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# Strategic Coordination with State Information at the Decoder

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**Abstract**—We investigate the coordination of autonomous devices with strategic and non-aligned utility functions. The encoder and the decoder of the point-to-point network choose their coding strategy in order to maximize their own utility function. This paper extends our previous results on strategic coordination by considering state information at the decoder. We study the connexion between Wyner-Ziv source coding and the problem of Bayesian persuasion in the economics literature.

## I. INTRODUCTION

In this paper, we investigate a point-to-point network of autonomous devices with non-aligned utility functions, see Fig. 1. Our study is based on notion of “Empirical Coordination” which characterizes the global behavior that can be implemented by local policies. Coming originally from the literature of Game Theory [1], [2], [3], [4], [5], the problem of Coordination has attracted a lot of attention in Information Theory [6], [7], [8], [9], [10], [11]. It consists in determining the minimal exchange of information required by autonomous devices in order to implement a coordinated behavior. More precisely, a target joint distribution is achievable if there exists a coding scheme whose empirical distribution of symbols converges to that target distribution. Then, it is possible to optimize any utility function - instead of the distortion - by considering the one-shot version of the problem instead of the problem by blocks of  $n$ -symbols. The notion of Empirical Coordination generalizes the “Rate-Distortion Theory” as well as “Channel coding result” and is strongly related to the joint source-channel coding with state information at both encoder and decoder [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22].

In this paper, we investigate the coordinated behavior of two devices with non-aligned utility functions, in the spirit of [23], [24]. Fig. 1 corresponds to the problem of Wyner-Ziv source coding with state information at the decoder [25], with a noisy channel. The only difference is that the encoder and decoder are players endowed with distinct utility functions  $\phi_1(u, v)$  and  $\phi_2(u, v)$ . If these utility functions were equal  $\phi_1(u, v) = \phi_2(u, v)$ , our solution would boil down to the classical result for noisy channel and Wyner-Ziv source.

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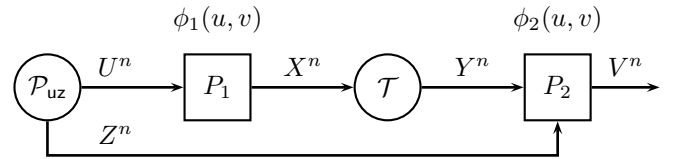


Fig. 1. The information source is i.i.d.  $\mathcal{P}_{uz}(u, z)$  and the channel  $\mathcal{T}(y|x)$  is memoryless. The encoder  $P_1$  and the decoder  $P_2$  are players that maximize their own utility functions  $\phi_1(u, v) \in \mathbb{R}$  and  $\phi_2(u, v) \in \mathbb{R}$ .

We consider a game in which the encoder and decoder are the players  $P_1$  and  $P_2$  that choose the encoding and decoding strategies in order to maximize their long-run utility. The equilibrium solution proposed by Stackelberg in [27] is more suited than the “Nash Equilibrium” [28], since the decoder  $P_2$  knows in advance the encoding strategy of  $P_1$ , i.e. the encoder  $P_1$  has “commitment power”. This problem is also related to the “Strategic Transmission of Information” in the literature of Game Theory [29], [30], [31], [32], [33], [34]. In fact, our problem is closely related to the problem of “Bayesian Persuasion” [35], [36], in which a sender wants to persuade a receiver to change her action. By sending some information, the encoder is able to control the posterior beliefs of the decoder, knowing that he will choose a best-reply action. The problem of strategic communication was investigated in the literature of Information Theory [37], [38], [39], [40] for Gaussian source and channel, and the quadratic distortion functions of [29].

