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Finite Element Simulation of Electrical Motors Fed by Current Inverters

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Abstract: This article presents a method of study for electric machines coupled to static converters respectively represented by a Finite Element and a circuit type model. This technique, based on a step by step process with respect to time for the simultaneous solution of the electrical circuit and electromagnetic field equations, is used for the working analysis of two permanent magnet motors fed by a load commutated current inverter. The calculation and experimental results are compared so as to validate the proposed method.

I. INTRODUCTION

Methods allowing the coupling of the field and electric circuit equations, have been presented and used to simulate the working of voltage inverter fed electric machines. In this case, the simulation is made by applying to the motor armature the known voltage waveshapes generated by this inverter type. The currents can be calculated by the simultaneous solution of the supply electric circuit and field equations [1].

The problem is more difficult if neither the currents in the windings nor the motor armature voltages are known. Works concerning this problem have been published over the last few years, but they remain relative to the case of generators associated with a diode rectifier bridge [2], [3], [4].

In this article, a simulation method is presented allowing the working analysis of electric machines fed by current inverters as shown in Figure 1. Current inverters are made of thyristors and the converter state is a function of the control angle of the thyristors and the machine currents and voltages.

Since, neither the currents in the winding, nor the applied machine voltages, nor even the different exterior circuit configurations presented by the inverter, are a priori known, a step by step process with respect to time must be used. At each time step, the circuit configuration must be determined and the unknown currents and voltages are calculated by the simultaneous solution of the magnetic field and the corresponding electric circuit equations.

II. LOAD COMMUTATED CURRENT SUPPLY PRINCIPLE

Current supply of the synchronous machine consists in imposing the current amplitude in the machine windings and its phase with respect to the electromotive force. The supply of the inverter is then ensured by a direct current source (Figure 1). This current source consists of a voltage source E connected in series with a high value inductance L_{ext} , to ensure a constant direct current. R_{ext} represents the resistance associated with the L_{ext} source inductance.

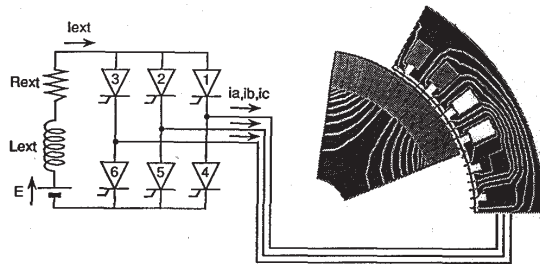


Figure 1: Permanent magnet motors fed by current inverters.

A rotor position sensor is generally used to detect the electromotive force phase angle. This electromotive force is then considered as a reference and the whole control makes it possible to impose the angle Ψ_a determining the thyristor ignition instants with respect to the zero crossing instants of the electromotive force [5].

For motor operation, the inverter must be load commutated. This is possible when the voltages of the machine are able to ensure the commutation of the currents and the thyristor turn off.

III. NUMERICAL MODEL

A. Field Equation with the Exterior Circuit and Taking into Account the Movement

The magnetic field is given by the diffusion equation (1):

$$\nabla \times (1/\mu) (\nabla \times \mathbf{A}) = \mathbf{J} - \sigma \partial \mathbf{A} / \partial t + \nabla \times (1/\mu_m) \mathbf{B}_m \quad (1)$$

where:

\mathbf{A} is the magnetic vector potential;

μ is the magnetic permeability;

\mathbf{J} is the current density;

σ is the electrical conductivity;

μ_m is the permanent magnet magnetic permeability;

\mathbf{B}_m is the magnet remanent magnetising vector.

Generally, the current density \mathbf{J} in the windings is unknown. Supposing the conductors which make up the windings present little cross sections, \mathbf{J} can be written in function of the currents in the coils i . Furthermore this assumption consisting in a constant current density, makes it possible to link up the applied voltage u to the current i by Kirchoff's equations:

$$\mathbf{u}(t) = \mathbf{R}(t)\mathbf{i}(t) + \partial/\partial t \{ \mathbf{L}(t)\mathbf{i}(t) \} + \partial/\partial t \mathbf{N}\Phi(t) \quad (2)$$

$\mathbf{u}(t)$ is the voltage vector applied on each phase;
 $\mathbf{R}(t)$ is the resistance diagonal matrix which must, necessarily, contain the r resistances of each phase;
 $\mathbf{L}(t)$ is the inductance diagonal matrix, which can contain the end winding inductances l which are not taken into account in a bidimensional model;
 $\mathbf{N}\Phi(t)$ is the flux matrix through the windings which is linked to the vector potential \mathbf{A} ;
 $\mathbf{i}(t)$ is the phase current vector.

