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Simultaneous Energy and Information Transmission: A Finite Block-Length Analysis

Samir M. Perlaza, Ali Tajer, and H. Vincent Poor

***Abstract*—In this paper, a non-asymptotic analysis of the fundamental limits of simultaneous energy and information transmission (SEIT) is presented. The notion of *information-capacity region*, i.e., the largest set of simultaneously achievable information and energy rates, is revisited in a context in which transmissions occur within a finite number of channel uses and strictly positive error decoding probability and energy shortage probability are tolerated. The focus is on the case of one transmitter, one information receiver and one energy harvester communicating through binary symmetric memoryless channels. In this case, the information-capacity region is approximated and the trade-off between information rate and energy rate is thoroughly studied.**

***Index Terms*—Information and Energy Transmission, Information and Power Transfer, Finite Block-Length Regime.**

I. Introduction

Simultaneous energy and information transmission (SEIT) refers to a communication system in which a set of transmitters aim to simultaneously carry on two tasks: information transmission to a set of information receivers (IRs); and energy transmission to a set of energy harvesters (EHs). The performance of SEIT is often measured by the information and energy transmission rates that can be simultaneously achieved under certain reliability constraints. Reliability can be measured by two metrics: decoding error probability (DEP); and energy shortage probability (ESP). The fundamental limits of SEIT consist of the largest set of information and energy rate tuples that can be simultaneously achieved. This set is often referred to as the *information-energy capacity region* [1]. Traditionally, information-energy capacity regions are calculated subject to the fact that both DEP and ESP must be arbitrarily close to zero. This strong reliability constraint leads to the unavoidable use of infinitely long communication blocks and thus, these fundamental limits are meaningful only under the assumption that the communication might last a very long time. Within this context, the case of one transmitter and one co-located IR and EH is studied in [2], [3], and [4]. The multi-user case is studied in [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], and references therein. Despite the existing literature, these approaches quickly lose relevance in scenarios in which communications must occur within a short period, e.g., the Internet of things.

More relevant fundamental limits, from an engineering perspective, are those that take into account that SEIT occurs within a finite number of channel uses and the system tolerates strictly positive DEP and ESP. This paper introduces this novel approach building upon the existing results on the fundamental limits of information transmission in the non-asymptotic block-length regime (c.f. [16] and [17]). The main result is the characterization of the information-energy capacity region of a SEIT system with one transmitter, one IR, and one EH communicating over binary symmetric memoryless channels. More specifically, for given number of channel uses and the biggest DEP and ESP that can be tolerated, the largest set of simultaneously achievable information and energy rates is characterized. The trade-off between information rate and energy rate, first reported in [2] and [3] in the asymptotic regime, is studied in the non-asymptotic regime. The system parameters for which information and energy transmission are conflicting tasks are also characterized. These results reveal the central role of the ESP in both the information transmission rate and the energy transmission rate.

The paper is organized as follows. Sec. II formulates the problem and introduces the notion of information-energy capacity region in the non-asymptotic block-length regime. Sec. III focuses on the case of binary symmetric memoryless channels and presents the main results. Sec. IV concludes this work.

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II. System Model

Consider a three-party communication system in which a transmitter aims at simultaneously sending information to an information receiver (IR) and energy to an energy harvester (EH) through a noisy communication medium.

Consider a random transformation $(\mathcal{X}, \mathcal{Y} \times \mathcal{Z}, P_{YZ|X})$ from an input alphabet $\mathcal{X}$ to an output alphabet $\mathcal{Y} \times \mathcal{Z}$ consisting in a transition probability kernel $P_{YZ|X}$. That is, given an input $x \in \mathcal{X}$, the output $(y, z) \in \mathcal{Y} \times \mathcal{Z}$ is observed with probability $P_{YZ|X}(y, z|x)$. With loss of generality, the noisy communication medium is represented by a memoryless channel. A memoryless channel is a random transformation

$$(\mathcal{X}^n, \mathcal{Y}^n \times \mathcal{Z}^n, P_{\boldsymbol{Y}\boldsymbol{Z}|\boldsymbol{X}}), \tag{1}$$