## II. STRATEGIC COORDINATION

### A. Problem Statement

We consider the problem of strategic coordination depicted in Fig. 1. Notations  $U^n, X^n, Y^n, Z^n, V^n$  stand for sequences of random variables of information source  $u^n = (u_1, \dots, u_n) \in \mathcal{U}^n$ , decoder’s state information  $z^n \in \mathcal{Z}^n$ , inputs of the channel  $x^n \in \mathcal{X}^n$ , outputs of the channel  $y^n \in \mathcal{Y}^n$  and decoder’s output  $v^n \in \mathcal{V}^n$ , respectively. The sets  $\mathcal{U}, \mathcal{Z}, \mathcal{X}, \mathcal{Y}, \mathcal{V}$  have finite cardinality. The set of probability distributions over  $\mathcal{X}$  is denoted by  $\Delta(\mathcal{X})$ . The notation  $\|Q - \mathcal{P}\|_1 = \sum_{x \in \mathcal{X}} |Q(x) - \mathcal{P}(x)|$  stands for the  $L_1$  distance

between the probability distributions  $\mathcal{Q}$  and  $\mathcal{P}$ . With a slight abuse of notation, we denote by  $\mathcal{Q}(x) \times \mathcal{Q}(v|x)$ , the product of distributions over  $\Delta(\mathcal{X} \times \mathcal{V})$ . Notation  $Y \ominus X \ominus U$  denotes the Markov chain property corresponding to  $\mathcal{P}(y|x, u) = \mathcal{P}(y|x)$  for all  $(u, x, y)$ . Player  $P_1$  observes a sequence of source symbols  $u^n \in \mathcal{U}^n$  and chooses at random a sequence of channel inputs  $x^n \in \mathcal{X}^n$ . Player  $P_2$  observes a sequence of channel outputs  $y^n \in \mathcal{Y}^n$  and state information  $z^n \in \mathcal{Z}^n$  before choosing at random a sequence of actions  $v^n \in \mathcal{V}^n$ .

### Definition II.1 (Strategies of both players)

• Player  $P_1$  chooses a strategy  $\sigma$  and player  $P_2$  chooses a strategy  $\tau$ , defined as follows:

$$\sigma: \mathcal{U}^n \longrightarrow \Delta(\mathcal{X}^n), \quad (1)$$

$$\tau: \mathcal{Y}^n \times \mathcal{Z}^n \longrightarrow \Delta(\mathcal{V}^n). \quad (2)$$

Both strategies  $(\sigma, \tau)$  are stochastic.

• A pair of strategies  $(\sigma, \tau)$  induces a joint probability distribution  $\mathcal{P}_{\sigma, \tau} \in \Delta(\mathcal{U}^n \times \mathcal{Z}^n \times \mathcal{X}^n \times \mathcal{Y}^n \times \mathcal{V}^n)$  over the  $n$ -sequences of symbols, defined by:

$$\begin{aligned} & \prod_{i=1}^n \mathcal{P}(U_i, Z_i) \times \mathcal{P}_\sigma(X^n | U^n) \\ & \times \prod_{i=1}^n \mathcal{T}(Y_i | X_i) \times \mathcal{P}_\tau(V^n | Y^n, Z^n). \end{aligned} \quad (3)$$

**Definition II.2 (Expected  $n$ -stage utilities)** The utilities of the  $n$ -stage game  $\Phi_1^n$  and  $\Phi_2^n$  are evaluated with respect to the marginal distribution  $\mathcal{P}_{\sigma, \tau}$  over the sequences  $(U^n, V^n)$  and the utility functions  $\phi_1(u, v) \in \mathbb{R}$ ,  $\phi_2(u, v) \in \mathbb{R}$ .

$$\begin{aligned} \Phi_1^n(\sigma, \tau) &= \mathbb{E}_{\sigma, \tau} \left[ \frac{1}{n} \sum_{i=1}^n \phi_1(U_i, V_i) \right] \\ &= \sum_{u^n, v^n} \mathcal{P}_{\sigma, \tau}(u^n, v^n) \cdot \left[ \frac{1}{n} \sum_{i=1}^n \phi_1(u_i, v_i) \right], \end{aligned} \quad (4)$$

$$\Phi_2^n(\sigma, \tau) = \sum_{u^n, v^n} \mathcal{P}_{\sigma, \tau}(u^n, v^n) \cdot \left[ \frac{1}{n} \sum_{i=1}^n \phi_2(u_i, v_i) \right]. \quad (5)$$