The time derivative can be written with Euler's scheme:

$$\mathbf{u}(t) = \{ \mathbf{R}(t) + [1/\Delta t] [2\mathbf{L}(t) - \mathbf{L}(t-\Delta t)] \} \mathbf{i}(t) - [1/\Delta t] \mathbf{L}(t)\mathbf{i}(t-\Delta t) + [1/\Delta t] \mathbf{N}\Phi(t) - [1/\Delta t] \mathbf{N}\Phi(t-\Delta t) \quad (3)$$

with Δt , the time calculation step.

During a static converter supply, the three machine phases are generally linked in a star connection. There is then an additional constraint: only two currents are linearly independent. Besides, in the case of a star connection without neutral wire, the voltages applied to the machine windings are voltages between phases (\mathbf{U}). The equation system corresponding to the supply circuit is then given by:

$$\mathbf{C2} \mathbf{U}(t) = \mathbf{C1} \{ \mathbf{R}(t) + [1/\Delta t] [2\mathbf{L}(t) - \mathbf{L}(t-\Delta t)] \} \mathbf{C1}^T \mathbf{i}^*(t) - \mathbf{C1} \{ [1/\Delta t] \mathbf{L}(t) \} \mathbf{C1}^T \mathbf{i}^*(t-\Delta t) + \mathbf{C1} \{ [1/\Delta t] \mathbf{N}\Phi(t) \} - \mathbf{C1} \{ [1/\Delta t] \mathbf{N}\Phi(t-\Delta t) \} \quad (4)$$

with

$$\mathbf{C1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \quad \mathbf{C2} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

and $\mathbf{i}^*(t)$ the vector of the two linearly independent currents.

Equation (1) is processed by means of the Finite Element method. First order elements are used. The resulting matrix system is coupled with the one of equation (4) and the whole system is solved in step by step with respect to time.

The rotor movement is taken into account by means of the Moving Band technique [6]. At each displacement step of the moving part, the elements belonging to the Moving Band are deformed until a re-meshing becomes necessary. With this process the number of study area nodes increases but a dynamic allocation of the periodicity or anti-periodicity conditions makes it possible not to increase the size of the matrix system to be solved.

This method was chosen because the calculation time is far shorter compared with other methods where the air-gap is not meshed [7], [8]. For a good representation of the air-gap, special quadrilateral elements have been adopted for its composition, as shown in Figure 2 [9].

B. Current Inverter Modelisation

When the machine is supplied by a static converter, the exterior supply circuit, seen by the motor, changes according to the state of the switches of the bridge. A

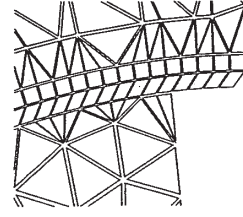


Figure 2: Airgap and Moving Band.

sequential method has been adopted for the inverter modelisation. Two essential sequences describe the inverter state, that is to say, conduction sequence and commutation sequence.

a) Conduction sequence: the configuration of the set supply circuit and machine, when the thyristors 1 and 6 are on is represented in Figure 3.

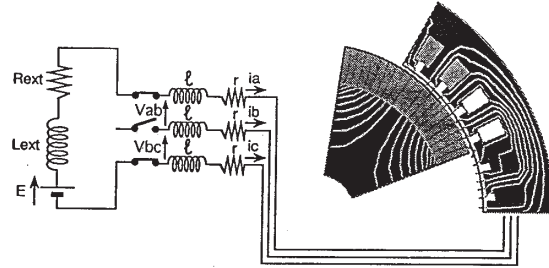


Figure 3: Conduction sequence.

It can be observed that $i_b=0$ and the voltage E is applied to the a and c phases. To simulate $i_b=0$, i.e., the phase b disconnection, the process adopted is the introduction of a high value of resistance, R_∞ , connected in series with r and l , respectively, phase resistance and end winding inductance. R_∞ is chosen equal to $1M\Omega$. This process makes it possible to keep the order of the matrix system constant. The terms corresponding to the matrix of equation (4), for this sequence, are presented in Table 1.

When the voltage on thyristor 5 becomes positive and the ignition order is given, the commutation sequence begins. The ignition instant, function of the Ψ_a angle which is constant if the speed is fixed, is calculated only once, at the beginning of the simulation.

