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An ordered metallic glass solid solution phase that grows from the melt like a crystal

Karena W. Chapman^a, Peter J. Chupas^a, Gabrielle G. Long^{a,b,*}, Leonid A. Bendersky^b,
 Lyle E. Levine^b, Frédéric Mompiou^{b,c}, Judith K. Stalick^d, John W. Cahn^{b,e}

^a X-Ray Science Division, Argonne National Laboratory, Argonne, IL 60439, USA

^b Metallurgy Division, National Institute of Standards and Technology, Gaithersburg, MD 20899, USA

[°] Centre d'Élaboration de Matériaux et d'Études Structurales, CNRS, 31055 Toulouse, France

^dNIST Center for Neutron Research, National Institute of Standards and Technology, Gaithersburg, MD 20899, USA

^e Department of Physics and Astronomy, University of Washington, Seattle, WA 98195, USA

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13 Abstract

We report structural studies of an Al–Fe–Si glassy solid that is a solid solution phase in the classical thermodynamic sense. We demonstrate that it is neither a frozen melt nor nanocrystalline. The glass has a well-defined solubility limit and rejects Al during formation from the melt. The pair distribution function of the glass reveals chemical ordering out to at least 12 Å that resembles the ordering within a stable crystalline intermetallic phase of neighboring composition. Under isothermal annealling at <u>305</u> °C the glass first rejects Al, which persists for approximately 1 h with no detectable change in structure, and finally is transformed by a first-order phase transition to a crystalline phase with a structure that is different from that within the glass. It is possible that this remarkable glass phase has a fully ordered atomic structure that nevertheless possesses no long-range translational symmetry and is isotropic.

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22 *Keywords:* Metallic glass; Phase transformation; Pair distribution function analysis; X-ray scattering

24 1. Introduction

Many ordered three-dimensional (3-D) arrangements of 25 points are neither periodic nor quasi-periodic [1], where 26 "order" means that the positions of the points are deter-27 mined. Physical realizations would be structures that are 28 neither crystalline nor quasi-crystalline. We recently 29 reported initial results on a new isotropic Al-Fe-Si alloy, 30 31 a metallic glass produced by rapid solidification of the melt, which may possess such a structure [2]. Here, we pres-32 ent the complete experimental results and analyses that 33 support this conclusion. 34

E-mail address: gglong@aps.anl.gov (G.G. Long).

Three general concepts are usually applied to glass 35 formation from metallic alloy melts. First, there is the 36 long-established view that glasses are frozen melts. Frozen 37 melt glasses have the same composition as the melt and 38 inherit the local ordering present in the melt at tempera-39 tures just above those where dynamic motion ceases. Sup-40 porting this concept is a large body of literature describing 41 many glass-forming systems. Second, there is the view that 42 metal alloy glasses are really polycrystalline with nanoscale 43 structural ordering. A third view is that metallic alloy 44 glasses can be composed of low energy, well-defined local 45 atomic structures. A consequence of this third view is the 46 expectation that local atomic arrangements in these glasses 47 closely approximate structures in crystalline phases of 48 similar composition. 49

These three concepts lead to different predictions about how the glass forms and what the final ordering will be,

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^{*} Corresponding author at: X-Ray Science Division, Argonne National
Q2 Laboratory, Argonne, IL 60439, USA. Tel.: +1 301 873 8365.

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52 and clearly no single approach describes all glasses. For example, the low temperatures reached by eutectic melts 53 occur because there are no stable crystalline phases at the 54 eutectic composition. Frozen eutectic melts nevertheless 55 56 are achieved, and this occurs not only because there is a large nucleation barrier, but also because crystal growth 57 58 is slowed by necessary diffusion [3]. The structure of such glasses would not approximate a crystalline phase. 59

We here present an example of a metallic alloy glass (the 60 "q-glass") that does not form as a frozen melt. Its forma-61 tion, guite remarkably, exhibits many characteristics more 62 typical of crystalline phases. In particular, the q-glass is a 63 solid phase that grows from the melt by means of a nucle-64 ation and growth mechanism; there is chemical partitioning 65 and atomic rearrangement at the glass-liquid interface in 66 the manner of a crystalline solid. Despite this classical 67 nucleation and growth, this phase is a glass in the sense 68 that it is isotropic (within measurement uncertainty) and 69 70 exhibits no translational ordering.

The existence of the q-glass raises many questions that 71 we address below. What is the local structure of the 72 73 q-glass? How does its structure compare with those of crys-74 talline phases of similar composition? Is it a true glass or a nanocrystalline aggregate? How stable is it? When it devi-75 trifies, what phases form? More fundamentally, is it a true 76 metastable phase in a thermodynamic sense? 77

78 2. Morphological character of the q-glass

79 We investigated specimens that are 100% q-glass, or nearly so, examined their formation mechanism, and pro-80 vide a detailed analysis of the q-glass atomic scale structure 81 to uncover possible similarities to neighboring phases. We 82 also followed the q-glass behavior during isothermal anneal-83 ing to test whether the q-glass might be nanocrystalline. 84

The first indication that the q-glass is not an under-85 cooled melt that has become kinetically frozen came from 86 87 the microstructural morphology of the formation of the q-glass [2]. q-Glass nodules are at the center of radiating 88 patterns of crystallization, indicating that the glass was first 89 to solidify from the melt [2,4]. This is in stark contrast to a 90 typical frozen melt glass, which would be the last phase to 91 form. As it grows the q-glass depletes the surrounding melt 92 93 of iron and silicon, indicating that the q-glass forms from the melt as a primary isotropic phase by nucleation fol-94 95 lowed by growth. During growth there is a partitioning of the chemical species at the interface between the q-glass 96 97 and the melt. These factors imply the coexistence of phases, 98 which is a defining feature of a first order phase transition in multicomponent systems. This remarkable behavior was 99 observed in a series of rapid solidification (melt spinning, 100 atomization and e-beam surface melting) studies of the for-101 102 mation of glassy Al–Fe–Si alloy compositions [3,5]. While 103 metallic glasses that are frozen melts appear to tolerate compositional variations as broad as those of the chosen 104 melt compositions, this Al-Fe-Si q-glass behaves instead 105 like a growing crystal, where atomic species that do not 106