**Definition II.3 (Decoder's best-replies)** For any strategy  $\sigma$  of  $P_1$ , we define the set of  $n$ -best-reply of  $P_2$  as follows:

$$BR_2^n(\sigma) = \left\{ \tau, \text{ s.t. } \Phi_2^n(\sigma, \tau) \geq \Phi_2^n(\sigma, \tilde{\tau}), \forall \tilde{\tau} \neq \tau \right\}. \quad (6)$$

**Definition II.4 (Characterization)** We consider an auxiliary random variable  $W$  with  $|\mathcal{W}| = \min(|\mathcal{V}|, |\mathcal{U}| + 1)$ . We define the set  $\mathbb{Q}_0$  of target probability distributions by:

$$\begin{aligned} \mathbb{Q}_0 &= \left\{ \mathcal{P}_{uz}(u, z) \times \mathcal{Q}(w|u), \text{ s.t.,} \right. \\ & \left. \max_{\mathcal{P}(x)} I(X; Y) - I(U; W) + I(Z; W) \geq 0 \right\}. \end{aligned} \quad (7)$$

We define the set  $\mathbb{Q}_2(\mathcal{Q}(u, z, w))$  of decoder's best-reply:

$$\mathbb{Q}_2(\mathcal{Q}(u, z, w)) = \operatorname{argmax}_{\mathcal{Q}(v|z, w)} \mathbb{E}_{\mathcal{Q}(u, z, w) \times \mathcal{Q}(v|z, w)} \left[ \phi_2(U, V) \right]. \quad (8)$$

The optimal utility  $\Phi_1^*$  of  $P_1$  is given by:

$$\Phi_1^* = \sup_{\mathcal{Q}(u, z, w) \in \mathbb{Q}_0} \min_{\substack{\mathcal{Q}(v|z, w) \in \\ \mathbb{Q}_2(\mathcal{Q}(u, z, w))}} \mathbb{E}_{\mathcal{Q}(u, z, w) \times \mathcal{Q}(v|z, w)} \left[ \phi_1(U, V) \right]. \quad (9)$$

We prove that the  $n$ -stage game of utility  $\Phi_1^n(\sigma, \tau)$  can be reformulated as a one-shot game in which the decoder chooses  $\mathcal{Q}(v|z, w)$ , knowing that the encoder has chosen  $\mathcal{Q}(w|u)$ .

**Theorem II.5 (Main Result)** The limit utility of  $P_1$  when  $P_2$  chooses any  $n$ -best-reply  $\tau \in BR_2^n(\sigma)$ :

$$\forall n \in \mathbb{N}, \forall \sigma, \min_{\tau \in BR_2^n(\sigma)} \Phi_1^n(\sigma, \tau) \leq \Phi_1^*, \quad (10)$$

$$\forall \varepsilon > 0, \exists \bar{n}, \forall n \geq \bar{n}, \exists \sigma, \min_{\tau \in BR_2^n(\sigma)} \Phi_1^n(\sigma, \tau) \geq \Phi_1^* - \varepsilon. \quad (11)$$

The proofs of the converse (10) and achievability (11) results are stated in Sec. IV and V.

### III. EXAMPLE WITH Z-STATE INFORMATION

The binary source  $U$  has probability  $\mathcal{P}(u_1) = p_0$  with  $p_0 \in [0, 1]$  and the state information  $Z$  is drawn through a Z-channel with parameter  $\delta \in [0, 1]$  as in Fig. 2. While observing the state

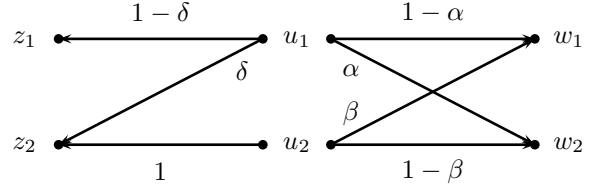


Fig. 2. Joint distribution  $\mathcal{P}(u, z)$  and the signaling strategy  $\mathcal{Q}(w|u)$ .

information  $Z$ , the decoder reactualizes his beliefs regarding the source:

$$q_1 = \mathcal{Q}(u_1|z_1) = \frac{p_0 \cdot (1 - \delta)}{p_0 \cdot (1 - \delta)} = 1, \quad (12)$$

$$q_2 = \mathcal{Q}(u_1|z_2) = \frac{p_0 \cdot \delta}{1 - p_0 \cdot (1 - \delta)}. \quad (13)$$

We denote by  $(q_1, q_2)$  the belief ex-ante, i.e. before the transmission of  $W$ .

