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1	Typology of the flow structure in dividing open channel flows
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11	

12 ABSTRACT

The present study reports the occurrence of a new recirculation structure taking place in the lateral branch of a 90° bifurcation flow. This recirculation structure is "helix-shaped" and strongly differs from the typical "closed" recirculation often reported in the literature. The aim of the study is to detail their characteristics using experimental and numerical approaches and to establish a typology, *i.e.* the flow conditions leading to each recirculation structure based on the upstream Froude number and the upstream aspect ratio.

Keywords: bifurcation, hydraulic parameters, open-channel flow, RANS model, recirculationstructures

21 **1 Introduction**

22 Bifurcations of open-channel flows are specific structures frequently encountered in sewer 23 systems or river delta. Bifurcation flows have been widely studied (Grace and Priest, 1958; 24 Shettar and Murthy, 1996; Hsu et al., 2002; Ramamurthy et al., 2007; Mignot et al., 2013; 25 Momplot et al., 2013). Hence, governing parameters of a bifurcation flows are well identified: 26 inlet discharge and discharge repartition in downstream channels. In the literature, the 27 principal challenge for bifurcations lies in the prediction of the flow distribution from the 28 incoming flow towards each outgoing flow. A review of analytical models developed to 29 access such prediction can be found in Rivière et al. (2007, 2011). The model proposed by the 30 authors is based on the momentum conservation law as proposed by Ramamurthy et al. 31 (1990), suitable stage-discharge relationships for the downstream controls in the outflow 32 channels and an empirical correlation obtained through experimental data.

Nevertheless, understanding the behavior of the flow structures within the bifurcationis also important, as they strongly impact pollutant or sediment transport and mixing

processes. The general pattern of a steady subcritical 3-branch bifurcation is described by Neary *et al.* (1999). A three-dimensional recirculating region develops in the lateral branch and secondary flows appear in both outlets. Mignot *et al.* (2014) detailed the mixing layer taking place at the frontier between the main flow and the recirculation zone in the lateral branch. Recirculation zones, also defined as bubbles, are encountered in various geometries as listed by Li and Djilali (1995).

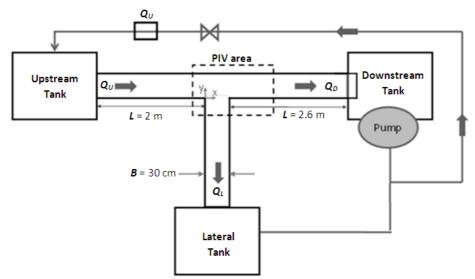
41 Regarding specifically the recirculation zone, authors such as Kasthuri and 42 Pundarikanthan (1987), Shettar and Murthy (1996) and Neary et al. (1999) sketch the 43 recirculation zone (in top view) as a closed semi-elliptic region developing along the 44 upstream wall of the lateral branch with maximum length and width at the free-surface and 45 minimum extensions in the near-bed region. Kasthuri and Pundarikanthan (1987) and Shettar 46 and Murthy (1996) respectively measure and compute the free-surface length and width of 47 this recirculation zone. The authors agree that as the relative lateral discharge increases, the dimensions of the recirculation zone decreases. Neary et al. (1999) then compute flow 48 49 configurations with varying channel width ratios and dimensionless water depths and exhibit varying characteristics of the recirculation zone pattern without clear explanation of the 50 51 different types of recirculation zones.

The aim of the present paper is twofold: first to describe the two main flow structures that can be observed in the lateral branch of a 90° bifurcation and second to determine the flow conditions for which each structure is observed. The paper is organized as follows: after presenting the experimental and numerical approaches, the characteristics of the different types of recirculation zones are described based on two measured and computed flows (F1 and F2) and finally a campaign of numerical simulations (including 16 different cases) is led in order to establish the flow typology.

59 2 Material and methods

60 2.1 Experimental set up

61 The experimental set-up (see Fig. 1) is a horizontal 3-branch equal width (B=30 cm) glass 62 open-channel bifurcation of 2 and 2.6 meters long channels for the inlet and both outlet 63 respectively. Boundary conditions are the inlet discharge Q_U (measured by a flow-meter in the 64 pumping loop) and the weir crest height C_D and C_L at the downstream end of each of the two 65 outlet channels. The water depths in the upstream, lateral and downstream branches are 66 defined as h_U , h_L and h_D respectively and are measured using a digital point gauge. The 67 discharge distribution Q_L/Q_U in the bifurcation is measured through an additional flow-meter 68 in the pumping loop. In the studied cases, flow conditions are sub-critical everywhere. Details 69 about this set-up are available in Mignot et al. (2013 and 2014).