	$R_{ji}(t)$	$L_{ji}(t)$
Phase a	$R_{ext} + r$	$L_{ext} + l$
Phase b	$R_\infty + r$	l
Phase c	r	l
$U_1(t)=U_{ab}=0$	$U_2(t)=U_{bc}=0$	$U_3(t)=U_{ca}=-E$

Table 1: Conduction sequence: terms of $\mathbf{R}(t)$, $\mathbf{L}(t)$ and $\mathbf{U}(t)$ for the conduction of thyristors 1 and 6.

b) Commutation sequence: during the commutation sequence, three thyristors are on at the same time. The equivalent circuit configuration, when the thyristors 1, 5 and 6 are on is presented in Figure 4. Two phases are short-circuited; the b and c phases in this example.

Equation 4 matrix terms for the showed commutation sequence are presented in Table 2.

The following conduction sequence happens when the thyristor 6 current, i_c , is cancelled. During the simulation, the time calculation step may not be small enough to detect the zero-current-crossing, which can lead to parasitic over-voltages due to the forced extinction of a current in the circuit inductances. To overcome this problem, when the current changes its sign, the process used consists in reversing in the time loop and forcing the current cancelling by introducing R_{∞} .

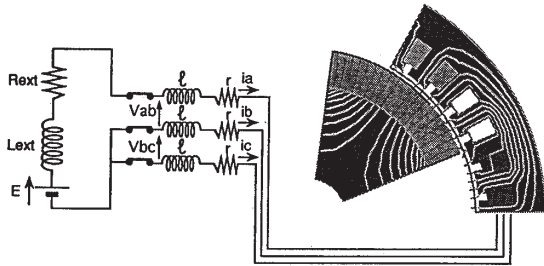


Figure 4: Commutation sequence.

	$R_{ji}(t)$	$L_{ii}(t)$
Phase a	$R_{ext} + r$	$L_{ext} + l$
Phase b	r	l
Phase c	r	l
$U_1(t)=U_{ab}=E$	$U_2(t)=U_{bc}=0$	$U_3(t)=U_{ca}=-E$

Table 2: Commutation sequence: terms of $R(t)$, $L(t)$ and $U(t)$ for the conduction of thyristor 1, 5 and 6.

For the other conduction and commutation sequences, equation 4 components are similarly determined.

IV. OBTAINED RESULTS

To validate the proposed model, two permanent magnet machines have been chosen. There are two motors without polar pieces and peripheral magnets on the rotor, both having the same stator. Because of the mechanical constraints, hoops binding the whole rotor had to be used. The main difference between the two prototypes is precisely due to the nature of the materials used for these hoops, one being conductive and the other not.

A. Non Conductive Hoop Machine

The hoop of this machine is made of fibre-glass and the interpolar wedges of resin, both non conductive materials the magnetic permeability of which is close to that of air. The magnets magnetisation are parallel to the symmetry axis of the magnet. In Figure 5, the structure is presented for a calculation position, with the associated no load magnetic field.

To ensure the best working possibilities of the proposed model, the calculation quality of the electromotive force is essential. That is why, the calculated and measured no load voltage curves are presented first in Figure 6, thus allowing the appreciation of the quality of the representation so obtained. The speed considered is equal to 3000 rpm. The no load calculation was made with values of R_{∞} added to the three machine phase resistances.

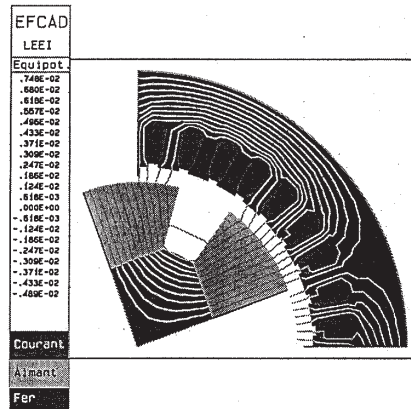
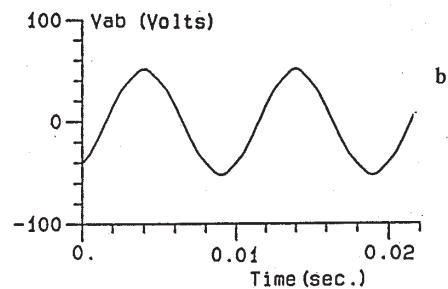
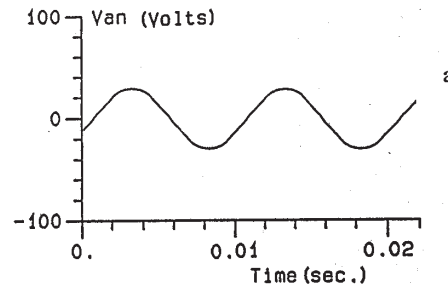
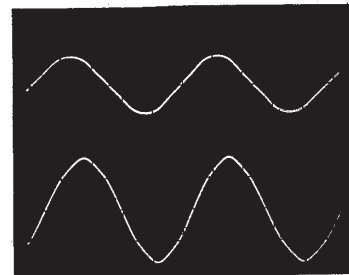


Figure 5: Non conductive hoop machine. Equipotential vector lines.



