the β reservoir. For a QD, we have $\tau_{\alpha\beta}(\varepsilon) = -\sqrt{\Gamma_{\alpha}\Gamma_{\beta}}G_{\sigma}^{r}(\varepsilon)$, and thus $\tau_{RL}(\varepsilon) = \tau_{LR}(\varepsilon)$. At low temperature, when only elastic scattering of electrons are considered, **S** is a unitary matrix, $\mathbf{SS}^{+} = \mathbb{1}$. Consequently the *T*-matrix must fulfill the optical theorem: $\mathbf{TT}^{+} = i[\mathbf{T}^{+} - \mathbf{T}]$, leading to the following relations

$$2\mathrm{Im}\{\tau_{LL}(\varepsilon)\} = \mathcal{T}_{LL}(\varepsilon) + \mathcal{T}_{LR}(\varepsilon) , \qquad (S19)$$

$$2\mathrm{Im}\{\tau_{RR}(\varepsilon)\} = \mathcal{T}_{RR}(\varepsilon) + \mathcal{T}_{LR}(\varepsilon) , \qquad (S20)$$

$$2\mathrm{Im}\{\tau_{LR}(\varepsilon)\} = \tau_{LR}^{*}(\varepsilon)\tau_{LL}(\varepsilon) + \tau_{LR}(\varepsilon)\tau_{RR}^{*}(\varepsilon)$$

$$= \tau_{LR}(\varepsilon)\tau_{LL}^*(\varepsilon) + \tau_{LR}^*(\varepsilon)\tau_{RR}(\varepsilon) , \qquad (S21)$$

where we have defined the transmission coefficient as $\mathcal{T}_{\alpha\beta}(\varepsilon) = |\tau_{\alpha\beta}(\varepsilon)|^2 = |t_{\alpha\beta}(\varepsilon)|^2$. Eqs. (S19) to (S21) can equivalently written in terms of transmission amplitudes as

$$2\operatorname{Re}\{t_{LL}(\varepsilon)\} = \mathcal{T}_{LL}(\varepsilon) + \mathcal{T}_{LR}(\varepsilon) , \qquad (S22)$$

$$2\operatorname{Re}\{t_{RR}(\varepsilon)\} = \mathcal{T}_{RR}(\varepsilon) + \mathcal{T}_{LR}(\varepsilon) , \qquad (S23)$$

$$2\operatorname{Re}\{t_{LR}(\varepsilon)\} = t_{LR}^{*}(\varepsilon)t_{LL}(\varepsilon) + t_{LR}(\varepsilon)t_{RR}^{*}(\varepsilon) = t_{LR}(\varepsilon)t_{LL}^{*}(\varepsilon) + t_{LR}^{*}(\varepsilon)t_{RR}(\varepsilon) .$$
(S24)

Note that these relations are automatically verified if the relation $G_{\sigma}^{r}(\varepsilon) - G_{\sigma}^{a}(\varepsilon) = -iG_{\sigma}^{r}(\varepsilon)(\Gamma_{L} + \Gamma_{R})G_{\sigma}^{a}(\varepsilon)$ holds.

It is easy to show that provided the optical theorem holds, the reflection coefficient $\mathcal{R}_{\alpha\alpha}(\varepsilon) = |r_{\alpha\alpha}(\varepsilon)|^2$ with $r_{\alpha\alpha}(\varepsilon) = 1 + i\tau_{\alpha\alpha}(\varepsilon) = 1 - t_{\alpha\alpha}(\varepsilon)$, reads as

$$\mathcal{R}_{LL}(\varepsilon) = \mathcal{R}_{RR}(\varepsilon) = 1 - \mathcal{T}_{LR}(\varepsilon) .$$
(S25)

2 – Matrix elements appearing in the noise

When the optical theorem holds, which is the case in the non-interacting limit or in the elastic scattering processes limit, the matrix elements involved in Table I of this Letter can be rewritten as shown below in Table II. Indeed, starting from the definition of the effective transmission coefficients, we have

$$\mathcal{T}_{LR}^{\text{eff},\alpha}(\varepsilon) = 2\text{Re}\{t_{\alpha\alpha}(\varepsilon)\} - \mathcal{T}_{\alpha\alpha}(\varepsilon) = 2\text{Im}\{\tau_{\alpha\alpha}(\varepsilon)\} - \mathcal{T}_{\alpha\alpha}(\varepsilon) = \mathcal{T}_{LR}(\varepsilon) , \qquad (S26)$$

thanks to Eqs. (S19) and (S20). Note that for symmetric couplings, i.e. $\Gamma_L = \Gamma_R$, the transmission amplitude does not depend on the reservoir index any longer, and is simply denoted $t(\varepsilon)$, which allows one to derive the matrix elements given in Table I of Ref. 1.