where $n \in \{1, 2, \ldots\}$ is the block length and $\boldsymbol{Y} = (Y_1, Y_2, \ldots, Y_n) \in \mathcal{Y}^n$, $\boldsymbol{Z} = (Z_1, Z_2, \ldots, Z_n) \in \mathcal{Z}^n$ and $\boldsymbol{X} = (X_1, X_2, \ldots, X_n) \in \mathcal{X}^n$ are n-dimensional vectors of random variables, such that given an input $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$, the output $(y_1, y_2, \ldots, y_n, z_1, z_2, \ldots, z_n)$ is observed with probability:

$$P_{\boldsymbol{Y}\boldsymbol{Z}|\boldsymbol{X}}(\boldsymbol{y}, \boldsymbol{z}|\boldsymbol{x}) = \prod_{t=1}^{n} P_{YZ|X}(y_t, z_t|x_t). \tag{2}$$

Within this context, two tasks are carried out by the transmitter: (a) the information transmission task; and (b) the energy transmission task.

A. *Information Transmission Task*

The message to be sent from the transmitter to the IR is a realization of a random variable that is uniformly distributed in $\{1, 2, \ldots, M\}$, with $1 \leqslant M < \infty$. To carry out this task within n channel uses, the transmitter uses an (n, M)-code.

Definition 1 ((n, M)-code): An (n, M)-code for the random transformation in (1) *is a system*

$$\{(\boldsymbol{u}(1), \mathcal{D}_1), (\boldsymbol{u}(2), \mathcal{D}_2), \ldots, (\boldsymbol{u}(M), \mathcal{D}_M)\}, \tag{3}$$

where for all $(i, j) \in \{1, 2, \ldots, M\}^2$, with $i \neq j$,

$$\boldsymbol{u}(i) = (u_1(i), u_2(i), \ldots, u_n(i)) \in \mathcal{X}^n, \tag{4a}$$
$$\boldsymbol{u}(i) \neq \boldsymbol{u}(j), \tag{4b}$$
$$\mathcal{D}_i \subset \mathcal{Y}^n \tag{4c}$$
$$\mathcal{D}_i \cap \mathcal{D}_j = \emptyset; \text{ and} \tag{4d}$$
$$\mathcal{D}_1 \cup \mathcal{D}_2 \cup \ldots \cup \mathcal{D}_M \subseteq \mathcal{Y}^n. \tag{4e}$$

Given the system in (3), to transmit the message index i, the transmitter inputs the symbol $u_t(i)$ to the channel at time $t \in \{1, 2, \ldots, n\}$. The IR observes at the end of channel use t, the output y_t. At the end of n channel uses, the IR states that the symbol i was transmitted if it satisfies the rule:

$$(y_1, y_2, \ldots, y_n) \in \mathcal{D}_i. \tag{5}$$

The probability of error associated with the transmission of message index i, denoted by $\lambda_i \in [0, 1]$, is

$$\lambda_i = \Pr\left[\boldsymbol{Y} \in \mathcal{D}_i^{\mathrm{c}} \mid \boldsymbol{X} = \boldsymbol{u}(i)\right], \tag{6}$$

where the probability is with respect to the marginal $P_{\boldsymbol{Y}|\boldsymbol{X}}$ of the joint probability distribution in (2). The average probability of error, denoted by λ, is

$$\lambda = \frac{1}{M} \sum_{m=1}^{M} \lambda_m. \tag{7}$$

Given a parameter $\epsilon \in [0, 1]$, the information transmission task is said to be reliably performed if one of the following criteria is satisfied:
(a) *The average decoding error probability is smaller than ϵ, i.e., $\lambda < \epsilon$*, or
(b) *The maximum decoding error probability is smaller than ϵ, i.e., for all $i \in \{1, 2, \ldots, M\}$, $\lambda_i < \epsilon$.*
An (n, M)-code that satisfies the first criterion is said to be an (n, M, ϵ)-code with average decoding error probability. Alternatively, an (n, M)-code that satisfies the second criterion is said to be an (n, M, ϵ)-code with maximal decoding error probability.