fit well into the structure are rejected. Specimens that are 107 100% q-glass (Fig. 1a) can only be grown with a composi-108 tion close to 15 at.% Fe. 20 at.% Si, which is close to the 109 compositions of the α -cubic Al-Fe-Si phase [6,7] and an 110 icosahedral phase. The α -cubic phase is stable in the Al-111 Mn-Si system, and metastable in the Al-Fe-Si system. 112 Further, we note that there is no nearby eutectic in the 113 phase diagram [8]. Even near the 100% q-glass composition 114 evidence for q-glass formation by a moving interface can be 115 seen in Fig. 1b. A trapped gas bubble between the melt and 116 the cooling wheel has slowed glass formation enough that 117 partitioning begins to occur midway through the foil. 118

3. Sample preparation and preliminary characterization

To find concentrations that yield 100% q-glass speci-120 mens of varying alloy composition were prepared from a 121 series of melts subjected to rapid solidification, after which 122 they were characterized by laboratory based X-ray diffrac-123 tion (XRD), transmission electron microscopy (TEM), and 124 selected area electron diffraction (SAED). The Al-Fe-Si 125 phase diagram showing the liquidus projections [8] is given 126 in Fig. 2. The red region indicates the region of stability for 127 the quasi-crystal plus Al, the yellow region indicates the 128 stable compositional range for the polycrystalline α -cubic approximating the quasi-crystal, and the blue region indicates the compositional range where the q-glass plus Al is stable. The composition of the q-glass specimen that is the subject of this paper is marked with a red star near the 15/20 composition. We note that the β -phase is designated β -Al_{4.5}FeSi in Fig. 2, in agreement with both this paper and other papers in the literature, such as Liu and Chang [9]. This phase is sometimes referred to as τ_6 .

For the research reported here rapidly solidified samples 138 were prepared by melt spinning in a helium environment 139 using a 40 cm diameter Cu wheel rotating at 3000 r.p.m. 140 The quenched ribbons were typically 5–10 mm long, by 141 5 mm wide, by $30-50 \mu$ m thick. Samples were screened by 142 means of XRD and TEM. XRD was used as a primary tool 143 to identify samples that were close to 100% of the phase of 144 interest. The samples that consisted of nearly 100% q-glass 145 (g-Al-Fe-Si) had the composition Al-15Fe-20Si, those of 146 nearly 100% a-cubic Al-Fe-Si (a-Al-Fe-Si) had the com-147 position Al-18Fe-8Si, and the predominantly icosahedral 148 phase (ico-Al-Fe-Si) had the composition Al-18Fe-22Si. 149 TEM and ultra-small angle X-ray scattering (in sector 150 32-ID at the Advanced Photon Source, Argonne National 151 Laboratory) analyses of the selected samples confirmed the 152 XRD measurements. 153

4. High energy X-ray scattering and determination of the static structure

To probe the short-range and medium-range order of 156 the q-glass we performed high energy X-ray scattering to 157 large values of momentum transfer Q. This enables accu-158 rate Fourier transformation of the data to recover the pair 159

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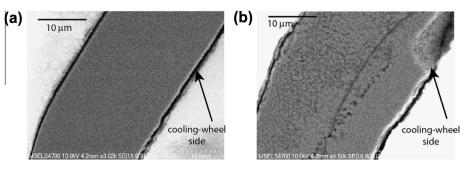


Fig. 1. (a) SEM image of a cross-section of a melt spun ribbon of the Al–15Fe–20Si at.% alloy showing a uniform glassy structure throughout the thickness and (b) SEM image of a cross-section of the same alloy near a bubble which slowed cooling, showing a uniform glassy structure on the cooling wheel (right) side, which develops into a heterogeneous glass + Al structure to the left of a clearly marked boundary.

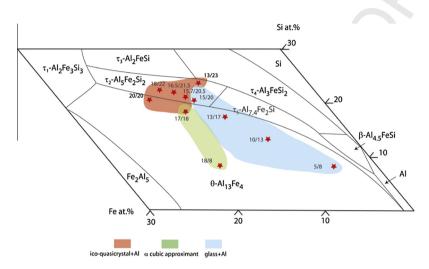


Fig. 2. Al–Fe–Si diagram showing the liquidus projections (after Ghosh [8]). The red region indicates where we find quasi-crystal + Al; the green region indicates where we find the polycrystalline α -cubic approximating the quasi-crystal; the blue region indicates where we find the q-glass + Al. The compositions that were studied are marked with red stars. The composition of the q-glass specimen (15/20) that is the subject of this paper is marked with the red star near the intersection of the red, green, and blue regions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

distribution function (PDF) [10]. The PDF formalism 160 161 enables the derivation of structural information in real space in the form of a probability distribution of atom-162 atom correlations. The method is widely used to study 163 the structure of glasses and other disordered materials, 164 where interest typically lies in the first, second and third 165 166 coordination shells of the structure, and the information extracted usually consists of bond lengths and coordination 167 numbers. Elucidating the nature of g-Al-Fe-Si requires 168 169 further identification of the intermediate-range structure, as will be seen in what follows. Fitting complex structural 170 models solely to one-dimensional PDF data can be chal-171 lenging and often must make use of careful analogies to 172 known crystalline forms. To address this we analyzed the 173 local structure of g-Al-Fe-Si, and investigated its relation-174 ship to α -Al-Fe-Si and ico-Al-Fe-Si. The samples are 175 176 listed in Table 1.

