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OFDM/OQAM Transmission over Time-Frequency Dispersive Channels: Interference Computation and Approximation

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Abstract—Orthogonal frequency division multiplexing based on offset QAM (OFDM/OQAM) scheme, which is a particular case of filter-bank multicarrier (FBMC) transmission technique, bypasses a major disadvantage of ordinary OFDM with cyclic prefix (CP) systems, namely the fact that FBMC with well time and frequency localized filters are more robust against the interference problem over frequency or time selective channel. In spite of that, the elimination of the CP increases the spectral efficiency of the system. In this paper, we evaluate the OFDM/OQAM system over radio-mobile channel by calculating the whole system equation. Terms of inter-symbol interference and inter-carrier interference are included in the equation of the received symbols. To simplify the calculation, we propose some approximation with some conditions related to the filter we use, and to the channel response. Finally we compare simulation results using different type of simple multipath channels.

Index Terms—OFDM Cyclic prefix, OFDM/OQAM, multipath channel, equalizer

I. INTRODUCTION

The increase in transmission rate or spectral efficiency and minimization of energy, were always the major issues that lead to the invention of modulation methods in digital communication systems, starting from baseband transmission, to single carrier modulation, and then to multicarrier modulation. The multicarrier modulation appears as a solution for reducing the complexity of the equalization step compared with single-carrier systems in order to reduce the inter-symbol interference (ISI) on multipath channels (frequency-selective). The principle is to use multiple carrier frequencies for data transmission.

Orthogonal frequency division multiplexing (OFDM) with cyclic prefix (CP-OFDM) modulation scheme is the most used today. Its popularity comes from its simple implementation structure and from that the addition of CP [1] allows one tap per sub-carrier equalization.

On the other side, adding the cyclic prefix causes a loss of spectral efficiency, and the rectangular waveform causes poor frequency localization. The OFDM with Offset Quadrature Amplitude Modulation (OFDM/OQAM), which is a particular case of filter bank based multicarrier (FBMC) systems, and has the same spectral efficiency as OFDM without CP, could be used instead of OFDM [2], [3]. The use of non-rectangular pulse-shapes, well localized in time and frequency, makes OFDM/OQAM more robust to the interference problem over time and frequency selective channels.

Indeed, authors in [4] have already well studied the criteria of adaptation to the transmission channel in the case OFDM/QAM (without offset). Moreover, there are also long-standing recommendations on how to adapt filters in the OFDM/OQAM case [5]. The specificity of our article is that we demonstrate the conditions of a good adaptation in the case OFDM/OQAM.

In section II, we recall continuous and discrete-time OFDM/OQAM formulations. The whole system equation for an OFDM/OQAM signal over a radio channel is calculated in section III. We also derive in this section the conditions under which simplifications may be assumed. The results obtained are simulated in section IV. And a conclusion is taken in section V.

II. OFDM/OQAM SYSTEM MODEL

A. Continuous-time modulation

OFDM/OQAM scheme is often developed and considered for high-speed transmissions over wired and wireless time and frequency selective channels [6], [7] instead of OFDM family thanks to its higher spectral efficiency, and ability to have a transmission without guard interval (GI) or CP. The idea is to take the principle of OFDM/QAM, but instead of transmitting a complex symbol $c_{m,n} = c_{m,n}^R + jc_{m,n}^I$ each T_0 , we transmit with a time lag equal to $T_0/2$, the real part or the imaginary part of the symbol $c_{m,n}$, while requiring that two adjacent symbols in time and frequency have a phase difference equal to $\pi/2$ [8], [9].

It requires more that the number of carriers is even: $M = 2N$. Let $g(t)$ a function belong to $L_2(\mathbf{R})$ (space of square integrally signals), which represents a prototype filter, T_0 the symbol time, $\tau_0 = T_0/2$, and $F_0 = 1/T_0$ the carrier spacing. According to C. Siclet [10], a continuous-time baseband modulated signal type OFDM/OQAM is written :

$$\begin{aligned} s(t) &= \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2N-1} a_{m,n} g_{m,n}(t) \\ &= \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2N-1} a_{m,n} j^{m+n} g(t - n\tau_0) e^{j2\pi m F_0 t} \quad (1) \end{aligned}$$

