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The Classical-Quantum Duality of Nature.

New Variables for Quantum Gravity

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Abstract: The classical-quantum duality at the basis of quantum theory is here extended to the Planck scale domain. The classical/semiclassical gravity (G) domain is dual (in the precise sense of the classical-quantum duality) to the quantum (Q) elementary particle domain: $O_Q = o_P^2 O_G^{-1}$, o_P being the Planck scale. This duality is *universal*. From the gravity (G) and quantum (Q) variables (O_G, O_Q) , we define new (QG) quantum gravity variables $O_{QG} = (1/2)(O_G + O_Q)$ which include all (classical, semiclassical and quantum gravity) domains passing by the Planck scale and the elementary particle domain. The QG variables are more *complete* than the usual (O_Q, O_G) ones which cover only one domain (Q or G). *Two* O_G or O_Q values (\pm) are needed for each value of O_{QG} (reflecting the *two* different and *dual* ways of reaching the Planck scale). We perform the complete analytic extension of the QG variables through analytic (holomorphic) mappings which preserve the lighth-cone structure. This allows us to reveal the *classical-quantum duality* of the Schwarzschild-Kruskal space-time: exterior regions are classical or semiclassical while the interior is totally *quantum*: its boundaries being the Planck scale. Exterior and interior lose their difference near the horizon: four Planck scale hyperbolae border the horizons as a *quantum dressing* or width: "*l'horizon habillé*". QG variables are naturally invariant under $O_G \rightarrow O_Q$. Space-time reflections, antipodal symmetry and PT or CPT symmetry are contained in the QG symmetry, which also shed insight into the global properties of the Kruskal manifold and its present renewed interest. Further results are presented in another paper.

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I. INTRODUCTION AND RESULTS

Nature is Quantum. That means that the real and complete laws of nature are those of quantum physics. Classical behaviours and domains are particular cases, limiting situations or approximations. Classical gravity, and thus successful General Relativity are incomplete (non quantum) theories and must be considered as a particular approximation from a more complete theory yet to achieve.

A complete quantum theory should include and account for the physics at the Planck scale and domain. As is known, the task to complete such a theory is far from being easy and follows in summary at least two different paths: **(i)** Starting from gravity, that is General Relativity and quantizing it (by applying the different quantization -perturbative and non perturbative- procedures, with the by now well known shortcomings and developpements and its rich bibliography -is not our aim here to review it-: canonical (hamiltonian) quantization, Wheeler-de Witt formalisms, path integral (euclidean) gravity, loop gravity, string theory....). **(ii)** Starting from Quantum theory and trying to extend it to the Planck scale domain. (instead of going from classical gravity to quantum gravity). Of course, in constructing the road (ii) many of the lessons from road (i) are most useful. One tractable and well posed piece of work is semiclassical gravity, in its several degrees of quantization: QFT in curved backgrounds and its (perturbative and non perturbative) back reaction (semiclassical Einstein Equations), and the path integral approach. Examples are the Hawking radiation,

the early universe inflation and the primordial quantum fluctuations, seeds of the structure in the Universe imprinted in the CMB temperature anisotropies and polarization. Moreover, the expanding quantum cosmological vacuum could be the source of the present acceleration of the universe (dark energy) compatible with a cosmological constant.

Let us recall that one of the starting milestones of quantum theory is the concept of classical-quantum (wave-particle) duality. In ref [1] we extended this concept to include the quantum gravity domain, ie wave-particle-gravity duality. We had set up the relevant scales characteristic of the classical gravity regime and related it to the semiclassical and quantum gravity regimes (being QFT or strings) refs [1], [2]. The quantum (Compton) wave length

$$L_Q = \frac{\hbar}{Mc}, \quad (1.1)$$

in the presence of gravity means

$$L_Q = \frac{l_P^2}{L_G}, \quad (L_G = \frac{G}{c^2} M) \quad (1.2)$$

L_G and l_P being the gravitational length and Planck scale length respectively. As the quantum behaviour is dual of the classical behaviour (through \hbar , c), the dual of the classical gravity regime is the quantum gravity regime (through \hbar , G , c), (in string theory, this classical-quantum duality is through \hbar , α' , c) refs [1],[2].

The *quantum mass* scale is the *dual mass*

$$M_Q = \frac{m_P^2}{M}, \quad (m_P \text{ being the Planck mass}). \quad (1.3)$$

Its corresponding temperature scale (energy in units of Boltzmann's constant) is precisely the Hawking temperature

$$T_{eQ} = \frac{1}{2\pi k_B} M_Q c^2, \quad (1.4)$$

which is a quantum temperature, measure of the Compton length (in units of k_B). The set of quantities $O_G = (L_G, M, \mathcal{K}_G, T_e)$ characteristic of the classical gravity regime, (here denoting for instance size, mass, surface gravity (or gravity acceleration), usual temperature respectively), and the corresponding set of quantities in the quantum regime $O_Q = (L_Q, M_Q, \mathcal{K}_Q, T_{eQ})$ are **dual** of each other, (in the precise sense of the wave-particle duality), ie:

$$O_G = o_P^2 O_Q^{-1} \quad (1.5)$$

This relation holding for each quantity in the set. O_G and O_Q are the same conceptual physical quantities in the different (classical/semiclassical and quantum) gravity regimes respectively. o_P being the corresponding quantity at the Planck scale purely depending of (\hbar, c, G) . O_Q standing for the concepts of quantum size L_Q , quantum mass M_Q , quantum acceleration $K_Q = c^2/L_Q$, and quantum temperature T_{eQ} for instance.

This is not an assumed or conjectured duality. As the wave-particle duality, this classical/semiclassical-quantum gravity duality does not relate to the number of dimensions, nor to a particular or imposed symmetry of the background manifold or space-time. These duality relations extend too to the microscopic density of states and entropies (ρ, S) ref [1].

[For the sake of completeness, (although we will not do string theory here) we recall that for strings we found similarly $O_s = o_s^2 O_Q^{-1}$ with α' instead of G/c^2 and o_s^2 purely depending on (\hbar, α', c) . $O_s = (L_s, M_s, \mathcal{K}_s, T_{es})$ denoting here the characteristic string size, string mass, string surface gravity or acceleration and string temperature of the system under consideration, ($T_s = \frac{1}{2\pi k_B} M_s c^2$), which are in general different from the usual flat space string expressions, (in particular they can be equal), refs [1], [2], [3-6]. Again, this is not an assumed or conjectured duality or a *at priori* imposed symmetry: The results of QFT and quantum string dynamics in curved backgrounds remarkably show these relations refs [1], [2]-[6].]

