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Elements of Theory for Electromagnetic Compatibility and Systems

Olivier Maurice

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Elements of Theory for Electromagnetic Compatibility and Systems

Olivier MAURICE

August 7, 2019

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To my sister Edith, to Robert Vallée and Jean-Claude Boudet,
not,
to my wife, my son, all my family,
and to all my friends & scientists interested by my researches.

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Chapter 1

Preamble

1.1 Foreword by Alain Reineix

The last years, the development of electronic systems has strongly increased. Then in the conception phases of systems it becomes difficult for the engineers to predict the best configurations that make the different functionalities in the best way. That's why if no tools can help the conceiver, no innovation will exist and all that can be done is just to make little modifications in an existing system. If we draw a state of art of the different approaches compatible with the conception of a system, first we can point out the numerical modeling tools. Even if the capabilities of modeling have strongly increased these last years, the complexity of systems has increased more rapidly than the

power of these tools. In this case, we can ask what is the best approach for the conception of systems? In fact, the experience of the engineer is very important because, in the most cases, his past experience will guide him to make reasonable choices in the future. That has been said is the importance of the experience but as I said above, the ideal solution is to have a tool capable to transcribe his experience so that he can make virtual experiment and immediately see the effect if he makes one parameter varying (or more than one). It will replace the famous rule of thumb and will give numerical values giving the effect of one parameter on an observable. It's in this context that Olivier Maurice adapted the Kron's method in the EMC domain. This approach will answer to the problem given above because we can sum up the approach by the necessity of the realization of a transcription of the problem into an equivalent circuit. But what is important in this transcription is that the more influential physical phenomena must be included in this model. For this, the experience of the engineer is very important; he has to observe the system under study and transcribe his vision into a model including the physical phenomena he thinks that will occur. At this level, we can conclude that there is not only one possible model, but if all the models take into account the physics the conclusions of different specialists will be the same. Said as it, the approach seems miraculous, but it is important to have a background on physics but also on mathematical objects that will be used by the engineer. This book is entirely dedicated to the different concepts that are included in the method proposed by Gabriel Kron and extended by Olivier Maurice. Each chapter will introduce mathematical concepts that will help the user

to have a really good master's of the approach. A progressive approach in the first chapters will help the reader to familiarize with the notations necessary for tensorial analysis and with the different steps necessary to the conception of a circuit. It appears that tensorial notations becomes an universal way to introduce different models from the physics and therefore will be the best to for multiphysics. As it is well known, the physical phenomena are the same in different referential, so the tensorial approach associated with the Einstein's notations, is really the best way to have to represent physical phenomena and the representation of physical quantities as flux in the primal space and efforts in the dual one make the approach suitable for mixing different physical phenomena as it is already made in in mecatronic. Du to the increasing complexity of system as already said above, it becomes necessary to mix different functionality but also different physics (mechanic, electromagnetic, ...) so the solution given in this book is really an interesting solution for future studies and in the conception of future systems. One of the most important point, among the different ones, recalled by the author and clearly exposed is the diakoptic approach. Its one of the most famous concept introduced by Gabriel Kron, with Diakoptics, he has found an original technic to decompose a complex system into substem and to give a tensor characteristic of each subsystem independently of the others which is a cornerstone of each complex system under study. Such a decomposition is essential to see a complex system as the connection between different sub-system, each one being dedicated to a particular functionality. In the decomposition, one other mathematical domain that is very important is the topology, the

book describes the way to translate this mathematical concept in the EMC domain, the manifolds and the second geometrization concept will give the mathematical representation of a local study and its extension to a large domain mainly for taking non linear phenomena into account. As made by Einstein for the geometrical representation of the gravity, this approach will give a graphical representation of the space curvature in the considered space of calculation. Such a concept will drive in the conception of system, and delimitate the regions of best behavior of the system and its sensitivity to a perturbation. The book is mainly dedicated to the EMC domain but the cyber physics is also addressed, this illustrates the universality of the proposed approach. So I hope dear reader that, when you will read the first page, you will dive into an interesting and exciting world. Good reading !!

1.2 Author's introduction

Dear reader,

the purpose of this book is to present various personal thinking about systems. Having worked in electromagnetic compatibility during more than thirty years, this has given me opportunities to search for efficient models for complex systems modeling.

I could have think in a particular summary and in a construct plan for this book. Rather than working classically, I prefer to give ideas and to speak of various subject in an order associates with some conductive wire. I think that every

scientist must construct his own ideas on physics. In parallel to classical courses, he can find in this kind of approaches other inspirations. As innovation comes from multidisciplinary works, it is indispensable to follow other tracks. I submit here one of these tracks, at least I hope to.

Coming from my own experience, my approach of the systems is principally based on electromagnetic compatibility. But this job includes electromagnetism as the mechanics, chemistry, biology and the human factor.