71 72 Figure 1. Experimental set up used for flow validation.

73 2.2 Modelling strategy

Numerical simulations are performed under the commercial software ANSYS Fluent version 14.0, following the modelling strategy proposed by Momplot *et al.* (2013) for computing bifurcation flow F0 (see Table 1), these simulations are confronted with PIV measurements of the horizontal velocity fields (see Fig. 2). Overall performances are fair. Additionally, simulated discharge repartition and measured discharge repartition shows fair agreement (differences are less than 10%).

I	Flow id.	Inlet discharge Q _U (L.s ⁻¹)	Weir crest height <i>h_{crest}</i> (m)	Discharge distribution (Q_L/Q_U)	Froude Number in upstream channel (-)	Upstream aspect ratio <i>B/h_U</i> (-)
	FO	4	0.12	0.51	0.102	2.5

80 Table 1. Characteristics of the validated flow.

81

82 The model solves the RANS (Reynolds Averaged Navier-Stokes) equations using the 83 Volume of Fluid - VOF -method for computing the free-surface curve and a Reynolds stress 84 model - RSM - as turbulence model for system closure (see Launder et al., 1975). Scalable 85 wall-functions (see Grotjans and Menter, 1998) are used for walls; a uniform velocity U_{Inlet} is 86 set at the inlet cross-section; atmospheric pressure P_0 is set at the top of the computational 87 domain and at outlets. Crest heights are explicitly represented in the mesh. After the weirs, 88 standard pressure outlet conditions are set. Discretisation scheme use for pressure is Body-89 Force Weighted and Second-Order Upwind for other variables. Pressure-velocity coupling 90 algorithm is PISO.

92 Mesh independency is verified using the grid convergence index (GCI) defined by 93 Roache (1994) as:

$$GCI_X = \frac{(X_F - X_C) \cdot r^p}{r^p - 1}$$

95

102

94

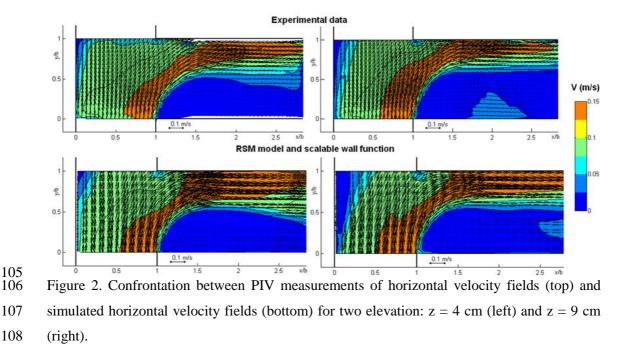
Equation 1 -GCI_X: the simulation error on variable X due to the fine mesh (dimension of 96 With : 97 X), X can be a discharge, a water depth, a velocity, etc. $-X_F$: the simulated variable X for the fine mesh (dimension of X) 98 99 $-X_C$: the simulated variable X for the coarse mesh (dimension of X) -*r*: the ratio $\frac{N_F}{N_C}$, N_F being the fine mesh number of cells and N_C the coarse 100

101 mesh number of cells (dimensionless)

-*p*: the discretisation schemes order (dimensionless).

103 The GCI_X value is an estimator of the error committed on the true variable X value

104 that is due to the mesh. Mesh independency is obtained when the relative error is below 5%.