177 High energy (80.725 keV, $\lambda = 0.15359$ Å) X-ray scatter-178 ing data suitable for PDF analysis were collected in the 179 APS beamline 1-ID-C [11] at the Advanced Photon 180 Source, Argonne National Laboratory, on samples of α-Al-Fe-Si, ico-Al-Fe-Si and g-Al-Fe-Si. Square X-ray 181 beams were defined by means of slits with unidirectional 182 focusing of the beam to enhance the flux at the smaller 183 beam sizes. Small beam sizes, $30 \times 30 \,\mu\text{m}$, are a good 184 match to the thickness of the samples ($\sim 40 \,\mu m$). Each 185 foil-like sample was mounted orthogonally to the beam 186 and step scanned along a square grid with dimensions 187 matching that of the beam size to allow diffraction data 188 to be collected at each point (see Fig. 3). The two-dimen-189 sional diffraction images (2048×2048 pixels) were mea-190 sured using an amorphous silicon-based area detector 191 [12] produced by General Electric Healthcare mounted 192 orthogonal to the beam path and centered on the beam. 193 The sample to detector distance and the tilt of the detec-194 tor relative to the beam were refined using a LaB_6 stan-195 dard within the analysis software Fit-2D [13]. The raw 196 images were corrected for gain, pixel efficiency, bad pixels 197 and dark current. The corrected two-dimensional (2-D) 198 images were integrated within Fit-2D to obtain the one-199 dimensional powder diffraction pattern, masking areas 200 obscured by the beam stop arm [14]. 201

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Table 1			
List of samples	and fitted	lattice	parameters.

Sample	Phase	Composition (at.%)	Lattice parameter a (Å)
a-Al-Fe-Si	Crystalline α-cubic	Al-18Fe-8Si	12.55
ico-Al-Fe-Si	Quasi-crystalline icosahedral	Al-18Fe-22Si	12.06
g-Al–Fe–Si	q-Glass	Al-15Fe-20Si	12.26

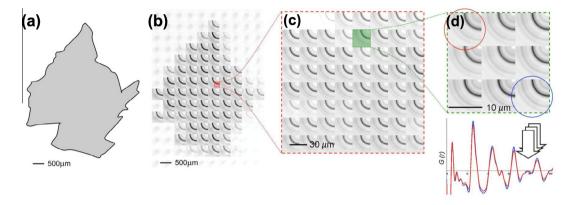


Fig. 3. The spatially resolved diffraction measurements performed on the g-Al-Fe-Si sample. (a) The shape of the g-Al-Fe-Si sample. Spatially resolved measurements using (b) a $500 \times 500 \,\mu\text{m}$ beam, (c) a $30 \times 30 \,\mu\text{m}$ beam, and (d) a $10 \times 10 \,\mu\text{m}$ beam. A quadrant of the 2-D diffraction pattern is shown in each square of the grids. Areas with texture are evident on length scales of 30 µm. We used the red and blue circled diffraction patterns on the 10 µm scale to extract the PDFs. Since these PDFs, shown below the 10 µm grid, are equivalent within error they demonstrate the uniformity of the sample on the 10 µm length scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

202 Preliminary X-ray measurements revealed that the α-Al-Fe-Si and the ico-Al-Fe-Si samples included a degree of 203 diffraction texture (i.e. made visible by the non-uniformity 204of the diffraction rings) over the 500 \times 500 and 30 \times 30 μ m 205 length scales, where these varied across the sample regions 206 examined. The g-Al-Fe-Si sample was not entirely uni-207 form, with isolated inclusions of face-centered cubic (fcc) 208 Al crystallites. Even within apparently uniform regions 209 such as that shown in Fig. 3c there was occasional evi-210 dence, in the form of slight diffraction texture, of very fine 211 non-uniformities. To identify the most uniform glassy 212 213 regions areas of the sample with minimal diffraction texture were selected in the 30 \times 30 μ m scans and used for the finer 214 scale $10 \times 10 \,\mu\text{m}$ scans. The spatially resolved mapping 215 allowed us to isolate diffraction patterns from each of the 216 samples that were free of significant texture and/or of sin-217 218 gle crystal spots, and it enabled us to obtain data suitable for PDF analysis for each of the three phases of interest. 219

The PDF analysis made use of the diffraction data col-220 lected with the smallest (10×10) µm beam. Contributions 221 to the scattering from the sample environment and back-222 ground were subtracted. Corrections for multiple scattering, 223 X-ray polarization, sample absorption, and Compton scatter-224 ing were applied to the diffraction patterns to obtain the struc-225 226 ture function S(Q). Direct Fourier transformation of the reduced structure function F(Q) = Q[S(Q) - 1] up to Q_{max} 227 $\approx 18 \text{ Å}^{-1}$ gave the PDFs G(r). Here $G(r) = 4\pi r [\rho(r) - \rho_{o}]$, 228 229 where $\rho(r)$ and ρ_0 are the instantaneous and average densities and r is the radial distance between atom pairs. The extraction 230 of G(r) and the structural refinement were conducted within 231 the analysis software PDFFIT [15]. 232

Particular care was required to quantitatively fit a struc-233 tural model to the experimental PDF data, due to the com-234 plexity of the α -cubic crystalline structure (*Pm*3, $a \approx 12.5$ Å, 235 11 Wyckoff positions (i.e. unique atom sites) and 138 atoms 236 per unit cell) compared with structures typically refined 237 using PDF methods. It proved useful to compare the fit 238 of the crystalline α -cubic phase model with the PDFs for 239 each sample applying crystallographic symmetry con-240 straints. While unconstrained refinement may yield a model 241 that appears to better fit the data, this is unlikely to offer a 242 unique solution and does not necessarily provide a reliable 243 representation of the structure. Refinement of the struc-244 tural model against the PDF data for α-cubic Al-Fe-Si 245 (Fig. 4a), refining the lattice parameter (a = 12.55 Å), 246 atomic displacement parameters, and atomic positions 247 within the symmetry constraints, yielded a reliable fit (with 248 a weighted profile R-factor, $R_{wp} = 13.9\%$ [16]. The quality 249 of this preliminary fit is comparable to reported refine-250 ments of single crystal diffraction data, although the 251 weighting of the PDF by a pre-factor Q (i.e. the Fourier 252 transform of F(Q) = Q[S(Q) - 1]), placing an emphasis 253 on high Q data, causes the numerical value of R_{wp} to be 254 inflated relative to a Bragg refinement of equal quality. 255 As crystalline Al may be within the scattering volume we 256 attempted to fit the experimental data with α -phase plus 257 crystalline Al, but we found no significant Al inclusions 258 in this sample. For comparison the α -cubic Al–Fe–Si model 259 (Fig. 4b) refined against data from the ico-Al-Fe-Si phase 260 yielded a less good fit ($R_{wp} = 28.5\%$, a = 12.06 A). 261