B. Energy Transmission Task

Let $g : \mathcal{Z} \rightarrow \mathbb{R}_+$ be a positive measurable function with respect to the largest sigma-algebra induced by the set $\mathcal{Z}$ and the Borel sigma-algebra in $[0, \infty)$. The amount of energy delivered to the EH by the channel outputs $\boldsymbol{z} = (z_1, z_2, \ldots, z_n)$, denoted by B_n, is:

$$B_n(\boldsymbol{z}) = \sum_{t=1}^{n} g(z_t). \tag{8}$$

The objective of the transmitter is to ensure that a minimum amount of energy b is harvested at the EH at the end of n channel uses, with $\frac{b}{n} > 0$, as the average energy transmission rate in energy units per channel use. An energy shortage is said to occur when the energy harvested at the EH is less than the minimum required at the end of the transmission. The probability of an energy shortage when transmitting the message index $i \in \{1, 2, \ldots, M\}$, denoted by θ_i, is:

$$\theta_i = \Pr\left[B_n(\boldsymbol{Z}) < b \mid \boldsymbol{X} = \boldsymbol{u}(i)\right], \tag{9}$$

where the probability is with respect to the marginal $P_{\boldsymbol{Z}|\boldsymbol{X}}$ of the joint probability distribution in (2). The average probability of energy shortage, denoted by θ, is

$$\theta = \frac{1}{M} \sum_{i=1}^{M} \theta_i. \tag{10}$$

Given a parameter $\delta \in [0, 1]$, the energy transmission task is said to be reliably performed if one of the following criteria is satisfied:
(c) *The average energy shortage probabilityis smaller than δ, i.e., $\theta < \delta$*, or
(d) *The maximum energy shortage probability is smaller than δ, i.e., for all $i \in \{1, 2, \ldots, M\}$, $\theta_i < \delta$.*
An (n, M, ϵ)-code that satisfies the first criterion is said to be an $(n, M, \epsilon, \delta, b)$-code with average energy shortage probability. Alternatively, an (n, M, ϵ)-code that satisfies the second criterion is said to be an $(n, M, \epsilon, \delta, b)$-code with maximal energy shortage probability.

C. Fundamental Limits

The non-asymptotic fundamental limits of SEIT are described by the notion of information-energy capacity region.

Definition 2 (Information-Energy Capacity Region): The information-energy capacity region $\mathcal{C} \in \mathbb{N}^2 \times [0, 1]^2 \times \mathbb{R}$ of the random transformation in (1) *is the set of all tuples $(n, M, \epsilon, \delta, b)$ for which there exists an $(n, M, \epsilon, \delta, b)$-code satisfying at least one of the reliability criterion pairs: (a, c); (a, d); (b, c); or (b, d).*

The information-energy capacity region $\mathcal{C}$ in Definition 2 is an element of a vector space over $\mathbb{R}^5$, which is difficult to characterize. In this work, given fixed parameters (n, ϵ, δ), the focus is on the subset $\mathcal{C}(n, \epsilon, \delta)$ that contains the pairs $(M, b) \in \mathbb{N} \times \mathcal{R}_+$ such that $(n, M, \epsilon, \delta, b) \in \mathcal{C}$. Therefore, if $(M, b) \in \mathcal{C}(n, \epsilon, \delta)$, then the information rate $\frac{\log_2(M)}{n}$ bits per channel use; and the energy rate $\frac{b}{n}$ energy units per channel use are jointly achievable within n channel uses with a (maximal or average) decoding error probability ϵ and (maximal or average) energy shortage probability δ.

The subset $\mathcal{C}(n, \epsilon, \delta)$ is an element of a vector space over $\mathbb{R}^2$. In order to emphasize the reliability criteria, when needed, the information-energy capacity region $\mathcal{C}(n, \epsilon, \delta)$ is often written as $\mathcal{C}_{(\mathrm{a,c})}(n, \epsilon, \delta)$, $\mathcal{C}_{(\mathrm{a,d})}(n, \epsilon, \delta)$, $\mathcal{C}_{(\mathrm{b,c})}(n, \epsilon, \delta)$ or $\mathcal{C}_{(\mathrm{b,d})}(n, \epsilon, \delta)$, respectively. A similar notation is used for the larger region $\mathcal{C}$.