I invite the reader to take some altitude in order to embrace the whole spirit of Kron's formalism. I call for tolerance of readers because I often forget some rigor for preferring speaking of an approach, of wide principles. The book must be treated by this way and no doubts that if some ideas I present may have some interest, they will be developed to gain in maturity. There may be some errors in this book, but I hope it is full of ideas.

When I have started to work on Kron's method in 1988, it was to be able to compute experiments in electromagnetic compatibility. I have discovered at this moment that Kron's formalism is unique to study theoretically a problem. All other methods: bond graphs, nodal methods, MNA, S parameters etc. dont give the same capacity to model . Clearly, complex systems involving relativistic movements can be modeled by the simplest way using Kron's formalism.

Unfortunately, my practice of the beautiful English language is weak. But I hope to be easy to read and understand, it will already be a great reward for me

Hope you will enjoy this discussion, *Olivier Maurice*

PS : in all the book¹ we use p for Laplace's operator.

¹The softwares used to make the draws were Qucs, Open-Office Draw and Open-Sankore. Some graphs was made using Yed. Computations were made using Python2.7. The book was written with L^AT_EX using TexMaker and TeXShop.

Chapter 2

On Systems

2.1 What is a system?

Somewhere we can find as many definition of what is a system than the system the more complex can have behaviors! In that case, we must remain as simple as possible. Starting from the Greek declinaison, we can say that a system is firstly a set. Nothing less, but nothing more. After this we can add adjectives to increase this simple property.

By studying systems we belong to the systemic community who studies the systems. We are systemicicians.

Some other definitions are very important. A collection is a group of objects. So, what can be the singularity of a system? Is a system a simple collection of objects?

We can consider that a system is a set in the mathematical sense of the word. It means that with the collection of objects, some relations between these objects are associated. It exists a structure in mathematics that seems to correspond to this kind of definition: the manifold. May we consider that a system is a manifold? In any ways we can try.

2.1.1 What is a manifold?

A mathematical manifold is the generalization of the concept of geometric forms. A curve is a manifold of dimension 1, a surface of dimension 2, etc.

There are two ways to abord the notion of manifold. A first method consists in connecting various parts step by step until to reach the whole system construction. A second method consists in defining a metric which defines the neigh-borough and by this technique, the proximity of the various parts of the system.

We will see that a simple technique to visualize this construction is to use graphs. Each branch of a graph can be a representation of a part of the system. Each of these branches is associated with the equation of a function. The construction of the whole graph corresponds to the one of the system and leads to a system of equations which is the mathematical representation of the physical system.

This system of equation, added of inequalities that define the domains of its parameters is a manifold following Poincare's definition. Once this manifold is created, the physical system can be forgotten and its study is replaced by the analyze of the manifold.

2.1.2 System of systems?

The system of systems is a quite recent notion created to describe sets of systems with interactions between them. It cannot be this time a simple set of systems because in that case, the system of systems is a system using the property of union of sets. If we want now to use the manifold definition for our systems, a system of system is not a so simple thing to define. Fortunately, it exists a quite simple way to abord these concepts: graphs. Through graphs we are able to define more precisely what is a system and after, a system of systems. But basis graphs have to be completed by a fundamental technique: cords^{1 2}.

Then, having all the material to represent systems or parts of systems, we will be able to model any kind of system with one more hypothesis: that $(\mathbb{M}, +)$ is an abelian group, \mathbb{M} being the set of manifolds.

2.1.3 Systems seen as graphs

Graphs are made of nodes, branches, meshes, faces, networks and for us, of cords. We can use them to represent objects and the interactions between objects. That's why they are so well adapted for the systems representation.

¹Maurice, O., Reineix, A., Durand, P., & Dubois, F. (2012, July). On mathematical definition of chords between networks. In European electro-magnetics, EuroEM 2012

²Olivier Maurice, Philippe Durand. Modélisation des systèmes complexes. : Présentation des grands principes de la méthode xTAN. Présentation effectuée lors de la réunion du groupe CESIR de l'AFSCET à l'ESIGELEC, le 19 décembre. 2015. hal-01246740.

Before to begin studying graphs, we can now give a new definition of what is a system by using words that will becomes clearer after: a system is represented by a graph. Each of its elements or objects are represented by a branch in the graph.

A system of systems is a graph made of more than one network. In other words, a sub-system is a network in a graph.

The branch is the fundamental part of graphs. To each branch will correspond an equation, and the networks allow to construct systems of equations which represent the systems. We will see that these systems of equations can be seen as manifolds. Studying systems consists after that in studying manifolds. It is a geometrical approach of systems using Kron's formalism and some others novelties.