In this paper we go further in the classical-quantum duality to include the Planck scale domain. Each of the sides of the duality above described, for ex. Eq (1.5) and previously treated accounts for only one domain separately: classical *or* quantum, O_G or O_Q but not for both domains together. And the Planck scale o_P is just a fixed scale and a appropriate unit, but not a variable. In this paper we define appropriated quantum gravity (QG) variables fully taking into account all domains, classical and quantum including gravity and their duality properties. From the variables (O_G, O_Q) , we construct QG variables O_{QG} which in units of the corresponding Planck scale magnitude o_P simply read:

$$O_{QG} = \frac{1}{2} (O_G + O_Q), \quad \frac{O_{QG}}{o_P} \equiv O = \frac{1}{2} (x + \frac{1}{x}), \quad x = \frac{O_G}{o_P} = \frac{o_P}{O_Q} \quad (1.6)$$

and which automatically endowe *the symmetry*:

$$O(1/x) = O(x), \quad \text{and} \quad O(x = 1) = 1 \text{ Planck scale.}$$

Fig 1 shows the QG variable $X \equiv L_{QG}/l_P$ against x . For comparison, each of its dual components (L_Q and L_G) are included too.

The QG variables are *complete*: they describe both the classical, semiclassical and quantum gravity domains passing by the Planck scale and the elementary particle domain as well. *Two* values x_{\pm} of the usual variables O_G or O_Q are necessary for each variable QG. The (+) and (-) branches precisely correspond to the two different and dual ways of reaching the Planck scale: from the quantum elementary particle domain ($0 \leq x \leq 1$) on one side and from the classical/semiclassical gravity domain ($1 \leq x \leq \infty$) on the other side. There is thus a *duality* between the quantum domain of elementary particles (without gravity) and the classical gravity domain of macroscopic masses (without the quantum theory).

In other words: The quantum domain of elementary particles is *dual* (in the precise meaning of the classical-quantum duality) of the classical gravity domain. This duality is *universal*. As the wave-particle duality, it does not relate to the number of dimensions or compactifications, nor to any particular imposed symmetry or manifold /space-time.

Similarly, for the QG momentum P and QG time T we have:

$$\frac{T_{QG}}{t_P} \equiv T = \frac{1}{2} \left(t - \frac{1}{t} \right), \quad t \equiv \frac{T_G}{t_P} \quad (1.7)$$

$$\text{which satisfy } T(1/t) = -T(t), \quad T(1) = 0 \text{ at the Planck scale.} \quad (1.8)$$

QG light-cone type variables can be defined and we perform the complete analytic extension of the QG variables through analytic (or holomorphic) mappings which preserve the light-cone type structure. Fig 2 shows the complete QG (X, T) manifold. Four patches I, II, II, IV are necessary to cover the full manifold QG. The (X, T) QG variables are Kruskal's type complete coordinates while the (x, t) variables are like the incomplete Schwarzschild's (or Rindler) ones. The analogy is even more manifest in terms of star coordinates $x = \exp(\kappa x^*)$ related to the Schwarzschild coordinates (r, t^*) by: $x = \sqrt{1 - 2\kappa r} \exp(\kappa r), t = \exp(\kappa t^*)$.

This allows us to reveal the *classical-quantum duality* of the Schwarzschild-Kruskal space-time. The regions which are exterior to the Planck scale hyperbolae $X^2 - T^2 = \pm 1$, (exterior regions I and III), are classical or semiclassical gravity domains while the interior regions are totally *quantum*, from the Planck scale $X^2 - T^2 = \pm 1$ hyperbolae passing the null horizons $X = \pm T$ (at $x = 0, 2\kappa r = 1$), till the Planck scale again (at $x = 1, r = 0$): $T^2 - X^2 = \pm 1$, (regions II and IV). The exterior region I and the interior region II are dual of each other

in the precise sense of the classical-quantum duality. This is also reflected in the duality symmetry $X(1/t) = X(t)$, $T(1/t) = -T(t)$ of the Kruskal manifold which shows up here.

The classical-quantum duality maps the internal region $x \in (0, 1)$ into the external classical domain $x \in (1, \infty)$ through the Planck scale ($x = 1, t = 1$) and conversely. The two identical halves or "worlds" (I, II) and (III, IV) which are space-time reflections and antipodally symmetric of each other are *classical-quantum duals* of each other. The QG duality symmetry includes the classical/semiclassical antipodal space-time symmetry and supports the antipodal identification which translates into the PT or CPT invariance of the quantum theory refs [7],[8]. In a different approach, 't Hooft required recently the antipodal identification for the black hole quantum theory and its unitarity refs [8],[9], and our QG results here support such results.

In the interior regions the Planck scale is reached at the loci $T^2 - X^2 = \pm 1$ ($r = 0$) which are bordering the null horizons $X = \pm T$ (at $2kr = 1$) and asymptotically reaching them for $T \rightarrow \pm\infty$. The *quantum region* extends beyond the horizons and is delimited by the Planck scale hyperbolae $X^2 - T^2 = \pm 1$ and $T^2 - X^2 = \pm 1$. These hyperbolae are acting like a *quantum dressing* or quantum width for the horizon, let us call it *l'horizon habillé* (dressed horizon). 1 is here l_p^2 . We see that *exterior and interior lose its difference near the horizons* $X = \pm T$. Although, is possible to use these words in scales larger than the Planck scale.

There are *two dual* ways of reaching the horizon and the Planck scale: from the exterior (classical/semiclassical) and from the interior (quantum) regions. This appears like a "*splitting*" or *shifting* of the null horizons $X = \pm T$ into the hyperbolae $X^2 - T^2 = \pm 1$. We find a similar result and other new related results when promoting (X, T) to quantum non-commutative coordinates as we do in ref [10].

Let us recall here that quantum field and string dynamics near horizons required quantum "shifts" or widths of the horizon of the order of the Planck scale, refs [11],[12],[13]. In QFT, the horizon shift appears as produced by the non-perturbative back reaction and the dynamics of the ingoing and outgoing modes crossing the horizon ref [11]. In string theory the horizon width appears as due to the non-zero (Planck scale) size of the string refs [12],[13]. We find interesting that in a different approach, and solely from our QG variables, a horizon "quantum dressing" or width does appear (and without searching for it) supporting the quantum nature of this space-time region. The QG variables include

naturally the Planck scale. Other related results are reported in ref [10].

Of course, the idea of a continuum space-time is a classical notion but it could be considered as induced from a QG state within the Planck scale domain. The QG variables or coordinates cover all the domains, classical and quantum, with and without gravity. In the classical and semiclassical gravity domains, they can be used directly as space-time variables or coordinates as we do here. In the QG Planck scale domain and beyond they can be considered as expectation values in a quantum state or they can be used to construct quantum operators as we do in ref [10].

