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SHALLOW MIXING LAYER DOWNSTREAM A SUDDEN EXPANSION

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ABSTRACT

The present paper aims at investigating the mixing layer located at the interface between the free stream and the recirculation zone downstream an open-channel, sudden, lateral expansion. Specific attention is paid on the interaction of the shallowness of the flow, characterized by the bed friction number, with the lateral confinement, due to the side wall. The velocity field for four flows, with the same geometry but very different bed friction numbers, is measured in detail in order to characterize the mean velocity fields and Reynolds stresses across the mixing layers and to evaluate the width of the mixing layers and their growth rates along with the typical oscillation frequencies. In the upstream region of the recirculation zone, the mixing layer characteristics for our configurations are analogous to the ones of classical - laterally unbounded - mixing layers. In this region, the shallowness modifies the shape of the streamwise velocity profiles, extends the mean velocity gradient magnitudes, lowers the Reynolds stress terms but hardly affects the mixing layer expansion rate. On the other hand, in the region near the flow reattachment, the mixing layer adopts a very different behavior, with an abrupt drop of the mixing layer expansion. This change in behavior is linked to the dynamics of the 2D vortices within the mixing layer. It is not due to a damping effect of the bed friction on these vortices as the local bed friction numbers remain much lower than the critical values reported in the literature. It is rather due to the interaction of the coherent structures with the side wall, the characteristics of this interaction being itself influenced

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21 by the flow shallowness. We additionally show that the damping effect due to bed friction is not
22 responsible for the huge variations reported on the recirculation zones length. This imposes to
23 distinguish between a "local mixing-layer shallowness" - derived using the mixing layer width as
24 length scale and governing the mixing layer characteristics - and a "global flow shallowness" -
25 derived using the expansion width and governing the recirculation length.

26 **Keywords:** Shallow, Mixing layer, Recirculation zone, Backward facing step, Vortex .

27 INTRODUCTION

28 As a flow separates from a side wall, a recirculation zone forms until the flow reattaches to the
29 wall further downstream. Li and Djilali (1995) listed the most common geometries giving birth to a
30 recirculation zone including upward and backward facing steps, sudden expansions or flows around
31 obstacles such as cylinders or plates. Within the recirculation zone, the net discharge is nil with
32 limited positive streamwise velocities near the separating streamline and negative reverse velocities
33 near the side wall. In natural streams, these regions have a major implication for transport of scalar
34 or sediment (O'Connor et al. (2010) and Engelhardt et al. (2004)) as the limited velocities make the
35 recirculation a privileged zone of material deposition. In the literature dealing with recirculation
36 zones, most attention is paid to the prediction of the recirculation length, *i.e.* the location of the
37 reattachment point (Li and Djilali (1995)).

38 On the other hand, along the interface between the free stream and the recirculation zone, the
39 strong transverse gradient of streamwise velocity leads to a vertical mixing layer, defined by Pope
40 as "a turbulent flow that forms between two uniform, nearly parallel streams of different velocity"
41 (Pope (2008)). A review of the most studied mixing layers configurations in open-channel flows
42 under deep conditions was established by Mignot et al. (2014a) and Mignot et al. (2014b). These
43 configurations exhibit the following behavior: the width of the mixing layer increases along the
44 streamwise axis from up- to downstream with the occurrence of advected turbulent eddies and a
45 maximum turbulence production occurs along this mixing layer due to the increased shear and ve-
46 locity gradient. These terms are thus maximum along the centerline of the mixing layer and rapidly
47 decrease on both sides, while their magnitude decreases towards downstream. Nevertheless, the

48 mixing layer considered herein develops at the interface between a main stream and a recirculation
49 zone downstream from a shallow sudden expansion (SSE). This mixing layer is not free as it ex-
50 hibits both lateral confinement due to the side wall and vertical confinement due to the impact of
51 the bed.

52 **Effect of the lateral confinement**

53 Among the geometries exhibiting a recirculation, the open-channel sudden lateral expansions
54 and the backward facing steps appear as similar configurations *a priori* with a similar lateral con-
55 finement. In both cases, the walls upstream and downstream from the step are parallel to each other
56 and the flow section suddenly increases towards downstream. The recirculation zone is thus con-
57 strained by two perpendicular walls and closed along the third side by the separating streamline.
58 Studies describing the characteristics of the separating mixing layer downstream backward facing
59 steps show that (see Chandrsuda and Bradshaw (1981), Jovic and Driver (1994), Kasagi and Mat-
60 sunaga (1995)) the axis of the mixing layer just downstream the separation is parallel to the inflow
61 while further downstream, the mixing layer approaches the downstream wall with an increasing
62 angle. As a consequence, in the upstream part of the recirculation the mixing layer strongly resem-
63 bles typical free mixing layers (see Bell and Mehta (1990) or Wygnanski and Fiedler (1970)) with
64 a maximum turbulent activity occurring along the centerline of the mixing layer. Further down-
65 stream, the mixing layer becomes affected by the side wall and the location of maximum Reynolds
66 stress and turbulent production is shifted from the side wall at a distance which is not discussed in
67 details in the papers cited above, but will be analyzed in the present paper.

68 **Effects of the vertical confinement**

69 The mixing layer which occurs in the open-channel sudden lateral expansions was far less in-
70 vestigated than the one in backward facing steps. The main difference is the vertical confinement
71 due to the limited water depth encountered in the expansion along the direction perpendicular to
72 the recirculation plane. A first effect (Uijtewaal and Booij (2000)) is that, with this geometri-
73 cal restriction of the water column, "eddies with dimensions larger than the water depth can only
74 move in the horizontal plane. The limited depth prohibits these eddies to be stretched in the ver-

75 tical direction and forces the large eddies into a quasi two-dimensional motion." Yet, the mixing
76 layer growth is connected to the behavior of the 2D coherent structures it contains. The dimen-
77 sions of these structures, generated in the upstream region of the mixing layer, increase towards
78 downstream (Uijttewaal and Booij (2000); Loucks and Wallace (2012)). However, the flow is ad-
79 ditionally confined between a stick condition at the bed and a zero vertical but free horizontal slip
80 velocity at the free-surface. This confinement creates a strong vertical gradient of velocity and thus
81 additional turbulence generated in the near wall region (Babarutsi et al., 1989). As a consequence,
82 the shallowness causes a limitation of the growth of the mixing layer width along its development:
83 while this growth rate is constant along the streamwise axis for a deep mixing layer, in shallow
84 conditions, the growth rate equals that of the deep conditions in the upstream region but it then
85 decreases when advancing towards downstream (see Uijttewaal and Booij (2000)).

86 To characterize the shallowness of the flow (*i.e.* the vertical confinement), a bed friction number
87 S (also referred in the literature as “stability parameter”) was introduced by Chu et al. (1983) based
88 on a linear stability analysis of the depth-averaged shallow water equations and later by Chu et al.
89 (1991) based on the turbulent kinetic energy analysis of the same equation where a perturbation
90 is introduced. This coefficient characterizes the ratio between the stabilizing effect due to the
91 bed friction (through dissipation of turbulent kinetic energy) and destabilizing effects due to the
92 transverse shear along the mixing layer (through turbulent kinetic energy production).

93 In simple shallow mixing-layer configurations (see Sukhodolov et al. (2010), Uijttewaal and
94 Booij (2000), Van Prooijen and Uijttewaal (2002)) corresponding to two parallel inflows with
95 different velocities suddenly released on the side of each other, the linear stability analysis proved
96 the existence of a critical bed friction number (Chu et al. (1983)), noted S_c so that: i) in shallow
97 conditions, S exceeds S_c and the bottom friction impedes the growth of instabilities while ii)
98 in deep conditions S remains lower than S_c and the bottom friction has negligible effect on the
99 growth of instabilities. Chu et al. (1983) propose $S_c \sim 0.12$ through an inviscid theory while
100 Alavian and Chu (1985) found $S_c = 0.06$ by additionally considering the turbulent motions and
101 later Chu et al. (1991) obtained $S_c \sim 0.12-0.145$. Based on measurements of the growth rate of

102 the shallow mixing layer width, Chu and Babarutsi (1988) found $S_c = 0.09$ while Uijttewaal and
103 Booij (2000) obtained $S_c = 0.08$. However, Van Prooijen and Uijttewaal (2002)) cast doubt on
104 the concept of a critical bed friction number itself. Indeed, the existence of a critical value S_c
105 is obtained by considering the most unstable mode in the linear stability. These authors consider
106 instead the stability of the dominant mode, *i.e.* of the wave number corresponding to the maximum
107 energy density or, in other words, corresponding to the coherent structures taking place in the flow.
108 Instead of the two regimes listed above (S_c acting as the threshold), Van Prooijen and Uijttewaal
109 (2002) consider three regimes: (i) growth of the dominant mode, (ii) decay of the dominant mode
110 with other modes still growing and (iii) decay of all modes for $S > S_c$. These three regimes are
111 well reproduced by their linear stability analysis, when compared with experimental measurements
112 and can explain the scattering values of S_c previously reported in the literature ($S_c=0.06-0.145$, see
113 above). Unfortunately, Van Prooijen and Uijttewaal (2002)'s analysis requires a function fitting the
114 transverse velocity profile along the flow, which is unavailable for the present sudden expansion
115 configuration. Thus, as the concept of critical bed friction number provides a fair overview of
116 the behaviour of the shallow mixing layer, regardless of the scattering on the value of S_c , it will
117 be used hereafter to identify our mixing layer regime, as still done in recent contributions for
118 vertically confined mixing layers by Constantinescu (2013). In the following, we thus consider
119 that the shallowness stabilizes the mixing layer if the local bed friction number reaches $O(0.1)$.