III. Memoryless Binary Symmetric Channels

This section focuses on the case in which $\mathcal{X} = \mathcal{Y} = \mathcal{Z} = \{0, 1\}$ and the random transformation in (1) is such that for all $\boldsymbol{x} \in \{0, 1\}^n$, the outputs $\boldsymbol{y} \in \{0, 1\}^n$ and $\boldsymbol{z} \in \{0, 1\}^n$ are observed with probability:

$$P_{\boldsymbol{Y}\boldsymbol{Z}|\boldsymbol{X}}(\boldsymbol{y}, \boldsymbol{z}|\boldsymbol{x}) = \prod_{t=1}^{n} P_{Y|X}(y_t|x_t) P_{Z|X}(z_t|x_t), \tag{11}$$

where for all $(x, y, z) \in \{0, 1\}^3$,

$$P_{Y|X}(y|x) = \alpha_1 \mathbb{1}_{\{x \neq y\}} + (1 - \alpha_1) \mathbb{1}_{\{x = y\}}, \tag{12}$$
$$P_{Z|X}(z|x) = \alpha_2 \mathbb{1}_{\{x \neq z\}} + (1 - \alpha_2) \mathbb{1}_{\{x = z\}}, \tag{13}$$

and $\alpha_1 \in [0, \frac{1}{2})$ and $\alpha_2 \in [0, \frac{1}{2})$ are the crossover probabilities of the channel. Let

$$b_0 = g(0) \geqslant 0 \text{ and } b_1 = g(1) \geqslant 0, \tag{14}$$

be the energy harvested when the output of the channel at the EH is 0 and 1, respectively. The case in which $b_0 = b_1$ is trivial, since the harvested energy is always $nb_1 = nb_0$ energy units, independently of the codebook. This implies that the information transmission task can be carried out without taking into account the energy transmission task. Hence, for avoiding the trivial cases, the following assumption is adopted without loss of generality:

$$b_1 < b_0. \tag{15}$$

For all $\boldsymbol{z} \in \{0,1\}^n$, it follows that

$$\begin{aligned} B_n(\boldsymbol{z}) &= b_0 N(0|\boldsymbol{z}) + b_1 N(1|\boldsymbol{z}) && (16)\\ &= (b_0 - b_1) N(0|\boldsymbol{z}) + nb_1, && (17) \end{aligned}$$

where $N(0|\boldsymbol{z})$ and $N(1|\boldsymbol{z})$ are the number of zeros and ones in the vector $\boldsymbol{z}$, respectively. Note that $B_n(\boldsymbol{z})$ is bounded for all $\boldsymbol{z} \in \mathcal{Z}^n$, i.e.,

$$nb_1 \leqslant B_n(\boldsymbol{z}) \leqslant nb_0. \tag{18}$$

The inequalities in (18) imply that there exists a case in which energy transmission might occur with zero (maximal or average) energy shortage probability at a given energy rate $\frac{b}{n} \leqslant b_1$. This is because, the event $B_n(\boldsymbol{Z}) < nb_1$ is observed with zero probability. Basically, transmitting any symbol, either zero or one, is indifferent from the energy transmission perspective. In this case, the information transmission task can be carried out independently of the energy transmission task given that nb_1 energy units can always be reliably transmitted in n channel uses. Alternatively, any energy transmission rate $\frac{b}{n} > b_0$ cannot be achieved with an average or maximal energy shortage probability strictly smaller than one. This is also due to (18).

Given an (n, M)-code described by the system in (3), let the empirical probability distribution of the channel input symbols induced by the codeword $\boldsymbol{u}(i)$, denoted by $\bar{P}_X^{(i)}$, be such that

$$\bar{P}_X^{(i)}(0) = 1 - \bar{P}_X^{(i)}(1) = \frac{1}{n}\sum_{t=1}^{n} \mathbb{1}_{\{u_t(i)=0\}}, \tag{19}$$

for all $i \in \{1, 2, \ldots, M\}$. Let also the empirical distribution of the channel input symbols jointly induced by all codewords, denoted by $\bar{P}_X(0)$, be such that

$$\bar{P}_X(0) = 1 - \bar{P}_X(1) = \frac{1}{nM}\sum_{i=1}^{M}\sum_{t=1}^{n} \mathbb{1}_{\{u_t(i)=0\}}. \tag{20}$$

Using these empirical distributions some upper bounds can be obtained on both the energy and information transmission rates.

