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# LOCALLY-ADAPTED CONVOLUTION-BASED SUPER-RESOLUTION OF IRREGULARLY-SAMPLED OCEAN REMOTE SENSING DATA

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## ABSTRACT

Super-resolution is a classical problem in image processing, with numerous applications to remote sensing image enhancement. Here, we address the super-resolution of irregularly-sampled remote sensing images. Using an optimal interpolation as the low-resolution reconstruction, we explore locally-adapted multimodal convolutional models and investigate different dictionary-based decompositions, namely based on principal component analysis (PCA), sparse priors and non-negativity constraints. We consider an application to the reconstruction of sea surface height (SSH) fields from two information sources, along-track altimeter data and sea surface temperature (SST) data. The reported experiments demonstrate the relevance of the proposed model, especially locally-adapted parametrizations with non-negativity constraints, to outperform optimally-interpolated reconstructions.

**Index Terms**— Super-resolution, convolutional model, irregular sampling, dictionary-based decomposition, non-negativity

## 1. INTRODUCTION

Image super-resolution or upscaling is a classical problem in image processing [1, 2]. Super-resolution techniques also apply to remote sensing image enhancement problems [3]. Contrary to the classical super-resolution setting, numerous satellite remote sensing applications do not only involve low-resolution images but also irregularly-sampled high-resolution information. The latter may be due to specific sampling patterns, such as along-track narrow-swath satellite data, as well as to partial occlusions caused by weather conditions [4, 5]. The availability of such partial high-resolution data supports locally-adapted super-resolution models, rather than models fully trained offline, with a view to accounting for the space-time variabilities of the monitored processes.

In this paper, we address such image super-resolution issues from irregularly-sampled high-resolution information. Following state-of-the-art super-resolution models [6–8], we consider locally-adapted convolution-based models. Our methodological contributions are two-fold: i) the proposed convolution-based models combine both a low-resolution image and a secondary image source, ii) we explore dictionary-based representations of the convolutional operators with different types of constraints, namely orthogonality, non-negativity and sparsity constraints [9, 10]. Such dictionary-based representations and constraints are particularly appealing to resort to locally-adapted super-resolution models calibrated from a

low number of high-resolution training data.

As case study, we apply the proposed framework to multi-source ocean remote sensing data, namely the reconstruction of high-resolution SSH (Sea Surface Height) images from satellite-derived along-track altimeter data, a high-resolution SST (Sea Surface Temperature) image and a low-resolution SSH image. We report numerical experiments, which demonstrate the relevance of the proposed super-resolution models, especially under non-negativity constraints, compared with optimally-interpolated SSH images.

The paper is organized as follows. In Section 2 we introduce the proposed super-resolution model along with the associated calibration schemes. In Section 3, we present the application to the reconstruction of satellite-derived SSH images and described experimental results. Finally, we report concluding remarks and discuss future work in Section 4.

## 2. MODEL FORMULATION

### 2.1. Problem statement

We aim at reconstructing a series of high-resolution images  $\{Y(t)\}_t$  at different times  $\{t_1, \dots, t_T\}$  from the corresponding series of low-resolution images  $\{Y_{LR}(t)\}_t$ . In the considered application setting, we are also provided with:

- a complementary source of high-resolution images  $\{X(t)\}_t$ , which may depict some local or global correlation with  $\{Y(t)\}_t$ ;
- an irregularly-sampled dataset of high-resolution point-wise observations  $\{\tilde{t}(k), \tilde{s}(k), \tilde{Y}(k)\}_k$ , with  $\tilde{t}(k)$ ,  $\tilde{s}(k)$  and  $\tilde{Y}(k)$  respectively the time, location and value of the  $k^{th}$  high-resolution observation.

Figure 1 reports an example of the considered sampling patterns. We let the reader refer to Section 3 for the detailed description of the considered application to ocean remote data.

