Power allocation Problem for Fading Channels in Cognitive Radio Networks

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***Abstract*—In this paper, we consider a wireless fading channel in Cognitive Radio (CR) systems. Two types of users try to access to the primary spectrum: the Primary users (PUs) and the Secondary users (SUs). In this context, the spectrum has been licensed to the primary users. The secondary users do not own the spectrum license; however, the secondary communication is allowed to coexist with the primary users (PUs) in the primary communication network provided that the SU does not interfere too much with the PU. In this spectrum sharing strategy, we study the power allocation problem in order to achieve the SU's capacity for different fading channel models. It is shown that fading for the channel between SU transmitter and PU receiver is usually a beneficial factor for ameliorating the SU channel capacities. Both, the average interference power constraint at the PU and the average transmit power constraint of the SU are considered in this power allocation problem. We provide a decoupling method, in order to compute the optimal power allocation policies in our optimization problem. Our method gives an optimal solution and reduces the complexity of the general problem using Lagrangian tools.**

***Index Terms*—Cognitive radio, power allocation problem, ergodic capacity, spectrum sharing, transmit power constraint, interference power constraint, fading channels.**

I. Introduction and Related Works

To deal with the spectrum under-utilization problem [1], scientific researchers [2], [3] have developed a promising and intelligent technology which is called *Cognitive Radio* (CR). In this context, there is two type of users : the primary users (PUs) and the secondary users (SUs) whose coexist in the spectrum which is originally allocated to the PUs. In order to improve the spectrum utilization efficiency, one can consider an opportunistic spectrum access by allowing the SUs to access to PU's spectrum when it is not used by any PU. However, it is not easy to detect [4] precisely the vacancies in the PU's spectrum. Thus, the CR network can allow a simultanoeus SU and PU's transmission such that the SU could transmit its power without interfering too much with the PU. This transmission strategy is called *spectrum sharing* [5]–[8].

In the literature, the fading channels' capacity was studied under several transmit power constraints in a non-CR context, by solving these problems, optimal and sub-optimal power allocation policies are given [9], [10]. For example, in [11], the authors minimize the outage probability in non-CR fading channels with the energy-harvesting constraints and channel distribution information at the transmitter.

Different from a non-CR network, in the CR context, power allocation problem of the SU should consider the interference caused to the PU by the SU in order to protect the quality of service (QoS) of the PU. For fading channels, the study of the power allocation problem in CR networks subject to an average or/and a peak interference power constraint at the PU receiver without considering a transmit power constraint, are made [6], [12], [13]. In [14], a PU outage probability constraint instead of power constraints is imposed to protect the QoS of the PU. In spectrum sharing CR scenario, the optimal power allocation strategies are presented under different power constraints (both transmit and interference power constraints) in [15], [16] and [17]. In [15], the authors focus on an energy-efficient optimal power allocation for fading channels. They studied the ergodic capacity, the outage capacity, and the minimum-rate capacity subject to constraints on the average interference power, and the peak/average transmit power constraint. It is shown that the SU's energy efficiency outperforms under the average transmit power constraint compared to the peak transmit power one.

In this paper, we study the SU ergodic capacity under both average transmit power constraint and average interference power constraint for different fading channel models.
The closest work to ours is [17], where the authors studied the ergodic capacity and the outage capacity under different types of power constraints. However, they do not give a rigorous proof for the case where average transmit power and average interference constraints are simultaneously considered.

Our contributions are met: First, we derive the optimal power allocation strategies for SU to achieve the maximum capacity. In addition to the average interference power constraint protecting the PU, we also consider the SU's average transmit power constraint [18]–[20]. Moreover, we study the achievable ergodic capacity under three different channel fading models: such that Rayleigh fading, Nakagami fading [21] and Log-normal fading. The novelty of this paper, it that we provide a decoupling method in order to solve our optimization problem. Our decoupling method reduces the complexity of the general problem and makes it more easier to solve.

The rest of the paper is organized as follows: In section

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II, we describe the system model and present the different fading channel models. In section III, a study of the achievable ergodic capacity is made and a decoupling method to calculate the optimal power allocation strategies is proposed. In section IV, we give and discuss the most significant and interesting simulation results that illustrate our study. Finally, we conclude the paper in section V.