By promoting the QG variables to quantum (non commutative) operators further insight into the quantum space-time structure is obtained and the antipodal identification of the two copies of the Kruskal global manifold is further supported by the quantum theory, ref [10].

QG variables can be also considered in phase-space with their full analytic extension to all values or copies. Comparison of the QG variables with the complete Q-variables of the harmonic oscillator is enlightening.

This paper is organized as follows: In Section II we set up the new QG variables for Quantum Gravity which fully take into account the classical-quantum (wave-particle) duality including gravity. Section III deals with the complete analytic extension of the QG variables, and their symmetry properties. Section IV deals with Schwarzschild-Kruskal black hole manifold, its classical-quantum duality properties, symmetries and extensions in this new context. In Section V we present our remarks and conclusions.

II. NEW VARIABLES FOR QUANTUM GRAVITY (QG) AND THEIR DUALITY PROPERTIES

In classical gravity, length and mass are proportional (through G and c), (gravitational length):

$$L_G = \frac{G}{c^2} M \tag{2.1}$$

In quantum physics, length and mass are inversely proportional (through, \hbar and c), (Compton length):

$$L_Q = \frac{\hbar}{cM} \tag{2.2}$$

The equality $L_G = L_Q$ yielding the Planck scale mass m_P (and length l_P) which satisfies by definition

$$\frac{l_P}{m_P} = \frac{G}{c^2} \quad , \quad l_P m_P = \frac{\hbar}{c}, \quad (2.3)$$

exhibiting the classical - quantum duality between the gravity tension G/c^2 , and the quantum action \hbar/c .

Each length separately L_G or L_Q , (and their respective associated energy scales) accounts only for one physical domain (classical gravity or quantum theory) but not for both of them. Planck scale l_P or m_P accounts for the quantum gravity domain but it is just a fixed scale and an appropriated unit, not a variable.

In the known elementary particle physics domain, all masses are too much smaller with respect to the Planck mass m_P , which is a enormously heavy mass for a elementary particle, L_Q is much larger than l_P , but L_G is much smaller than l_P for the elementary particles although not being quantum gravity objects:

$$\textit{Elementary Particles, } L_G < L_Q : 0 \leq M < m_P, \quad 0 \leq L_G < l_P, \quad l_P < L_Q \leq \infty \quad (2.4)$$

On the other hand, in the classical gravitational domain, masses are much larger than m_P which is a ridiculously small mass for the macroscopic bodies, L_G is larger than l_P but their L_Q is inside l_P , although not being quantum gravitational objects:

$$\textit{Classical Gravity, } L_G > L_Q : m_P < M < \infty, \quad l_P < L_G < \infty, \quad 0 \leq L_Q < l_P \quad (2.5)$$

An appropriated length variable for quantum gravity (QG) fully taking into account *both domains*, classical and quantum is the following:

$$L_{QG} = \frac{1}{2} (L_G + L_Q), \quad \text{that is:} \quad \frac{L_{QG}}{l_P} = \frac{1}{2} \left(\frac{M}{m_P} + \frac{m_P}{M} \right) \quad (2.6)$$

L_{QG} contains *both* L_G and L_Q and their duality through the Planck scale:

- For $m_P < M \leq \infty$: $L_{QG} \simeq L_G, \quad L_G > L_Q$
- For $0 \leq M < m_P$: $L_{QG} \simeq L_Q, \quad L_Q > L_G$
- For $M = m_P$: $L_{QG} = 1 = L_Q = L_G = l_P$

Or, in dimensionless variables:

$$X = \frac{1}{2} \left(x + \frac{1}{x} \right), \quad X \equiv \frac{L_{QG}}{l_P}, \quad x \equiv \frac{M}{m_P} \quad (2.7)$$

These are pure numbers, all magnitudes are defined with respect to their corresponding Planck scale expressions, and the variable X is invariant under the transformation $x \rightarrow x^{-1}$:

$$X(x^{-1}) = X(x), \quad X(x \gg 1) = x, \quad X(x \ll 1) = x^{-1}, \quad X(x = 1) = 1 \quad (2.8)$$

X endows both the classical ($x \gg 1$) and quantum ($x \ll 1$) domains and the Planck domain ($x \approx 1$) as well. The physical classical-quantum duality intrinsically manifests in the QG variable X . Similarly, we have for the QG mass variable M_{QG} :

$$\frac{M_{QG}}{m_P} = \frac{1}{2} \left(\frac{M}{m_P} + \frac{m_P}{M} \right) = \frac{1}{2} \left(x + \frac{1}{x} \right) \quad (2.9)$$

A full set of QG-variables, as Energy $E_{QG} = M_{QG} c^2$, Temperature $T_{eQG} = M_{QG} c^2 / 2\pi k_B$, (k_B being the Boltzmann constant), surface gravity (or gravity acceleration) $K_{QG} = c^2 / X_{QG}$ (or other relevant magnitudes) and their corresponding dimensionless expressions follow in analogous way:

$$\frac{L_{QG}}{l_P} = \frac{M_{QG}}{m_P} = \frac{K_{QG}}{k_P} = \frac{T_{eQG}}{t_{eP}} = \frac{1}{2} \left(x + \frac{1}{x} \right) \quad (2.10)$$

κ_P and t_{eP} being the Planck acceleration (Planck surface gravity) and the Planck temperature respectively:

$$\kappa_P \equiv \frac{c^2}{l_P}, \quad t_{eP} \equiv \frac{1}{2\pi k_B} m_P c^2 \quad (2.11)$$

Several comments are in order here:

- QG variables contain as limiting cases, the quantum (wave-particle) domain $x \ll 1$, the classical/semiclassical gravity ($x \gg 1$) domain and the Planck ($x \sim 1$) domain as well.
- QG-variables are more "complete" than the usual incomplete variables x which only describe one domain (Q or G) and there are two values x_{\pm} for each X_{QG} .

The completion of the complete manifold of QG variables requires several "patches" or analytic extensions to cover the full sets $X \geq 1$ or $X \leq 1$:

$$x_{\pm} = X \pm \sqrt{X^2 - 1}, \quad X \geq 1; \quad x_{\pm} = X \pm \sqrt{1 - X^2}, \quad X \leq 1 \quad (2.12)$$

The two ($X \geq 1$), ($X \leq 1$) domains being the classical and quantum domains respectively with their two (\pm) branches each, and when $x_+ = x_- : X = 1$, $x_{\pm} = 1$, (Planck scale).