The reconstruction of high-resolution image  $Y(t)$  given low-resolution image  $Y_{LR}(t)$  is stated according to the following convolution-based model:

$$Y(t) = Y_{LR}(t) + H_Y * Y_{LR}(t) + H_X * X(t) + N(t) \quad (1)$$

where  $N$  is a space-time noise process.  $H_Y$  (resp.  $H_X$ ) is the two-dimensional impulse response of the  $Y_{LR}$  (resp.  $X$ ) component of the proposed convolutional model.  $H_Y$  and  $H_X$  are characterized by  $(2W_p + 1) \times (2W_p + 1)$  discrete representations onto the considered high-resolution grid. Importantly,  $H_Y$  and  $H_X$  are space-and-time-varying operators and capture the space-time variabilities

of  $(Y, Y_{LR})$  and  $(Y, X)$  relationships. This model can be regarded as a patch-based super-resolution approach where high-resolution image  $Y$  at a given location is computed as a linear combination of  $(2 * W_p + 1) \times (2 * W_p + 1)$  patches of images  $X$  and  $Y_{LR}$  centered at the same location. Parametrization  $H_X = 0$  clearly relates to regression-based super-resolution models [6, 7].

## 2.2. Unconstrained model calibration

The calibration of model (1) amounts to the estimation of the  $(2W_p + 1) \times (2W_p + 1)$  matrix representations of operators  $H_Y$  and  $H_X$  at any space-time location. The availability of the irregularly-sampled dataset  $\{\tilde{t}(k), \tilde{s}(k), \tilde{Y}(k)\}_k$  provides the means for this locally-adapted calibration. It may be noted that, in classical image super-resolution issue, such models are trained offline or involve nearest-neighbor techniques using a training dataset of joint low-resolution and high-resolution image patches [6, 7]. Here, we proceed as follows. For a given space-time location  $(t_0, s_0)$ , we regard all data such that  $\tilde{t}(k) \in [t_0 - D_t, t_0 + D_t]$  and  $\|\tilde{s}(k) - s_0\| \leq D_s$  as observations for model (1) at location  $(t_0, s_0)$ . Parameters  $D_t$  and  $D_s$  state respectively the spatio-temporal extent of the considered neighborhood around location  $(t_0, s_0)$ . Given the irregular sampling of the high-resolution dataset, no guarantees exist that sampling locations  $\tilde{s}(k)$  will lie within the considered  $X/Y_{LR}$  grid, and thus  $(2W_p + 1) \times (2W_p + 1)$  high-resolution  $X$  patches and low-resolution  $Y_{LR}$  patches need to be interpolated around spatio-temporal locations  $(\tilde{s}(k), \tilde{t}(k))$ . Local impulse responses  $H_X$  and  $H_Y$  are then fitted by minimizing the mean square reconstruction error  $\mathcal{E}(H_X, H_Y)$  for the high-resolution detail  $dY = Y - Y_{LR}$  at irregularly-sampled dataset positions  $(\tilde{s}(k), \tilde{t}(k))$ :

$$\mathcal{E}(H_X, H_Y) = \sum_k \left\| d\tilde{Y}(k) - \widehat{d\tilde{Y}}(k) \right\|^2 \quad (2)$$

$$\text{where } \widehat{d\tilde{Y}}(k) = H_Y * Y_{LR}(\tilde{t}(k), \tilde{s}(k)) + H_X * X(\tilde{t}(k), \tilde{s}(k)) \quad (3)$$

Assuming the number of observations is high-enough, minimization (2) resorts to a least-square estimation of operators  $H_Y$  and  $H_X$ .

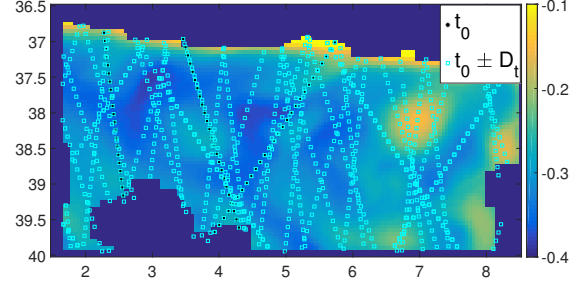
## 2.3. Dictionary-based decompositions

A critical aspect of the above least-square minimization is the number of available training data points and the underlying balance between locally-adapted and robust parametrizations. With a view to improving estimation robustness as well model interpretability, we explore dictionary-based decomposition approaches. They resort to the following decomposition of operators  $H_X$  and  $H_Y$ :