For an ideal channel, the expression of the demodulation can be simplified by bringing the real scalar product of $L_2(\mathbf{R})$ with a receiving base

$$\check{g}_{m,n}(t) = \check{g}(t - n\tau_0)e^{j2\pi m F_0 t} j^{m+n}. \quad (2)$$

the received symbol is then:

$$\hat{a}_{m,n} = \langle \check{g}_{m,n}, s \rangle_{L_2(\mathbf{R}), \mathbf{R}} = \Re \left\{ \int_{-\infty}^{+\infty} \check{g}_{m,n}^*(t) s(t) dt \right\} \quad (3)$$

B. Discrete-time modulation

The signal $s(t)$ occupies a band $B = (2N - 1)F_0 + B_g$ we can approximate by $B \approx 2NF_0$ if N is high. it is then possible to sample the transmitted signal, without loss of information by choosing the critical sampling rate $F_s = B$. The sampling period is then $T_s = \frac{T_0}{2N}$ [11].

According Siclet [10] the transmitted discrete signal is of the form:

$$\begin{aligned} s[k] &= \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2N-1} a_{m,n} g_{m,n}[k] \\ &= \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2N-1} a_{m,n} g[k - nN] e^{j\frac{2\pi}{2N} m(k-D/2)} j^{m+n} \end{aligned} \quad (4)$$

where D is an integer value parameter introduced in order to take causality into account. And as a result we can get an estimate $\tilde{a}_{m,n}$ of symbols sent $a_{m,n}$ by:

$$\hat{a}_{m,n} = \langle \check{g}_{m,n}, s \rangle_{l_2(\mathbf{Z}), \mathbf{R}} = \Re \left\{ \sum_{k=-\infty}^{+\infty} \check{g}_{m,n}^*[k] s[k] \right\}, \quad (5)$$

where $l_2(\mathbf{Z})$ is the space of square summable sequences.

III. OFDM/OQAM ON RADIO CHANNEL

Removal of the cyclic prefix increases the spectral efficiency of the system. But despite the good time frequency localization of the prototype filters that maybe used in this modulation scheme, an end to inter-symbol and inter-carrier interference appears, and the more the channel is selective in time and/or frequency, the more this term becomes important.

A. Continuous-time analysis

Using the formula of the baseband continuous-time transmitted signal modulated of OFDM/OQAM type $s(t)$ given by equation (1), and the input output equation over radio channel transmission:

$$r(t) = \sum_{i=1}^I \alpha_i(t) s(t - \tau_i) + \eta(t), \quad (6)$$

we have :

$$r(t) = \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2N-1} a_{m,n} j^{m+n} \sum_{i=1}^I \alpha_i(t) e^{-j2\pi m F_0 \tau_i} \times g((t - \tau_i) - n\tau_0) e^{j2\pi m F_0 t} + \eta(t). \quad (7)$$

The expression of the estimated symbol is:

$$\begin{aligned} \tilde{a}_{p,q} &= \sum_{n=-\infty}^{+\infty} \sum_{m=0}^{2N-1} a_{m,n} \Re \left\{ \sum_{i=1}^I e^{-j2\pi m F_0 \tau_i} j^{(m+n)-(p+q)} \right. \\ &\times \left. \int_{-\infty}^{+\infty} \alpha_i(t) \check{g}^*(t - q\tau_0) g((t - \tau_i) - n\tau_0) e^{j2\pi(m-p)F_0 t} dt \right\} + B_{p,q}. \end{aligned} \quad (8)$$

where

$$B_{p,q} = \langle g_{p,q}, \eta \rangle_{L_2(\mathbf{R}), \mathbf{R}} \quad (9)$$

Note $I_{m,n,p,q}$ the interference between $a_{m,n}$ and $a_{p,q}$. Then :

$$\begin{aligned} I_{m,n,p,q} &= (-1)^{q(m-p)} \Re \left\{ \sum_{i=1}^I e^{-j2\pi m F_0 \tau_i} j^{(m+n)-(p+q)} \right. \\ &\times \left. \int_{-\infty}^{+\infty} \alpha_i(t + q\tau_0) \check{g}^*(t) g(t - (n-q)\tau_0 - \tau_i) e^{j2\pi(m-p)F_0 t} dt \right\}. \end{aligned} \quad (10)$$