II. System and Channel Model

We consider a spectrum-sharing system in which a secondary user (SU) is allowed to use the spectrum licensed to a primary user (PU), as long as the amount of interference power inflicted at the receiver of the PU is within predefined constraints on the average and peak powers.

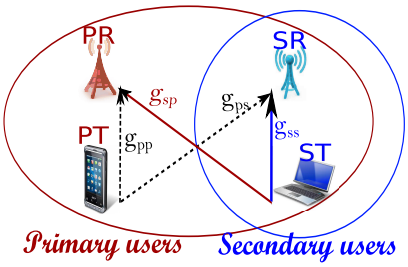

Fig. 1. System model for spectrum sharing in cognitive radio network

As illustrated in Fig. 1, we consider a spectrum sharing network with one PU and one SU. We denote by the subscripts s and p to refer to SU, and PU, respectively. The link between the SU transmitter (ST) and the PU receiver (PR) is assumed to be a flat fading channel with instantaneous channel power gain $g_{ss}[n]$. The SU's channel between the ST and the SU receiver (SR) is also a flat fading channel characterized by instantaneous channel power gain $g_{sp}[n]$. The received signal $y_s[n]$ at the receiver of the SU depends on the transmitted signal $x_s[n]$ according to:

$$y_s[n] = \sqrt{g_{ss}[n]}x_s[n] + z_s[n], \tag{1}$$

where n indicates the time index, $z_s[n]$ represents the AWGN. The noise z_s is assumed to be independent random variables with the distribution $\mathcal{CN}(0, N_0)$ (circularly symmetric complex Gaussian variable with mean zero and variance N_0). For reasons of clarity, hereafter, the time index n is dropped. We also assume that g_{ss} and g_{sp} are independent and identically distributed (i.i.d.). with probability density function $f(g_{ss})$, and $f(g_{sp})$, respectively. Perfect channel state information (CSI) on g_{ss} and g_{sp} is assumed to be available at ST. Furthermore, it is assumed that the interference from the PT to the SR, called here g_{ps}, can be ignored or considered in the AWGN at the SR.

In the following, we study different fading channel models by defining the probability density function in each model.

A. Rayleigh fading

For Rayleigh fading, the channel power gains g_{ss} and g_{sp} are exponentially distributed. Assume g_{ss} and g_{sp} are unit-mean and mutually independent. Then, in this channel model, the probability density function of a random variable x is expressed as:

$$f_{Rayl}(x) \triangleq \frac{x}{\sigma^2}\exp(\frac{-x^2}{2\sigma^2}),\ x \geq 0, \tag{2}$$

where σ^2 represents the mean of the power gains variables.

B. Log-Normal fading

In probability and statistical theory, a random variable x is following a log-normal law of parameters β and σ^2 if the variable $y = \ln(x)$ follows a normal distribution of mean β and variance σ^2. In the literature, it is usually denoted by $Log\mathcal{N}(\beta, \sigma^2)$. The Log-normal law of parameters β and σ admits for density of probability:

$$f_{LogN}(x) \triangleq \frac{1}{x\sigma\sqrt{2\pi}}\exp(\frac{-(\ln(x)-\beta)^2}{2\sigma^2}),\ x > 0. \tag{3}$$

C. Nakagami fading

The Nakagami law [22] is a continuous two-parameter probability law. The parameter $m > 0$ is a form parameter, the second parameter $\omega > 0$ is used to control the propagation.

$$f_{Nak}(x) \triangleq \frac{2m^m}{\Gamma(m)\omega^m}\exp(\frac{-m}{\omega}x^2),\ x > 0, \tag{4}$$

where Γ is related to the gamma law.

III. Optimization Problem Characterisation

In this paper, we consider two type of constraints: average transmit power constraint and average interference constraint.

The average transmit power constraint can be represented by:

$$\mathbb{E}[P(g_{ss}, g_{sp})] \leq P_{av}, \tag{5}$$

where, P_{av} represents the average transmit power limit, $P(g_{ss}, g_{sp})$ represents the instantaneous transmit power at ST for the channel gain pair (g_{ss}, g_{sp}) and we denote by $\mathbb{E}[.] = \mathbb{E}_{(g_{ss},g_{sp})}[.]$ the expectation taken over (g_{ss}, g_{sp}).