$$H_{\{X, Y\}} = \sum_{k=1}^K \alpha_k D_k^{\{X, Y\}} \quad (4)$$

where  $D_k^Y$  (resp.  $D_k^X$ ) is the  $k^{th}$  component of the dictionary of operators for operator  $H_Y$  (resp.  $H_X$ ) and  $\alpha_k$  is the  $k^{th}$  scalar coefficient that states the decomposition of operator  $H_Y$  (resp.  $H_X$ ) onto dictionary element  $D_k^Y$  (resp.  $D_k^X$ ). It should be noted that a joint dictionary-based representation is considered in our study, so that decomposition coefficients  $\alpha_k$  are shared by the two convolutional operators  $H_Y$  and  $H_X$ .

Following classical dictionary-based settings [11], we explore their



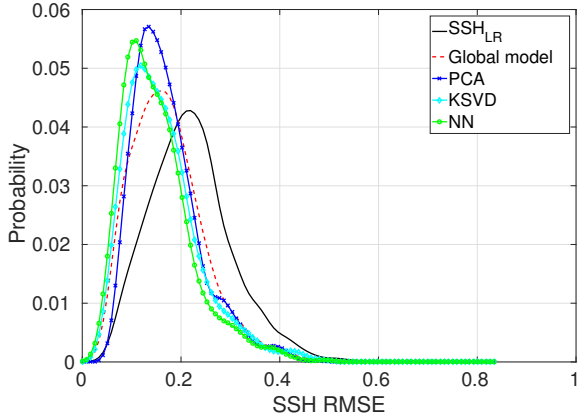
**Fig. 1:** Illustration of the irregular sampling of high-resolution observations associated with ocean remote sensing data: sea surface height image with the sampled along-track positions by satellite altimeters (cyan squares) in a  $\pm 10$ -day time window around April 20<sup>th</sup>, 2012.

applications to convolution operators. We investigate three different types of constraints for dictionary elements  $\{D_k^Y\}$  and decomposition coefficients  $\{\alpha_k\}$ : namely orthogonality, sparsity and non-negativity constraints. The calibration of these dictionary-based settings first involve the estimation of dictionary elements  $\{D_k^Y\}$  using training data. We here assume we are provided with a representative dataset of unconstrained estimates of operators  $H_Y$  and  $H_X$  from (2), denoted by  $\{H_Y^n, H_X^n\}_n$ . More precisely, the considered dictionary-based decompositions are as follows:

- **Orthogonality constraint:** under this constraint, dictionary elements  $\{D_k^Y\}$  form an orthonormal basis with no other constraints onto coefficients  $\{\alpha_k\}$ . This decomposition relates to the application of principal component analysis (PCA) [12] to dataset  $\{H_Y^n, H_X^n\}_n$ . Given the trained dictionary, the estimation of decomposition coefficients  $\{\alpha_k\}$  comes to the projection of the unconstrained operator estimates onto dictionary elements  $\{D_k^Y\}$ .
- **Sparsity constraint:** the sparse dictionary-based decomposition [13] resorts to complementing MSE criterion (2) with the  $L_1$  norm of coefficients  $\{\alpha_k\}$ . We apply a KSVD scheme to dataset  $\{H_Y^n, H_X^n\}_n$  to train dictionary elements  $\{D_k^Y\}$ . Given the trained dictionary, we proceed similarly to kSVD and use orthogonal matching pursuit [14] for the sparse estimation of decomposition coefficients  $\{\alpha_k\}$  for any new unconstrained operator estimate.
- **Non-negativity constraint:** the non-negative dictionary-based decomposition constrains coefficients  $\{\alpha_k\}$  to be non-negative. Given dataset  $\{H_Y^n, H_X^n\}_n$ , the training of dictionary elements  $\{D_k^Y\}$  resorts to the minimization of reconstruction error (2) under non-negativity constraints for the decomposition coefficients. We exploit an iterative proximal operator-based algorithm [15]. Given the trained dictionary, the estimation of decomposition coefficients  $\{\alpha_k\}$  comes to a least-square estimation under non-negativity constraints.