109 Flow structure description 3

110 Table 2. Cases F1 and F2 studied experimentally and numerically exhibiting respectively a

"helix-shaped" recirculation and a "closed" recirculation structure in the lateral branch. 111

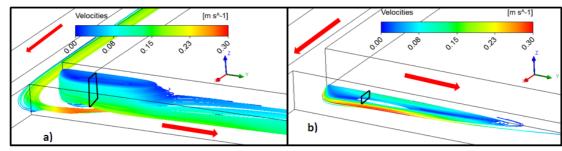
Flow id.	Inlet discharge Q _U (L.s ⁻¹)	Weir crest height <i>h_{crest}</i> (m)	Discharge distribution (Q _L /Q _U)	Froude Number in upstream channel (-)	Upstream aspect ratio <i>B/h_U</i> (-)
F1	8	0.09	0.47	0.218	2.50
F2	4	0.025	0.50	0.448	6.69

112 113

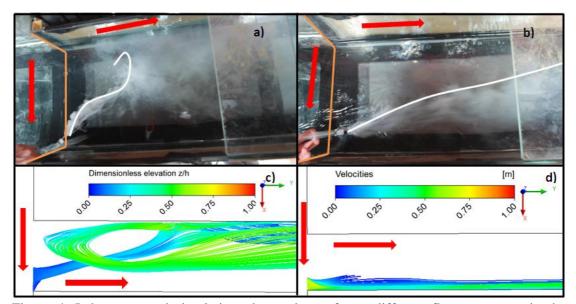
	F1 - h_{crest} = 9 cm Q_U = 8 L/s-					
	Base mesh	GCI mesh	Absolute mesh error	Relative mesh error		
Mesh size (cells)	738000	461500	-	-		
<i>Q</i> _D (m ³ .s ⁻¹)	4.320	4.352	0.053	0.012		
<i>Q</i> [⊥] (m ³ .s ⁻¹)	3.743	3.706	0.061	0.016		
<i>U_{moy-U}</i> (m.s ⁻¹)	0.237	0.24	0.005	0.021		
<i>U_{moy-L}</i> (m.s ⁻¹)	0.1104	0.1108	0.001	0.006		
	F2 - h_{crest} = 2.5 cm Q_U = 4 L/s-					
	Base mesh	GCI mesh	Absolute mesh error	Relative mesh error		
Mesh size (cells)	756000	412500	-	-		
<i>Q</i> _D (m ³ .s ⁻¹)	2.276	2.343	0.110	0.048		
<i>Q</i> [⊥] (m ³ .s ⁻¹)	1.762	1.744	0.030	0.017		
<i>U_{moy-U}</i> (m.s ⁻¹)	0.388	0.379	0.015	0.038		
<i>U_{moy-L}</i> (m.s ⁻¹)	0.211	0.21	0.002	0.008		

114 Table 3. GCI analysis for flow F1 and F2.

115



a)
Figure 3. Two flow structures in the lateral branch of a bifurcation flow: a) a helix-shaped
recirculation for flow F1 and, b) a closed recirculation for F2. Drawn streamlines are the ones
going through the white plane located in the lateral branch at a distance equal to 1B from the
entry section, covering the whole water depth and extending transversally from the left bank
to the streamlines that separates at the corner between the upstream and lateral branches. This
plane permits to enclose the whole recirculation. Red arrows indicate the main flow
directions.



124

Figure 4. Laboratory and simulation observations of two different flow structures in the lateral branch of two bifurcation flows: F1 (a-c) and F2 (b-d). (a-b) show experiments, (c-d) show numerical streamlines colored by the dimensionless elevation z/h_L (from blue, being the bottom of the channel; to red, being the free-surface). For (a-b), a white dye tracer is injected in the downstream corner of the lateral channel, near the bottom, the white line represents the limit of the tracer extension, red arrows indicate flow directions and orange lines mark the inlet section of the lateral branch.

132

133 Among all studied flows, two recirculation structures could be observed. Table 2 presents two 134 flow cases investigated numerically and experimentally, each one exhibits a different 135 recirculation structure: a so-called "helix shaped" for F1 and a "closed" one for F2. Fig. 4 136 shows numerical results for these flow cases and particularly the behaviour of some selected 137 streamlines. Fig. 4 compares experimental observations for the two flow cases regarding the 138 transport of white dye tracer in the lateral branch and numerical streamlines obtained through 139 RANS simulations. In both cases, injection takes place near the bottom part of the 140 downstream corner region. These results confirm the fair agreement between simulated and 141 measured flow patterns. Additionally, results of the GCI analysis displayed in Table 3 142 indicates that meshes used to compute both flows are efficient for the prediction of discharge 143 and bulk velocities in the lateral branch. Surprisingly, the mesh for flow F2 is less efficient 144 for discharge and velocities representation in the upstream branch than the mesh for flow F1.