On the other hand, given that transmissions pertaining to the secondary user should not harm the signal quality at the PR, we impose constraints on the received-power at the primary's receiver. Let Q_{av} denotes the average received power limit at the PR. The average interference power constraint can be written as follows:

$$\mathbb{E}[g_{sp}P(g_{ss}, g_{sp})] \leq Q_{av}. \tag{6}$$

Recently, from a cognitive radio point of view, the SU's channel capacity has attracted a lot of attention. For fading channels, ergodic capacity is defined in [17] as "the maximum achievable rate averaged over all the fading blocks". In the following, we use a similar approach as in [9] and [17], then we obtain the ergodic capacity of the secondary user link by solving the following optimization problem:

$$C_{er} = \begin{cases} \max\limits_{P(g_{ss},g_{sp})\geq 0} & \mathbb{E}\left[\log_2\left(1+\frac{g_{ss}P(g_{ss},g_{sp})}{N_0}\right)\right] \\ \text{subject to} & \mathbb{E}[P(g_{ss},g_{sp})] \leq P_{av} \\ & \mathbb{E}[g_{sp}P(g_{ss},g_{sp})] \leq Q_{av} \end{cases} \tag{7}$$

At the optimum, the maximum achievable capacity can be given by:

$$C_{er} = \mathbb{E}\left[\log_2\left(1+\frac{g_{ss}P^*(g_{ss},g_{sp})}{N_0}\right)\right]$$

$$= \int_{g_{ss}}\int_{g_{sp}} \log_2\left(1+\frac{g_{ss}}{N_0}P^*(g_{ss},g_{sp})\right) f(g_{ss})f(g_{sp})dg_{ss}dg_{sp}.$$

where the probability density functions over power gains (g_{ss}, g_{sp}) depend on the fading channel model defined in (2),(3) and (4).

As we can see, it is very difficult to calculate analytically this expression of the maximum achievable ergodic capacity. So, in order to calculate the ergodic capacity in each fading channel model, we will give next the optimal power solution $P^*(g_{ss}, g_{sp})$ analytically.

A. Optimal power allocation

Since our optimization problem defined in (7) is strictly concave, we can use the Lagrangian method [23] and we obtain the following Lagrangian function:

$$\mathbb{L}(P(g_{ss},g_{sp}),\lambda,\mu) = -\mathbb{E}\left[\log_2\left(1+\frac{g_{ss}P(g_{ss},g_{sp})}{N_0}\right)\right]$$
$$+\lambda\left(\mathbb{E}[P(g_{ss},g_{sp})]-P_{av}\right)+\mu\left(\mathbb{E}[g_{sp}P(g_{ss},g_{sp})]-Q_{av}\right),$$

where λ and μ represent the non negative Lagrangian multipliers associated to the average transmit power constraint and the average interference power constraint, respectively. According to the Karush Kuhn Tucker (KKT) conditions, the optimal solution needs to satisfy the following equations:

$$\begin{cases} \lambda\left(\mathbb{E}[P(g_{ss},g_{sp})]-P_{av}\right)=0 \\ \mu\left(\mathbb{E}[g_{sp}P(g_{ss},g_{sp})]-Q_{av}\right)=0 \\ \dfrac{\partial\mathbb{L}}{\partial P^*}=0 \Leftrightarrow -\dfrac{Kg_{ss}}{g_{ss}P^*(g_{ss},g_{sp})+N_0}+\lambda+\mu g_{sp}=0, \end{cases}$$

where, $K=\log_2(e)$ is the constant caused by the derivation of the logarithmic function in the base 2. It is easy to observe that λ is either equal to zero or determined by solving the average transmit power equality $\mathbb{E}[P(g_{ss},g_{sp})] = P_{av}$. Moreover, μ is either equal to zero or determined by solving the average interference power equality: $\mathbb{E}[g_{sp}P(g_{ss},g_{sp})] = Q_{av}$.