## 2.4. Locally-adapted dictionary-based convolutional models

The application of the proposed dictionary-based decompositions to the super-resolution of irregularly-sampled high-resolution images involves the following main steps. For a given dictionary-based decomposition, we first train the associated dictionaries  $\{D_k^X, D_k^Y\}$ . Considering the entire image time series, we proceed to the unconstrained estimation of operators  $H_X$  and  $H_Y$  from (2) for a variety



**Fig. 2:** Probability distribution for the relative root mean square reconstruction error (RMSE) for daily high-resolution SSH images  $\{Y(t)\}_t$ , for a global convolutional model and for locally-adapted decompositions of a global convolutional model using principal component analysis (PCA) [12], KSVD [13] and non-negative decomposition (NN) and considering  $K = 10$  classes. The probability distribution of the RMSE for daily low-resolution SSH images  $\{Y_{LR}(t)\}_t$  is given as reference (noted as  $SSH_{LR}$ ).

of spatio-temporal neighborhoods with given parameters  $D_s^{Tr}$  and  $D_t^{Tr}$ . Parameters  $D_s^{Tr}$  and  $D_t^{Tr}$  are set such that the number of high-resolution observations is high enough to solve for least-square criterion (2). We typically sample around 1500 neighborhoods to build a representative dataset of operators  $H_X$  and  $H_Y$ . Given the trained dictionaries, we proceed to the super-resolution of an image at a given date  $t^*$  as follows. For any given spatial location  $s^*$ , we first estimate the associated decomposition coefficients  $\{\alpha_k\}$  from the subset of high-resolution observations in a spatio-temporal neighborhood of space-time location  $(t^*, s^*)$  with parameters  $D_s^{SR}$  and  $D_t^{SR}$ . The later parameters typically define smaller spatio-temporal neighborhoods than training neighborhoods with parameters  $D_s^{Tr}$  and  $D_t^{Tr}$ . As such, estimated coefficients  $\{\alpha_k\}$  come to the projection of more local convolutional operators onto the subspace spanned by the estimated dictionaries, thus yielding a more locally-adapted model (1). This calibrated model is then applied to the reconstruction of image  $Y$  in a neighborhood of location  $(t^*, s^*)$ . To reduce the computational time, we perform this calibration of locally-adapted models for a regular subsampling of the image grid, typically  $D_s^{SR}/2$ , and use a spatial averaging of overlapping local reconstructions to obtain a single high-resolution reconstruction of image  $Y$ .

### 3. EXPERIMENTS

As case study, we consider an application to ocean remote sensing data, more particularly to the reconstruction of sea-surface height (SSH) image time series from along-track altimeter data. Satellite altimeters are narrow-swath sensors such that high-resolution altimeter data is only acquired along the satellite track path [16], resulting in an particularly scarce and irregular sampling of the ocean surface as illustrated in Fig. 1. Interestingly, numerous studies have pointed out the potential contribution of high-resolution sea surface temperature (SST) images to the reconstruction of SSH images, as they share common geometrical patterns associated with the underlying