Recent attempts to model the PDF of nanoparticle sam-262 ples with uniform size distributions have applied damping 263

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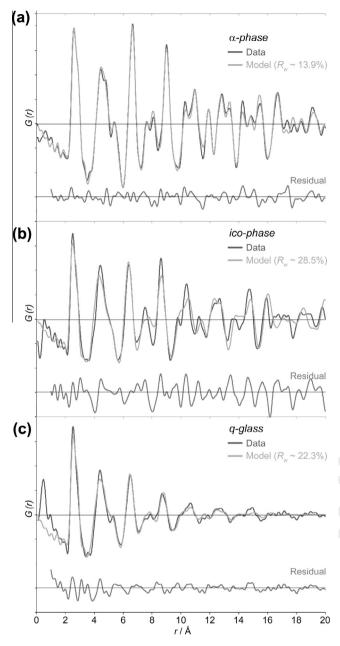


Fig. 4. The refinement profiles of the fits applying the α -cubic crystalline structure to the PDFs of the three samples. The experimental G(r) (solid line), calculated G(r) (dotted line) and the corresponding difference (black lower line) are shown for (a) α -Al–Fe–Si (lattice parameter a = 12.55 Å), (b) ico-Al–Fe–Si (a = 12.06 Å) and (c) g-Al–Fe–Si (a = 12.26 Å).

factors to G(r) that decrease the magnitude of the correla-264 tions at increasing values of r, such that the maximum cor-265 relation distinguishable in G(r) is consistent with the 266 diameter of the nanoparticles [17]. This approach can sim-267 ulate the PDFs of materials that are ordered on length 268 scales from 10 to several tens of Å. We have applied a sim-269 ilar approach to model the PDF of the g-Al-Fe-Si sample, 270 271 since barely discernible correlations were evident to ~ 20 Å. 272 Refinement of a structural model for the α -cubic approximating phase against the PDF from the glass (Fig. 4c) 273 yielded an agreement factor of $R_{wp} = 22.3\%$, and a lattice 274

parameter a = 12.26 Å. This indicates a strong similarity 275 between the local structure of the α-Al-Fe-Si sample and 276 the q-glass, indicative of the similarity between the local 277 structures of the two samples. The q-glass structure fits less 278 well to the neighboring icosahedral phase and not at all 279 well to an Al-rich β -phase, as will be discussed later. 280 Fig. 5a shows the S(q) values of the three phases, offset ver-281 tically for clarity. Fig. 5b shows the total PDF G(r) of the 282 three phases overlaid for comparison and Fig. 5c displays 283 G(r) for the α - and the β -phases. The data in Fig. 5b have 284 been scaled (by 1.024r) for the q-glass and (by 1.041r) for 285 the icosahedral phase to improve the overlay of the general 286 features of the PDFs and to account for the composition-287 dependent mean atomic distances. For PDF analysis these 288 distances are referred to as the refinement lattice parame-289 ters, shown in Table 1, of the different phases. The data 290 in Fig. 5c cannot be scaled, as the two phases are unrelated. 291

The first correlation, evident at ~ 2.7 Å, has slight asym-292 metry and increased width relative to that associated with 293 the Debye temperature alone. Such asymmetry usually 294 indicates that this peak contains contributions from differ-295 ent first shell correlations centered about this distance. 296 From 4 to 6 Å more significant differences in the PDFs 297 become apparent. In both the α -cubic Al–Fe–Si and the 298 g-Al-Fe-Si phases a correlation at 4.7 Å is resolved from 299 a less intense feature at 5.7 Å. In the ico-Al-Fe-Si phase 300 there is a single prominent feature at 4.7 Å with a slight 301 shoulder at higher r. We deduce from the partial pair cor-302 relations in the cubic α -Al–Fe–Si phase (see Fig. 6) that this 303 feature is due to an Al-Al correlation. At 8 Å there is a 304 double peak structure for α -Al-Fe-Si and g-Al-Fe-Si, 305 but only a single peak for ico-Al-Fe-Si. 306

The peaks in the PDF for the crystalline α -cubic phase and for the icosahedral phase continue to high *r*, consistent with long-range order. In contrast, the intensity of the atom-atom correlations in the g-Al-Fe-Si phase PDF decrease with increasing *r*, with no well-defined features beyond ~2 Å. This damping is consistent with the absence of long-range translational and rotational symmetry in a glassy material, although the peaks in this case persist to significantly longer length scales than for typical oxidebased or metallic glasses [18]. We note that *G*(*r*) for from the q-glass decreases in amplitude around 12 Å, which indicates that the size of the structural units comprising the glass are comparable in size to the icosahedral clusters in the α -cubic phase.