Therefore, by applying the KKT conditions, we get the optimal power as follows: [1]

$$P^*(g_{ss},g_{sp}) = \left(\frac{K}{\lambda+\mu g_{sp}}-\frac{N_0}{g_{ss}}\right)^+. \tag{8}$$

B. Our decoupling method

Since it is difficult to solve analytically λ and μ where the average transmit power and the average interference power constraints are satisfied with equalities. We decouple the original problem into two sub-problems, which are easily solved individually.

[1]We denote by $(x)^+ = \max(0,x)$.

- Sub-Problem 1:

$$(SP_1)\begin{cases} \max\limits_{P(g_{ss},g_{sp})\geq 0} & \mathbb{E}\left[\log_2\left(1+\dfrac{g_{ss}P(g_{ss},g_{sp})}{N_0}\right)\right] \\ s.t. & \mathbb{E}\left[P(g_{ss},g_{sp})\right]\leq P_{av}. \end{cases}$$

This problem is equivalent to the problem (7) without average interference power constraint. In this case, the optimal power allocation for (SP_1), denoted by $P^{(1)}(g_{ss}, g_{sp})$, is given by:

$$P^{(1)}(g_{ss},g_{sp}) = \left(\frac{K}{\lambda}-\frac{N_0}{g_{ss}}\right)^+, \tag{9}$$

where λ satisfy the average transmit power constraint with equality : $\mathbb{E}[P^{(1)}(g_{ss}, g_{sp})] = P_{av}$. Therefore, we obtain the non-negative dual variable λ as follows:

$$\lambda = \frac{K}{P_{av}+\frac{N_0}{g_{ss}}} \tag{10}$$

- Sub-Problem 2:

$$(SP_2)\begin{cases} \max\limits_{P(g_{ss},g_{sp})\geq 0} & \mathbb{E}\left[\log_2\left(1+\dfrac{g_{ss}P(g_{ss},g_{sp})}{N_0}\right)\right] \\ s.t. & \mathbb{E}\left[g_{sp}P(g_{ss},g_{sp})\right]\leq Q_{av}. \end{cases}$$

This problem is equivalent to to the problem (7) without average transmit power constraint. In this case, the optimal power allocation for (SP_2), denoted by $P^{(2)}(g_{ss}, g_{sp})$, is given by:

$$P^{(2)}(g_{ss},g_{sp}) = \left(\frac{K}{\mu g_{sp}}-\frac{N_0}{g_{ss}}\right)^+, \tag{11}$$

where μ satisfy the average interference power constraint with equality : $\mathbb{E}[g_{sp}P^{(2)}(g_{ss}, g_{sp})] = Q_{av}$. Therefore, we obtain the non-negative dual variable μ as follows:

$$\mu = \frac{K}{Q_{av}+\frac{N_0 g_{sp}}{g_{ss}}}. \tag{12}$$

The question now is how we can solve the general problem and calculate $P^*(g_{ss}, g_{sp})$ from the two optimal allocations corresponding to the sub-problems (SP_1) and (SP_2)?

Theorem 1. *Both constraints on average transmit power $\mathbb{E}[P^{(2)}(g_{ss},g_{sp})] \leq P_{av}$ and on average interference power $\mathbb{E}[g_{sp}P^{(1)}(g_{ss},g_{sp})] \leq Q_{av}$ can not be simultaneously satisfied.*

Proof. We will prove this theorem by the absurd method: Let R_1 and R_2 denote the feasible region for Sub-Problem (SP_1) and Sub-Problem (SP_2) respectively. We suppose that the optimal power for the second Sub-Problem $P^{(2)}(g_{ss}, g_{sp})$ satisfies the average transmit power constraint $E[P^{(2)}(g_{ss}, g_{sp})] \leq P_{av}$. Then $P^{(2)}(g0, g1)$ must be in R_1. Since $P^{(1)}(g_{ss}, g_{sp})$ is the optimal value for (SP_1), then the following inequality must hold:

$$C_{er}(P^{(1)}(g_{ss},g_{sp})) > C_{er}(P^{(2)}(g_{ss},g_{sp})). \tag{13}$$

On the other hand, we suppose that the optimal power for the first Sub-Problem $P^{(1)}(g_{ss}, g_{sp})$ satisfies the average interference power constraint $E[g_{sp}P^{(1)}(g_{ss}, g_{sp})] \leq Q_{av}$. Then