120 **Specific case of the shallow sudden lateral expansion SSE**

121 To the authors' knowledge, the only analyses of the mixing layers developing downstream a
122 SSE were performed, using experimental approach, by Babarutsi et al. (1989) using a 1D velocity
123 probe along the streamwise axis and more recently by Talstra (2011) through Surface-PIV. These
124 studies show that the SSE mixing layers exhibit high similarities with the more classical mixing
125 layers: notably, the streamwise velocity fluctuation is maximum along the separating streamline
126 and decreases on both sides. The main differences between a laterally-free mixing layer and a SSE
127 mixing layer are that for a SSE: the velocity on the slow flow side (*i.e.* in the recirculation zone)
128 is quite negligible and the curvature of the mixing layer centerline increases towards downstream,

129 until reaching the reattachment point.

130 Babarutsi et al. (1989) propose to extrapolate this knowledge at the local, mixing layer, scale
131 to predict the length L of the recirculation zone downstream a SSE (see Fig.1) at the integral scale.
132 With this purpose, Babarutsi et al. (1989) adopt as integral transverse length scale the expansion
133 width (d) and as vertical length scale the water depth at the expansion section (h_0), as depicted in
134 Fig.1. The (integral) bed friction number, noted S_d , finally reads:

$$135 \quad S_d = \frac{c_{f0}d}{2h_0} \quad (1)$$

136 with c_{f0} the friction coefficient at the expansion section. Using again the concept of critical bed
137 friction number, Babarutsi et al. (1989) define two flow regimes separated by a critical value of the
138 integral bed friction number $S_d=0.05-0.1$: for $S_d<0.05$ the flow is referred to as « deep water flow
139 » and the relative recirculation length is $L/d = 8$; for $S_d>0.1$ the flow is called a « shallow water
140 flow » and the relative recirculation length is $L/d = 0.6/S_d$. Later, Chu et al. (2004) instead name
141 these two regimes, respectively, « non-frictional » and « frictional » modes, and, with additional
142 experimental results, propose $L/d = 0.7/S_d$ for the shallow water flows (frictional mode).

143 Through recent experiments in the Laboratory of Fluid Mechanics and Acoustics (LMFA) at the
144 University of Lyon (INSA-Lyon, France), Chatelain et al. (2014) observe quite a different behavior
145 depicted by Fig.2 that plots a series of 16 non-dimensional recirculation lengths configurations with
146 increasing S_d values with $R_b = (B - d)/B = 0.75$. The “bell” shaped curve on Fig.2 differs from
147 the two asymptotic regimes proposed by Babarutsi et al. (1989) or Chu et al. (2004): it indeed
148 exhibits a maximum non-dimensional length L/d for $S_d \sim 0.01$ and decreasing lengths both for
149 lower and higher S_d values. However, this work confirms that the macroscopic (integral) behavior
150 of the recirculating zone (typically its length) is a function of the integral bed friction number S_d .

151 Talstra (2011)’s study explores experimentally the classical mixing layer features (velocity pro-
152 files, Reynolds stress profiles, mixing layer width, coherent structures with the mixing layer) for
153 three different SSE mixing layers. It shows that the initial mixing layer growth is similar to the one
154 for a free mixing layer and that interactions exist between the main recirculation and the vortex

155 shedding. Numerical simulations supplement the experiments to explore the role of secondary cur-
156 rents and of upstream disturbances. However, the expansion ratio R_b varies from one experiment to
157 another while this parameter was shown to influence the flow downstream the expansion (Riviere
158 et al. (2008) and Chatelain et al. (2014)). This prevents from concluding regarding the influence of
159 the shallowness alone on the SSE flow.

160 **Scientific issue**

161 To summarize, it appears firstly that 1) laterally bounded deep mixing layers and 2) laterally
162 unbounded shallow mixing layers are well described in the literature. Laterally bounded deep
163 mixing layers studies report 3D turbulence with emphasis on the reattachment point at the wall.
164 Shallow mixing layers studies report 2D turbulence with emphasis on the influence of the shal-
165 lowness on the growing rate of the mixing layer width. Conversely, little attention was devoted
166 to the "combined" case of both laterally bounded and shallow mixing layers as the one occurring
167 in the shallow open-channel sudden lateral expansions. In particular, the combined effects of the
168 shallowness and of the lateral wall on the mixing layer are expected to interact with each other
169 at the reattachment point and this interaction is still to be studied. Secondly, two parameters are
170 used in the literature to quantify the shallowness. The local bed friction number S uses the mixing
171 layer width as horizontal length scale and governs the evolution of the mixing layer width and of
172 the associated 2D large scale turbulent structures. The integral bed friction number S_d uses the
173 expansion width as length scale and characterizes the recirculation length. The connection of S_d
174 to S , *i.e.* the connection of the recirculation length to the adjacent mixing layer, remains to be
175 confirmed.

176 Therefore, the first objective of the present work is to measure and characterize the shallow
177 mixing layer downstream a sudden expansion and then to emphasize its differences with more
178 simple configurations in order to finally sort the combined effects of the shallowness and of the
179 lateral confinement on the mixing layer, especially in the region of the reattachment point. The
180 second objective is to examine if the phenomena at the mixing layer scale (notably the coherent
181 structures behavior) can explain the behavior of the recirculation zone at a macroscopic scale,

182 notably the evolution of the recirculation length as a function of S_d . To achieve both objectives,
183 four cases corresponding to the two macroscopic regimes (shallow and deep) were selected for a
184 detailed analysis.

185 The paper is organized as follow. A first section details the experimental facility. A second
186 section is devoted to select the four configurations corresponding to different macroscopic behav-
187 iors of the recirculation zone and to different mean flow properties. The connection with the local
188 phenomena is enabled by the third section which details the properties of the mixing layers and
189 compares them to more classical mixing layers. As these properties are linked to the 2D turbulent
190 eddies behavior, the latter are studied in a fourth section. From all these results, the discussion
191 sums-up the new knowledge on both objectives of the present paper: (i) the combined effect of
192 the shallowness and of the lateral wall on the mixing layer and (ii) the connection of the (integral)
193 recirculation behavior with the local mixing layer properties. Note that for the sake of clarity when
194 comparing the present SSE mixing layers with more simple configurations from the literature,
195 the following terminology is used: “deep” configurations refer to configurations with negligible
196 vertical confinement, “unbounded” configurations refer to configurations with negligible lateral
197 confinement and finally “free” mixing layers are both deep and unbounded.

198 **EXPERIMENTS**

199 **Experimental facility**

200 The experiments are conducted in an open-channel flume located in the Laboratory of Fluid
201 Mechanics and Acoustics (LMFA) at the University of Lyon (INSA-Lyon, France) sketched in
202 Fig.1. The flume is $L_t=8$ m long, straight, with a constant streamwise slope of 0.18% and a sym-
203 metrical rectangular cross-section of width $B=0.8$ m. The flume is PVC made with a typical rough-
204 ness height $\varepsilon \sim 5 \times 10^{-5}$ m. A rectangular shape impervious block of width $d=0.2$ m is included
205 along the upstream part of the right bank over a length $L_b = 3.56$ m. The expansion ratio thus
206 equals $R_b = (B - d)/B = 0.75$. The axis system is set as depicted on Fig.1 with the center lo-
207 cated at the expansion section along the side wall, x the streamwise axis and y the crosswise axis.
208 The discharge Q is measured in the pumping loop using one of the two available electromagnetic

209 flowmeters (Endress-Hauser): the first one in the range $Q=5-40$ L/s with an uncertainty 0.2 L/s,
210 the other for $Q=0-5$ L/s with an uncertainty of 0.025 L/s. The upstream boundary condition con-
211 sists of a grid buffer and a honeycomb with small mesh (0.5 cm alveolus) in order to stabilize the
212 inflow. Moreover, a float board made of extruded polystyrene lies on the free surface in order to
213 suppress the potential free surface oscillations. The downstream boundary condition consists of an
214 adjustable tailgate, preceded by a stilling basin, allowing to precisely adjust the downstream water
215 depth.