If we assume that the time support $\text{Supp}\{\check{g}(t)\} = [-\frac{\Delta t}{2}, \frac{\Delta t}{2}]$, then :

$$\begin{aligned} I_{m,n,p,q} &= (-1)^{q(m-p)} \Re \left\{ \sum_{i=1}^I e^{-j2\pi m F_0 \tau_i} j^{(m+n)-(p+q)} \right. \\ &\times \left. \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \alpha_i(t + q\tau_0) \check{g}^*(t) g(t - (n-q)\tau_0 - \tau_i) e^{j2\pi(m-p)F_0 t} dt \right\}. \end{aligned} \quad (11)$$

Using approximations resulting from the mean value theorem:

$$\alpha_i(t + q\tau_0) = \alpha_i(q\tau_0) + \varepsilon_{i,q}(t) \quad (12)$$

and

$$g(t - n\tau_0 - \tau_i) = g(t - n\tau_0) + \varepsilon_n^{(g)}(\tau_i) \quad (13)$$

where

$$\begin{aligned} |\varepsilon_{i,q}(t)| &\leq \frac{\Delta t}{2} M_{i,q} \\ \text{with } M_{i,q} &= \max_{|t| < \frac{\Delta t}{2}} |\alpha_i'(t + q\tau_0)| \end{aligned} \quad (14)$$

and

$$\begin{aligned} |\varepsilon_n^{(g)}(\tau_i)| &< \tau_{\max} \times M_n^{(g)} \\ \text{with } M_n^{(g)} &= \max_{|t| < \frac{\Delta t}{2}, 0 \leq \tau \leq \tau_{\max}} |g'(t - n\tau_0 - \tau)| \end{aligned} \quad (15)$$

where $\alpha'(t)$ and $g'(t)$ are the time derivations of $\alpha(t)$ and $g(t)$ respectively. Then, the interference term (11) becomes:

$$\begin{aligned} I_{m,n,p,q} &= (-1)^{q(m-p)} \Re \left\{ \sum_{i=1}^I e^{-j2\pi m F_0 \tau_i} j^{(m-p)+(n-q)} \right. \\ &\times \left. (J_{i,m,n,p,q}^1 + J_{i,m,n,p,q}^2 + J_{i,m,n,p,q}^3 + J_{i,m,n,p,q}^4) \right\} \end{aligned} \quad (16)$$

with:

$$J_{i,m,n,p,q}^1 = \alpha_i(q\tau_0) \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \check{g}^*(t) g(t - (n-q)\tau_0) e^{j2\pi(m-p)F_0 t} dt, \quad (17)$$

$$J_{i,m,n,p,q}^2 = \alpha_i(q\tau_0) \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \check{g}^*(t) \varepsilon_{n-q}^{(g)}(\tau_i) e^{j2\pi(m-p)F_0 t} dt, \quad (18)$$

$$J_{i,m,n,p,q}^3 = \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \varepsilon_{i,q}(t) \check{g}^*(t) g(t - (n-q)\tau_0) e^{j2\pi(m-p)F_0 t} dt, \quad (19)$$

$$J_{i,m,n,p,q}^4 = \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} \varepsilon_{i,q}(t) \varepsilon_{n-q}^{(g)}(\tau_i) \check{g}^*(t) e^{j2\pi(m-p)F_0 t} dt. \quad (20)$$

Also, we have:

$$|J_{i,m,n,p,q}^2| \leq |\alpha_i(q\tau_0)| \tau_{\max} \times M_{n-q}^{(g)} \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} |\check{g}^*(t)| dt, \quad (21)$$

$$|J_{i,m,n,p,q}^3| \leq \left| \frac{\Delta t}{2} M_{i,q} \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} |\check{g}^*(t) g(t - (n-q)\tau_0)| dt, \quad (22)$$

$$|J_{i,m,n,p,q}^4| \leq |\alpha_i(q\tau_0)| \tau_{\max} \frac{\Delta t}{2} M_{i,q} \times M_{n-q}^{(g)} \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} |\check{g}^*(t)| dt. \quad (23)$$