The two domains precisely account for the two different and duals ways of approaching the Planck mass: The elementary particle domain $0 \leq x \leq 1$, and the macroscopic gravity domain $1 \leq x \leq \infty$. These two domains are duals of each other in the precise sense of the classical-quantum duality through the "Planck scale duality" Eqs (1.5), (1.6). In other words:

Quantum theory has $L_Q \gg l_P$ and $L_G \ll l_P$

Classical gravity has $L_Q \ll l_P$ and $L_G \gg l_P$

Quantum Gravity has L_{QG} and any value of L_G and L_Q , and the Planck domain as well.

This is reflected in the QG duality satisfied intrinsically by the QG variables L_{QG} and M_{QG} as well as in the "Planck duality" relation E.(1.5) for both Lengths L_G and L_Q separately and for the *dual (quantum) Mass* M_Q :

$$M_Q = \frac{m_P^2}{M} \quad (2.13)$$

The elementary particle domain of masses and lengths Eq.(2.4) is mapped respectively through Eq.(1.5) into the macroscopic mass and length domain of classical gravitational objects Eq.(2.5) and conversely. More generically: A set of classical gravitational quantities

$$O_G = (L_G, M, K_G, T_e)$$

characteristic of the classical gravity regime, (here denoting length, mass, gravity acceleration, temperature respectively), and the corresponding set in the quantum regime

$$O_Q = (L_Q, M_Q, K_Q, T_{eQ})$$

are **dual** of each other (in the precisely sense of the classical- quantum duality), ie

$$O_G = o_P^2 O_Q^{-1} \quad (2.14)$$

This relation holding for any quantity in the set. O_G and O_Q being the same conceptual physical quantities in the different (classical and quantum) regimes including gravity. o_{Pl} are the corresponding Planck scale magnitudes depending only of (\hbar, c, G) . O_Q stands for the

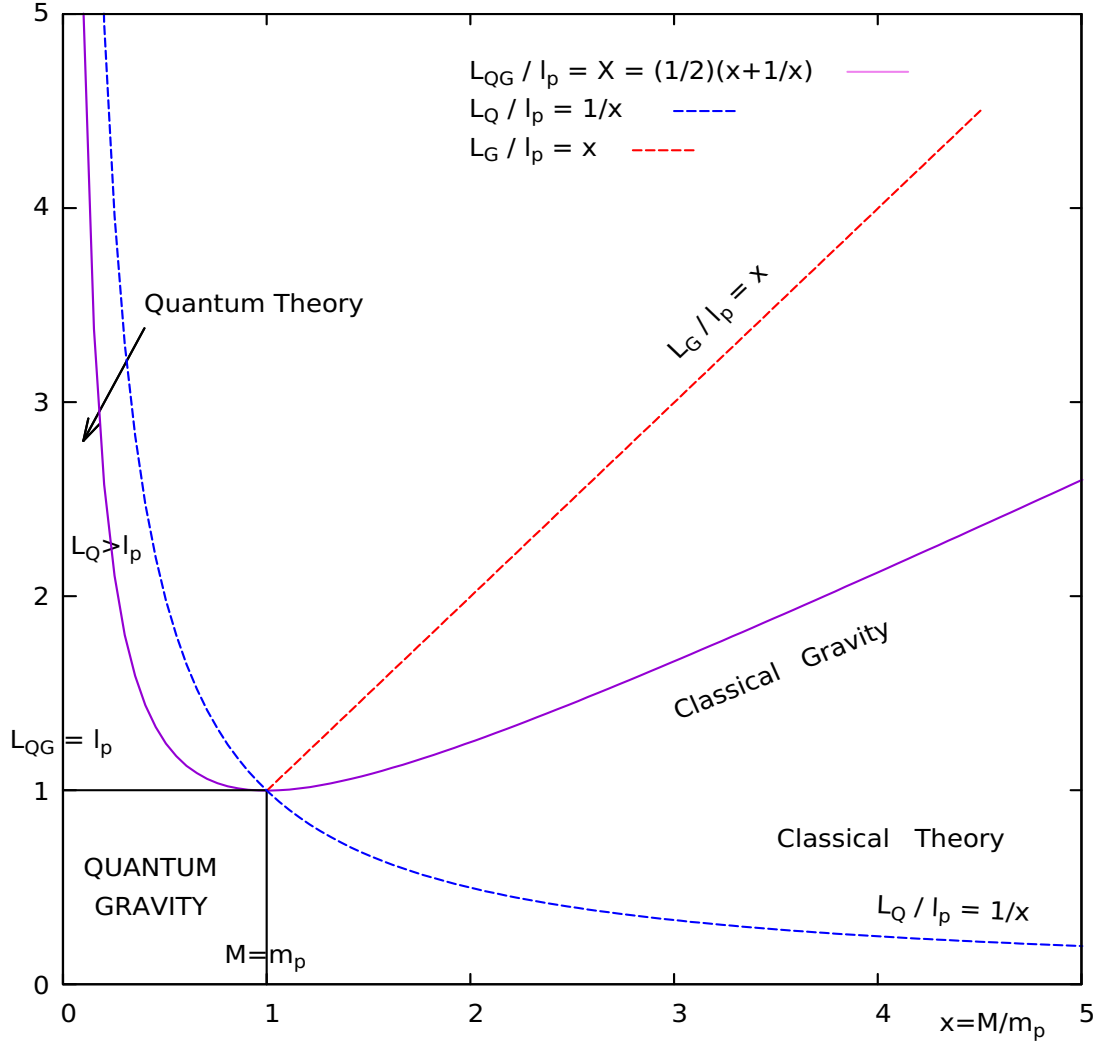


FIG. 1. The dimensionless QG variable $X = (1/2)(x + 1/x)$, $X \equiv L_{QG}/l_P$ and its classical and quantum components against the dimensionless mass $x = M/m_p$ (m_P and l_P stand for the Planck mass and Planck length). For completeness, each component of L_{QG}/l_P : the dimensionless classical gravity length $L_G/l_P = x$ and the quantum length $L_Q/l_P = 1/x$ are also displayed. $X = 1, x = 1$ at the Planck scale delimitates the QG Planck scale domain.

quantum magnitudes: quantum size L_Q , quantum mass M_Q , quantum surface acceleration \mathcal{K}_Q and quantum temperature T_{eQ} :

$$T_{eQ} = \frac{1}{2\pi k_B} M_Q c^2$$

The energy $M_Q c^2$ has naturally associated a quantum temperature (in units of k_B), which is

also a measure (in units of k_B) of the quantum length L_Q , and $K_Q = c^2/L_Q$. In particular, for semiclassical black holes, T_{eQ} is the Hawking temperature, but T_{eQ} is a much more generic concept.