216 The velocity field is measured using a Vectrino+ Nortek side-looking ADV (Acoustic Doppler
217 Velocimeter) mounted on an automatic displacement and recording carriage connected to a PC
218 computer through LabVIEW software. In shallow conditions, this device permits to access the
219 two horizontal velocity components u , v along the streamwise (x) and transverse (y) directions
220 respectively, with $u = \bar{u} + u'$ where the overline refers to time-averaged and prime to fluctuating
221 velocity component. Hollow glass spheres ($50\mu\text{m}$) and micro-bubbles are added to the water in
222 the upstream tank to improve the acoustic backscattering. Spikes included in the raw ADV data
223 are removed using the now classical Phase-Space Thresholding Method developed by Goring and
224 Nikora (2002). Time convergence of all terms is ensured with measurements at a frequency of 30
225 Hz during 180 seconds. Moreover, when estimating the velocity variances (for the estimation of the
226 normal Reynolds stresses) noise is removed using the spectral method proposed by Voulgaris and
227 Trowbridge (1998). The measurement grid is composed of about one thousand points in the region
228 downstream from the expansion ($0 < x < 1.3L$ with L the recirculation length and $0 < y < 3.5d$)
229 with a spatial resolution of 2×4 cm in the recirculation zone ($y < 1.5d$) and 4×4 cm in the free
230 stream ($y > 1.5d$), as shown in Fig.3. The velocity field is measured at an elevation equal to $0.4h_0$
231 with h_0 and U_0 the water depth and mean streamwise velocity measured at the expansion section
232 ($x=0$) at a transverse distance of $y = 1.5d$. Finally, the water depth is measured using an ultrasonic
233 probe (Baumer electric, uncertainty 0.15 mm) also located on the automatic displacement carriage.

234 FLOW DESCRIPTION

235 Selection of the experimental configurations

236 Fig.2 plots a series of 16 non-dimensional recirculation lengths configurations with increasing
237 S_d values presented by Chatelain et al. (2014). The recirculation length L is defined as the distance
238 from the expansion where the mean streamwise velocity along the side wall changes sign from
239 negative (towards upstream) in the recirculation to positive (towards downstream) further down-
240 stream. The specific “bell” shape presented by Chatelain et al. (2014), which differs from the fitting
241 curves proposed by Chu et al. (2004), exhibits a maximum non-dimensional length L/d obtained
242 for $S_d \sim 0.01$ and decreasing lengths for lower and higher S_d values. Four configurations (F1 to F4)
243 are selected here. Two configurations, F1 and F2, are specifically selected for the detailed analysis
244 of the mixing layer characteristics. They are located on both sides of the “bell” with compara-
245 ble recirculation lengths: F1 is a deep configuration ($S_d < 0.01$) and F2 a shallow configuration
246 ($S_d > 0.01$). Two additional configurations are also considered for comparison: F3 located on the
247 “bell” top and referred as a transitional flow and F4 an additional shallow configuration obtained
248 with a larger bottom roughness (by addition of aluminum tear plates). All flow characteristics are
249 detailed in Table 1.

250 Mean flow properties

251 As a first step, the full development of the flow in the upstream channel ($x < 0$) is verified (not
252 shown here) so that the water depth, mean horizontal velocity components and Reynolds stresses
253 along the center line of the incoming channel do not evolve when approaching the expansion. The
254 mean velocity fields measured downstream from the expansion at $z=0.4h_0$ elevation are plotted in
255 Fig.3. The flow separates at $x/L=0$ and $y/d=1$, with a velocity vector almost parallel to x axis
256 and reattaches at $x/L=1$ where the mean streamwise velocity close to the wall changes sign from
257 negative to positive. Note however that the reattachment point is known to vary in time due to
258 the passing of coherent structure (*e.g.* Riviere et al. (2011)). A main recirculation cell forms for
259 $y/d < 1$ and $x/L < 1$ and a small secondary cell near the corner at $x=y=0$ is also present (Chu et al.
260 (2004)); as could be seen on a zoom of Fig.3 (not shown here). In the outer region ($y/d > 1.5$), the
261 velocity field is deflected towards the recirculation region and decelerates along x axis.

262 In addition, the streamwise mean velocity normalized by U_0 is plotted along several cross-
263 sections in Fig.4 along with the separating streamline. First, it appears that the two separating
264 streamlines differ as the shallow mixing layer (F2) approaches the side wall more rapidly than the
265 deep one (F1). Moreover, while the velocity profiles for both configurations are very similar to
266 each other within the recirculation zone and resemble measurements by Talstra (2011) (see their
267 figure 3.11), they strongly differ in the outer region. For the shallow F2 configuration, the velocity
268 profiles rapidly increase across the separating streamline and become almost uniform for $y/d > 1.1$.
269 Oppositely, for the deep F1 configuration, the mean streamwise velocity increases more gently
270 along y axis until $y/d > 1.5$ and no uniformity is observed in the plotted region. This difference of
271 behavior is in agreement with data from Babarutsi et al. (1989) when comparing their deep (in their
272 figures 3 and 4) and shallow (figures 6 and 7) cases. The mean streamwise velocity profiles then
273 appear to strongly differ from those self-similar reported for unbounded mixing layers (Bell and
274 Mehta (1990)) and appear to be highly impacted by the lateral confinement and the shallowness.

275 Fig. 5 then plots along the same sections the dimensionless transverse gradient of streamwise
276 mean velocity. This figure reveals that the maximum gradient is located along the separating
277 streamline for $x/L < 0.8 - 0.9$ and that further downstream it remains far from the side wall
278 while the separating streamline reattaches. Moreover, the maximum gradient is measured at the
279 expansion section and it decreases towards the reattachment point. The region of high gradient
280 then spreads laterally when advancing towards downstream. The shape of velocity gradient herein
281 is in agreement with all mixing layer configurations mentioned in the introduction section and
282 thus confirms the existence of a mixing-layer along the separating streamline. Nevertheless, strong
283 differences can be observed between the two flow configurations considered herein: the maximum
284 gradient is stronger for the shallow case (F2) while the transverse extension of high gradient is
285 higher for F1.

286 **Reynolds stress tensor**

287 The components of the 2D Reynolds stress tensor $\overline{u'^2}$, $\overline{v'^2}$ and $-\overline{u'v'}$ are plotted in dimen-
288 sionless form in Fig. 6: the distributions are quite similar for the 3 terms. First, the stresses are

289 maximum along the separating streamline (for $y/d < 0.8-0.9$) and vanish on both sides (in the re-
290 circulation zone and the outer main flow). Moreover, the stress magnitudes are increasing in the
291 upstream region of the mixing layer, reach a maximum at $x/L \sim 0.3$ to 0.5 and decrease down-
292 stream. These behaviors are in agreement with data from the SSE in the literature (Babarutsi et al.
293 (1989) and Talstra (2011)) and with deep or shallow-unbounded mixing layers. Finally, when ap-
294 proaching the reattachment point, the maximum stresses are measured away from the side wall
295 (near $y/d \sim 0.3$ to 0.5).

296 In order to compare the magnitude of the Reynolds stresses between both F1 and F2 config-
297 urations and with data from the literature, Tab.2 summarizes some available maximum Reynolds
298 stresses magnitudes selected in a free and in a shallow unbounded mixing layers, in a backward
299 facing step and in available shallow sudden expansions. For all configurations, the maximum
300 streamwise normal stress exceeds both the crosswise normal stress and the shear stress: the cross-
301 wise normal stress equals about 40-50% and the shear stress about 35% of the streamwise normal
302 stress except for some measurements from Talstra (2011) which are slightly out of these ranges.
303 Moreover, for the sudden expansions (from Babarutsi et al. (1989) and present configurations), the
304 maximum stress values decrease as S_d increases. Globally, the Reynolds stress magnitudes appear
305 to behave similarly as for deep and for unbounded mixing layers.

306 Fig. 7 plots the location of maximum velocity gradient and Reynolds stresses at each measured
307 section along with the separating streamline. It confirms that the centerline of the mixing layer,
308 defined here as the location of maximum velocity gradient, is also the location of maximum stresses
309 and of the separating streamline as long as it remains far from the side wall ($x/L < 0.8$), i.e.
310 as long as the effect of the lateral confinement is negligible. The mixing layer centerline and
311 separating streamline then separate at $x/L \approx 0.8$ where the centerline remains apart from the wall
312 (as previously reported for backward facing steps, see Chandrsuda and Bradshaw (1981)), at a
313 distance $y/d=0.6$ for F1 and $y/d=0.3$, shorter, for F2. This result is a first hint that the shallowness
314 affects the lateral confinement of the mixing layer: this difference of distance to the lateral wall at
315 the reattachment section is connected to the wider mixing layer for F1, due to a weaker vertical

316 confinement, as studied in next section.

317 MIXING-LAYER CHARACTERISTICS

318 Definitions

319 The width of the mixing layer is defined as the maximum slope thickness:

$$320 \quad \delta(x) = \frac{\Delta U(x)}{\left| \frac{\partial \bar{u}(x)}{\partial y} \right|_{max}} \quad (2)$$

321 This definition is commonly used in the literature regarding shallow mixing layers (*e.g.* Chu and
322 Babarutsi (1988); Uijttewaal and Booij (2000); Van Prooijen and Uijttewaal (2002); Talstra (2011);
323 Constantinescu (2013)) and, unlike for other definitions, does not require velocity plateaus on
324 both sides of the mixing layer. This definition requires however the outer velocity difference
325 $\Delta U(x) = U_1(x) - U_2(x)$ where $U_1(x)$ and $U_2(x)$ are the outer velocity magnitudes, respectively
326 in the main flow and the recirculation zone.