145

The two figures permit to describe both recirculation structures:

The closed recirculation observed for flow F2 is a 2D semi-elliptic closed region
developing along the upstream wall of the lateral branch as described in the literature: no flow
enters or leaves this region and it is of larger streamwise and transverse extension at the freesurface than near the bed (Fig. 3b). Consequently, the streamlines starting from the

downstream corner of the intersection remain quite parallel to the banks of the lateral channeltowards downstream and do not interact with the recirculation zone (Fig. 4b).

152 - The helix-shaped recirculation observed for flow F1 is a 3D ascendant flow (see Fig. 153 3a and 4c): i. supplied by the bottom flow of the upstream channel, ii. entering the lateral 154 branch near the bottom part of the downstream corner area, iii. approaching the opposite 155 (upstream) wall of the lateral branch, iv. raising towards the free-surface, first in the direction 156 of the intersection (towards upstream) and then towards downstream along center of the 157 branch. v. escaping towards downstream in the upstream half of the branch. Consequently, the 158 streamlines starting from the downstream corner of the intersection enter the recirculation 159 structure (Fig. 4a). These two distinct flow structures corroborate the different streamlines 160 plots near the bottom reported by Neary et al. (1999) in their figure 10 and the pathlines in 161 their figure 11c.

162 **4 Flow typology**

163 In order to establish the flow conditions for which each recirculation structure is observed, a 164 flow typology is established following the parameters obtained through dimensional analysis. 165 Using the same approach as Mignot et al. (2013) and assuming (as for the authors) that the flow is turbulent (see Table 10) and smooth, the 8 parameters governing the flow 166 167 characteristics are: the three discharges (Q_U, Q_L, Q_D) , the three water depths (h_U, h_L, h_D) and the two weir crest heights (C_D , C_L). Mass conservation equation ($Q_U = Q_L + Q_D$) permits to 168 169 remove one parameter (Q_L) ; both known stage discharge equations $(h_D = f(Q_D, C_D)$ and $h_L =$ 170 $f(Q_L, C_L)$ permit to remove h_L and C_D ; the momentum equation introduced by Ramamurthy et 171 al. (1990) permits to remove h_D ; the empirical closure equation introduced by Rivière et al. 172 (2007) permits to remove Q_D ; finally, the following simplification considered in the present 173 work $C_D = C_L$ permits to remove C_L . We end up with the two remaining parameters Q_U and h_U , 174 which can be transformed (see Mignot *et al.*, 2013) as dimensionless independent parameters: 175 upstream Froude number Fr_U and upstream aspect ratio B/h_U . Note that the present 176 simplification $C_D = C_L$ is responsible for the reduction of independent parameters from 3 in 177 Mignot et al. (2013) to 2 in the present work and leads to a discharge distribution Q_L/Q_U of 178 about 45 to 55%.

The aim of the part being a flow typology assessment for a recirculation zone, a quick look at others recirculation zone typologies is needed. There is many situations leading to a recirculation zone (listed by Li and Djilali, 1995). For each of these situations, a flow typology can be established by studying specific forces configurations. For example, Chu et al. (2004) establish a typology for a recirculation zone where confinement and friction are the determining forces. Dufresne et al. (2010) also establish a recirculation zone typology in rectangular shallow reservoirs, where inertial forces and pressure/water depth gradients are determining forces. These two cases lead to two different typologies. In the present case, the suspected determining forces are centrifugal force and pressure force: when the centrifugal force effect is significant, we can observe the helix-shaped recirculation because of the pressure force induce by the centrifugal force (see Fig. 5). It is similar to the tea-leaves effect.