$P(1)(g_{ss}, g_{sp})$ must be in R_2. Since $P(2)(g_{ss}, g_{sp})$ is the optimal value for (SP_2), then the following inequality must hold:

$$C_{er}(P^{(2)}(g_{ss}, g_{sp})) > C_{er}(P^{(1)}(g_{ss}, g_{sp})). \quad (14)$$

Obviously, from (13) and (14) this forms contradiction and Theorem 1 is then proved. □

Theorem 2. *If $E[g_{sp}P^{(1)}(g_{ss}, g_{sp})] \leq Q_{av}$ holds, then the optimal power allocation $P^{(1)}(g_{ss}, g_{sp})$ must be the global optimal value for the original problem.*
Similarly, if $E[P^{(2)}(g_{ss}, g_{sp})] \leq P_{av}$ holds, then the optimal power allocation $P^{(2)}(g0, g1)$ must be the global optimal value.

Remark III.1. *If Theorem 2 is not satisfied then, we use the Lagrangian method [20] and [23] we calculate numerically the global optimal power in* (8)*.*

Based on these Theorems, we can solve the original problem using the following algorithm:

Algorithm 1: Our decoupling method algorithm under average transmit power and average interference power constraints

1) Solve the (SP_1) problem, calculate λ in (10) and calculate the power allocation $P^{(1)}(g_{ss}, g_{sp})$ as follows:

$$P^{(1)}(g_{ss}, g_{sp}) = \left(\frac{K}{\lambda} - \frac{N_0}{g_{ss}}\right)^+$$

if $\mathbb{E}[g_{sp}P^{(1)}(g_{ss}, g_{sp})] \leq Q_{av}$ **then**
  $P^*(g_{ss}, g_{sp}) = P^{(1)}(g_{ss}, g_{sp})$
else
  2) Solve (SP_2) problem, calculate μ in (12) and calculate the power allocation $P^{(2)}(g_{ss}, g_{sp})$ given by:

$$P^{(2)}(g_{ss}, g_{sp}) = \left(\frac{K}{\mu g_{sp}} - \frac{N_0}{g_{ss}}\right)^+$$

  if $\mathbb{E}[P^{(2)}(g_{ss}, g_{sp})] \leq P_{av}$ **then**
    $P^*(g_{ss}, g_{sp}) = P^{(2)}(g_{ss}, g_{sp})$
    else
      3) Use Lagrangian method and calculate $P^*(g_{ss}, g_{sp})$ as follows:

$$P^*(g_{ss}, g_{sp}) = \left(\frac{K}{\lambda + \mu g_{sp}} - \frac{N_0}{g_{ss}}\right)^+.$$

    end
  end
end

This decoupling method reduces the complexity of the initial problem by replacing the study of this general problem to two decoupled problems which are more easier to solve. In practice, we remark that, in most cases $P^{(1)}(g_{ss}, g_{sp})$ in (9) or $P^{(2)}(g_{ss}, g_{sp})$ in (11) is the optimal solution for our general problem.

IV. Numerical Results

In this section, we evaluate our proposed method via numerical simulations. All observations below have been verified via extensive Monte-Carlo simulations with generic parameters. We have selected only a few of the most illustrative and interesting scenarios to be presented next. In the following, we consider the flat fading CR system with a single active PU and one SU. The noise and interference power is normalized as $N_0 = 0.1W$. The SU's power gains g_{ss} and g_{sp} are generated by random distribution $g_s \backsim \mathcal{N}(0, \sigma_s^2)$.

A. Fading Channels

For additive white Gaussian noise (AWGN) channels, the SU's capacity in spectrum sharing strategy was studied in [24] under a received power constraint. It is shown that in an AWGN channel, this is equivalent to considering a transmit power constraint. However, this become quite different in the case of fading channels.