The binary auxiliary random variable  $W \in \{w_1, w_2\}$  is drawn with distribution  $\mathcal{Q}(w|u)$  and parameters  $\alpha \in [0, 1], \beta \in [0, 1]$  as in Fig. 2. After receiving the symbol  $W$ , the decoder reactualizes his posterior beliefs denoted by  $(p_1, p_2, p_3, p_4)$ :

$$\mathcal{Q}(u_1|w_1, z_1) = p_1 = \mathcal{Q}(u_1|w_2, z_1) = p_3 = 1, \quad (14)$$

$$\mathcal{Q}(u_1|w_1, z_2) = \frac{p_0 \cdot (1 - \alpha) \cdot \delta}{p_0 \cdot (1 - \alpha) \cdot \delta + (1 - p_0) \cdot \beta} = p_2, \quad (15)$$

$$\mathcal{Q}(u_1|w_2, z_2) = \frac{p_0 \cdot \alpha \cdot \delta}{p_0 \cdot \alpha \cdot \delta + (1 - p_0) \cdot (1 - \beta)} = p_4. \quad (16)$$

(15)-(16) reformulate into the signaling strategy  $\mathcal{Q}(w|u)$ :

$$\alpha = \frac{p_4 \cdot (p_2 \cdot (1 - p_0(1 - \delta)) - p_0 \cdot \delta)}{p_0 \cdot \delta \cdot (p_2 - p_4)}, \quad (17)$$

$$\beta = \frac{(1 - p_2) \cdot (p_0 \cdot \delta - p_4 \cdot (1 - p_0(1 - \delta)))}{(p_2 - p_4) \cdot (1 - p_0)}. \quad (18)$$

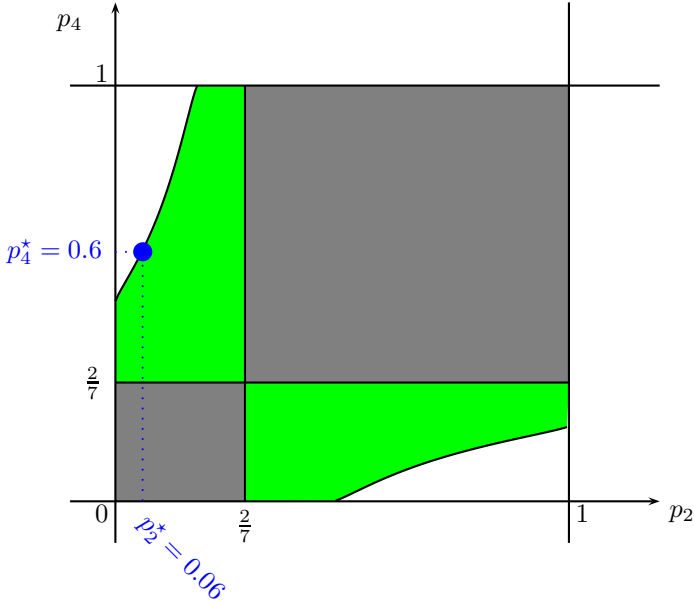


Fig. 3. Regions of posterior beliefs  $(p_2, p_4)$  satisfying information constraint  $C - I(U; W) + I(Z; W) \geq 0$  for  $p_0 = 0.5$ ,  $C = 1 - H(0.25)$ ,  $\delta = 0.4$ . The threshold  $\frac{2}{7}$  corresponds to the beliefs ex-ante  $q_2$  given by (13), induced by the symbol  $z_2$ .

**Lemma 1** A pair of posterior beliefs  $(p_2, p_4)$  is feasible if and only if  $p_2 < q_2 < p_4$  or  $p_4 < q_2 < p_2$ .