A. An Upper Bound on the Energy Rate of (n, M)-Codes

Define $Q : \mathbb{R} \to [0, 1]$ as the complementary cumulative distribution function of the standard normal distribution, and define $Q^{-1} : (0, 1) \to \mathbb{R}$ as the functional inverse of Q. Using this notation, the following lemma provides an upper bound on the number of energy units b that can be reliably delivered by any given (n, M)-code with an average or maximal energy shortage probability $\delta \in (0, 1)$.

Lemma 1: Given any (n, M)-code of the form in (3) *for the random transformation in* (11) *satisfying* (15)*, with maximal energy shortage probability δ, it holds that for all $i \in \{1, 2, \ldots, M\}$, the energy rate b must satisfy:*

$$\begin{aligned} b<&n\Big((b_0 - b_1)\Big(\left(1 - 2\alpha_2\right)\bar{P}_X^{(i)}(0) + \alpha_2\Big) + b_1\Big)\\ &-\sqrt{n(b_0 - b_1)^2\alpha_2(1-\alpha_2)}Q^{-1}(\delta)\\ &-\sqrt{\frac{\pi}{2}}(b_0 - b_1)\Big((1-\alpha_2)^2 + \alpha_2^2\Big). \end{aligned} \tag{21}$$

whereas with average energy shortage probability δ, the energy rate b must satisfy:

$$\begin{aligned} b<&n\Big((b_0 - b_1)\Big(\left(1 - 2\alpha_2\right)\bar{P}_X(0) + \alpha_2\Big) + b_1\Big)\\ &-\sqrt{n(b_0 - b_1)^2\alpha_2(1-\alpha_2)}Q^{-1}(\delta)\\ &-\sqrt{\frac{\pi}{2}}(b_0 - b_1)\Big((1-\alpha_2)^2 + \alpha_2^2\Big). \end{aligned} \tag{22}$$

Proof of Lemma 1: Note that taking into account the reliability criterion (d), it follows that b must satisfy, for all $i \in \{1, 2, \ldots, M\}$,

$$b < \sup\left\{a > 0 : \forall i \ \Pr\left[B_n(\boldsymbol{Z}) < a \,|\, \boldsymbol{X} = \boldsymbol{u}(i)\right] < \delta\right\}, \tag{23}$$

where the probability operator in (23) applies with respect to the marginal $P_{\boldsymbol{Z}|\boldsymbol{X}}$ of the joint distribution in (11). Note also that the random variable $B_n(\boldsymbol{Z})$ in (23) is the sum of the n binary random variables $\mathbb{1}_{\{Z_t=0\}}$, where Z_t follows the distribution $P_{Z|X=u_t(i)}$. That is,

$$\Pr\left[B_n(\boldsymbol{Z}) < a \,|\, \boldsymbol{X} = \boldsymbol{u}(i)\right] = \Pr\left[\sum_{t=1}^{n} \mathbb{1}_{\{Z_t=0\}} < \left(\frac{a - nb_1}{b_0 - b_1}\right)\right].$$

Hence, from the Berry-Essen theorem [18], it follows that for all $i \in \{1, 2, \ldots, M\}$:

$$\begin{aligned} \Pr\left[B_n(\boldsymbol{Z}) < a \,|\, \boldsymbol{X} = \boldsymbol{u}(i)\right] &\leqslant \frac{(1-\alpha_2)^2 + \alpha_2^2}{2\sqrt{n\alpha_2(1-\alpha_2)}}\\ &+Q\left(\frac{n\left((1-2\alpha_2)\bar{P}_X^{(i)}(0) + \alpha_2\right) - \frac{a - nb_1}{b_0 - b_1}}{\sqrt{n\alpha_2(1-\alpha_2)}}\right). \end{aligned} \tag{24}$$

Note that the right-hand side of (24) is monotonically increasing with a. Hence, the supremum on the right-hand side of (23) can be approximated by the value of $a > 0$ that satisfies

$$\begin{aligned} &\frac{n\left((1-2\alpha_2)\bar{P}_X^{(i)}(0) + \alpha_2\right) - \frac{a - nb_1}{b_0 - b_1}}{\sqrt{n\alpha_2(1-\alpha_2)}}\\ &= Q^{-1}\left(\delta - \frac{(1-\alpha_2)^2 + \alpha_2^2}{2\sqrt{n\alpha_2(1-\alpha_2)}}\right). \end{aligned} \tag{25}$$