327 The streamwise growth of the mixing-layer width is classically defined as a function of the
328 ratio between the outer velocity difference and the center velocity $U_c(x) = (U_1(x) + U_2(x))/2$,
329 reading:

$$330 \quad \frac{d\delta}{dx} = \alpha \frac{\Delta U(x)}{U_c(x)} \quad (3)$$

331 where α is the spreading rate coefficient ; for free mixing layers, $\alpha=0.06-0.11$ (Pope (2008)) or
332 $\alpha=0.085$ (Lesieur (1997)).

333 Finally, the mixing layer width is connected to the local bed friction number $S(x)$ (Chu et al.
334 (1983)) as (using Eq.2)

$$335 \quad S(x) = \frac{c_f U_c(x)}{2h \left| \frac{\partial \bar{u}(x)}{\partial y} \right|_{max}} = \frac{c_f U_c(x)}{2h \Delta U(x)} \delta(x) \quad (4)$$

336 Reported streamwise evolution of the mixing layer width from the literature

337 Eq.3 reveals that the streamwise evolution of the mixing layer width $\delta(x)$ can be derived from
338 the streamwise evolution of the outer velocity difference $\Delta U(x)$ and the center velocity $U_c(x)$,
339 these evolutions being strongly affected by the characteristics of the mixing layer.

340 A first case corresponds to the unbounded shallow mixing layer. Under specific assumptions,
341 the depth-averaged momentum equations lead to $\Delta U(x)$ decreasing exponentially with the stream-
342 wise distance x (Chu and Babarutsi (1988) ; Van Prooijen and Uijttewaai (2002)). Van Prooijen
343 and Uijttewaai (2002) consider additionally a constant value of $U_c(x)$ and then obtain an exponen-
344 tial decay of the mixing layer width $\delta(x)$. This tendency confirms the stabilization of the mixing
345 layer width observed in the literature on unbounded shallow mixing layers and is in satisfactorily
346 agreement with their experimental data.

347 A second case corresponds to the field experiments of Sukhodolov et al. (2010) in a straight
348 shallow river reach with an initial crosswise velocity gradient. The authors report a linear decrease
349 of the outer velocity difference $\Delta U(x)$. Considering a constant value of the center velocity U_c , as
350 for authors cited for the first case, leads to a parabolic evolution of the mixing layer width $\delta(x)$.
351 These authors consider that the agreement is satisfactory with their field measurements using a
352 fixed spreading rate coefficient $\alpha=0.11$. Note that Booij and Tukker (2001) obtained alternatively
353 an exponential decay (see their cases A and D) and a linear decay (see their cases B and C) of the
354 outer velocity difference $\Delta U(x)$.

355 A third case, the curved mixing layer observed in an open-channel confluence (Mignot et al.
356 (2014b)) exhibits a linear decay of the outer velocity difference and a linear increase of the center
357 velocity. The mixing layer width is thus described by a logarithmic law ; a reasonable agreement
358 with experimental values is obtained, considering $\alpha=0.09$.

359 The present shallow sudden expansions then consist in a fourth case. Regarding the outer ve-
360 locity difference, the faster outer velocity $U_1(x)$ in the free stream is measured at $y/d=1.5$ for each
361 section while $U_2(x)$ is the outer velocity on the recirculation side. In their study around a square
362 cylinder, Lyn and Rodi (1994) define U_2 as the minimum velocity (negative) measured within the
363 recirculation region along each profile. In his sudden expansion, Talstra (2011) considers $U_2(x)=0$,
364 *i.e.* the mean streamwise velocity in the zero-discharge recirculation region. This definition is used
365 in the present paper so that the outer velocity difference reads $\Delta U(x) = U_1(x) - U_2(x) = U_1(x)$
366 and the center velocity reads $U_c(x) = (U_1(x) + U_2(x))/2 = U_1(x)/2$ so that the ratio $\frac{\Delta U(x)}{U_c(x)} = 2$ is

367 constant along the whole mixing layer. Hence, the longitudinal growth of the mixing layer in Eq.3
 368 obeys to:

$$369 \quad \frac{d\delta}{dx} = \alpha \frac{\Delta U(x)}{U_c(x)} = 2\alpha \quad (5)$$

370 Please note that Talstra (2011) considers a different definition: $U_c(x) = U_1(x = 0)/2 \approx U_0/2$
 371 and thus does not end up with the same expression as Eq.5.

372 **Present streamwise evolution of the mixing layer width**

373 The streamwise evolution of the measured mixing layer width δ is plotted in Fig. 8. For
 374 the four cases, the flow development upstream from the expansion leads to similar mixing layer
 375 widths at the expansion, $\delta_0/d = \delta(x = 0)/d \approx 0.12$. However, further downstream, the mixing
 376 layer expands differently for the different cases. For the deep configuration F1, δ increases linearly
 377 (as predicted by Eq.5) with an increasing rate up to $x/L = 0.7$ corresponding to $\alpha=0.07$. Further
 378 downstream, the linear spreading rate suddenly reduces, as demonstrated by the linear regressions
 379 on Fig. 8. For the shallow cases F2, F3 and F4, δ also increases linearly up to $x/L = 0.7$ (as
 380 predicted by Eq.5), with rates smaller than for F1 and which minimum is obtained for the shallower
 381 cases (F2 and F4): $\alpha \approx 0.06$. Again, the spreading rate suddenly reduces at $x/L=0.5$ to 0.7 , but
 382 much more than F1 as the curves experience a plateau: $\delta(x)$ becomes constant.

383 The sudden decrease of this growth rate cannot be attributed to the bed friction. Indeed, as
 384 in our case $\frac{\Delta U(x)}{U_c(x)} = 2$, the local bed friction number from Eq.4 becomes $S(x) = \delta(x)c_f/4h$.
 385 Moreover, assuming that both the friction coefficient and the water depth are almost constant along
 386 the mixing layer, as for Babarutsi et al. (1989), see Eq.1, the final expression used herein reads:

$$387 \quad S(x) = \frac{\delta(x)c_{f0}}{4h_0} \quad (6)$$

388 The abrupt change of spreading rate occurs for $S(x)=0.012$ for case F1 and the plateaus ($\alpha \approx 0$)
 389 occur for $S(x)=0.003-0.025$ (referred to as S_{max} in Tab.1) for F2, F3, F4 (see Fig. 8). All these
 390 values are significantly smaller than the critical values proposed in the literature ($S_c=O(0.1)$), see

391 the introduction section). Hence, no damping of the size of the coherent structures (correspond-
392 ing whether to the dominant or the most unstable mode, see introduction) due to bed friction is
393 expected.

394 The rupture in the spreading rate of the mixing layer width, is rather attributed to the interaction
395 of the mixing layer with the side wall ($y = 0$), *i.e.* to the lateral confinement, itself being influenced
396 by the shallowness, as discussed above (see Fig. 7). In Fig.9, the streamwise evolution of the
397 centerline of the mixing layer (y_c , the location of maximum velocity gradient) is plotted along with
398 the half width $\delta(x)/2$ extension on both sides, assuming a symmetrical extension distribution. It
399 shows that, both for shallow and deep cases, the downstream end of the recirculation corresponds
400 to the intersection of the mixing layer boundary with the side wall: $(y_c - \delta/2) \sim 0$ at $x/L=1$. And
401 as, at the reattachment point, δ is much larger for deep than for shallow cases, the centerline of the
402 mixing layer ends up further from the wall. Note that Biancafiore et al. (2011) already revealed
403 through DNS calculations for a laminar transversely-confined mixing layer that the increasing
404 width of the mixing layer suddenly stops at a given streamwise axis due to the lateral confinement.

405 To conclude, the SSE mixing layers behave as free mixing layers in the upstream region
406 ($x/L \leq 0.6 - 0.7$), with a constant spreading rate, and as laterally-bounded mixing layers further
407 downstream, with an abruptly decreasing spreading rate that differs for deep and shallow cases.
408 This sudden decrease is thus governed by the combined effects of shallowness (which imposes 2D
409 large turbulent eddies) and of the interaction of these eddies with the lateral wall - which itself
410 depends on the shallowness.