Calculation



Measurement

Figure 6: No load voltages calculated and measured for the non conductive hoop machine at 3000 rpm.
a) Neutral-phase voltage.
b) Phase-phase voltage.

The simulation results obtained for the motor working at 3000 rpm fed by a current inverter, are presented in Figure 7, with the corresponding experimental results. The voltage V_{ab} between phases a and b is calculated with relation (5).

$$V_{ab}(t) = r[i_a(t) - i_b(t)] + [1/\Delta t] [i_a(t) - i_b(t)] + [1/\Delta t] [N\Phi_a(t) - N\Phi_b(t)] - [1/\Delta t] [i_a(t-\Delta t) - i_b(t-\Delta t)] - [1/\Delta t] [N\Phi_a(t-\Delta t) - N\Phi_b(t-\Delta t)] \quad (5)$$

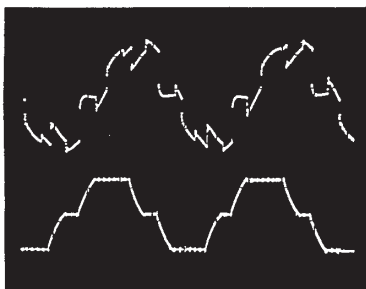
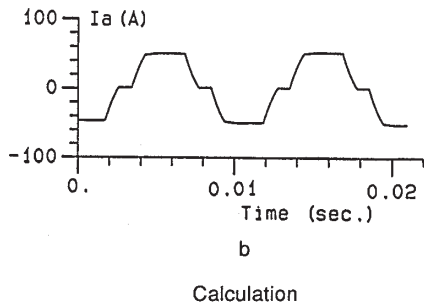
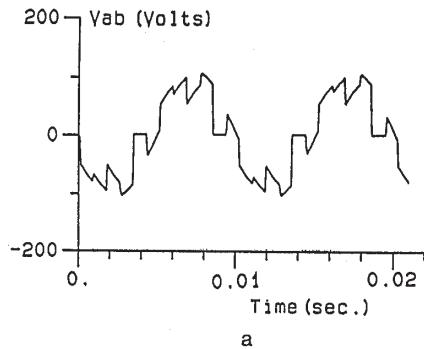


Figure 7: Calculation and measurement results for the non conductive hoop machine fed by current inverter at 3000 rpm
a) V_{ab} voltage.
b) i_a current.

B. Conductive Hoop Machine

For this machine, the hoop and the interpolar wedges are made of aluminium. The magnets present, as for the first

machine, a parallel magnetisation. The magnetic structure is shown in Figure 8. The density of the current induced in the conductive parts for one of the calculation instants is shown in Figure 9.

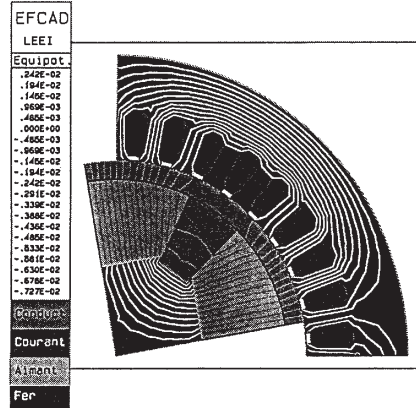


Figure 8: Conductive hoop machine. Equipotential vector lines.

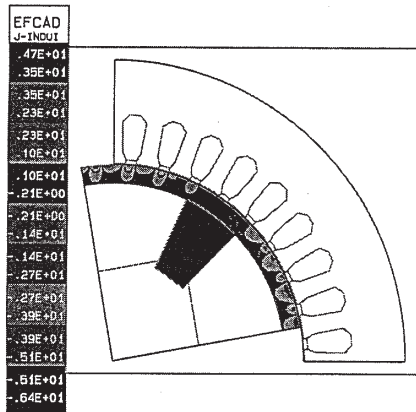


Figure 9: Conductive hoop machine. Density of the rotor induced currents.