The expression above yields (21) by solving for a. A similar procedure can be performed to prove (22). This completes the proof. ■

For fixed parameters $(n, \alpha_2, \delta, b_0, b_1)$, let $\rho^* : \mathbb{R}_+ \to \mathbb{R}_+$ be defined as

$$\rho^*(b) = \min\left(1, \rho^+(b)\right), \tag{26}$$

with

$$\rho^{+}(b) = \frac{\left(\frac{b-nb_1}{n(b_0-b_1)}-\alpha_2+\sqrt{\frac{\alpha_2(1-\alpha_2)}{n}}Q^{-1}(\delta)+\sqrt{\frac{\pi}{2}}\left(\frac{(1-\alpha_2)^2+\alpha_2^2}{n}\right)\right)^{+}}{1-2\alpha_2}.$$

Using this notation, the following corollary is an immediate result from Lemma 1.

Corollary 1: Consider two $(n, M, \epsilon, \delta, b)$-codes of the form (3) *for the random transformation in* (11)*, one with average energy shortage probability and another one with maximal energy shortage probability. Then, the empirical input distribution $\bar{P}_X$ of the former satisfies*

$$\bar{P}_X(0) \geqslant \rho^*(b), \tag{27}$$

whereas the empirical input distribution $\bar{P}_X^{(i)}$ of the latter with respect to codeword $\boldsymbol{u}(i)$ satisfies for all $i \in \{1, 2, \ldots, M\}$,

$$\bar{P}_X^{(i)}(0) \geqslant \rho^*(b), \tag{28}$$

where $\rho^*(b)$ is defined in (26)*.*

Corollary 1 leads to interesting conclusions by noticing that $\rho^*(b)$ is a lower bound on the fraction of zeros in each codeword (maximal energy shortage probability) or the fraction of zeros among all codewords (average energy shortage probability) when energy is transmitted at an average energy rate $\frac{b}{n}$.

First, note that the input distribution that achieves the largest information transmission rate is the uniform distribution [16]. That is, $\bar{P}_X(0) = 1 - \bar{P}_X(1) = \frac{1}{2}$. Hence, from Corollary 1, it follows that for fixed parameters $(n, \delta, \alpha_2, b_0, b_1)$, when $\rho^*(b) \leqslant \frac{1}{2}$, there always exists an $(n, M, \epsilon, \delta, b)$-code for which there is no penalty in terms of information rate due to energy transmission. The following corollary describes this observation.

Corollary 2: Given the random transformation in (11)*, fixed parameters $(n, \delta, \alpha_2, b_0, b_1)$ satisfying* (15)*, and*

$$b \leqslant n\left(\frac{b_0+b_1}{2}\right) - \sqrt{n\,(b_0-b_1)^2\,\alpha_2(1-\alpha_2)}Q^{-1}(\delta) - \sqrt{\frac{\pi}{2}}(b_0-b_1)\left((1-\alpha_2)^2+\alpha_2^2\right), \tag{29}$$

there always exists an $(n, M, \epsilon, \delta, b)$-code for which there is no trade-off between energy and information rates.

Essentially, Corollary 2 determines a threshold on the number of energy units b below which the energy transmission task is no longer conflicting with the information transmission task. Interestingly, this holds independently of whether the average or maximal energy shortage probability is considered. That is, under the conditions of Corollary 2, $(n, M, \epsilon, \delta, b)$-codes with average or maximal energy shortage probability can be designed by exclusively considering the information transmission performance.

Secondly, for fixed parameters $(n, \delta, \alpha_2, b_0, b_1)$, if $\rho^*(b) = 1$, no information transmission is feasible at strictly positive rate. This is basically because there is only one feasible codeword: an n-dimensional vector with zero entries. Moreover, when $\rho^*(b) > 1$, either information or energy transmission at a positive rate is impossible because even the codeword with all zero entries is unable to meet the number of energy units b with (maximal or average) energy shortage probability δ strictly smaller than one. The following corollary describes this observation.