411 **ROLE OF THE COHERENT STRUCTURES IN THE MIXING LAYER**

412 **Coherent structures in the mixing layer**

413 The aim of the present section is to estimate the peak frequency associated to the coherent
414 structures advected along the mixing layer. Fig.10 plots the energy spectrum of the transverse
415 velocity component v in the mixing layer at $x/L=0.2$ and $y = y_c$ while Fig.11 plots similar spectra
416 every $x = L/13$ distance (at $y = y_c$), in a similar manner as Hertzberg and Ho (1995) or White and

417 Nepf (2007) for other mixing layer configurations, with the peak indicated by a symbol for each
418 spectrum.

419 For averaged frequencies ($f=0.7$ to 7Hz), the main inertial range characterized by the $-5/3$
420 cascade is observed. In the low frequency range, the peak frequency f_p corresponding to the dom-
421 inating vortex passing frequency appears to differ as a function of the shallowness: at $x/L=0.2$,
422 $f_p \sim 0.6\text{Hz}$ for the shallow F2 case, while $f_p \sim 0.3\text{Hz}$ for the deep F1 case (see Fig.10). For F1,
423 this peak frequency decreases towards downstream from $f_p=0.3\text{Hz}$ to $f_p=0.1\text{Hz}$ at $x/L=0.5$ and an
424 even lower frequency at the reattachment section ($x/L=1$). For F2, the peak frequency decreases
425 less rapidly from $f_p=0.6\text{Hz}$ to $f_p=0.4\text{Hz}$ at $x/L=0.5$ and $f_p=0.2\text{Hz}$ at $x/L=1$. The decreasing peak
426 frequency behaviors along the streamwise axis are in agreement with observations from Talstra
427 (2011) and Uijttewaal and Booij (2000). Moreover, the vortex passing frequency appears to de-
428 crease less rapidly for the shallow case than for the deep case and this is connected to the dynamic
429 of large eddies (see below).

430 The eddies can be directly observed using time-exposure photographs of floating sawdust (see
431 Riviere et al. (2011)). Photographs for the shallow case F2 in Fig.12 exhibit 2D vortices of vertical
432 axis, with an increasing size during their advection. The behavior appears quite reproducible from
433 one vortex to another. The reattachment location can be identified, and varies from one photograph
434 to another around the $x/L = 1$ location, depending on the position of the vortices. On Fig.13, the
435 deep case F1 appears more complex: again, vortices of vertical axis are clearly visible, with bigger
436 sizes, but with less reproducible trajectories and sizes. The flow reattachment cannot be located as
437 easily as for the shallow case F2.

438 In order to obtain more quantitative vortex characteristics, the auto-correlation function of the
439 transverse velocity fluctuation R_{vv} is computed along the centerline of the mixing layer ($y = y_c$),
440 at $x/L=0.2, 0.5, 0.8$ and 1 (see Fig.14). The characteristic time scale of the large scale coherent
441 structures is considered as the time shift between two successive peaks, as defined by Constanti-
442 nescu (2013). The signal is more complex for the deep case F1, confirming that eddies are less
443 reproducible on the photographs, but the two cases exhibit similar trends (see Fig.14). Firstly, the

444 time shift τ between two successive eddies increases as x/L increases, indicating that the 2D ed-
445 dies grow in the streamwise direction. Secondly, the coherence of the signal is worse for $x/L=0.2$
446 than further downstream, corresponding to the initial development of eddies. Thirdly, the time shift
447 for the shallow case F2 is much shorter than for the deep condition F1, corroborating the difference
448 of peak frequencies observed through the spectrum analysis (Figs. 10 -11).

449 Tab.3 compares the results from both techniques (data from the photographs and from the ADV,
450 including the autocorrelation function) for two x/L values, for the shallow case F2. Estimates of
451 the transverse (along y) length scale of vortices from photographs compare well with the mixing
452 layer width derived from the mean velocity fields. Estimates of the convection velocity by mea-
453 suring the vortices center displacement between two successive photographs compare well with
454 the measured local mean velocity (for the latter, two values are provided, measured on both sides
455 of the maximum gradient location). Estimates of the vortex passing frequencies from photographs
456 compare well with the spectra peak frequencies at the same location. Finally, estimates of the
457 longitudinal length (along x) of vortices from photographs compare well with the integral length
458 Λ obtained by multiplying the local mean velocity with the characteristic times from the auto-
459 correlation R_{vv} on both sides of the maximum gradient location (Uijtewaal and Booij (2000)).
460 The same comparison is performed for the deep case F1 at $x/L=0.5$, in Tab.4. The agreement is
461 slightly lower (due to the difficulty in estimating accurate values from the photographs) than for
462 case F2 but the conclusions are alike: the specific tendency of the mixing layer when approaching
463 the reattachment point, attributed to the width of the mixing layer can actually be attributed to the
464 impact of the coherent vortices with the lateral wall.

465 To correlate further the role of vortices on the behavior of the mixing layer and on the flow
466 reattachment, the dimensionless mixing layer width δ/d is plotted in Fig.15 as a function of x/L ,
467 along with the integral longitudinal length scale of vortices Λ/d . The behaviors of δ/d and of Λ/d
468 are similar: linear increase with a slope shortage for the deep case F1 and linear increase followed
469 by a plateau for the shallow case F2. Considering the two different vertical axes, the mixing layer
470 width is about 3.5 times smaller than the longitudinal integral length scale. This ratio corresponds

471 to the longitudinal/transverse diameters ratio observed in the photographs (Fig.12 and Tab.3) and
472 to the ratios obtained experimentally (Uijttewaal and Booij (2000)) or numerically (Cheng and
473 Constantinescu (2014)) for an unbounded shallow mixing layer of larger scale. In his experimental
474 study, Talstra (2011) reports a ratio of about 2 ; the only explanation for this discrepancy is that he
475 used a spatial correlation based on the vector potential function, instead of a time autorrelation.

476 To summarize, vortices appear to increase in terms both of streamwise and transverse extension
477 along the development of the mixing layer. For the case where the coherent structures are strongly
478 2D (F2), the streamwise evolution of their streamwise length is in particularly fair agreement
479 with the streamwise evolution of the mixing layer width: a linear increase in the upstream region
480 followed by a plateau when approaching the reattachment point, due to their interaction with the
481 lateral wall. The deeper case F1 suffers from a huge scattering in the measurements of coherent
482 structure streamwise length: this prevents from emitting so definite conclusions.

483 To conclude, the shallowness, accounted by the integral bed friction number S_d , influences the
484 vortices size in the same way as for the mixing layer width (see above). In the upstream part of
485 the mixing layer ($x/L \leq 0.6 - 0.7$), both the mixing layer and the large scale vortices evolve
486 similarly for F1 and F2. Further downstream ($x/L > 0.6 - 0.7$), both the mixing layer and the
487 vortices stop growing for the shallow cases (F2, F3, F4) whereas they both go on growing - though
488 more gently - for the deep case F1. Moreover, the behavior of the mixing layer when approaching
489 the wall is a consequence of the interaction of the 2D large scale vortices with the lateral wall. This
490 interaction is affected by the shallowness and could be linked to the weaker 2D character of the
491 vortices in the deep case, as the water depth increases by a factor 7.4 from F2 to F1.

492 **DISCUSSION AND CONCLUSIONS**

493 This paper characterized experimentally the mixing layer that develops at the interface between
494 the main flow and the recirculation zone created by a shallow sudden expansion (SSE). These
495 mixing layers are confined vertically between the channel bed and the free-surface (shallowness)
496 and bounded laterally by the wall (lateral confinement). The first objective was to identify the
497 combined influence of the shallowness and of the lateral confinement on the SSE mixing layers, by

498 checking their differences with simpler mixing layers available in the literature, i.e. either shallow
499 and unbounded or deep and laterally confined mixing layers. The second objective was to check
500 the link between the macroscopic behavior of the recirculation (mainly its length) and the local
501 behavior of the mixing layer. Hence, four experimental configurations with identical geometry but
502 varying shallowness were carefully selected to cover all regimes of behavior of the recirculation:
503 F1 referred to as deep, F3 as transitional, F2 as shallow and F4 as shallow with a high bed friction
504 number value.

505 The first conclusion is related to the role of the shallowness and of the lateral confinement
506 on the mixing layer compared to simpler mixing layers. In its upstream region, the SSE mixing
507 layer exhibits classical features of all mixing layers. It is the location of high velocity gradients
508 and Reynolds stresses, both with a maximum magnitude at the centerline of the mixing layer
509 and a rapidly decreasing magnitude on both sides. This centerline merges into the separating
510 streamline originating from the upstream corner. The width of the mixing layer increases linearly
511 towards downstream from an initial magnitude at the separation corner as for any free (deep and
512 unbounded) mixing layer. In its downstream region, this behavior changes: the expansion of the
513 mixing layer width stops for the shallow cases while it is reduced for the deep case. The damping
514 of the coherent structures by the bed friction appears not to be responsible of this specific behavior,
515 as the local bed friction numbers remain too small in our whole experimental range. The reason
516 for this sudden reduction of width expansion is rather the interaction of the lateral wall with the
517 coherent turbulent structures in the vicinity of the reattachment region. This interaction appears to
518 differ for the shallow and the deep flows, maybe connected to weakening of the 2D character of
519 coherent structures in the mixing layer when increasing the water depth.