The proof of Lemma 1 comes from the constraints  $\alpha \in [0, 1]$ ,  $\beta \in [0, 1]$  in (17) and (18). The pair of posterior beliefs  $(p_2, p_4)$  cannot belong to the grey regions of Fig. 3. We

	$v_1$	$v_2$
$u_1$	10	4
$u_2$	0	9

Fig. 4. Utility  $\phi_2(u, v)$  of  $P_2$

	$v_1$	$v_2$
$u_1$	1	0
$u_2$	1	0

Fig. 5. Utility  $\phi_1(u, v)$  of  $P_1$

consider that the channel capacity is fixed equal to  $C = \max_{\mathcal{P}(x)} I(X; Y) = 1 - H(0.25)$ . The information constraint of  $\mathbb{Q}_0$  writes:

$$C - I(W; U) + I(W; Z) \geq 0 \quad (19)$$

$$\iff H(U|W, Z) \geq H(U|Z) - C \quad (20)$$

$$\iff \mathcal{P}(w_1, z_2)H(p_2) + \mathcal{P}(w_2, z_2)H(p_4) \geq H(U|Z) - C \quad (21)$$

$$\iff \frac{p_0 \cdot \delta - p_4 \cdot (1 - p_0(1 - \delta))}{p_2 - p_4} H(p_2) + \frac{p_2 \cdot (1 - p_0(1 - \delta)) - p_0 \cdot \delta}{p_2 - p_4} H(p_4) \geq H(U|Z) - C. \quad (22)$$

The green regions of Fig. 3 represent the pairs of posterior beliefs  $(p_2, p_4)$  that satisfy the information constraint. We consider the utility functions of the encoder  $\phi_1(u, v)$  and of the decoder  $\phi_2(u, v)$ , given by Fig. 4 and 5. The player  $P_2$  holds a belief  $\mathcal{P}(u_1)$  regarding the source of information  $U$ .

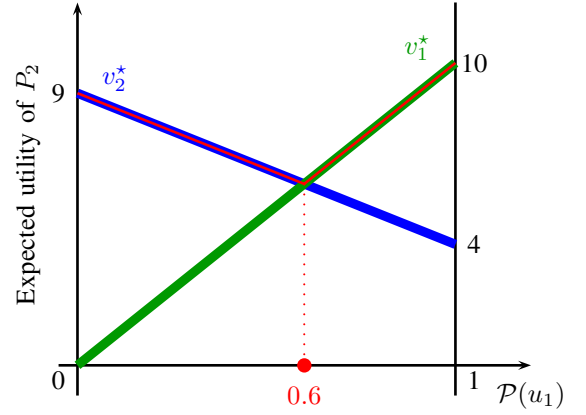


Fig. 6. The best-reply action  $v^*$  of  $P_2$  depends on his belief  $\mathcal{P}(u)$  regarding the source  $U$ : if  $\mathcal{P}(u_1) \in [0, 0.6]$  he plays  $v_2^*$  and if  $\mathcal{P}(u_1) \in [0.6, 1]$  he plays  $v_1^*$ .

He chooses a best-reply action  $v_1^*$  or  $v_2^*$  depending on the interval  $[0, 0.6]$  or  $[0.6, 1]$  in which lies the belief  $\mathcal{P}(u_1)$ , see Fig. 6. The utility of player  $P_1$  only depends on the action

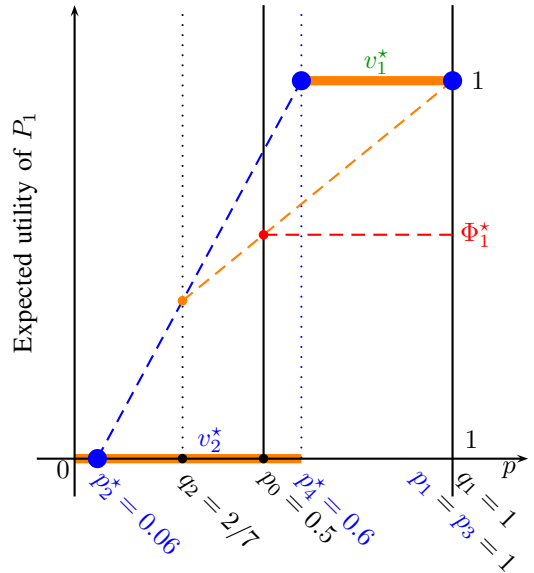


Fig. 7. The expected utility of player  $P_1$  depending on the belief  $\mathcal{P}(u_1)$  of player  $P_2$ . The optimal posterior beliefs  $(p_2^*, p_4^*)$  satisfy the information constraint  $C - I(U; W) + I(Z; W) \geq 0$ , for  $p_0 = 0.5$ ,  $C = 1 - H(0.25)$ ,  $\delta = 0.4$ .