Corollary 3: Given the random transformation in (11)*, fixed parameters $(n, \delta, \alpha_2, b_0, b_1)$ satisfying* (15)*, and*

$$b \geqslant n\big((1-\alpha_2)b_0+\alpha_2 b_1\big) - \sqrt{n\,(b_0-b_1)^2\,\alpha_2(1-\alpha_2)}Q^{-1}(\delta) - \sqrt{\frac{\pi}{2}}(b_0-b_1)\left((1-\alpha_2)^2+\alpha_2^2\right), \tag{30}$$

there does not exist an $(n, M, \epsilon, \delta, b)$-code with $M > 1$. If the inequality is strict, there does not exist an $(n, M\epsilon, \delta, b)$-code with $\delta < 1$.

Finally, for the parameters that do not satisfy the conditions in Corollary 2 or Corollary 3, both information and energy can be simultaneously and reliably transmitted with a particular trade-off. The following sections explore this conflicting interaction.

B. An Upper Bound on the Information Rate of (n, M)-Codes

The following proposition describes a bound on M the number of different codewords in a given system of the form in (3). This bound does not take into account the decoding error probability and thus, it might be loose in some cases. However, for parameters that do not satisfy the conditions in Corollary 2 and Corollary 3, this bound might be tight.

Proposition 1: Given the random transformation in (11)*, fixed parameters $(n, \delta, \alpha_2, b_0, b_1)$ satisfying* (15)*, and b not satisfying* (29) *and* (30)*, it holds that for any (n, M)-code with either reliability criterion pair* (a, d) *and* (b, d)*:*

$$M \leqslant \binom{n}{\lceil n\rho^*(b)\rceil} 2^{(n-\lceil n\rho^*(b)\rceil)}, \tag{31}$$

where $\rho^*(b)$ is defined by (26)*.*

Proof of Proposition 1: Corollary 1 provides an approximation to the minimum number of zeros in each codeword in any given $(n, M, \epsilon, \delta, b)$-code with average or maximal probability of energy shortage. That is, for all $i \in \{1, 2, \ldots, M\}$ if follows that:

$$N(0|\boldsymbol{u}(i)) \geqslant \lceil n\rho^*(b)\rceil. \tag{32}$$

This immediately provides the upper-bound on M the number of different codewords in (3), given that all codewords must contain at least $\lceil n\rho^*(b)\rceil$ zeros. Hence, the right-hand side of (31) is the maximum number of codewords of length n for which at most $n - \lceil n\rho^*(b)\rceil$ symbols can be ones. This completes the proof. ■

Note that when $\rho^*(b) \in (\frac{1}{2}, 1]$, the upper-bound (31) on M is monotonically decreasing with b. The following section provides the joint analysis of both information and energy transmission rates.

C. An Approximation to the Information-Energy Capacity Region

This section provides an approximation to the information-energy capacity region $\mathcal{C}(n, \epsilon, \delta)$ for fixed parameters (n, ϵ, δ). The approximation consists of the description of two regions, i.e., an achievable region $\underline{\mathcal{C}}(n, \epsilon, \delta)$ and a converse region $\overline{\mathcal{C}}(n, \epsilon, \delta)$, that satisfy the following inclusion:

$$\underline{\mathcal{C}}(n, \epsilon, \delta) \subseteq \mathcal{C}(n, \epsilon, \delta) \subseteq \overline{\mathcal{C}}(n, \epsilon, \delta). \tag{35}$$

Let the probability mass function of a multinomial distribution with k possible outcomes with probabilities $p_1, p_2, \ldots, p_k$ and n trials be denoted by $f_{k,n}(x_1, x_2, \ldots, x_k; p_1, p_2, \ldots, p_k)$. Let the function $\psi : \mathbb{N}^2 \times [0, 1] \rightarrow [0, 1]$ be defined as

$$\psi(n, \ell, \alpha) = \sum_{\ell_0=0}^{\ell-1} \binom{n}{\ell_0} \alpha^{n-\ell_0}(1-\alpha)^{\ell_0}. \tag{36}$$

$$\phi(n, M^*, \rho, \alpha_1) =\min\Bigg[1, \big(M^*-1\big) \sum_{\ell_0=0}^{n} \sum_{\ell_1=0}^{\ell_0} \sum_{\ell_2=0}^{n-\ell_0} \sum_{\ell_3=0}^{\ell_1+\ell_2} \sum_{\ell_4=0}^{\ell_0} \binom{\ell_0}{\ell_4}\binom{n-\ell_0}{\ell_3-\ell_4}\rho^{n-\ell_0-\ell_3+2\ell_4}(1-\rho)^{\ell_0+\ell_3-2\ell_4}$$