There is thus a *duality* between the quantum domain of elementary particles (without gravity) and the classical gravity domain of macroscopic masses (without the quantum theory). In other words: The quantum domain of elementary particles (at the level of the set of fundamental variables as masses, lengths, temperatures, acceleration) is *dual* (in the precise sense of classical-quantum duality) of the classical gravity domain. This is not an assumed or conjectured duality and does not relate to the number of dimensions or space-time compactification, nor to any imposed symmetry.

III. COMPLETE ANALYTIC EXTENSION OF THE QG VARIABLES

Another QG variable, a "time"- type variable T_{QG} can be formed from the difference of L_G and L_Q (and similarly, a QG momentum with M_G and M_Q):

$$\begin{aligned} T_{QG} &= \frac{1}{2} (T_G - T_Q), & T_G &= \frac{L_G}{c}, & T_Q &= \frac{L_Q}{c} = \frac{t_P^2}{T_G} \\ T &= \frac{1}{2} \left(t - \frac{1}{t}\right), & T &\equiv \frac{T_{QG}}{t_P}, & t &\equiv \frac{T_G}{t_P}, \end{aligned} \quad (3.1)$$

t_P being the Planck time scale $t_P^2 = \hbar G/c^5$. The QG variables X and T satisfy:

$$\begin{aligned} X(x) &= X(x^{-1}), & X(-x) &= -X, & X(\pm\infty) &= \pm\infty, & X(0) &= \infty, & X(1) &= 1 \\ T(t) &= -T(t^{-1}), & T(-t) &= -T, & T(\pm\infty) &= \pm\infty, & T(0) &= -\infty, & T(1) &= 0 \end{aligned}$$

These properties of the (X, T) variables are more apparent in terms of the "star- coordinates" (x^*, t^*) :

$$x = \exp(\kappa x^*), \quad t = \exp(\kappa t^*), \quad \text{which imply} \quad X = \cosh(\kappa x^*), \quad T = \sinh(\kappa t^*).$$

The parameter κ is a pure number introduced for convenience of the sequel, it plays the role of the dimensionless surface gravity (gravity acceleration) allowed by the mapping. (In particular, at the Planck scale: $\kappa = 1$).

This leads us to consider 'null-type' QG variables and a more general exponential mapping yielding the QG variables (X, T) , in terms of (x^*, t^*) :

$$X \pm T = \exp \kappa(x^* \pm t^*), \quad U = X - T, \quad V = X + T, \quad (3.2)$$

which maps characteristic lines into characteristic lines, ie the null type structure of (X, T) and (x^*, t^*) is preserved, it is analytic and holomorphic if continued to complex variables. In the (x, t) variables, this mapping simply reads:

$$U = x t^{-1}, \quad V = x t$$

The QG variables (X, T) are complete coordinates fully covering the analytic extension of the QG manifold, while (x^*, t^*) or (x, t) cover incomplete domains. (X, T) are "Kruskal"-like variables, while (x^*, t^*) or (x, t) are "Schwarzschild"-like or "Rindler"-like since they are bounded, local or covering incomplete domains. Still, the comparison with the Schwarzschild r -coordinate is more apparent by singling out the role of $x = 1$ (Planck scale) in this context:

$$x^* = r + \frac{1}{\kappa} \log \sqrt{2\kappa r - 1}, \quad 1 < 2\kappa r \leq \infty, \quad -\infty \leq x^* \leq \infty$$

$$\text{Thus, } x = \sqrt{2\kappa r - 1} \exp(\kappa r), \quad 0 \leq x \leq \infty, \quad \text{and } 1 \leq t \leq \infty, \quad -\infty \leq t^* \leq \infty$$

Four charts I, II, III, IV are necessary to fully describe the complete QG domain. It follows:

$$X = \exp(\kappa x^*) \cosh(\kappa t^*) = \sqrt{2\kappa r - 1} \exp(\kappa r) \cosh(\kappa t^*) \quad (3.3)$$

$$T = \exp(\kappa x^*) \sinh(\kappa t^*) = \sqrt{2\kappa r - 1} \exp(\kappa r) \sinh(\kappa t^*) \quad (3.4)$$

$$UV = X^2 - T^2 = e^{(2\kappa x^*)} = (2\kappa r - 1) e^{(2\kappa r)}, \quad \frac{V}{U} = e^{(2\kappa t^*)}, \quad \frac{T}{X} = \tanh(\kappa t^*)$$

Or, in terms of the (x, t) variables, these equations read:

$$X = \frac{x}{2} \left(t + \frac{1}{t} \right), \quad T = \frac{x}{2} \left(t - \frac{1}{t} \right) \quad (3.5)$$

$$UV = x^2, \quad \frac{V}{U} = t^2, \quad \frac{T}{X} = \frac{t - t^{-1}}{t + t^{-1}} \quad (3.6)$$

Equations (3.3)-(3.4) correspond to the domain (I): $|X| > T$, $x > 0$, $(2\kappa r > 1)$ and its mirror copy $|X| < T$, (region III). . These are the "exterior" regions $1 < x \leq \infty$, that is L_G and M larger than l_P and m_P respectively, i.e the classical and semiclassical regions. They include the known classical gravity General Relativity domain, the semiclassical gravity and quantum theory (without quantum gravity) domain of the incomplete variables (x, t) . The complete QG (X, T) variables can take all values from 0 to $\pm \infty$ passing by 1 without any "boundary" or restriction.

The "past" and "future" QG *interior regions* $T > |X|$ and $T < |X|$ are precisely the *full quantum* domains where $x \leq 1$, ie $L_G \leq l_P$, $M \leq m_P$ (regions II and IV respectively). In