520 The second conclusion is related to the connection of S_d with $S(x)$. It was shown herein
521 and in the literature that the global flow shallowness (S_d) strongly influences the macroscopic
522 behavior of the recirculation, in particular its dimensionless length L/d . It is then tempting to relate
523 this macroscopic effect to the degree of damping of the coherent structures in the mixing layer.
524 Nevertheless, as already mentioned above, a bed friction damping was not observed in the present

525 mixing layers. No evident relationship can be obtained between the global flow shallowness and
526 the local mixing layer shallowness. In other words, a critical value of the integral bed friction
527 number S_d (Eq. 1), separating the "deep water flow" and the "shallow water flow" behaviors in
528 Fig. 2, cannot be related to a critical value of the local bed friction number $S(x)$ (Eq. 6), indicating
529 the appearance of a noticeable damping of the coherent structures by the bed friction. For all that,
530 the global flow shallowness (S_d) jointly affects the recirculation length and the coherent structures
531 evolution and size. Future work, using different techniques such as PIV, should be devoted to
532 characterize these coherent structure dynamics more in details all along their advection within the
533 mixing layer, notably in the vicinity of the reattachment point so that where their exact interaction
534 with the side wall could be better understood, for different values of the global flow shallowness.
535 It would then be possible to definitely establish if the large scale vortices play a role in the huge
536 variation of the recirculation length as a function of the shallowness downstream expansions.

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NOTATION

S = bed friction number;

S_d = global bed friction number;

S_c = critical bed friction number;

c_f = friction coefficient;

d = expansion width (m);

h = water depth (m);

B = total width of channel (m);

R_b = expansion ratio $= (B - d)/B$;

u = streamwise velocity (m/s);

v = transverse velocity (m/s);

617 U_1 and U_2 = outer velocities (m/s);

U_c = mean center velocity (m/s);

ΔU = outer velocity difference (m/s);

f_p = peak frequency corresponding to the dominating vortex (Hz);

Q = inlet discharge (l/s);

L = length of the recirculation zone (m);

L_b = length of the upstream flow region (m);

L_t = total channel length (m);

δ = width of the mixing layer (m);

α = grow rate of the mixing layer width;

Λ = integral length scale (m);

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TABLE 1. Flow characteristics for all studied configurations

Test	Q (l/s)	U_0 (m/s)	h_0 (m)	c_{f0}	S_d	L (m)	L/d	S_{max}	Fr_0	Re_0
F1 (deep)	20	0.23	0.156	0.0049	0.0032	1.29	6.45	-	0.19	9.4×10^4
F2 (shallow)	4.05	0.36	0.021	0.0068	0.032	1.16	5.80	0.009	0.79	2.8×10^4
F3 (transitional)	16.15	0.55	0.050	0.0050	0.01	1.99	9.95	0.003	0.79	9.4×10^4
F4 (shallow)	2.363	0.20	0.022	0.0195	0.089	0.86	4.3	0.025	0.43	1.6×10^4

TABLE 2. The maximum measured Reynolds stresses compared with literature data (with ΔU_0 the outer velocity difference at the upstream section where the mixing layer initiates). Note: FML means free mixing layer (White and Nepf (2007)), BFS means backward facing step (Chandrsuda and Bradshaw (1981)), SUM means shallow unbounded mixing layer (Uijtewaal and Booij (2000)) and SSE means shallow sudden expansion with SSE1 from Babarutsi et al. (1989), SSE2 from Talstra (2011) and SSE3 are present data.

	FML	BFS	SUM	SSE1		SSE2			SSE3	
						Case 1	Case 2	Case 3	F1	F2
S_d				0.0098	0.098	0.022	0.015	0.007	0.0032	0.032
$\overline{u'^2}/\Delta U_0^2(\%)$	3.1	2.7	2.97	5.4	2.25	0.5	0.9	1.2	2.38	1.2
$\overline{v'^2}/\Delta U_0^2(\%)$	1.9	1.5	1.36	-	-	0.3	0.7	0.4	0.88	0.6
$-\overline{u'v'}/\Delta U_0^2(\%)$	0.925	1	1.04	-	-	0.2	0.4	0.2	0.74	0.5
$\overline{v'^2}/\overline{u'^2}(\%)$	61	56	46	-	-	80	78	25	37	50
$-\overline{u'v'}/\overline{u'^2}(\%)$	30	37	35	-	-	40	44	17	31	42

TABLE 3. Characteristics of vortices and of the mixing layer for case F2 at two locations

	$x/L \sim 0.4$		$x/L \sim 0.7$	
	Photograph	ADV	Photograph	ADV
<i>Transverse size</i> (m)	0.09	$\delta=0.07$	0.12	$\delta=0.11$
$U_{advection}$ (m/s)	0.25	0.14-0.24	0.2	0.13-0.2
<i>Frequency</i> (Hz)	0.59	0.6	0.44	0.47
<i>Streamwise size</i> (m)	0.3	$\Lambda=0.24-0.36$	0.36	$\Lambda=0.31-0.42$

TABLE 4. The characteristics of the vortices passing along the mixing layer for F1

	$x/L \sim 0.5$	
	Photograph	ADV
<i>Thickness</i> (m)	0.2	$\delta=0.13$
$U_{advection}$ (m/s)	0.06	0.08-0.12
<i>Frequency</i> (Hz)	0.13	0.18
<i>Streamwise size</i> (m)	0.25-0.44	$\Lambda=0.35-0.61$

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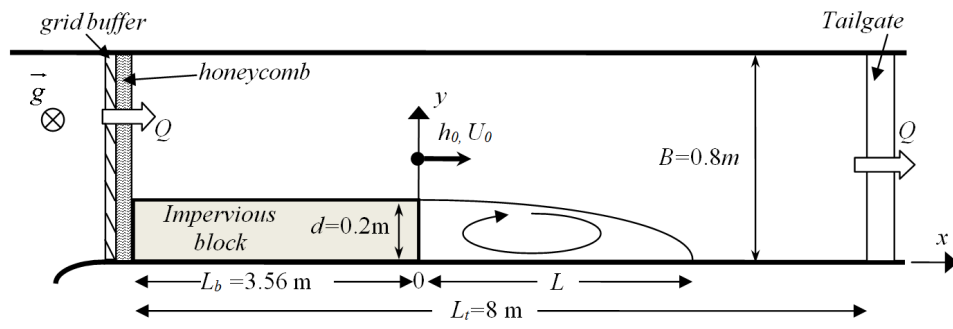


FIG. 1. Plan view of the experimental set-up

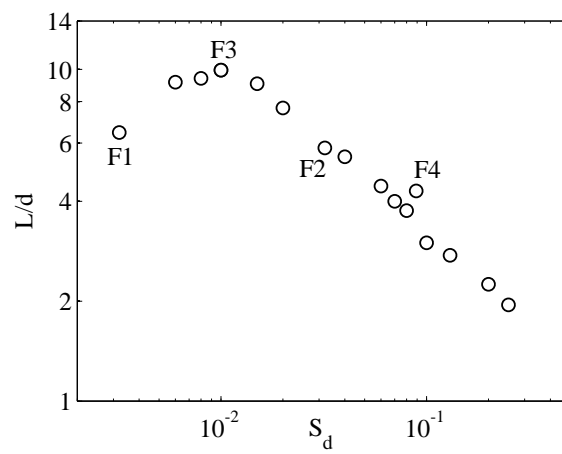


FIG. 2. Evolution of the non-dimensional recirculation length as a function of the bed friction number with the selected configurations

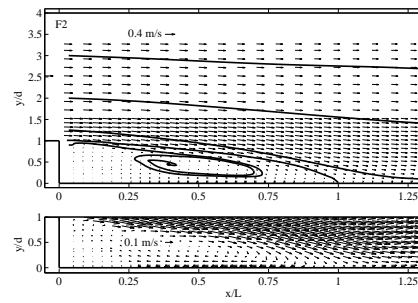
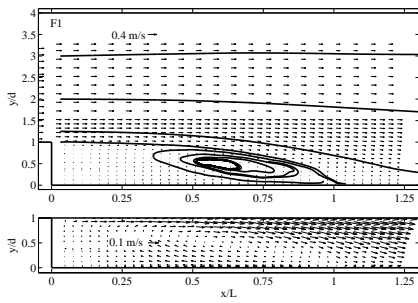


FIG. 3. Time-averaged velocity field along with selected streamlines (plain lines—), the bottom plot is a zoom of the top plot in the recirculation zone where $y/d < 1$