of player  $P_2$  and is represented by the orange line in Fig. 7. The encoder would like to send some information in order to modify the posterior beliefs of  $P_2$  such that  $p_4$  belongs to the interval  $p_4 \in [0.6, 1]$ . Then the best-reply action of  $P_2$  would be  $v_1^*$  that rewards player  $P_1$ . The optimal solution is to fix  $p_4^* = 0.6$  and to find  $p_2^*$  that satisfies the information constraint

with equality:

$$C - I(W; U) + I(W; Z) = 0 \quad (23)$$

$$\begin{aligned} &\Leftrightarrow \frac{p_0 \cdot \delta - p_4^* \cdot (1 - p_0(1 - \delta))}{p_2^* - p_4^*} H(p_2^*) \\ &+ \frac{p_2^* \cdot (1 - p_0(1 - \delta)) - p_0 \cdot \delta}{p_2^* - p_4^*} H(p_4^*) = H(U|Z) - C. \end{aligned} \quad (24)$$

Hence the optimal solution is  $p_2^* \simeq 0.06$  and the pair  $(p_2^*, p_4^*)$  lies at the border of the green region of Fig. 3. This solution induces the conditional entropies  $H(U|W, z_2) \simeq 0.7146$  and  $H(U|W, Z) = H(U|Z) - C \simeq 0.5747$ . The optimal utility for player  $P_1$  is  $\Phi_1^* \simeq 0.5925$ .

#### IV. CONVERSE PROOF OF THEOREM II.5

We consider an arbitrary strategy  $\sigma$  of length  $n \in \mathbb{N}$ . We denote by  $T$  the uniform random variable  $\{1, \dots, n\}$  and we introduce the auxiliary random variable  $W = (Y^n, Z^{-T}, T)$  whose joint probability distribution  $\mathcal{P}_\sigma(u, z, w)$  with  $(U, Z)$  is defined by:

$$\begin{aligned} \mathcal{P}_\sigma(u, z, w) &= \mathcal{P}_\sigma(u_T, z_T, y^n, z^{-T}, T) \\ &= \mathcal{P}(T = i) \cdot \mathcal{P}_\sigma(u_T, z_T, y^n, z^{-T} | T = i) \\ &= \frac{1}{n} \cdot \mathcal{P}_\sigma(u_i, z_i, y^n, z^{-i}). \end{aligned} \quad (25)$$

This identification ensures that the Markov chain  $W \text{---} U_T \text{---} Z_T$  is satisfied. We now prove that the distribution  $\mathcal{P}(u, z, w)$  satisfies the information constraint of the set  $\mathbb{Q}_0$ .

$$0 \leq I(X^n; Y^n) - I(U^n, Z^n; Y^n) \quad (26)$$

$$\leq \sum_{i=1}^n H(Y_i) - \sum_{i=1}^n H(Y_i | X_i) - I(U^n; Y^n | Z^n) \quad (27)$$

$$\leq n \cdot \max_{\mathcal{P}(x)} I(X; Y) - \sum_{i=1}^n I(U_i; Y^n | Z^n, U^{i-1}) \quad (28)$$

$$= n \cdot \max_{\mathcal{P}(x)} I(X; Y) - \sum_{i=1}^n I(U_i; Y^n, Z^{-i}, U^{i-1} | Z_i) \quad (29)$$

$$\leq n \cdot \max_{\mathcal{P}(x)} I(X; Y) - \sum_{i=1}^n I(U_i; Y^n, Z^{-i} | Z_i) \quad (30)$$