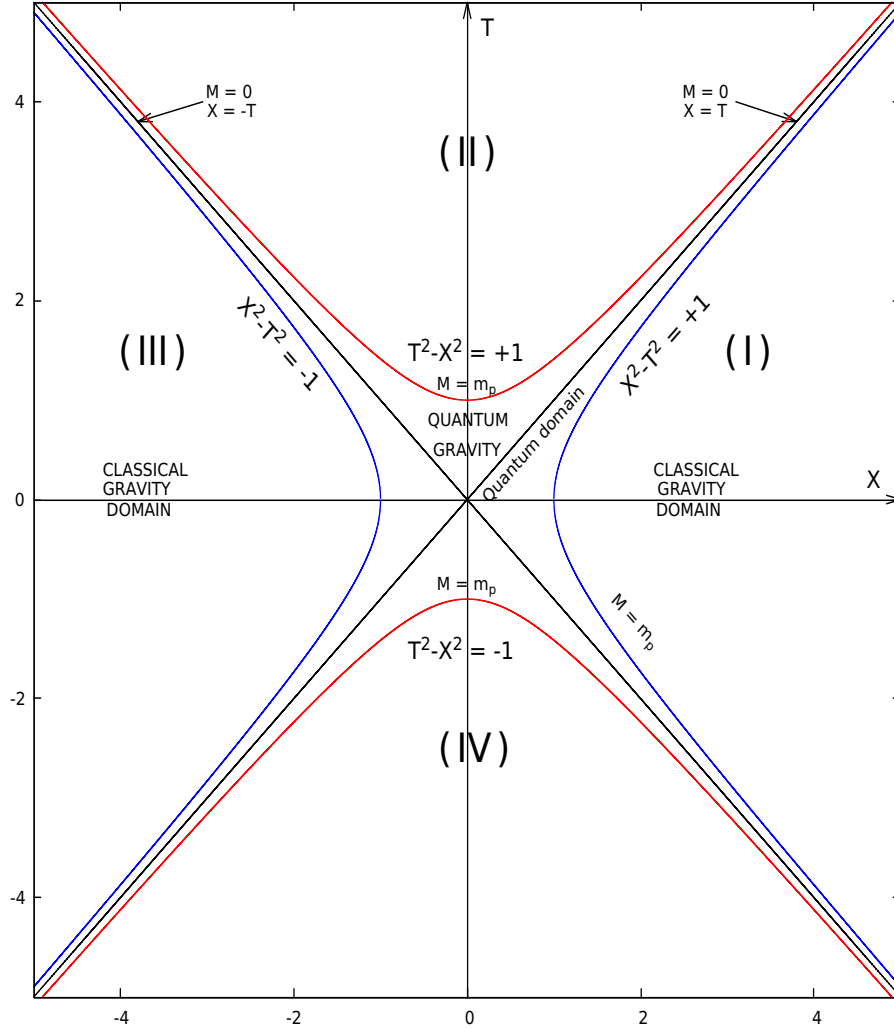


FIG. 2. **The complete analytic extension of the QG variables** (X, T) . Four regions are needed to cover the full manifold QG. (X, T) are complete variables covering all the classical and quantum domains and the quantum gravity domain as well, while (x, t) or (x^*, t^*) are incomplete variables, covering only one region (classical or quantum) without covering the quantum gravity domain. The four Planck scale hyperbolae $X^2 - T^2 = \pm 1$, $T^2 - X^2 = \pm 1$ (in Planck units) delimitate the quantum region including the quantum gravity domain (quantum range of masses -as elementary particles- passing by zero to the Planck mass). 1 is here l_P^2 . The Planck mass is reached from the classical side of large masses including the largest astronomical objects (region I and its mirror III) and from the quantum side of small masses (regions II and IV), as it must be: (classical-quantum duality at work through the Planck scale).

these QG domains, (X, T) are given by the "interior" variables, which are the analogues of Eqs (3.3)-(3.4) with X and T interchanged, and x^* (or x) given by:

$$x^* = r + \frac{1}{\kappa} \log \sqrt{1 - 2\kappa r}, \quad x = \sqrt{1 - 2\kappa r} \exp(\kappa r), \quad 0 \leq 2\kappa r \leq 1, \quad 0 \leq x \leq 1$$

$$X = \exp(\kappa x^*) \sinh(\kappa t^*) = \frac{1}{2} \sqrt{1 - 2\kappa r} e^{(\kappa r)} \left(t - \frac{1}{t}\right) \quad (3.7)$$

$$T = \exp(\kappa x^*) \cosh(\kappa t^*) = \frac{1}{2} \sqrt{1 - 2\kappa r} e^{(\kappa r)} \left(t + \frac{1}{t}\right) \quad (3.8)$$

$$UV = T^2 - X^2 = e^{(2\kappa x^*)} = x^2, \quad V/U = e^{(2\kappa t^*)} = t^2 \quad (3.9)$$

The fixed mass values $x = \text{const.}$ follow the trajectories $X^2 - T^2 = \text{const.}$ The straight lines $X = \pm T$, ($U = 0$ or $V = 0$) for $x^* = -\infty$, (that is $2\kappa r = 1$) are at the null values $x = 0$: $L_G = 0, M = 0$, (null (future or past) horizons: $t^* = \pm\infty$, (being $t = +\infty$ or 0)).

The Planck scale $x = 1$ (ie $M = m_P$) corresponds to the hyperbolae $T^2 - X^2 = \pm 1$ at $r = 0$ (ie $x^* = 0$): $x = 1$ in the future (+), and past (-) interior regions. The correspondence with the Kruskal manifold is manifest.

IV. CLASSICAL-QUANTUM DUALITY OF THE SCHWARZSCHILD-KRUSKAL SPACE-TIME

Let us now consider the Schwarzschild black hole:

$$dS^2 = -(1 - 2\kappa/r) dt^{*2} + (1 - 2\kappa/r)^{-1} dr^2 + d\Omega_{\perp}^2$$

Here (r, t^*) are the usual Schwarzschild space-time coordinates, $x^* = r + \frac{1}{\kappa} \log \sqrt{1 - 2\kappa r}$ and $\kappa = c^4/(2r_{BH}) = c^4/(4GM_{BH})$, M_{BH} being the black hole mass.

The black hole Kruskal mapping is the same as Eq (3.2) and the space-time Kruskal coordinates have identical expressions to Eqs (3.3)-(3.4):

$$X \pm T = \exp \kappa(x^* \pm t^*), \quad U = X - T, \quad V = X + T \quad (4.1)$$

In the interior black hole regions $0 \leq 2\kappa r \leq 1$: $x^* = r + \kappa^{-1} \log \sqrt{1 - 2\kappa r}$, (X, T) are interchanged and are given by Eqs (3.7)-(3.9).

In terms of the (x, t) coordinates, Kruskal (U, V) coordinates read simply

$$U = x t^{-1}, \quad V = x t, \quad UV = x^2, \quad \frac{V}{U} = t^2 \quad (4.2)$$

As is known, $X^2 - T^2 = 0$ are the past and future null horizons $2\kappa r = 1$, ($x^* = \infty$), here at $x = 0$. $T^2 - X^2 = \pm 1$ are the past (-) and future (+) classical space-time singularities at $r = 0$, ($x^* = 0$), here at $x = 1$.