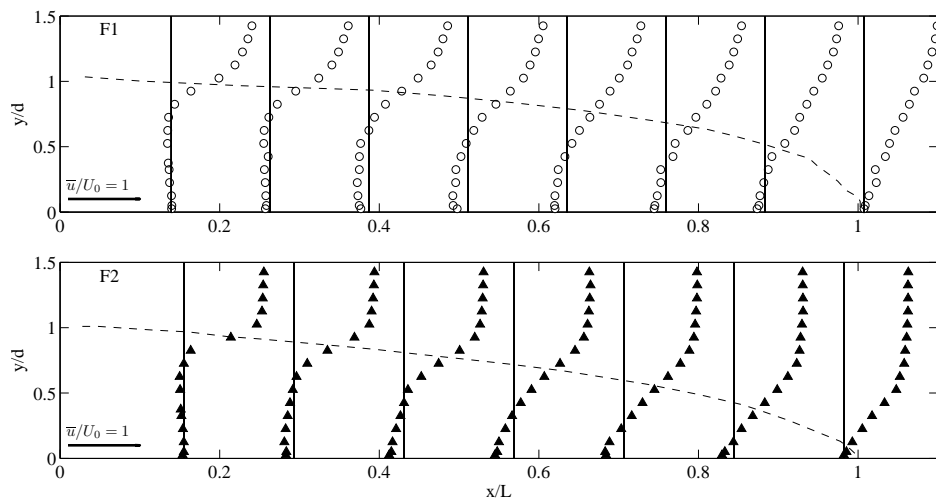


FIG. 4. Evolution of transverse profiles of mean streamwise velocity along a few selected sections (\circ refers to F1 and \blacktriangle refers to F2) with the separating streamlines plotted as dash lines

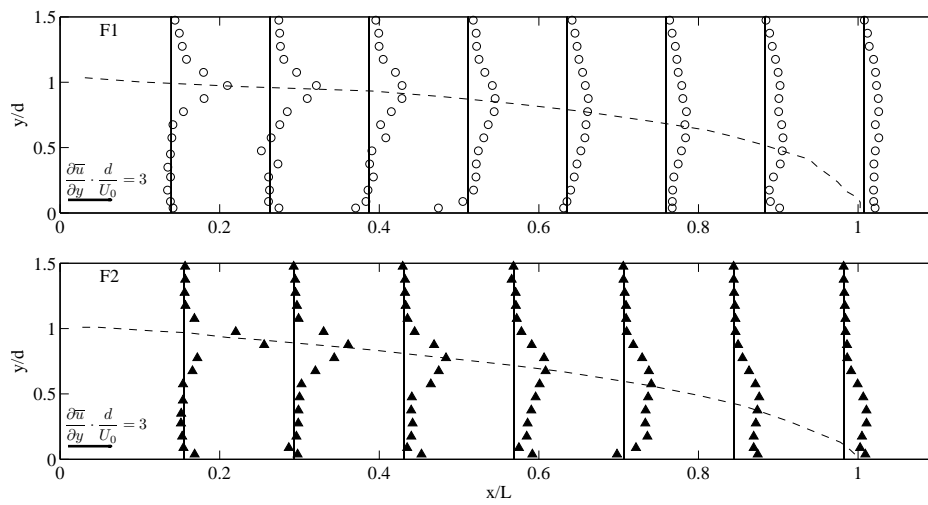


FIG. 5. Evolution of the transverse gradient of mean streamwise velocity with the separating streamlines plotted as dash lines

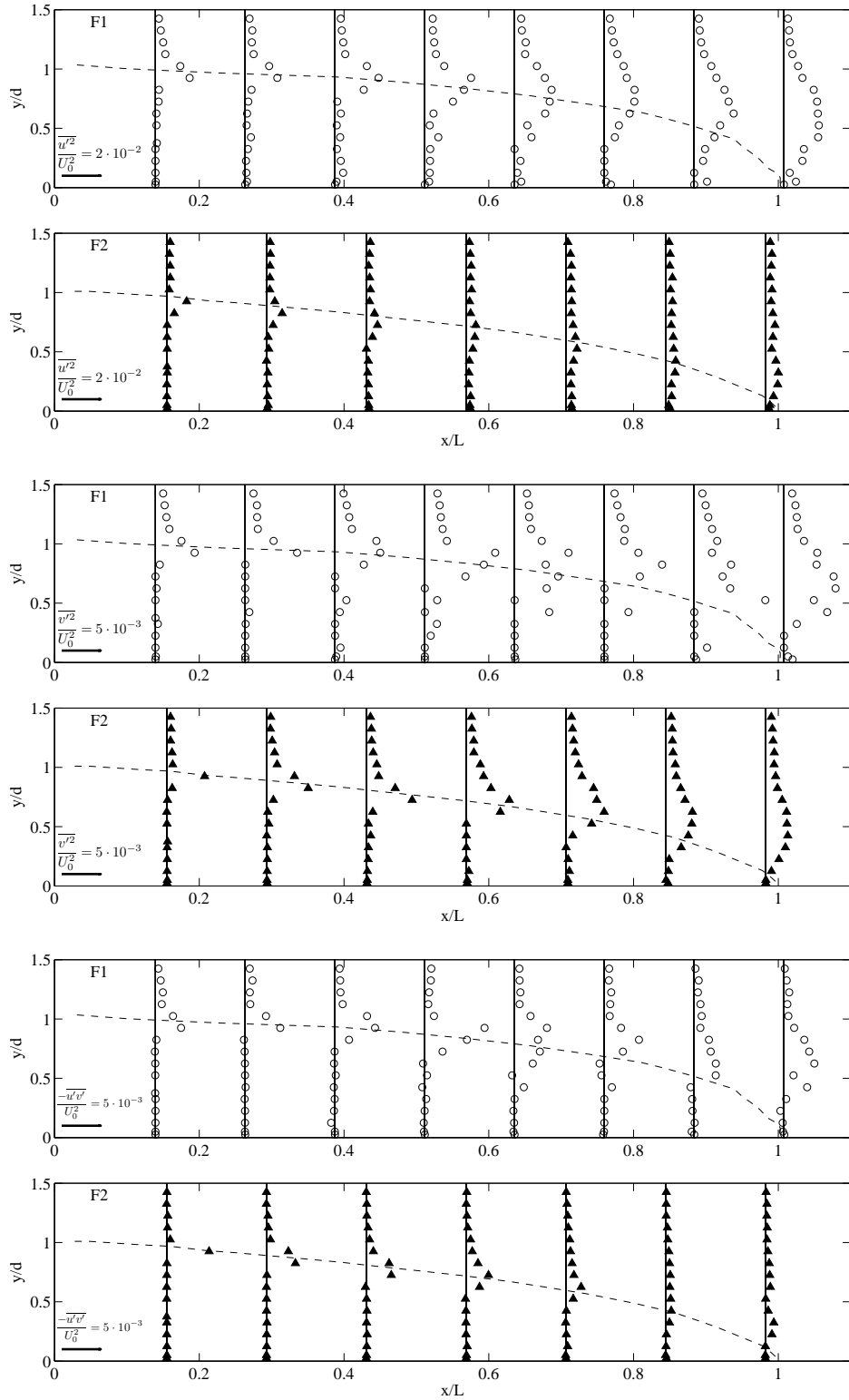


FIG. 6. Evolution of the Reynolds stress terms with the separating streamlines plotted as dash lines. Note: the different scales for the streamwise normal term and both other terms)

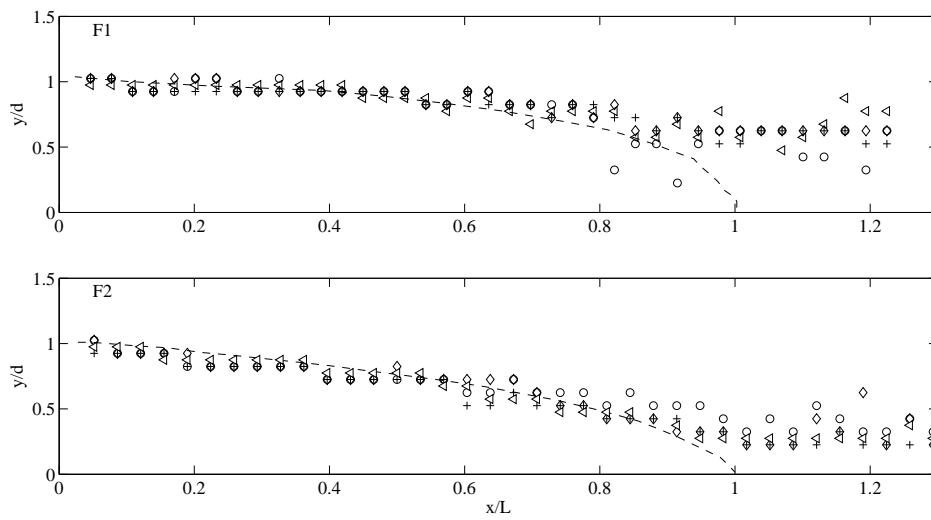


FIG. 7. Location of maximum velocity gradient (\triangle), $\overline{v_l^2}$ (\circ), $\overline{v_l^2}$ ($+$) and $-\overline{u_l v_l}$ (\diamond) along with the separating streamline plotted as dash lines