5. Heat treatment and phase changes in the solid state

Having determined the local structural ordering in the 322 q-glass, we now turn to differentiating between a glass 323 and a nano- or microcrystalline structure [19,20]. We join 324 Chen and Spaepen in defining a microcrystalline solid as 325 a material in which the grain structure coarsens upon 326 annealing with no phase transition involved. To measure 327 the phase development and their volume fractions we mon-328 itored the presence of diffraction peaks, linking directly to 329

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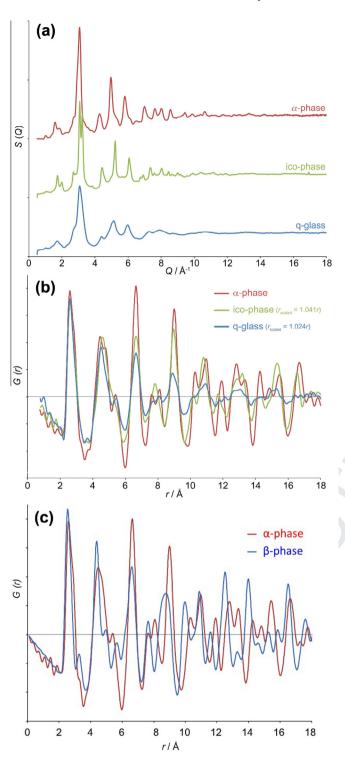


Fig. 5. (a) S(Q) of the α -phase, icosahedral phase and glass phase, offset vertically for clarity. (b) G(r) of the α -phase, icosahedral phase and glass phase overlaid for comparison. To improve the overlay of the general features of the PDFs and to account for the different lattice parameters of the different phases the data have been scaled (by 1.024r) for the glass and (by 1.041r) for the icosahedral phase. (c) G(r) of the α -phase and the β phases overlaid for comparison.

the International Union of Crystallography definition of 330 crystallinity [21]. Isothermal anneals of nano- or microcrys-331 talline structures would show a simple narrowing of the 332

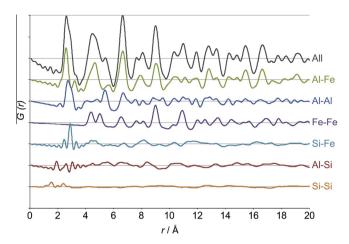


Fig. 6. Partial pair distributions of the six possible interatomic pairings in α-cubic Al-Fe-Si. The topmost shows a sum of all of the pairings. The known partial pair distributions within the α-cubic Al-Fe-Si phase enables the comparison of the total G(r) of this phase with the total G(r) of g-Al-Fe-Si to serve as a guide to the likely atomic pairings in the q-glass.

broadened diffraction peaks toward sharp Bragg peaks as 333 the grains coarsen. A structure that is not crystalline will 334 show evidence of a phase transformation, a period of coexistence with crystalline diffraction peaks growing as the residual broad glass peaks lose intensity without narrowing as the growing crystals consume the glass.

Appropriate temperatures for the in situ isothermal Xray measurements were identified by conducting prelimin-340 ary isothermal calorimetric measurements on the q-glass 341 at 320 °C, 330 °C, and 335 °C. To monitor the series of 342 structural transitions during annealing high energy 343 (~58 keV, $\lambda \approx 0.213$ Å) X-ray scattering data suitable for 344 PDF analysis were collected in beamline 11-ID-B at the 345 Advanced Photon Source, Argonne National Laboratory. 346 The sample was mounted orthogonal to a $200 \times 200 \,\mu\text{m}$ 347 X-ray beam within a temperature controlled sample envi-348 ronment (DSC Thermatica System) from Linkam Scientific 349 Instruments. Data were collected at ambient temperature 350 and during annealing of the q-glass at 330 °C (3 h) and 351 305 °C (16 h). Data were collected during 1 min exposures 352 at approximately 2 min intervals in the case of the short 353 anneal, beginning immediately after a rapid ramp in tem-354 perature to 330 °C. Data were collected during 5 min expo-355 sures at approximately 6 min intervals during the 16 h 356 isothermal hold at 305 °C, after which there was an addi-357 tional rapid ramp to 345 °C, where we took the last 4 h 358 of isothermal data. Every tenth image was dedicated to 359 measuring the dark field. Additional data were collected 360 upon recovery of the annealed sample, with greater than 361 2θ resolution (longer sample to detector distance), to assist 362 in phase identification. 363

For each of the three preliminary isothermal measure-364 ments a first-order phase transition was observed. The pha-365 ses(s) present during this transformation were identified 366 from the in situ synchrotron XRD data obtained during 367 the 330 °C isothermal anneal as described above. We found 368 that the q-glass persisted for about 40 min, after which it 369

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370 was transformed via a first-order phase transition 371 to β-Al_{4 5}FeSi [22]. The β-Al_{4 5}FeSi structure is a monoclinic pseudo-tetragonal double lavering of double-capped 372 (Al,Si) square antiprisms with shared edges, with periodic 373 374 stacking faults along the *c*-axis. The 10-coordinated Fe atoms lie at the center of the antiprisms. The PDF results 375 376 shown in Fig. 5c highlight the differences between this phase and the α -phase. 377

During an extended isothermal anneal at 305 °C the q-378 glass exhibited more complex behavior. Fig. 7 shows the 379 appearance of reflections indicative of fcc Al in the early 380 in situ data. Since we required a $200 \times 200 \ \mu m \ X$ -ray beam 381 for in situ annealing data collection, compared with the 382 $10 \times 10 \,\mu\text{m}$ X-ray beam for static data collection, the pres-383 ence of fine scale Al texture could not be entirely avoided in 384 the q-glass data. By following the growth of the intensity of 385 the crystalline Al diffraction peaks as a function of time we 386 discovered that q-glass rejects excess Al, producing fcc Al. 387 This is the solid-solid equivalent of the solid-liquid process 388 during rapid solidification, and indicates that the glass has 389 solubility limits that are temperature dependent. After the 390 391 q-glass rejected Al this two-phase mixture persisted for 392 approximately 1 h until a new phase, again β-Al_{4.5}FeSi, grew in, consuming both the remaining q-glass and most, 393 but not all, of the fcc Al. This is a classical peritectoid reac-394 tion in which two phases, in our case the fcc Al and the 395 q-glass, react to form another phase, β -Al₄ ₅FeSi. We see 396 in Fig. 8a, where the crystalline Al and β -Al₄ ₅FeSi peak 397 intensities are plotted as a function of time, that after this 398 peritectoid phase transformation is complete there is 399 another stable period during which polycrystalline Al and 400 β-Al_{4.5}FeSi coexist. Analyses of the product by means of 401 PDF methods, where the scans from the long stable period 402