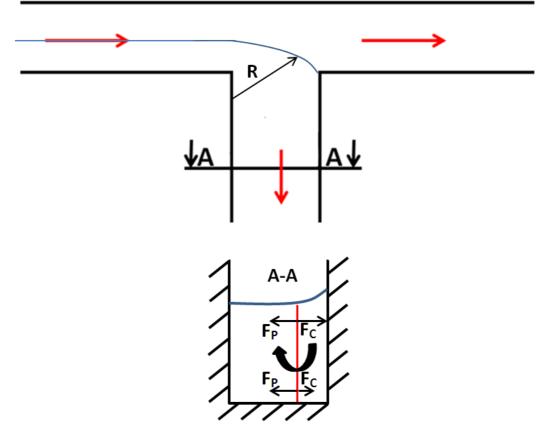


Figure 5. Forces balance in a lateral branch cross-section. Red arrows indicate flow directions. F_C is the volume centrifugal force (N.m⁻³), F_P the volume pressure force (N.m⁻³). Blue line indicates the free surface in the cross-section and R is the curvature radius of the separation.

195

196

Two forces are defined in the lateral branch cross-section A-A (see Fig. 5) as follow:

 $F_P = \rho g$

197
$$F_C = \rho \frac{U_U^2}{R}$$

Equation 2

199 200

198

Equation 3

200		
201	Where:	- F_C : the centrifugal force (N.m ⁻³)
202		- U_U : the mean velocity in the upstream channel (m.s ⁻¹)
203		-R: the curvature radius of the separation (m)
204		- F_P : the pressure force (N.m ³)
205		-g: the acceleration of gravity (= 9.81 m.s^{-2})
206		

207 A comparison between the two defined forces gives:

208
$$\frac{F_C}{F_P} = \frac{\rho \frac{U_U^2}{R}}{\rho g} = \frac{U_U^2}{Rg}$$

209

210

It is possible to assimilate *R* to the channel width *B*. Equation 4 becomes:

Equation 4

Equation 5

211
$$\frac{F_C}{F_P} = \frac{U_U^2}{Bg} = \frac{U_U^2}{\frac{B}{h_U} \cdot gh_U} = \frac{Fr_U^2}{\frac{B}{h_U}}$$

212

213 With: $-h_U$: the water depth in the upstream channel (m)

Equation 5 indicates that a relationship between the squared Froude number in the upstream channel FrU² and the upstream aspect ratio B/hU can determine the flow topology.

Table 4 presents the numerical campaign, comprising 16 flow cases, led to establish the flow typology. For each case, Froude number is upstream channel Fr_U is defined as:

219
$$Fr_U = \frac{U_U}{\sqrt{g \cdot h_U}} \in [0.035; 0.558]$$

220 Reynolds number in upstream channel ReU is defined as:

221
$$Re_U = \frac{4 \cdot U_u \cdot B \cdot h_U}{v \cdot (B + 2 \cdot h_U)} \in [7400; 103900]$$

222With : $-U_U$: the mean velocity in the upstream channel (m.s⁻¹)223-B: the channel width (m)224 $-h_U$: the upstream channel water depth (m)225All tested cases are thus subcritical and turbulent (see Table 4).226For this campaign, the crest height in both outlet branches C_D and $C_L (C_D = C_L)$ and

227 the upstream discharge Q_U are the two varying boundary conditions which permit to vary the

228 two independent parameters: Fr_U and B/h_U .

229

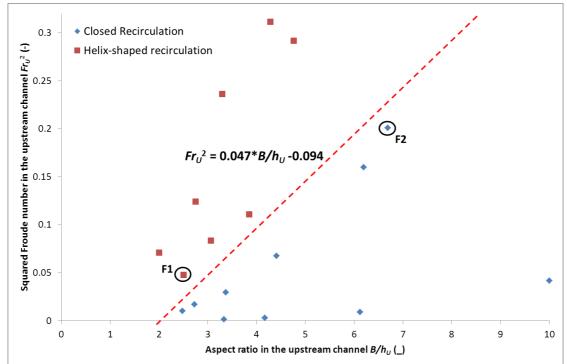
Table 4. Simulated cases for the typology numerical campaign.