In Fig. 2, we plot the ergodic capacity under average transmit and average interference power constraints for fixed $Q_{av} = 5$ dB. It is observed that when $P_{av} = 0$, the ergodic capacities for the four curves shown in this figure are almost the same [2]. This means that the average power threshold P_{av} restricts the performance of the network. However, when P_{av} is sufficiently high compared to Q_{av}, then, the ergodic capacities become different. In this scenario, when g_{ss} models the AWGN channel, the SU's capacity when g_{sp} also models the AWGN channel is lower than that when g_{sp} models the Rayleigh fading channel. Moreover, when g_{ss} models the Rayleigh fading channel, the SU's capacity when g_{sp} models the AWGN channel is lower than that when g_{sp} models the Rayleigh fading channel. Thus, from this figure, we illustrate that fading of the channel between the ST and the PR is a beneficial factor in terms of maximizing the achievable ergodic capacity of SU channel.

Now, we consider the Rayleigh fading channel model for both power gains g_{ss} and g_{sp}:
In Fig. 3, we plot the ergodic capacity versus P_{av} under average transmit and average interference power constraints for different channel models [2][3]. We remark that the curve for Rayleigh fading channel outperforms the AWGN channel, the Log-normal fading channel.

Therefore from both figures, we remark that the fading for the channel between secondary user transmitter and PU receiver is usually a beneficial factor for ameliorating the secondary user channel's capacity.

[2]For Rayleigh fading channels, the channel power gains (exponentially distributed) are assumed to be unit mean $\sigma^2 = 1$. For AWGN channels, the channel power gains are also assumed to be one $\sim \mathcal{N}(0, 1)$.

[3]For Log-Normal fading channels, the channel power gains are assumed to be null mean $\mu = 0$ and unit variance $\sigma^2 = 1$.

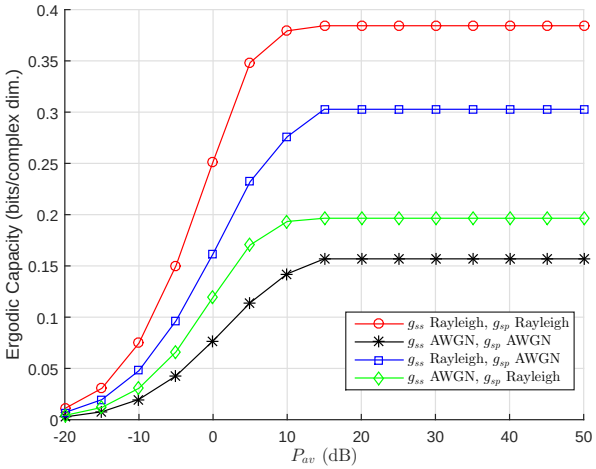

Fig. 2. The maximum achievable capacity of the SU vs. average transmit power P_{av} where $Q_{av} = 5$ dB for AWGN and Rayleigh fading channel models.

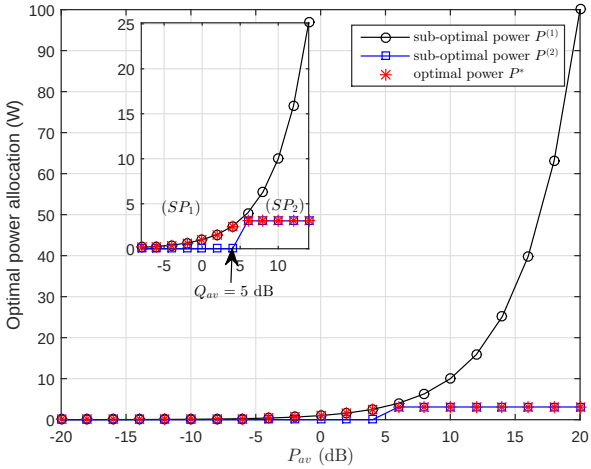

Fig. 4. The global optimal power allocation $P^*(g_{ss}, g_{sp})$ vs. average power P_{av} where $Q_{av} = 5$ dB for a Rayleigh Fading channel.