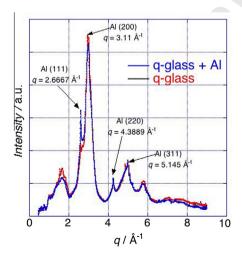


Fig. 7. Diffraction from g-Al–Fe–Si (red) and g-Al–Fe–Si with Al inclusions (blue). Note that some of the Al diffraction peaks fall on the broad g-Al–Fe–Si rings. A small amount of nano-crystalline Al texture is evident in the q-glass measurement due to the larger slit size required for the isothermal annealing measurements. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

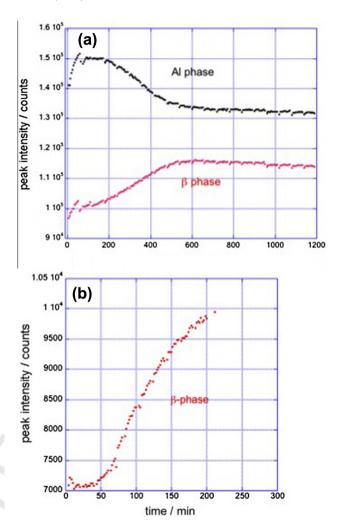


Fig. 8. Peak intensities for the crystalline phases as a function of time during the (a) 305 °C and (b) 330 °C thermal anneals show the increase in β -phase and increase and later decrease in the fcc Al phase at 305 °C. As a result of a declining X-ray beam intensity the plateau that start at 600 min are actually flat and indicate that equilibrium has been reached.

are summed, and Rietveld refinement of a higher resolution diffraction pattern, agree to within 1% that the product is 90 wt.% β -Al_{4.5}FeSi and 10 wt.% fcc Al (Fig. 9). Clearly, the fcc Al is not entirely consumed, and furthermore no additional phases are formed. We next subtracted the Al to derive the structure of the "relaxed" glass. The result is shown in Fig. 10, where we show the structure of the q-glass before the isothermal anneal compared with that of the in situ relaxed q-glass during isothermal annealing. The two structures are nearly identical.

Since there is no evidence in the $330 \,^{\circ}$ C data for the formation of fcc Al (Fig. 8b) and there is no indication of a transformation to a more stable q-glass, we conclude that the q-glass has a larger compositional stability range at the higher temperature. However, even when there is no evolution of the q-glass to a relaxed structure, as during the $305 \,^{\circ}$ C anneal, the product is the same as that of the higher temperature anneal, namely β -Al_{4.5}FeSi, which is quite different from the structure of the original q-glass.

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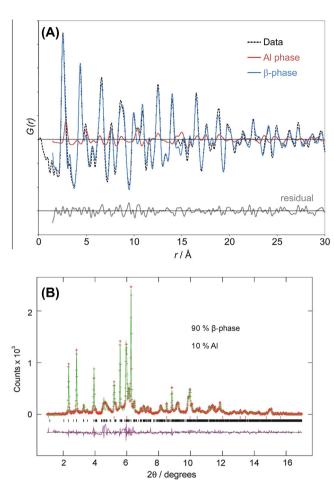


Fig. 9. Analysis of the product of the 305 °C anneal. (A) Model of the summed PDF data from 600 to 1200 min, indicating an excellent fit to 11 wt.% Al and 89 wt.% β -Al_{4.5}FeSi. (B) Rietveld refinement of XRD results for the product of the 305 °C anneal. Tick marks indicate calculated peak positions for β -phase (lower marks) and Al (upper marks). $\lambda = 0.2128$ Å. The analysis finds 90 wt.% β -Al_{4.5}FeSi and 10 wt.% crystalline Al. The data prove that β -Al_{4.5}FeSi and Al remain at the end of anneal.

422 6. Discussion and conclusions

The q-glass is an isotropic solid obtained by rapid cool-423 ing of a melt. It forms by nucleation and growth along a 424 moving interface between solid and melt with partitioning 425 of the chemical elements, thereby proving that this solid 426 is not a frozen melt. These features appear in the micro-427 graph of an Al-rich specimen [2], where we see the forma-428 429 tion of a two phase lamellar composite that is a eutectic solid composed of fcc Al and q-glass. We obtained some 430 evidence for partitioning from an energy dispersive X-ray 431 spectroscopy (EDS) analysis of the compositions of the iso-432 tropic solid and the eutectic composite. Additional evi-433 dence of partitioning comes from the narrow ($\sim 10 \text{ nm}$) 434 white band around each nodule. This is a diffusion zone 435 436 rich in rejected aluminum with a thickness that is approx-437 imately D/v, where D is a diffusion coefficient and v is the nodule interface growth velocity. Assuming a typical liquid 438 phase diffusion constant of $D \approx 10^{-9} \text{ m}^2 \text{ s}^{-1}$ implies that 439

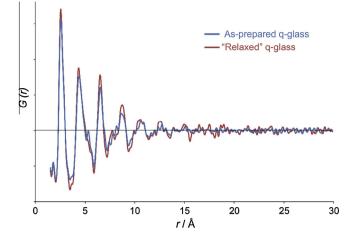


Fig. 10. The as-prepared q-glass G(r) was derived from an ex situ measurement before the 305 °C thermal anneal, where crystalline Al was not subtracted from these data. The annealed "relaxed" q-glass result was derived by subtracting Al from the summed scans measured during the plateau in Al intensity at 100–160 min during the in situ 305 °C thermal anneal. In this case G(r) above r = 15 Å was fitted to crystalline Al, and the result was subtracted from the total G(r) to give the relaxed q-glass component.