The Kruskal-Schwarzschild manifold is endowed with several discrete symmetries

$$U \rightarrow \pm V, \quad V \rightarrow \pm U, \quad \text{ie} \quad X = -X, \quad T = -T, \quad (4.3)$$

which can be accompanied by corresponding changes in the transverse (angular) coordinates and their antipodal ones

$$\Omega_{\perp} \rightarrow \pm \Omega_{\perp}. \quad (4.4)$$

The *duality symmetry* of the Kruskal space-time (hidden by the usual Schwarzschild coordinates (r, t^*)) shows up clearly in the (x, t) variables:

$$X(t^{-1}) = X(t), \quad T(t^{-1}) = -T(t) \quad (4.5)$$

$$U(t^{-1}) = V(t), \quad V(t^{-1}) = U(t) \quad (4.6)$$

This duality under $t \rightarrow t^{-1}$ means that the time domains: $0 \leq t \leq 1$ and $1 \leq t \leq \infty$ are mapped into $\infty \leq t \leq 1$ and $1 \leq t \leq 0$ respectively, or equivalently, into $-\infty \leq t^* \leq 0$ and $0 \leq t^* \leq \infty$. Recall here that t is dimensionless (in units of the Planck time t_P) and arises from a physical classical gravitational time T_G as in Eq.(3.1), or a classical length L_G which is in turn connected to the quantum length L_Q and thus to the quantum dual time T_Q as given by Eqs(3.1). Thus, in this context, the duality transform $t \rightarrow t^{-1}$ changes the classical time t into the quantum time t^{-1} and conversely, but the Kruskal time T as the quantum gravity (QG) time remains invariant. The external regions (I) and (III) are classical gravity and semiclassical domains while the interior regions are quantum. These regions are *dual* of each other in the precise sense of the classical-quantum duality.

The classical-quantum duality transform in the Kruskal space-time maps the internal quantum region $x \in (0, 1)$ into the external classical domain $x \in (1, \infty)$ through the Planck scale ($x = 1, t = 1$) and conversely. The halves (I, II) and (III, IV), ie the classical/semiclassical wedge (I) and its corresponding quantum (future) interior II on one side, and the copy III and its corresponding (past) region IV on the other side, are antipodally identical of each other. This supports the *antipodal identification* of the two worlds ref [7], refs [8],[9]. By promoting the QG variables to quantum (non commutative) coordinates,

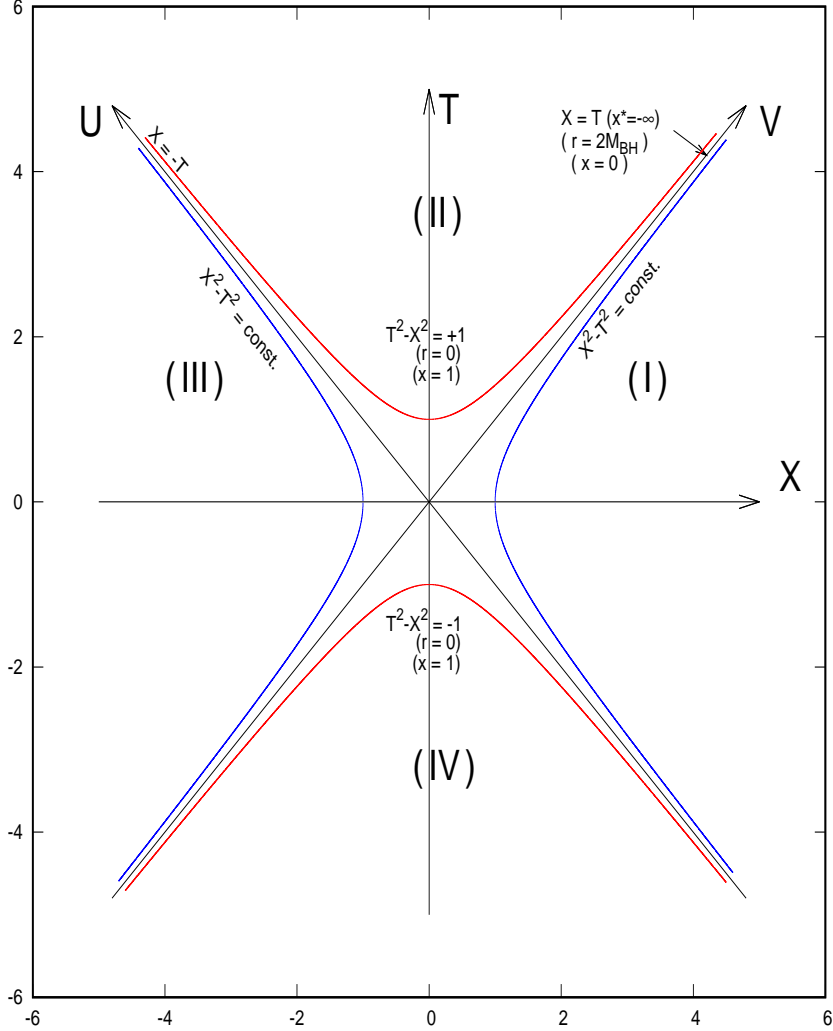


FIG. 3. **The complete Schwarzschild-Kruskal space-time and its classical-quantum duality.** Kruskal (X, T) coordinates are QG variables. Exterior (I) and (III) regions are classical or semiclassical. The interior regions (II) and (IV) are fully *quantum*. The four Planck scale hyperbolae $X^2 - T^2 = \pm 1$, $T^2 - X^2 = \pm 1$ (in Planck units) delimitate the quantum gravity region *including* the horizons $X = \pm T$. Exterior and interior lose their difference near the horizon: The horizons are bordered by the four Planck scale hyperbolae which act like a *quantum dressing* or width, we call it *l'horizon habillé* (dressed horizon) . 1 is here l_P^2 . The Planck scale is reached from the exterior: $X^2 - T^2 = \pm 1$ and from the interior: $T^2 - X^2 = \pm 1$ (at $r = 0$). The antipodal space-time symmetry and the PT or CPT symmetry are contained in the QG symmetry. Other properties and symmetries of the manifold as classical-quantum duality mappings are discussed in the text.

further insight into the quantum space-time structure is obtained and the identification of the two Kruskal copies is further supported by the quantum theory ref [10].

In the interior regions the Planck scale is reached at the loci $T^2 - X^2 = \pm 1$ ($r = 0$) which are bordering the null horizons $X = \pm T$ (at $2kr = 1$) and asymptotically reaching them for $T \rightarrow \pm\infty$, Fig 3. The *quantum region extends beyond the horizons* and is delimited by the Planck scale hyperbolae $X^2 - T^2 = \pm 1$. These hyperbolae are acting like a *quantum dressing* or quantum width for the horizon, let us call it *l'horizon habillé* (dressed horizon). 1 is here l_P^2 . We see that exterior and interior lose its difference near the horizons $X = \pm T$. Although, is possible to use these words in scales larger than the Planck scale.

There are *two* dual ways of reaching the horizon and the Planck scale: from the exterior (classical/semiclassical) and from the interior (quantum) regions. This appears like a *"splitting" or shifting* of the null horizons $X = \pm T$ into the hyperbolae $X^2 - T^2 = \pm 1$, $X = \pm\sqrt{T^2 \pm 1}$ being 1 the (dimensionless) Planck scale. We find a similar result when promoting (X, T) to quantum non-commutative coordinates as we do in ref [10].