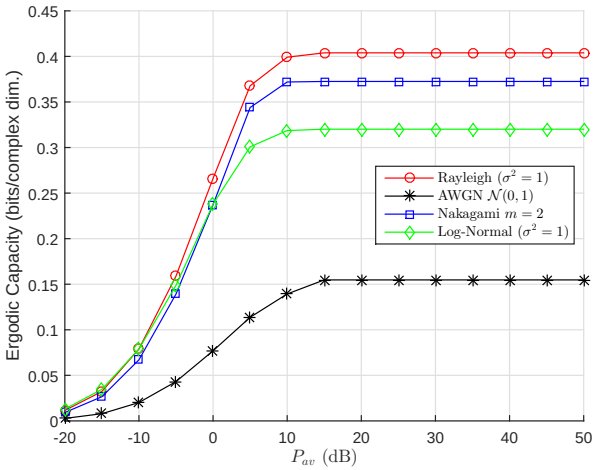

Fig. 3. The maximum achievable capacity of the SU vs. average power P_{av} where $Q_{av} = 5$ dB for different types of Fading channels.

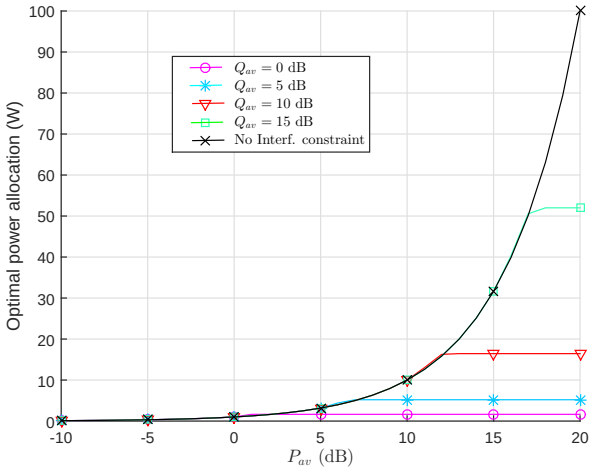

Fig. 5. The optimal power allocation $P^*(g_{ss}, g_{sp})$ vs. average power P_{av} for different value of Q_{av}.

B. Decoupling method

In Fig. 4, we plot the optimal power allocation $P^*(g_{ss}, g_{sp})$ of our optimization problem (calculated via Algorithm 1). In the same figure, we plot also the optimal solution of the Sub-Problem (SP_1) and the optimal solution of the Sub-Problem (SP_2). We remark that, when P_{av} is small ($P_{av} << Q_{av}$), the optimal solution of (SP_1) is the general optimal power allocation $P^*(g_{ss}, g_{sp}) = P^{(1)}(g_{ss}, g_{sp})$. When P_{av} is high ($P_{av} >> Q_{av}$), the optimal solution of (SP_2) is the general optimal power allocation $P^*(g_{ss}, g_{sp}) = P^{(2)}(g_{ss}, g_{sp})$.

We remark that, in all cases, either the sub-optimal solution $P^{(1)}(g_{ss}, g_{sp})$ in (9) or sub-optimal solution $P^{(2)}(g_{ss}, g_{sp})$ in (11) is the optimal solution for our general problem and there is no intermediary value.

Fig. 5 shows the optimal power allocation $P^*(g_{ss}, g_{sp})$ under average transmit and average interference power constraints. For comparison, the black curve with $Q_{av} = +\infty$ (i.e. no interference power constraint) is also shown. It is observed that when P_{av} is small compared to Q_{av} ($P_{av} << Q_{av}$), the optimal power allocations for different Q_{av} do not vary much. However, when P_{av} is sufficiently high compared to Q_{av} (i.e., $P_{av} >> Q_{av}$), the power allocations become flat. This means that the transmit power constraint P_{av} becomes the dominant constraint in this case.

V. Conclusion

In this paper, a fading cognitive radio channel is considered. In a spectrum sharing context, the optimal power allocation strategies are studied in order to achieve the ergodic capacity of the SU under different fading channel models. It is shown that fading for the channel between SU transmitter and PU receiver is usually a beneficial factor for ameliorating the

SU channel capacities. Both the average interference power constraints at PU and the average transmit power constraints of SU were considered. We have provided a decoupling method, in order to solve the power allocation problem under average transmit power and average interference power constraints. It was shown that our method reduces the complexity of the general problem and makes it more easier to solve. Future works can include the case where several SUs coexist in the primary networks, so one will provide a joint scheduling spectrum and power allocation problem [20] .

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