 $v \approx 0.1 \text{ m s}^{-1}_{\perp}$. As the nodules surrounded by such diffusion zones grow they become unstable [23], where the onset of this instability is seen in the undulations in the interface that provide additional evidence for partitioning. The transverse section in Fig. 1b provides evidence that the nearly 100% q-glass samples also form by nucleation and growth. 440 441 443 444 445

An alternative explanation for the observed separation 447 into two components is phase separation into two liquids 448 of different composition that, on further cooling, become 449 the q-glass and a eutectic. However, we eliminated this pos-450 sibility in earlier experiments by varying the composition of 451 the alloy melt and observing that as the overall Al content 452 was reduced the isotropic solid continued to be the first 453 phase to form and eventually became the major phase 454 [2,4]. This ruled out liquid phase separation. 455

It is always possible to find local atomic arrangements in 456 liquids and glasses that match pieces of crystals and even of 457 quasi-crystals. Whether such pieces are tiny crystals and 458 the glass is a polycrystalline aggregate cannot be decided 459 from structural data. We define a microcrystalline solid 460 as a material in which the grain structure coarsens upon 461 annealing with no phase transition involved. During 462 in situ annealing we found instead that the broad XRD 463 rings from the q-glass remained broad and gradually 464 diminished in intensity, while sharp diffraction peaks from 465 unrelated crystalline phases appeared and grew during the 466 anneal. This is consistent with a first order phase transition. 467

During the 305 °C anneal the q-glass persisted for more 468 than 1 h before gradual nucleation and growth of a β -phase. This demonstrated that the q-glass is a metastable 470 phase in the classical sense, i.e. stable to small perturbations and unstable only to the appearance of β -phase 472

"nuclei" particles that are large enough to overcome capil-lary barriers from the interfaces that have to be created.

475 The q-glass has strong local chemical ordering, which is common in intermetallic crystalline phases of the same 476 477 three elements. This ordering should depend upon composition and temperature. We looked for, but could not find, 478 479 diffraction evidence of relaxation of the q-glass structure (Fig. 10). At 305 °C we observed a steady growth of fcc 480 Al peaks in the diffraction pattern (Fig. 8a) as Al is rejected 481 by the q-glass. As expected for a two phase equilibrium the 482 rate of fcc Al phase formation slows until it is almost unde-483 tectable as metastable equilibrium is approached. 484

⁴⁸⁵ During the $\underline{330}$ °C anneal (Fig. 8b) the q-glass persisted ⁴⁸⁶ for ~40 min before gradual nucleation and growth of the ⁴⁸⁷ β -phase. During this anneal there was no diffraction evi-⁴⁸⁸ dence of Al rejection. We conclude that the solubility of ⁴⁸⁹ aluminum in the q-glass is broad enough at this tempera-⁴⁹⁰ ture to incorporate the Al present.

491 Eventually annealing at both temperatures resulted in the nucleation and growth of a distinct β -phase that con-492 sumed the q-glass. Because of the symmetry change and 493 494 the lack of structural similarity to the original motifs within 495 the q-glass this is also a first order phase transition. 496 Because the q-glass is composed of three elements it can coexist at constant pressure with up to two other 497 phases over a range of temperatures. During growth of 498 the β -phase at 305 °C the q-glass coexisted with both the 499 β -phase and the rejected fcc Al. This difference between 500 the two anneals indicates that the solvus of the β -phase 501 for aluminum is also temperature dependent. 502

Like all thermodynamic phases, the q-glass can partici-503 pate in heterogeneous coexistence with other phases along 504 505 interfaces where, away from critical points, the composition and structure change over atomic scale distances. In 506 general such interfaces move to reduce the free energy as 507 phases expand or shrink, compositions adjust, and the area 508 of interfaces changes. The presence of interfaces establishes 509 510 that the adjoining regions are of different phase. If adjoining regions were of the same phase there would be no inter-511 face, only a spreading diffusion zone. We have identified 512 three phases that form interfaces with the q-glass. During 513 formation upon rapid cooling the q-glass forms interfaces 514 515 with the initial melt and with fcc Al in eutectic structures. 516 During an isothermal anneal the glass forms interfaces with fcc Al precipitates and the β -phase. These last two are also 517 518 possible with a glass that is a frozen melt, but the coexisting compositions of these phases in the q-glass are different 519 from what is expected with a frozen melt. 520

521 We have demonstrated that the q-glass is a solid solution phase in the thermodynamic sense. It is neither a fro-522 523 zen melt nor nanocrystalline. The large ordering distance of ~ 12 Å shown in the PDF results in Fig. 5b demonstrates 524 525 that a large fraction of the atoms maintain a well-defined 526 positional relationship that, within this range, is consistent 527 with an ordered solid. Nevertheless, the phase is unquestionably homogeneously disordered, e.g. with no preferred 528 growth or structural directionality. To our knowledge this 529

combination of traits has never before been reported, hence the impetus to understand the underlying structural properties of this unique phase.

The existence of a growth front with the melt implies that the atomic surface of the q-glass forms the template for successive growth. We need to understand why, during growth of the q-glass, the directional order that typifies crystals and quasi-crystals is lost. This behavior must also be reconciled with the high degree of local ordering demonstrated by the PDF results.

The PDF data indicate that the local structure of the glass closely resembles that of the α -cubic Al–Fe–Si phase, which is composed of concentric shell Mackay icosahedra that maintain their orientation by packing along specific 3-fold icosahedral directions [23]. Similar clusters are found with a variety of different packing configurations [24] in other systems. Additional evidence of unexpected directional order of these structural units has been observed in TEM studies of polycrystalline aggregates. Although the axes of adjacent crystals differ by 5-fold rotations the structural units maintain the same orientation [26].

One possible explanation for the loss of orientational order is the presence of local frustration during growth. If incompatible packing configurations grow from the same seed cluster local frustration would likely disrupt the longrange ordering. Such disordered packings of Mackaybased clusters in two dimensions have been observed using TEM in the Ti–Mn–Si system [27].

A more intriguing possibility is that the q-glass is a fully ordered atomic structure that possesses no long-range translational symmetry and is isotropic. This too is consistent with the PDF results. As mentioned in the Introduction, there are many 3-D configurations of points that exhibit these properties [1]. The aperiodic zoo discussed in the Appendix describes some 2-D examples. The Appendix also offers additional theoretical considerations regarding fully ordered isotropic structures. The quasi-crystalline phases taught us that perfect atomic ordering does not require translational symmetry. If the q-glass turns out to be fully ordered then the requirement for finite rotational symmetry in an ordered solid would also prove incorrect.