It appears that J^2 , J^3 and J^4 may be neglected if $g(t)$ is time concentrated around 0 with small derivative. Thus, $g(t)$ should also be frequency concentrated around 0, since

$$g'(t) = j2\pi \int_{-\infty}^{+\infty} \nu G(\nu) e^{j2\pi\nu t} dt, \quad (24)$$

with $G(\nu)$ the Fourier transform of $g(t)$. Then, term interference term in (11) may be approached by:

$$\begin{aligned} \hat{I}_{m,n,p,q} &= (-1)^{q(m-p)} \Re \left\{ \sum_{i=1}^I \alpha_i(q\tau_0) e^{-j2\pi m F_0 \tau_i} \right. \\ &\times \left. j^{(m-p)+(n-q)} \int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} g^*(t) g(t - (n-q)\tau_0) e^{j2\pi(m-p)F_0 t} dt \right\}. \end{aligned} \quad (25)$$

B. Discrete time analysis

As we showed in the first section, to respect the Nyquist-Shannon theorem, sampling rate should be $B = 2NF_0$, then

$$\tau_0 = \frac{1}{2F_0} = \frac{N}{B} = NT_s. \quad (26)$$

And under this condition, we can show that for $\tau = lT_s$:

$$\int_{\mathbf{R}} g^*(t) g(t - \tau) e^{j2\pi f t} dt = \sum_{k=-\infty}^{+\infty} g^*[k] g[k-l] e^{j2\pi f k/B} \quad (27)$$

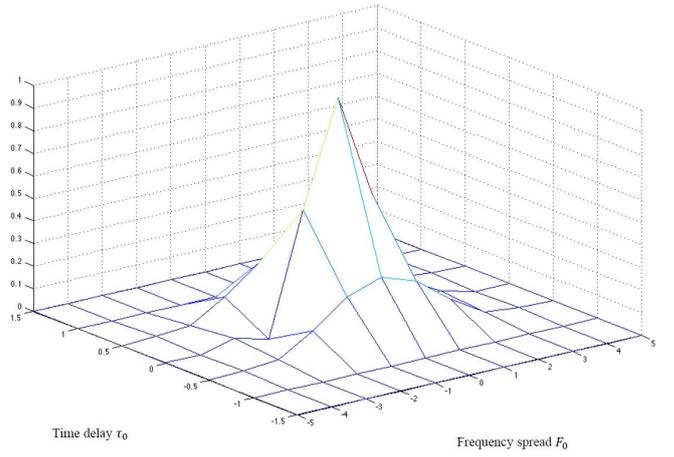


Figure 1. Standard of the auto-ambiguity of the prototype filter g for $M=1024$ and $L=1024$.

so

$$\begin{aligned} \int_{\mathbf{R}} g^*(t) g(t - (n-q)\tau_0) e^{j2\pi(m-p)F_0 t} dt &= \\ \sum_{k=-\infty}^{+\infty} g^*[k] g[k - (n-q)N] e^{j2\pi(m-p)F_0 k/B} &= \\ \sum_{k=-\infty}^{+\infty} g^*[k] g[k - (n-q)N] e^{j2\pi(m-p)F_0 k/(2NF_0)} &= \\ \sum_{k=-\infty}^{+\infty} g^*[k] g[k - (n-q)N] e^{j2\pi(m-p)k/(2N)}. \end{aligned} \quad (28)$$

Let L_g be the length of the prototype filter used, the interference approximation formula becomes:

$$\begin{aligned} \hat{I}_{m,n,p,q} &= (-1)^{q(m-p)} \Re \left\{ \sum_{i=1}^I \alpha_i(q\tau_0) e^{-j2\pi m F_0 \tau_i} \right. \\ &\times \left. j^{(m-p)+(n-q)} \sum_{k=0}^{L_g} g^*[k] g[k - (n-q)N] e^{j2\pi(m-p)k/(2N)} \right\} \\ &= (-1)^{q(m-p)} \Re \left\{ H(mF_0, q\tau_0) \right. \\ &\times \left. j^{(m-p)+(n-q)} A_g((m-p)F_0, (n-q)\tau_0) \right\}, \end{aligned} \quad (29)$$

where $A_g(f, \tau)$ is the auto-ambiguity of the filter used, and $H(f, \tau)$ the sub-channel response.