$M^{\gamma\delta}_{\alpha\beta}(\varepsilon,\nu)$	$\gamma=\delta=L$	$\gamma=\delta=R$	$\gamma = L, \delta = R$	$\gamma=R,\delta=L$
$\alpha = L$	$\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-h\nu)$	$\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-h\nu)$	$\left[1 - \mathcal{T}_{LR}(\varepsilon)\right] \mathcal{T}_{LR}(\varepsilon - h\nu)$	$\mathcal{T}_{LR}(\varepsilon)[1-\mathcal{T}_{LR}(\varepsilon-h\nu)]$
$\beta = L$	$+ t_{LL}(\varepsilon)-t_{LL}(\varepsilon-h\nu) ^2$			
$\alpha = R$	$\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-h\nu)$	$\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-h\nu)$	$\mathcal{T}_{LR}(\varepsilon)[1-\mathcal{T}_{LR}(\varepsilon-h\nu)]$	$[1 - \mathcal{T}_{LR}(\varepsilon)]\mathcal{T}_{LR}(\varepsilon - h\nu)$
$\beta = R$		$+ t_{RR}(\varepsilon)-t_{RR}(\varepsilon-h\nu) ^2$		
$\alpha = L$	$t_{LR}(\varepsilon)t_{LR}^*(\varepsilon-h\nu)$	$t_{LR}^*(\varepsilon)t_{LR}(\varepsilon-h u)$	$t_{LR}(\varepsilon)t_{LR}(\varepsilon-h\nu)$	$t_{LR}^*(\varepsilon)t_{LR}^*(\varepsilon-h\nu)$
$\beta = R$	$\times [r_{LL}^*(\varepsilon)r_{LL}(\varepsilon - h\nu) - 1]$	$\times [r_{RR}(\varepsilon)r_{RR}^*(\varepsilon-h\nu)-1]$	$\times r_{LL}^*(\varepsilon)r_{RR}^*(\varepsilon-h\nu)$	$\times r_{RR}(\varepsilon)r_{LL}(\varepsilon - h\nu)$
$\alpha = R$	$t_{LR}^*(\varepsilon)t_{LR}(\varepsilon-h\nu)$	$t_{LR}(\varepsilon)t_{LR}^*(\varepsilon-h\nu)$	$t_{LR}^*(\varepsilon)t_{LR}^*(\varepsilon-h\nu)$	$t_{LR}(\varepsilon)t_{LR}(\varepsilon-h\nu)$
$\beta = L$	$\times [r_{LL}(\varepsilon)r_{LL}^*(\varepsilon - h\nu) - 1]$	$\times [r_{RR}^*(\varepsilon)r_{RR}(\varepsilon-h\nu)-1]$	$\times r_{LL}(\varepsilon)r_{RR}(\varepsilon - h\nu)$	$\times r_{RR}^*(\varepsilon)r_{LL}^*(\varepsilon-h\nu)$

TABLE II: Expressions of the matrix elements $M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\nu)$ for a non-interacting QD, or in the presence of elastic scattering processes only.