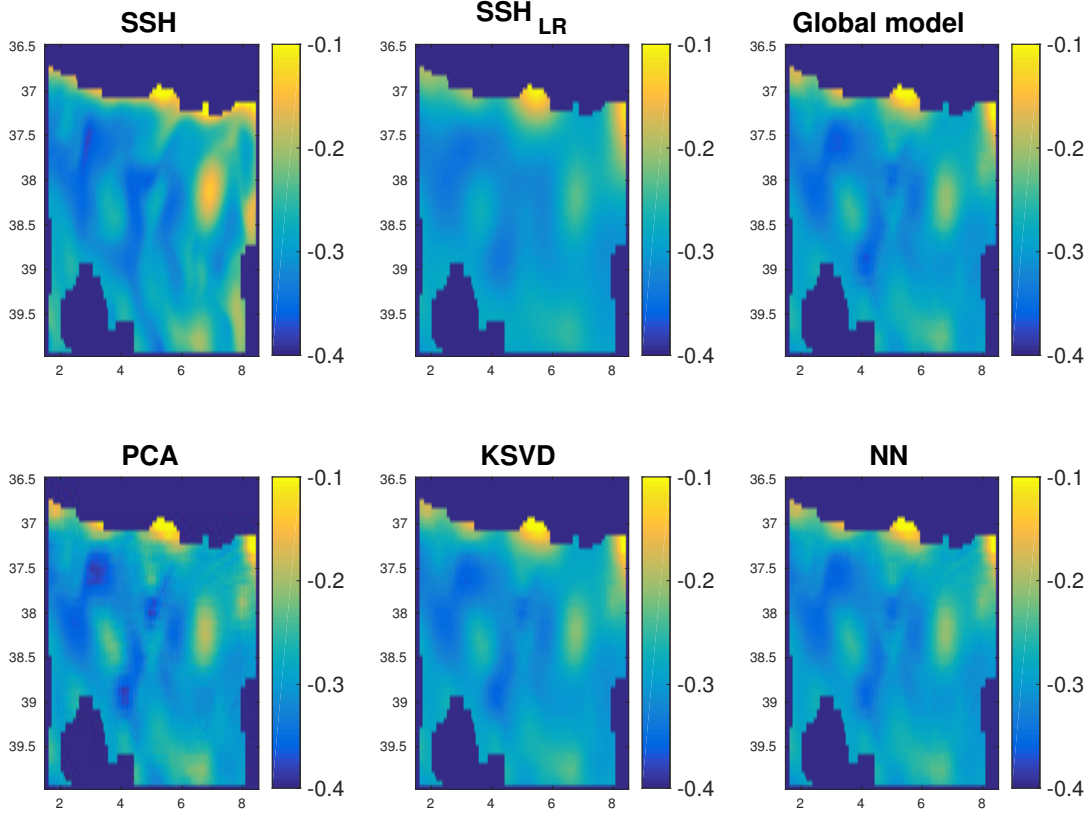
**Table 1:** Relative root mean square reconstruction error (RMSE) for daily high-resolution SSH images  $\{Y(t)\}_t$ , for a global convolutional model and for locally-adapted decompositions of a global convolutional model using principal component analysis (PCA) [12], KSVD [13] and non-negative decomposition (NN), considering  $K = 2$ ,  $K = 5$  and  $K = 10$  classes. The RMSE value for daily low-resolution SSH images  $\{Y_{LR}(t)\}_t$  is given as reference (noted as  $SSH_{LR}$ ). Best results for each number of classes  $K$  considered are presented in bold. Results that outperform a global convolutional model are underlined.

	$K = 2$	$K = 5$	$K = 10$
PCA	0.1823	<u>0.1732</u>	<u>0.1717</u>
KSVD	<u>0.1629</u>	<u>0.1629</u>	<u>0.1629</u>
NN	<b><u>0.1562</u></b>	<b><u>0.1521</u></b>	<b><u>0.1519</u></b>
Global model			0.1733
$SSH_{LR}$			0.2201

upper ocean dynamics [17, 18]. In addition, optimally-interpolated products [16] provide a low-resolution reconstruction of the SSH image. Overall, the reconstruction of high-resolution SSH image time series resorts to a super-resolution issue from irregularly-sampled high-resolution information as stated in Section 2. It may be stressed that this case study involves a scaling factor of about 10 between the low-resolution and high-resolution data, which makes it particularly challenging compared with classical image super-resolution issues. In our experiments, we exploit a ground-truthed dataset using an observing system simulation experiment for a case study region in the Western Mediterranean Sea ( $36.5^\circ N$  to  $40^\circ N$ ,  $1.5^\circ E$  to  $8.5^\circ E$ ). A high-resolution numerical simulation of the WMOP model [19] is used to generate daily high-resolution SSH images from 2009 to 2013 for a  $1/20^\circ$  grid. The along-track dataset is simulated by sampling the SSH images at real along-track positions issued from from multiple altimetry missions in 2014 and 2015 (see Figure 1). Given the simulated along-track dataset, optimally-interpolated SSH fields [16], referred to as low-resolution SSH images  $Y_{LR}$ , are computed for a  $1/8^\circ$  grid resolution. The calibration of the proposed convolutional operators is performed by considering  $W_p = 1$ , which corresponds to  $3 \times 3$  convolutional masks. We use the following parameter setting for spatio-temporal neighborhoods:  $t_0 \pm D_t$ -day time windows with  $D_t = 10$ , and  $D_s \times D_s$  spatial neighborhoods with  $D_s^{Tr} = 7^\circ$  for the training step and  $D_s = 2^\circ$  for the locally-adapted calibration steps.