To simplify the equalization calculations, we will restrict to the highest interference term values. The response of the subchannel $H(mF_0, q\tau_0)$ depends on the selectivity of the channel, and can be maximized by $\sum_i \max_k |\alpha_i(k)|$. So it is enough to interest ourself the auto-ambiguity function amplitude. Our aim is obviously to use a filter with the highest time-frequency localization as possible. That is why we have used the prototype filter given in [12]. It leads to the auto-ambiguity function depicted in Figure 1. This leads us to take

into consideration the interference terms for $-1 \leq m-p \leq 1$ and $-1 \leq n-q \leq 1$. Let:

$$J_{a,b} = \hat{I}_{p+a,q+b,p,q} \\ = (-1)^{qa} \Re \left\{ H(mF_0, q\tau_0) j^{a+b} A_g(aF_0, b\tau_0) \right\} \quad (30)$$

then

$$\hat{a}_{p,q} \approx \sum_{a=-1}^{+1} \sum_{b=-1}^1 J_{a,b} a_{p+a,q+b}. \quad (31)$$

IV. SIMULATIONS

Let $a_{m,n}$ be the transmitted symbol, $\tilde{a}_{m,n}$ the received symbol obtained by the simulation of the system on a multipath channel, and $\hat{a}_{m,n}$ obtained by the simulation of the approximated interference equation.

A. Multipath channel models

Let $h(\tau)$ be a multipath channel, I is the final path considered, so:

$$h(\tau) = \sum_{i=1}^I \alpha_i \delta(t - \tau_i) \quad (32)$$

In our simulation we used two models of the multiple channels, where α_i follows different functions. In addition, we assume that τ_i is a multiple of the sampling period T_s , and let τ_{\max} the delay of last path included (I). Then:

$$\tau_i = i \times T_e$$

$$\text{and } \tau_{\max} = I \times T_e.$$

The two multipath channel models are :

- with exponential decreasing gain :

$$\alpha_i = A \times e^{-5i/I} \text{ for } 0 \leq i \leq I, \text{ and } 0 \text{ otherwise;}$$

- with linear gain :

$$\alpha_i = A \times \left(1 - \frac{i}{I+1}\right) \text{ for } 0 \leq i \leq I, \text{ and } 0 \text{ otherwise;}$$

with A such that $\sum_{i=0}^I \alpha_i^2 = 1$.

B. Simulation results

To find out if the approximations used are acceptable or not, we are interested in the simulation of the transmission chain of an OFDM/OQAM signal on a multipath channel to obtain the actual received symbols $\tilde{a}_{m,n}$, and simulation of the equivalence of the chain using the interference formula for the received symbols of the estimator $\theta(\tilde{a}_{m,n}) = \hat{a}_{m,n}$. Let $\bar{a}_{m,n}$ be the equalized symbol resulting from $\tilde{a}_{m,n}$ using the approximate interference equation, the estimator is chosen such that:

$$\text{if } \tilde{a}_{m,n} = \hat{a}_{m,n}$$

$$\text{then } \bar{a}_{m,n} = a_{m,n}$$

The simulation studies two characteristics:

- the bias of the estimator:

$$\text{Bias}(\theta) = E\{\tilde{a} - \hat{a}\}. \quad (33)$$

- the variance of the estimator:

$$\text{Var}(\theta) = \frac{E\{|\tilde{a} - \hat{a}|^2\}}{E\{|\tilde{a}|^2\}} \quad (34)$$

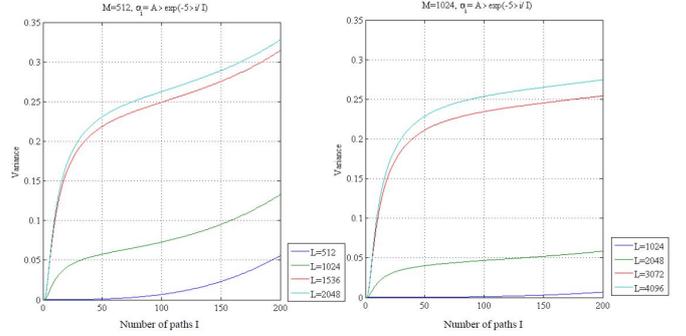


Figure 2. Variance of the estimator over a channel with exponential decreasing gain.