$$= n \cdot \max_{\mathcal{P}(x)} I(X; Y) - n \cdot I(U_T; Y^n, Z^{-T} | Z_T, T) \quad (31)$$

$$= n \cdot \max_{\mathcal{P}(x)} I(X; Y) - n \cdot I(U_T; Y^n, Z^{-T}, T | Z_T) \quad (32)$$

$$= n \cdot \max_{\mathcal{P}(x)} I(X; Y) - n \cdot I(U; W | Z) \quad (33)$$

$$= n \cdot \left( \max_{\mathcal{P}(x)} I(X; Y) - I(U; W) + I(Z; W) \right). \quad (34)$$

(26) comes from the Markov chain  $Y^n \text{---} X^n \text{---} (U^n, Z^n)$ .

(27) comes from the memoryless property of the channel and from removing the positive term  $I(U^n; Z^n) \geq 0$ .

(28) comes from taking the maximum  $\mathcal{P}(x)$  and chain rule.

(29) comes from the i.i.d. property of the source  $(U, Z)$  that implies  $I(U_i, Z_i; Z^{-i}, U^{i-1}) = I(U_i; Z^{-i}, U^{i-1} | Z_i) = 0$ .

(30) comes from removing  $I(U_i; U^{i-1} | Y^n, Z^{-i}, Z_i) \geq 0$ .

(31) comes from the uniform random variable  $T \in \{1, \dots, n\}$ .

(32) comes from the independence between  $T$  and the source  $(U, Z)$ , that implies  $I(U_T, Z_T; T) = I(U_T; T | Z_T) = 0$ .

(33) comes from the identification  $W = (Y^n, Z^{-T}, T)$ .

(34) comes from the Markov chain  $W \text{---} U_T \text{---} Z_T$ . This proves that the distribution  $\mathcal{P}_\sigma(u, z, w)$  belongs to the set  $\mathbb{Q}_0$ .

For any strategies  $(\sigma, \tau)$ , we reformulate the long-run utilities with the auxiliary random variable  $W = (Y^n, Z^{-T}, T)$ .

$$\Phi_1^n(\sigma, \tau)$$

$$= \sum_{u^n, z^n, y^n} \mathcal{P}_\sigma(u^n, z^n, y^n) \sum_{v^n} \mathcal{P}_\tau(v^n | y^n, z^n) \cdot \frac{1}{n} \sum_{i=1}^n \phi_1(u_i, v_i) \quad (35)$$

$$= \sum_{i=1}^n \sum_{\substack{u_i, z_i, \\ z^{-i}, y^n}} \frac{1}{n} \cdot \mathcal{P}_\sigma(u_i, z_i, y^n) \sum_{v_i} \mathcal{P}_\tau(v_i | y^n, z^n) \cdot \phi_1(u_i, v_i) \quad (36)$$

$$= \sum_{\substack{u_i, z_i, z^{-i}, \\ y^n, i}} \mathcal{P}_\sigma(u_i, z_i, z^{-i}, y^n, i) \sum_{v_i} \mathcal{P}_\tau(v_i | y^n, z^{-i}, z_i, i) \cdot \phi_1(u_i, v_i) \quad (37)$$

$$= \sum_{u, z, w} \mathcal{P}_\sigma(u, z, w) \sum_v \mathcal{P}_\tau(v | w, z) \cdot \phi_1(u, v). \quad (38)$$

(35) - (37) are reformulations valid also for  $\phi_2(u, v)$ .

(38) comes from replacing the random variables  $(Y^n, Z^{-T}, T)$  by  $W$ , whose distribution is stated in (25).