We know of no experimental measurement that can distinguish between these two models. Diffraction is a standard probe of atomic structure and the presence of sharp diffraction peaks proves the existence of long-range ordering. However, we are faced with the possibility that the reverse may not be true, that glass-like scattering may result from highly ordered atomic arrangements.

Disclaimer

Certain commercial instruments are identified in this paper to foster understanding. Such identification does not imply recommendation or endorsement by the Department of Commerce or the National Institute of Standards and Technology, nor does it imply that the equipments identified are necessarily the best available for the purpose. 584

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585 **7. Uncited reference**

586 Q3 [25].

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608 Appendix A

609 A.1. The aperiodic zoo

The International Union of Crystallography definition 610 of crystals is based on the existence of sharp diffraction 611 peaks and includes both crystals and guasi-crystals. It 612 would also include layered structures such as pyrolytic 613 graphite that are stacked periodically, even though the lay-614 ers are randomly oriented, while most 2-D and 3-D isotro-615 616 pic structures would be excluded. Within the limit the pinwheel tiling described by Radin [28] includes all orienta-617 tions and is an example of an ordered 2-D arrangement of 618

points that is neither periodic nor quasi-periodic, and even 619 within the limit is not isotropic. The quasi-periodic 2-D til-620 ing with a 19-fold axis shown in Fig. A1A approximates an 621 isotropic structure and its diffraction pattern in Fig. A1B 622 shows glass-like rings, although it is composed of Bragg 623 peaks. We believe that within the limit of infinite rotations 624 the diffraction from such a structure would approximate 625 that from a 2-D glass. Techniques other than diffraction 626 would be required to establish the degree of ordering in 627 such a system. 628

Computer simulations have identified transitions from 629 liquids to ordered isotropic phases. In 1992 Dzugutov 630 [29] proposed an artificial potential for a single component 631 system in which non-crystalline structures would have 632 lower energies than crystalline structures. Simulations 633 found a first order reversible phase transition between a 634 melt and a non-crystalline isotropic solid that is highly 635 structured and has low entropy. Mendelev [30] found a 636 similar first-order reversible transition in a molecular 637 dynamics simulation that used an early embedded atom 638 potential for Al. He found a melting temperature where 639 the two phases coexist along an interface. Above this tem-640 perature melting of the solid occurred heterogeneously by 641 motion of the interface, and not by homogeneous processes 642 throughout the solid. Below the melting temperature the 643 interface moved into the melt. Mendelev [30] reported a 644 latent heat, a volume change and a 10⁴ change in diffusiv-645 ity. Thus our observation that an isotropic solid phase can 646 grow from the melt has some theoretical basis. These sim-647 ulation studies prove that this can occur with some atomic 648 potential. Our observations prove that it can occur in real 649 systems. 650

A.2. Fundamental theoretical considerations

For a three component melt there are generally several 652 temperatures at which solids are in equilibrium with melts. 653 At the liquidus temperature a small amount of solid can 654 coexist with the melt. Generally this will be a solid with a 655 composition different from the melt and its growth will 656

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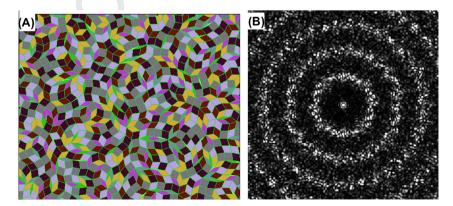


Fig. A1. (A) The quasi-periodic 2-D tiling with a 19-fold axis approximates an isotropic structure. (B) Its diffraction pattern shows glass-like rings, although it is composed of Bragg peaks. In the limit of infinite rotations the diffraction from such a structure would approximate that from a 2-D glass. (Calculated results from Steffen Weber's website at http://jcrystal.com/steffenweber/JAVA/jtiling/jtiling.html.)

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change the composition of the remaining melt. At and
below the solidus temperature the melt has been fully
replaced by solids. Generally there will be a mixture of solids, unless there is a solid with the same composition as
that of the melt.

The liquidus and solidus temperatures coincide for spe-662 663 cial compositions. "Congruent" compositions occur where a melt and a single solid of the same composition are in 664 equilibrium at a congruent temperature, usually at temper-665 ature maxima. Eutectic compositions occur when at the liq-666 uidus a melt can decompose fully into a mixture of two or 667 more solids. Eutectic temperatures are at minima. Of spe-668 cial interest for this research is the case when a melt that 669 is not of a eutectic composition encounters a liquidus on 670 cooling. The growth of that first phase will change the melt 671 composition, sometimes towards a eutectic. This is what is 672 seen in Fig. 1. The q-glass nodules are the first phase to 673 form. They differ in composition from the melt, and reject 674 675 aluminum into it. When the melt reaches the eutectic composition two phases, whose volume-averaged composition 676 matches that of the remaining melt, grow outward from 677 678 the nodules.

679 A microcrystalline model of metallic glasses has been 680 proposed by one us (L.B.) based on an experimental study of how grain size depends on cooling rate. Isotropic fea-681 tureless specimens were found where the grain size 682 approached the atomic scale, raising the question of 683 whether these were metallic glasses or polycrystalline 684 aggregates with a grain size of less than 1 nm. Their energy 685 would exceed that of large crystals by the presence 686 of $\sim 10^9 \,\mathrm{m}^2$ grain boundaries, having an energy of 687 $\sim 10^8 \, \mathrm{J \, m^{-3}}$ specimen, and comparable with the heat 688 of fusion. We suggest that it is unreasonable to assume a 689 model where the crystals have sizes close to or even smaller 690 than the unit cells. Some claim even to have distinguished 691 between microcrystals and quasi-microcrystals on this 692 scale. If the specimen is indeed microcrystalline no further 693 694 phase change is needed for coarsening of the grains on heating to sizes where their crystallinity is not in doubt. 695 Such a process can be observed in XRD spectra as broad 696 glassy peaks narrow and become resolved as individual 697 698 crystalline peaks.

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