_		Crest height in downstream branches h _{crest} (m)	Inlet discharge Qu (L.s ⁻¹)	Flow distribution (<i>Qı/Qu</i>)	Reynolds Number in upstream branch <i>Re_U</i> (-)	Froude Number in upstream branch Fr _U (-)	Upstream aspect ratio <i>B/h</i> _U (-)
-	Case 1	0.03	1	0.485	11066	0.204	10.00
	Case 2	0.03	4	0.45	40240	0.400	6.20
	Case 3	0.03	8	0.454	75340	0.540	4.76
	Case 4	0.03	12	0.345	88130	0.558	4.29
	Case 5	0.05	1	0.505	9700	0.095	6.12
	Case 6	0.05	4	0.481	39810	0.260	4.41

Case 7	0.05	8	0.457	79070	0.333	3.84
Case 8	0.05	12	0.433	103903	0.486	3.30
Case 9	0.07	1	0.511	8504	0.052	4.16
Case 10	0.07	4	0.472	35950	0.172	3.37
Case 11	0.07	8	0.459	67260	0.289	3.06
Case 12	0.07	12	0.555	80810	0.352	2.75
Case 13	0.09	1	0.521	7400	0.035	3.33
Case 14	0.09	4	0.488	34310	0.130	2.73
Case 15	0.09	8	0.468	63070	0.218	2.50
Case 16	0.09	12	0.474	96800	0.266	2.00

231

232 Fig. 6 shows the distribution of each recirculation structure, according to two parameters: squared upstream Froude number Fr_U^2 and upstream aspect ratio B/h_U . Both regions are 233 234 clearly separated from each other: a linear - at least with the present set of experiments oblique boundary separates the two types. For low Fr_U^2 and high B/h_U values, the closed 235 recirculation takes place whilst for high Fr_U^2 and low B/h_U values, the helix-shaped 236 237 recirculation occurs.





238 239 Figure 6. Flow typology in the lateral branch. Flow F1 and F2 are circled. The red-dashed line 240 represents the boundary between a classic recirculation in lateral branch and a helix-shaped 241 one.

242 5 **Conclusions and perspectives**

The present papers aimed at defining the flow patterns occurring in the lateral branch of an 243 244 open-channel bifurcation. A typology comprising two flow structures was established based 245 on the characteristics of the recirculation zone. The two structures are named i. "closed 246 recirculation", similar to the flow pattern previously described in the literature and ii. "helix-247 shaped recirculation" for which the flow pattern strongly differs and is described in the 248 present paper. Both structures were observed using both experimental and numerical 249 approaches. Following the assumptions of a smooth and turbulent flow regime and equal weir 250 crest heights at both outlets, the typology is based on the comparison between centrifugal 251 force effect and pressure force effect by the mean of squared upstream Froude number Fr_{U^2} and upstream aspect ratio B/h_U and exhibits two clear regions of recirculation structure 252 253 occurrence.

- As perspectives, bed friction effect should be investigated, as well as Reynolds number effect.
- As the define flow topology depends on curvature radius of the separation, the effect of channel width ratio and discharge repartition will have an effect and should also be investigated.

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264 Notations

- 265 B = channel width (m)
- 266 C_L = crest height in the lateral branch (m)
- 267 C_D = crest height in the downstream branch (m)
- 268 F_C = centrifugal force (N.m⁻³)
- 269 F_P = pressure force (N.m⁻³)
- 270 Fr_U = Froude number in upstream channel (-)
- 271 $GCI_X =$ grid convergence index value for variable X (dimension of variable X)
- 272 h_U = water depth in the upstream channel (m)
- 273 h_L = water depth in the lateral branch (m)
- 274 h_D = water depth in the downstream branch (m)
- 275 $h_{crest} = \text{crest height for case F1 and F2 (m)}$
- 276 k = wall roughness (m)
- 277 N_C = number of cells of the coarse mesh (-)
- 278 N_F = number of cells of the fine mesh (-)
- 279 P_0 = atmospheric pressure (Pa)
- 280 Q_U = upstream discharge (L.s⁻¹)
- 281 Q_L = lateral branch discharge (L.s⁻¹)
- 282 Q_D = downstream branch discharge (L.s⁻¹)
- 283 r = cell number ratio between fine mesh and coarse mesh (-)
- 284 R = curvature radius of the separation zone (m)
- 285 Re_U = Reynolds number in upstream branch (-)

- 286 U_{Inlet} = numerical velocity set at the inlet cross-section of the upstream channel (m.s⁻¹)
- 287 U_U = mean velocity in the upstream channel (m.s⁻¹)
- 288 X_C = value of variable X for coarse mesh (variable)
- 289 X_F = value of variable X for fine mesh (variable)
- 290 z = elevation (m)
- 291 $v = viscosity of water (= 1.10^{-6} m^2.s^{-1})$

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