$$f_{4,n}\left(\ell_1, \ell_0-\ell_1, n-\ell_1-\ell_2-\ell_3, \ell_2; \rho(1-\alpha_1), (1-\rho)\alpha_1, \alpha_1\rho, (1-\alpha_1)(1-\rho)\right)\Bigg] \quad (33)$$

$$\chi(n, \rho, \alpha, B^*, b_0, b_1)=\min\left[1, \sum_{t=1}^{n}\binom{n}{t}\rho^t(1-\rho)^{n-t} Q\left(\frac{(1-2\alpha_2)t+n\alpha_2-\frac{B^*-nb_1}{b_0-b_1}}{\sqrt{n\alpha_2(1-\alpha_2)}}\right)+\frac{(1-\alpha_2)^2+\alpha_2^2}{2\sqrt{n\alpha_2(1-\alpha_2)}}\right] \quad (34)$$

Using this notation, the following theorems describe the sets $\underline{\mathcal{C}}(n,\epsilon,\delta)$ and $\overline{\mathcal{C}}(n,\epsilon,\delta)$.

Theorem 1 (A Converse Region): Given fixed parameters (n,ϵ,δ)*, the information-energy capacity region* $\mathcal{C}(n,\epsilon,\delta)$ *of the random transformation in* (11)*, with* b_0 *and* b_1 *satisfying* (15)*, is contained into the set*

$$\bar{\mathcal{C}}(n,\epsilon,\delta) = \left\{(M,b) \in \mathbb{N}\times\mathbb{R}_+ : M < M(b) \text{ and } b < B^+\right\}, \quad (37)$$

where

$$B^+=n\Big((1-\alpha_2)b_0+\alpha_2 b_1\Big) -\sqrt{n(b_0-b_1)^2\alpha_2(1-\alpha_2)}Q^{-1}(\delta) -\sqrt{\frac{\pi}{2}}(b_0-b_1)\Big((1-\alpha_2)^2+\alpha_2^2\Big). \quad (38)$$

and

$$M(b) = \min\left[\frac{1}{(1-\mu)\psi(n,L,\frac{1}{2})+\mu\psi(n,L+1,\frac{1}{2})}, \quad (39)\right.$$

$$\left.\binom{n}{\lceil n\rho^*(b)\rceil}2^{(n-\lceil n\rho^*(b)\rceil)}\right] \quad (40)$$

with $\mu\in[0,1]$ *and* $L\in\mathbb{N}$ *chosen to satisfy*

$$(1-\mu)\psi(n,L,\alpha_1)+\mu\psi(n,L+1,\alpha_1) = 1-\epsilon. \quad (41)$$

Theorem 2 (An Achievable Region): Given fixed parameters (n,ϵ,δ)*, the information-energy capacity region* $\mathcal{C}(n,\epsilon,\delta)$ *of the random transformation in* (11)*, with* b_0 *and* b_1 *satisfying* (15)*, contains the set*

$$\underline{\mathcal{C}}(n,\epsilon,\delta) = \left\{(M,b) \in \mathbb{N}\times\mathbb{R}_+ : M < M^* \text{ and } b < B^*\right\}, \quad (42)$$

where M^* *is the largest natural that satisfies*

$$\phi(n, M^*, \rho, \alpha_1) < \epsilon, \quad (43)$$

and B^* *is the largest real that satisfies*

$$\chi(n,\rho,\alpha_2,B^*,b_0,b_1) < \delta, \quad (44)$$

for some $\rho\in[0,1]$. *The functions* ϕ *and* χ *are defined in* (33) *and* (34)*, respectively.*

IV. Conclusions

In this paper, the fundamental limits of SEIT have been studied under the assumption that the transmission occurs during a finite number of channel uses at the expense of strictly positive DEP and ESP. From this perspective, a non-asymptotic fundamental limit has been introduced: the information-energy capacity region. That is, the largest set of jointly achievable energy and information rates. The focus is on the case of one transmitter, one IR and one EH communicating via binary symmetric memoryless channels. In this case, the information-energy capacity region is approximated and the information-energy transmission trade-off is thoroughly studied.

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