D – NUMERICAL CALCULATION OF $G_{\sigma}^{r}(\varepsilon)$ IN THE PRESENCE OF COULOMB INTERACTIONS

When Coulomb interactions U are present in the QD, it is necessary to take the spin degree of freedom into account. Indeed the Hamiltonian describing our system is the single-site Anderson Hamiltonian including on-site Coulomb interaction and reads as

$$H = \sum_{k,\alpha \in (L,R),\sigma} \varepsilon_{k\alpha\sigma} c_{k\alpha\sigma} c_{k\alpha\sigma} + \sum_{\sigma} \varepsilon_0 d_{\sigma}^{\dagger} d_{\sigma} + U n_{\uparrow} n_{\downarrow} + \sum_{k,\alpha \in (L,R),\sigma} (V_{\alpha} c_{k\alpha\sigma}^{\dagger} d_{\sigma} + h.c.) , \qquad (S27)$$

where d^{\dagger}_{σ} $(c^{\dagger}_{k\alpha\sigma})$ and d_{σ} $(c_{k\alpha\sigma})$ are the creation and annihilation operators of an electron in the QD (α reservoir) respectively, and $n_{\sigma} = d^{\dagger}_{\sigma} d_{\sigma}$. Following Refs. 5–7, we numerically calculate the retarded Green function using the expression

$$G_{\sigma}^{r}(\varepsilon) = \frac{1 - \langle n_{\bar{\sigma}} \rangle}{\varepsilon - \varepsilon_{0} - \Sigma_{\sigma}^{0}(\varepsilon) - \Pi_{\sigma}^{(1)}(\varepsilon)} + \frac{\langle n_{\bar{\sigma}} \rangle}{\varepsilon - \varepsilon_{0} - U - \Sigma_{\sigma}^{0}(\varepsilon) - \Pi_{\sigma}^{(2)}(\varepsilon)},$$
(S28)

with $\Sigma^0_{\sigma}(\varepsilon) = -i\Gamma_{\sigma}(\varepsilon)$ and $\Gamma_{\sigma}(\varepsilon) = \sum_{\alpha=L,R} \Gamma_{\alpha\sigma}(\varepsilon)$. In the FWBL, $\Sigma^0_{\sigma}(\varepsilon)$ is independent of ε and takes the value $-i\Gamma_{\sigma}$. $\Pi^{(1)}_{\sigma}(\varepsilon)$ and $\Pi^{(2)}_{\sigma}(\varepsilon)$ are defined as

$$\Pi_{\sigma}^{(1)}(\varepsilon) = -U \frac{\Sigma_{\sigma}^{(1)}(\varepsilon) - (\varepsilon - \varepsilon_0) \Sigma_{\sigma}^{(4)}(\varepsilon)}{\varepsilon - \varepsilon_0 - U - \Sigma_{\sigma}^{(3)}(\varepsilon) + U \Sigma_{\sigma}^{(4)}(\varepsilon)} , \qquad (S29)$$

$$\Pi_{\sigma}^{(2)}(\varepsilon) = U \frac{\Sigma_{\sigma}^{(2)}(\varepsilon) + (\varepsilon - \varepsilon_0 - U)\Sigma_{\sigma}^{(4)}(\varepsilon)}{\varepsilon - \varepsilon_0 - \Sigma_{\sigma}^{(3)}(\varepsilon) + U\Sigma_{\sigma}^{(4)}(\varepsilon)} , \qquad (S30)$$

where for $i \in [1, 4]$,

$$\Sigma_{\sigma}^{(i)}(\varepsilon) = \sum_{k,\alpha} |V_{\alpha}|^{2} \left[\frac{\mathcal{A}_{k\alpha\sigma}^{(i)}}{\varepsilon + \widetilde{\varepsilon}_{\sigma} - \widetilde{\varepsilon}_{\sigma} - \varepsilon_{k\alpha} + i\widetilde{\gamma}_{\sigma}} + \frac{\mathcal{A}_{k\alpha\sigma}^{\prime(i)}}{\varepsilon + \widetilde{\varepsilon}_{k\alpha} - \widetilde{\varepsilon}_{\sigma} - \widetilde{\varepsilon}_{\sigma} - U + i\widetilde{\gamma}_{D}} \right],$$
(S31)