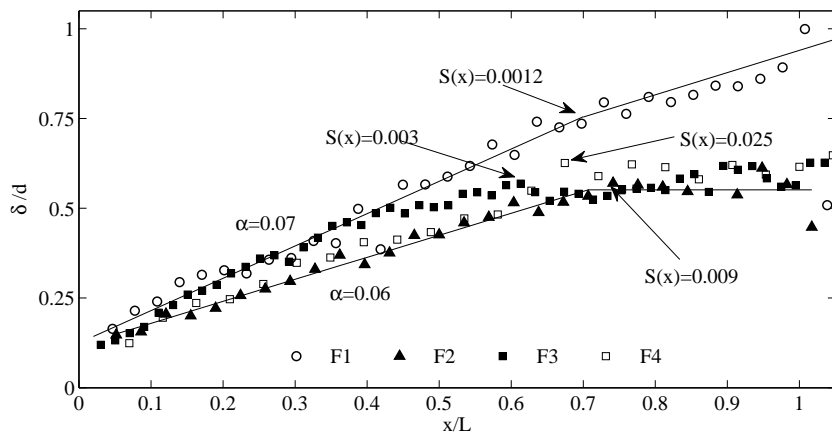


FIG. 8. Streamwise evolution of the measured (symbol) and fitted (plain lines —) non-dimensional mixing layer width

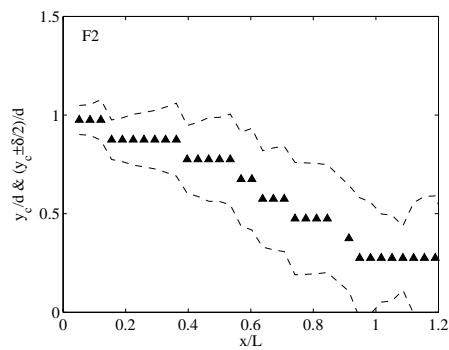
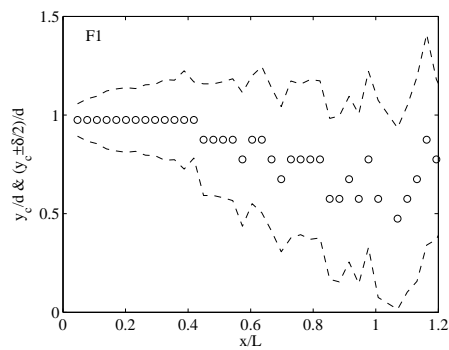


FIG. 9. Evolution of the mixing layer centerline (symbols) and the estimated mixing layer boundaries (dash lines —)

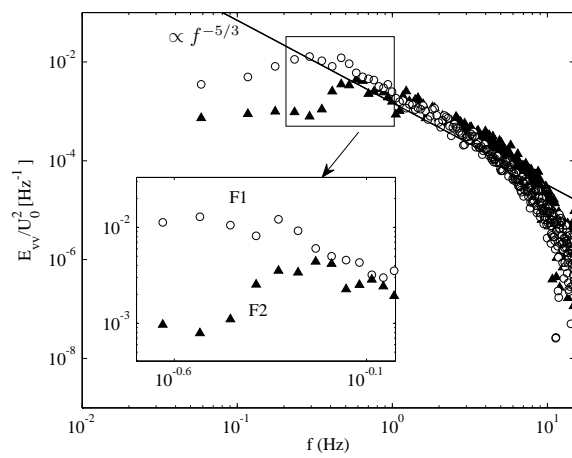


FIG. 10. Energy spectrum of the transverse velocity component v along the center-line of the mixing layer at $x = 0.2L$.

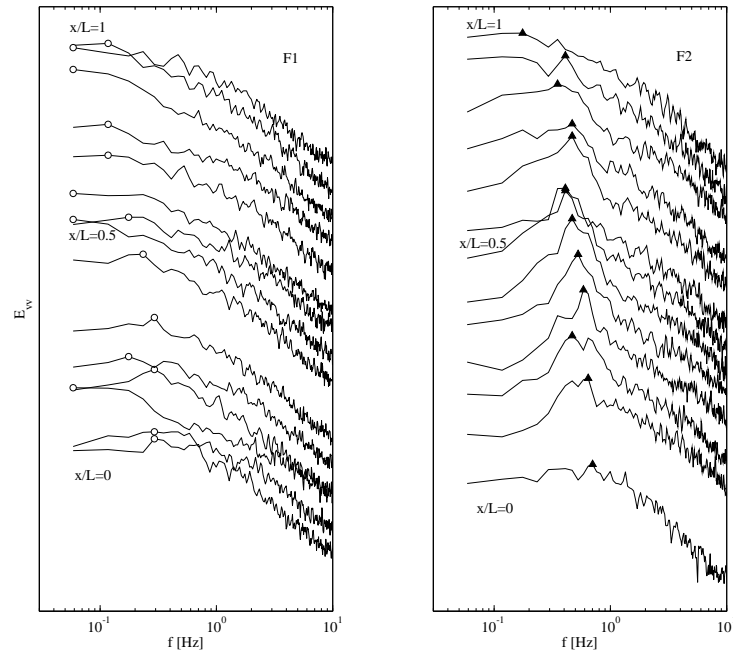


FIG. 11. Streamwise evolution of the energy spectrum of transverse velocity along the mixing layer centerline from $x/L=0$ to $x/L=1$. Note: the symbols presents the peak value of each x/L section.

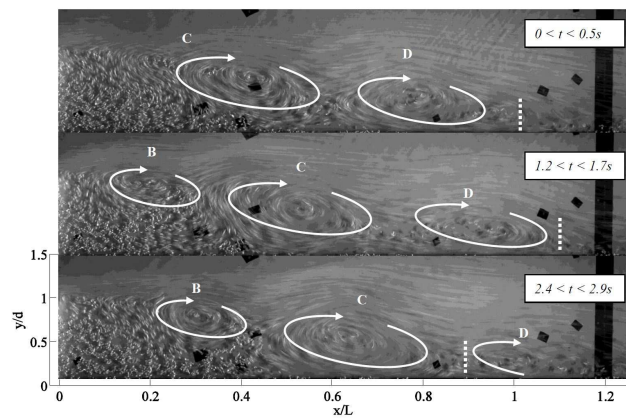


FIG. 12. Successive 0.5s exposure photographs for case F2. Dotted line $\dot{}$ marks the location of the instantaneous reattachment

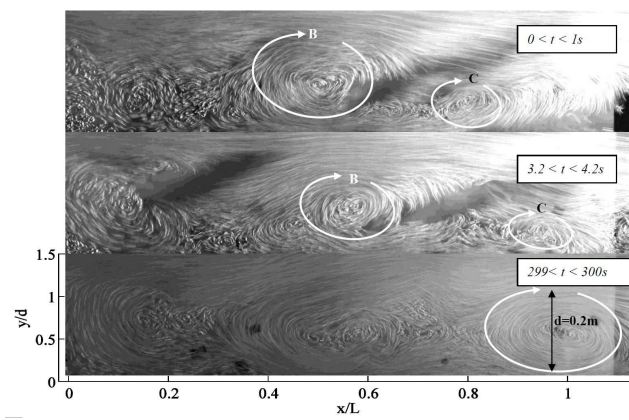


FIG. 13. Successive 1s exposure photographs for case F1

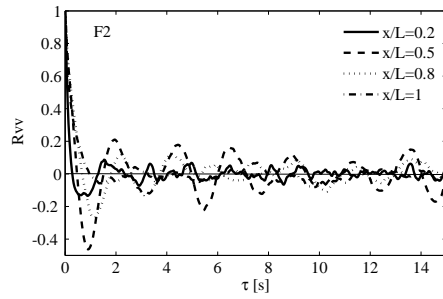
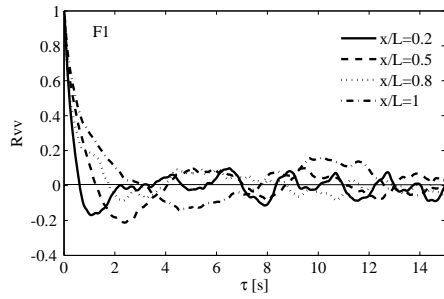


FIG. 14. Auto-correlation function of the transverse velocity fluctuation R_{vv} in different section of x/L along the centerline of the mixing layer

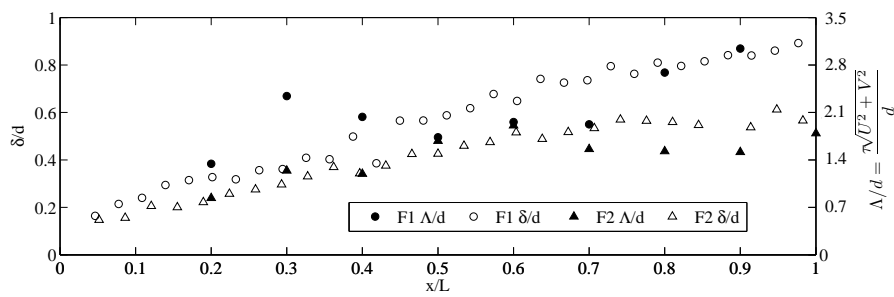


FIG. 15. The comparison between the mixing layer width (definition of Eq.2, left axis with symbols \circ and \triangle) and longitudinal length scale of vortices (derived from the autocorrelation, right axis with symbols \bullet and \blacktriangle), along the mixing layer centerline from $x/L=0$ to $x/L=1$