Let us recall here that quantum field or string dynamics near horizons require quantum "shifts" or widths of the horizon of the order of the Planck scale, refs [11], [12],[13]. For quantum fields, the quantum shift appears as produced by the back reaction and the dynamics of ingoing and outgoing modes crossing the horizon ref [11]. For quantum strings, the horizon width is required by the size (of the order of the Planck scale) of the string . We find interesting that in a different approach, and *solely* from our QG variables, a horizon *quantum dressing* or width does appear, (and without searching for it) supporting the quantum nature of this space-time region. The QG variables naturally contain the Planck scale and in some sense the horizon splitting due to the *two* valued and dual covering (like Kruskal coordinates). Other related results are reported in ref [10].

V. CONCLUDING REMARKS

- We have provided here new quantum gravity (QG) variables which extend the known gravity (G) and quantum (Q) variables and reduce to them in each of the limiting sectors $G \gg Q$ or $Q \gg G$.

The QG variables contain together both classical and quantum domains including

quantum gravity and the elementary particle sector passing by the Planck scale. They generalize the gravity (G) and quantum (Q) variables of each classical and quantum domain, and contain their duality properties. They are invariant under the quantum (Q) and gravity (G) duality through the Planck scale: $Q \rightarrow G^{-1}$ duality, and conversely $G \rightarrow Q^{-1}$.

- The dual of classical behaviour is quantum behaviour: "de Broglie duality" with \hbar , "Compton duality" with \hbar and c , our "Planck duality" with \hbar, c and G : $O_Q = o_P^2 O_G^{-1}$, and "QG duality" here which extends all of them:
 $O_{QG}(O_G) = O_{QG}(O_Q)$, ie $O_G \rightarrow O_Q$ and conversely.

The QG variables $O_{QG} = (1/2)(O_G + O_Q)$ naturally contain all the above dualities. This duality is *universal*. As the wave-particle duality, it does not relate to the number or the kind of dimensions nor to any imposed symmetry or compactification. QG variables are automatically dual symmetric, $O_G \rightarrow O_Q$:

$$O_{QG}(1/x) = O_{QG}(x), \quad x = O_G/o_P = o_P/O_Q.$$

- The complete analytic extension of QG variables allowed us to reveal the classical-quantum duality of the space-time.

The regions I and III (Fig 2): exterior to the Planck scale hyperbolae $X^2 - T^2 = \pm 1$ contain both classical and semiclassical behaviours, depending on whether the classical or the quantum component or regime dominates, that is whether O_G , or O_Q are dominant, while the interior regions II and IV (inside the four Planck scale hyperbolae) are totally *quantum* in the range $[0, l_P]$.

- This shedded light on new quantum properties of the Schwarzschild-Kruskal black hole structure.

The four Planck scale hyperbolae $X^2 - T^2 = \pm 1$, $T^2 - X^2 = \pm 1$ (in Planck units) delimitate the quantum gravity region. The horizons $X = \pm T$ are now bordered by the four Planck scale hyperbolae which are like a *quantum dressing* or quantum width for the horizon, *l'horizon habillé* ("dressed horizon"). 1 is here l_P^2 .

The Planck scale is reached from the exterior $X^2 - T^2 = \pm 1$ and from the interior: $r = 0$ is the Planck scale $T^2 - X^2 = \pm 1$.

The exterior and interior regions thus appear "*dressed*" and *redefined*: they acquire a Planck scale structure. Near the horizons $X = \pm T$, exterior and interior *lose their difference*.

We have already discussed in the Introduction and along the paper the main new features of the paper and will not include all of them again here. We refer to section I for a summary of the results.

- The antipodal identification of space-time ("aist" in short) is not the purpose nor the subject of this paper, but as already mentioned, our results here support aist in the quantum theory, and our new quantum space-time structure results ref [10] imply it. We include then here some remarks to update the issue given the new context here and the recent refs [8],[9]. In refs [7], we investigated aist in a semiclassical QFT description, imposing such a boundary condition to the vacuum in a "strong way" stressing that it would give a zero norm state in a global spatial section. But in each half it is perfectly consistent to construct antipodally symmetric (or antipodally antisymmetric) states with non-zero norm. (Also, the classical global Cauchy section would not be such that in full quantum gravity). Importantly, in aist the space-time topology changes and the resulting manifold (coset or quotient space) is projective, another characteristic manifestation (non trivial topology) of quantum gravity. In aist the normalized antipodally symmetric (or antisymmetric) theory has *two* times more probability than the non aist symmetric theory. These differences of factors two in the aist and non-aist theories are *not* misleading, they are totally correct and perfectly understandable in the two-fold covering of the global manifolds. The QG variables and QG symmetry encloses all that.
- The size of the black hole is the gravitational length L_G in the classical regime, it is the Compton length L_Q in the semiclassical regime, it is the Planck size (or the string size L_s) in the full quantum gravity regime. Similarly, the horizon acceleration (surface gravity) of the black hole is the Planck acceleration κ_P in the QG regime. The Hawking temperature T_{eQ} (measure of the surface gravity or of the Compton length) becomes the Planck temperature in the full quantum gravity regime. The gravitational thermal features as Hawking radiation are typical of the semiclassical phase. The end of evaporation is non thermal and purely quantum. For masses smaller than the

Planck mass the black hole is not *anymore* a black hole but a elementary particle state. Moreover, the quantum mass spectrum we found recently for *all* masses confirms this picture ref [10].

- Space-time can be parametrized by *masses* ("mass coordinates"), just related to length and time as QG variables, on the same footing of space and time. In Planck units, any of these variables (or another convenient set) can be used. This reveals particularly interesting for mass quantization ref [10].

More generally, other analytic mappings for full analytic extensions in other manifolds could be considered, ref[14]. In QG, functions of the QG variables could satisfy QG duality symmetry: $f(x, t) = \pm \text{const. } f(x^{-1}, t^{-1})$. QG variables can be also considered in phase-space with their full analytic extension to all values or patches.

- In summary, *QG variables* or coordinates cover all the domains, classical and quantum, with and without gravity. They turn out to be the classical-quantum duals of each other, in the precise sense of the wave-particle duality, here extended to the quantum gravity (Planck) domain: wave-particle-gravity duality: *QG duality* in short. QG variables can be used directly as space-time variables or coordinates as we do here, they can be considered as the result of expectation values in a quantum state or they can be used to construct appropriate quantum operators as we do in ref [10].
- The idea of a continuum space-time is a classical (non-quantum) approximation. A step forward in which the space-time itself is quantized confirms the results presented here and yields more radical new results presented in another paper ref [10].

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