with $\tilde{\varepsilon}_{\sigma} = \varepsilon_0 + \operatorname{Re}\{\Sigma_{\sigma}^{(1)}(\tilde{\varepsilon}_{\sigma})\}, \ \mathcal{A}_{k\alpha\sigma}^{(1)} = \sum_{k'} \langle c_{k'\alpha\bar{\sigma}}^{\dagger} c_{k\alpha\bar{\sigma}} \rangle, \ \mathcal{A}_{k\alpha\sigma}^{(2)} = 1 - \sum_{k'\alpha} \langle c_{k'\alpha\bar{\sigma}}^{\dagger} c_{k\alpha\bar{\sigma}} \rangle, \ \mathcal{A}_{k\alpha\sigma}^{(3)} = 1, \text{ and } \mathcal{A}_{k\alpha\sigma}^{(4)} = \langle d_{\sigma}^{\dagger} c_{k\alpha\bar{\sigma}} \rangle / V_{\alpha}. \ \mathcal{A}_{k\alpha\sigma}^{\prime(i)} = (\mathcal{A}_{k\alpha\sigma}^{(i)})^* \text{ for } i \in [1,3], \text{ and } \mathcal{A}_{k\alpha\sigma}^{\prime(4)} = -(\mathcal{A}_{k\alpha\sigma}^{(4)})^*. \ \tilde{\gamma}_{\sigma} \text{ and } \tilde{\gamma}_D \text{ are calculated by using the Fermi golden rule up to the fourth order with } V_{\alpha} [6,7]. The numerical calculations are performed self-consistently.$

E – DIFFERENTIAL CONDUCTANCE IN A KONDO QD

In Fig. S1, we report the color-plot of the differential conductance, G = dI/dV, for an interacting QD as a function of the level energy ε_0 and bias voltage V. The results are shown for both symmetric and asymmetric couplings, and both symmetric and asymmetric bias voltage profiles. In the presence of Coulomb interactions (U = 3 meV), we observe a Kondo ridge in the n = 1 conductance valley (n being the QD occupation) characteristic of the Kondo effect which manifests itself at low temperature (see plots in Fig. S1 at V = 0). Here T = 80 mK is lower than the Kondo temperature, $T_K \approx 4.38$ K, estimated from the Haldane formula: $k_B T_K \approx \sqrt{U\Gamma/2} \exp(\pi \varepsilon_0(\varepsilon_0 + U)/2U\Gamma)$, with $\Gamma = \Gamma_L + \Gamma_R$. Moreover, one observes a Coulomb blockade structure: a Coulomb diamond (shown in violet in the center of the Fig. S1(a)) when the bias voltage profile is symmetric, or two parallel branches $(eV = \varepsilon_0 \text{ and } eV = \varepsilon_0 + U)$ when the bias voltage profile is asymmetric (see Fig. S1(c)). The introduction of an asymmetry in the couplings (weakening Γ_R over Γ_L , keeping $\Gamma_R + \Gamma_L$ constant) induces the following two changes in the conductance: (i) the value of the conductance decreases due to the reduced transmission through the QD, as it can be clearly seen by comparing the intensity along the Kondo ridge; (ii) at a given value of ε_0 , the relative height of the two broad peaks is changed, e.g., in case of asymmetric bias voltage profile (Fig. S1(d)), the two sub-branches, $eV = \varepsilon_0 + U < 0$ and $eV = \varepsilon_0 > 0$, becomes more prominent than the two other sub-branches $eV = \varepsilon_0 + U > 0$ and $eV = \varepsilon_0 < 0$.



FIG. S1: Color-plot of the differential conductance G of an interacting QD (in units of e^2/h) as a function of the level energy ε_0 and bias voltage V, for T = 80 mK and U = 3 meV. (a) and (c): symmetric couplings $\Gamma_{L,R} = 0.5$ meV (a = 1). (b) and (d): asymmetric couplings $\Gamma_L = 0.8$ meV, $\Gamma_R = 0.2$ meV (a = 4). (a) and (b): symmetric bias voltage profile $\mu_L = -\mu_R = -eV/2$. (c) and (d): asymmetric bias voltage profile $\mu_L = 0$ and $\mu_R = eV$.

Additional remark: the profile of the bias voltage through the QD and the asymmetry of the left and right couplings are closely linked. It is generally expected that the profile is symmetric at a = 1, and asymmetric at $a \neq 1$. However only the consideration of the electrodynamics of the whole system can help in determining the link. Here, we rather consider all the possible cases in Fig. S1, but choose to focus on an asymmetric bias voltage profile to be able to compare the curves of Fig. 2 given in the main text of this Letter with experiments of Ref. 8.

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