In Table 1, we report the average root mean square reconstruction error (RMSE) for daily high-resolution SSH images  $\{Y(t)\}_t$ , for a global convolutional model and for locally-adapted convolutional models, using principal component analysis (PCA) [12], KSVD [13] and non-negative dictionary-based decomposition (NN) and considering  $K = 2$ ,  $K = 5$  and  $K = 10$  elements in the dictionaries. The reconstruction RMSE for daily low-resolution SSH images  $\{Y_{LR}(t)\}_t$  (noted as  $SSH_{LR}$ ) is given as reference.

From Table 1, locally-adapted convolutional models clearly outperform global models (with the exception of the PCA-based decomposition for a small number of classes  $K$ ), which can be explained by the improved local adaptation to local spatio-temporal variabilities through locally-adapted decomposition coefficients. In this respect, the non-negative decomposition outperforms alternative approaches, with a maximum relative gain (with respect to optimally-interpolated low-resolution SSH images  $\{Y_{LR}(t)\}_t$ , at  $K = 10$ ) of 30.99% for NN, 25.99% for KSVD, 21.99% for PCA and 21.26% for a global



**Fig. 3: High-resolution SSH image  $Y$  reconstruction, April 20<sup>th</sup>, 2012:** first row, from left to right, real high-resolution SSH image  $Y$ , low-resolution SSH image  $Y_{LR}$  (noted as  $SSH_{LR}$ ), reconstruction of high-resolution SSH image  $Y$  using global convolutional model (1); second row, reconstruction of high-resolution SSH image  $Y$  using a 10-class locally-adapted decomposition (4) of global convolutional model (1) using, from left to right, principal component analysis (PCA) [12], KSVD [13] and non-negative decomposition (NN).

convolutional model.

These results are further illustrated by the reconstruction of high-resolution SSH image  $Y$  for sample date April 20<sup>th</sup>, 2012 presented in Figure 3 and by the probability distributions of daily reconstruction root mean square error for high-resolution SSH images  $\{Y(t)\}_t$ , computed for the global convolutional model and for each one of the considered locally-adapted models with  $K = 10$ , presented in Figure 2. Visually, the proposed super-resolution models clearly improve the reconstruction of finer-scale details compared to the low-resolution image. The model using non-negativity constraints seem to involve slightly sharper the gradients compared with the unconstrained and sparsity-based model. PCA-based model appear visually less relevant.

#### 4. CONCLUSION

In this paper, we addressed the multimodal super-resolution of irregularly-sampled high-resolution images. This issue arises in a number of remote sensing applications, where several sensors associated with different regular and irregular sampling patterns may contribute to the reconstruction of a given high-resolution image. As a case study, we considered an application to the reconstruc-

tion of high-resolution sea surface height (SSH) images. From a methodological point of view, we complement previous convolution-based super-resolution models [7, 8] with the evaluation of different dictionary-based decompositions and the use of a complementary high-resolution image source. Dictionary-based decompositions are regarded as a means to better account for spatio-temporal variabilities through more locally-adapted model calibrations. Our numerical experiments support the selection of non-negativity constraints to achieve a better local adaptation. They demonstrate the relevance of the proposed approach to achieve a better reconstruction of higher-resolution details, compared with the optimally-interpolated fields. Future work includes non-local extensions of the proposed model to combine spatio-temporal and similarity-based neighborhoods as considered in regression-based super-resolution models [7, 8]. Non-linear dictionary-based decomposition seems particularly appealing to combine non-linear mapping, for instance CNN-based models [20], and locally-adapted models. As far as ocean remote sensing applications are considered, applying the proposed models to different sampling patterns, for instance along-track narrow-swath satellite data vs. wide-swath satellite data, appears to be of interest, the later possibly enabling the modeling of higher-order geometrical details.

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