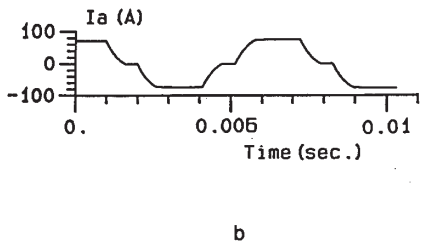
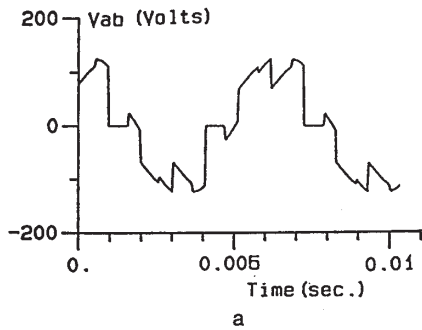
The results obtained during the current inverter supply are shown in figure 10, for a 5000 rpm speed.

C. Comparison and Analysis of the Two Machine Simulation Results

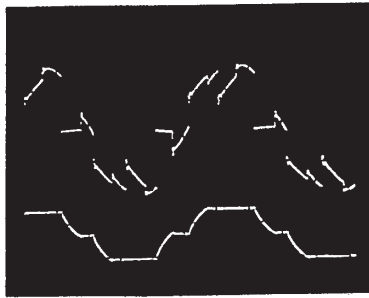
The simulated wave shapes at a constant 3000 rpm speed with a 60° self-piloted angle for both machines are presented in figure 11. Comparing them shows that, in relation to the non conductive hoop machine, the conductive hoop, in particular, has the following effects:

- a voltage drop during the sequence between commutation;
- a lower commutation time, in spite of the previous voltage drop, which shows a decrease in the commutation inductance.

These phenomena well-known from an experimental and theoretical point of view, are the typical expression of a damping effect, introduced here by the conductive hoop and

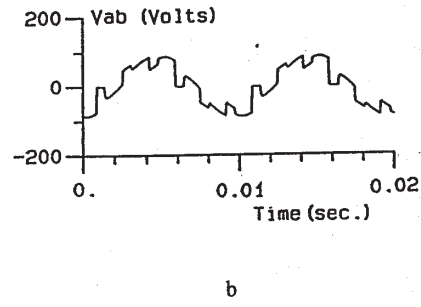
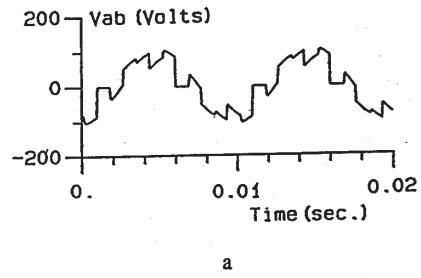


Calculation

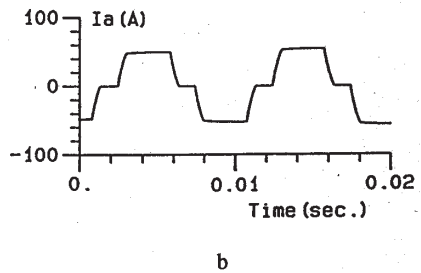
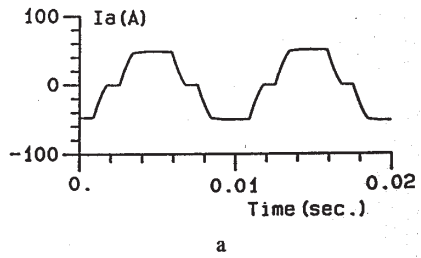


Measurement

Figure 10: Calculation and measurement results for the conductive hoop machine fed by current inverter at 5000 rpm.
a) V_{ab} voltage.
b) i_a current.



V_{ab} voltage.



i_a current.

Figure 11: Comparison of calculated results (3000 rpm).
a) Non conductive hoop machine.
b) Conductive hoop machine.

the induced currents that flow through it [5]. The modelisation used allows a quantitative estimation of this effect in direct relation to the nature of the hoop and dimensions.

CONCLUSION

In this work, a method allowing the coupling between the field equations and the supply electronic circuit equations has been presented. This method has been used to simulate permanent magnet synchronous machines fed through a load commutated current inverter. To simplify the realisation, a sequential method has been used to describe the state of the inverter bridge, in function of the thyristor firing angle and the machine currents and voltages. The rotor motion is taken into account through the Moving Band technique. The comparison of the simulation results with the experimental studies, brings out the sharpness of the model representation and a very good concordance of the results validating the proposed method. Its realisation, then gives information which is hardly obtainable by simpler models or by experience, such as the induced current distribution in the massive pieces of the machines and their influence on the working.

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