By replacing  $W = (Y^n, Z^{-T}, T)$ , the set of  $n$ -best-reply  $\text{BR}_2^n(\sigma)$  is equal to the set  $\mathbb{Q}_2(\mathcal{P}_\sigma(u, z, w))$ :

$$\text{BR}_2^n(\sigma)$$

$$\begin{aligned} &= \operatorname{argmax}_{\mathcal{P}_\tau(v^n | y^n, z^n)} \sum_{\substack{u^n, z^n, \\ x^n, y^n}} \mathcal{P}_\sigma(u^n, z^n, x^n, y^n) \\ &\quad \times \sum_{v^n} \mathcal{P}_\tau(v^n | y^n, z^n) \cdot \frac{1}{n} \sum_{i=1}^n \phi_2(u_i, v_i) \end{aligned} \quad (39)$$

$$= \operatorname{argmax}_{\mathcal{P}_\tau(v | w, z)} \sum_{u, z, w} \mathcal{P}_\sigma(u, z, w) \sum_v \mathcal{P}_\tau(v | w, z) \cdot \phi_2(u, v) \quad (40)$$

$$= \mathbb{Q}_2(\mathcal{P}_\sigma(u, z, w)). \quad (41)$$

We conclude the proof of (10) in Theorem II.5.

$$\begin{aligned} \min_{\tau \in \text{BR}_2^n(\sigma)} \Phi_1^n(\sigma, \tau) &= \min_{\tau \in \text{BR}_2^n(\sigma)} \sum_{u^n, z^n, y^n} \mathcal{P}_\sigma(u^n, z^n, y^n) \\ &\quad \times \sum_{v^n} \mathcal{P}_\tau(v^n | y^n, z^n) \cdot \frac{1}{n} \sum_{i=1}^n \phi_1(u_i, v_i) \quad (42) \\ &= \min_{\substack{\mathcal{P}_\tau(v | w, z) \in \\ \mathbb{Q}_2(\mathcal{P}_\sigma(u, z, w))}} \sum_{u, z, w} \mathcal{P}_\sigma(u, z, w) \sum_v \mathcal{P}_\tau(v | w, z) \cdot \phi_1(u, v) \end{aligned} \quad (43)$$

$$\leq \sup_{\mathcal{Q}(u, z, w) \in \tilde{\mathbb{Q}}_0} \min_{\substack{\mathcal{Q}(v | w, z) \in \\ \mathbb{Q}_2(\mathcal{Q}(u, z, w))}} \mathbb{E}_{\mathcal{Q}(u, z, w) \times \mathcal{Q}(v | w, z)} \left[ \phi_1(U, V) \right] = \Phi_1^*. \quad (44)$$

(42) comes from the definitions.

(43) comes from (41) that identifies the set of  $n$ -best-reply  $\text{BR}_2^n(\sigma)$  with the set  $\mathbb{Q}_2(\mathcal{P}_\sigma(u, z, w))$  of definition II.4.

(44) comes from (34) that shows the distribution  $\mathcal{P}_\sigma(u, z, w)$  belongs to the set  $\mathbb{Q}_0$ .

The cardinality bound  $|\mathcal{W}| = \min(|\mathcal{V}|, |\mathcal{U}| + 1)$  follows from Caratheodory's Lemma and Markov chain  $Z \leftrightarrow U \leftrightarrow W$ .

## V. SKETCH OF ACHIEVABILITY OF THEOREM II.5

We denote by  $\mathcal{Q}(u, z, w) \in \mathbb{Q}_0$  the distribution that is optimal for (9). We consider the concatenation of Wyner-Ziv source coding [25] with a channel code [26]. Empirical Coordination results [14] ensures that the sequences of symbols  $(U^n, Z^n, W^n)$  are jointly typical for  $\mathcal{Q}(u, z, w)$  with large probability. Following the same lines as in [23], [24], we prove that the beliefs  $\mathcal{P}_\sigma(u_i|y^n, z^n)$  induced by the strategy  $\sigma$  are close to the target belief  $\mathcal{Q}(u|z, w)$ :

$$\mathbb{E}_\sigma \left[ \frac{1}{n} \sum_{i=1}^n D \left( \mathcal{P}_\sigma(U_i|Y^n, Z^n) \parallel \mathcal{Q}(U_i|W_i, Z_i) \right) \right] \leq \varepsilon. \quad (45)$$

This provides a lower bound on the utility of  $P_1$ :

$$\min_{\tau \in \text{BR}_2^*(\sigma)} \Phi_1^n(\sigma, \tau) \geq \Phi_1^* - \varepsilon. \quad (46)$$

The full version of the proof is in [41].

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