Simulation results show that the bias does not exceed 0.015 for a standardized signal amplitude, ie not more than 1.5 %. It then can be assumed that this estimator is unbiased. On the other hand, the simulation of the variance $\text{Var}(\theta)$, shows that the equalization may be used on a frequency selective channel (multipath) for a prototype filter of length equal to the number subcarriers ($L = M$) and under certain conditions on the channel response.

For the exponential decreasing gain channel, the equalizer can be adopted for a number of paths $I \leq 50$ if the number of carriers is $M = 512$, and for $I \leq 100$ if the number of carriers is $M = 1024$. On the other hand, for the linear decreasing gain channel, the use of the proposed equalizer is acceptable for $I \leq 30$ if we use $M = 512$ subcarriers, and for $I \leq 60$ if using $M = 1024$ subcarriers.

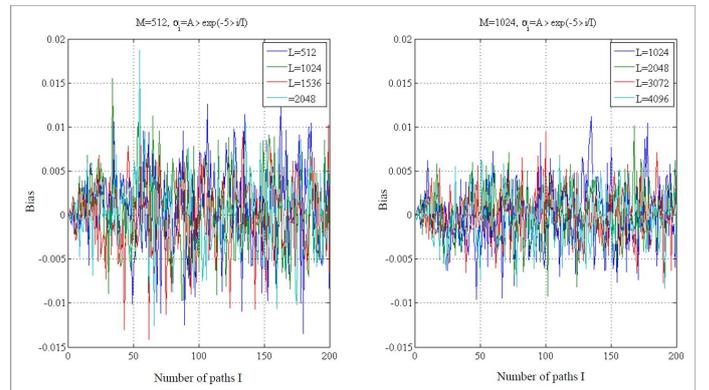


Figure 3. Bias of the estimator over a channel with exponential decreasing gain.

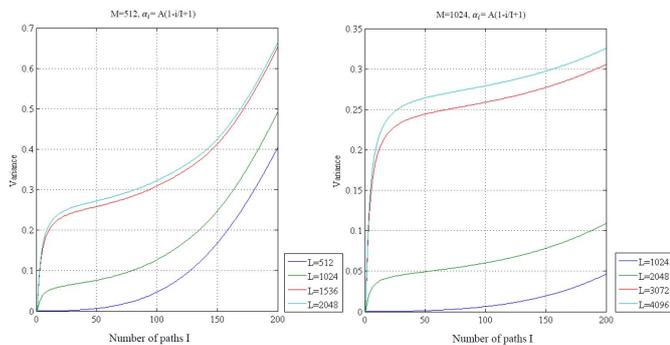


Figure 4. Variance of the estimator over a channel with linear decreasing gain.

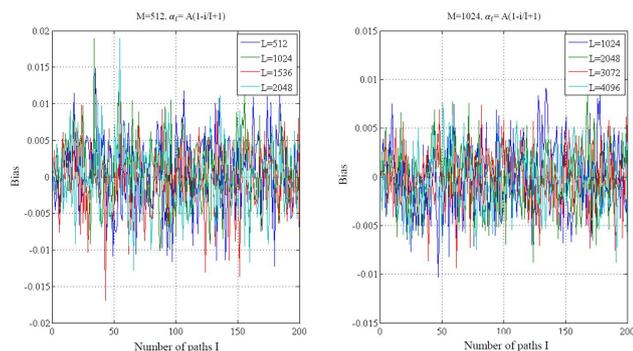


Figure 5. Bias of the estimator over a channel with linear decreasing gain.

V. CONCLUSION

In this paper, we studied the behavior of an OFDM/OQAM signal over a time and frequency selective channel. Terms of inter-symbols interference and inter-carrier interference (ISI and ICI) are added in received signal, depending on the parameters of the transmission channel, and on the auto-ambiguity of the prototype filter used. In order to have a simple equalizer, we carried approximations of the interference function to simplify the calculation. These approximations are valid if one chooses a filter matched to the transmission channel. The simulation of the transmission channel is compared with the results of the input-output equation with the function of interference approximated using prototype filters of different lengths, and for a different number of carriers. Results of the simulation model for multipath channels, show that the use of the proposed approximation, and hence the equalizer is limited by the selectivity of the channel (frequency), and using the best filters for the case studied.

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