2.1.4 To conclude this chapter

A real, cyber-physical system is represented by a graph and is a mathematical manifold. A system of systems is a manifold made of various subparts represented by a graph constructed from a set of graphs. Cords add relations between these graphs.

Chapter 3

Graphs

Graphs are used for real objects as for abstract things. They are a precious help to understand topological relations as systems behaviors and particularities.

Fundamental element of graphs is the branch. Nodes are more difficult to understand, contrarily to what we could think. Defining a branch is relatively easy. A first property very important is that the object represented by a branch must have two borders, like the branch. Nodes are the border of a branch. Each branch has two nodes. The connection to the object can be made through two access which are the two borders in relation with the two nodes. These access are used by the major observables in relation with the major use of the object. For example in electricity, a resistance has two access. This doesn't mean that other ports doesn't exist. In particular, ambient tem-

perature can affect the resistance value. But this temperature is not connected to the resistance like a generator. It comes from the environment of the resistance and acts on its junction. To transmitt the temperature to the object (here an electrical component) we need another sort of link. That's one reason for creating cords.

Using an object in a system implies to know how it works in various conditions. The operation of characterization is a major step in the system modeling. For modeling a system, we need to know how work its parts. This characterization uses the inputs available on the object to apply various sollicitations in order to measure its responses. This operation cannot be applied directly on a node that has only one input. That's why real objects are represented at least by a branch.

3.1 Branches

A branch has two borders. The concept of border is not so simple to understand. What is more simple to understand, is the dual of the border operator. If we consider a state variable q pointing out some physical entity that exists all along a way where it moves. Integrating this entity on the way and dividing by the time taken to make this trip gives the flux f that belong to the branch. We can write:

$$f = \frac{1}{\Delta t} \int_t^{t+\Delta t} dq = \frac{q(t + \Delta t) - q(t)}{\Delta t} \quad (3.1)$$

The elementary entity, the state variable q , belongs to the

zero dimension space of the components of graphs: i.e. nodes. The flux, coming from the integration of the state variable along the way associated with the branch, belongs to the one dimension space of the components of graphs: the branches.

But to make the entity q moving, we need to apply energy. That's the fundamental process of characterization. Basic principle of this process is to apply a stimulus to a structure and to measure the flux created by this stimulus in the same structure.

We can associate with any branch some properties and the corresponding equation. Any branch can receive an electromotive or mechanical force. If such a source is applied to the branch when it is open, no flux can flow on it. Because in that case, the branch has two borders, so any flux is stopped on these limits. If we close the branch, i.e. if we connect the two nodes together, the borders disappear and the flux can flow all along the branch, even browse it more than one time. The question now is to know how we can close the branch. Following Maxwell's equations, a closed circulation of field is something we can define. A closed circulation of magnetic field being linked with a central current that doesn't exist here, we can only refer to a closed circulation of electric field.

If the branch is characterized by an operator acting on the flux, the circulation of the electric field becomes (see figure 3.1):

$$e = \oint_C d\vec{C} \cdot \vec{E} = \mathcal{H}(i) + U \quad (3.2)$$

On the figure, the presence of the operator \mathcal{H} is symbolized by a rectangle drawn on the figure.

The equation $e = \mathcal{H}(i) + U$ where i is the flux and U the potential difference across the branch is called Kirchoff's branch.

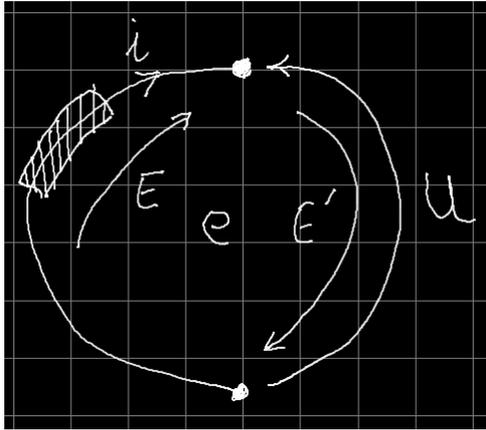


Figure 3.1: Characterization of a branch

We can make a first very simple and fundamental experience on the previous branch. If we connect both nodes together, we suppress the potential difference. How may we do that ?

The incidence matrix gives the relation between the potentials at the nodes ψ and the potential differences U . In the circuit figure 1, we note now U_1 the potential difference across a first branch 1 and U_2 across a second branch 2, ψ_1 the potential on the node 1 and ψ_2 on the node 2. The incidence matrix A_m^k defined by $U_m = A_m^k \psi_k$ is here given by:

$$A_m^k = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad (3.3)$$

We can connect the two nodes 1 and 2, writing in a transformation that they are both equal to a single node 1. This means that the two nodes are made in correspondence with the node 1 through the transformation matrix Λ :

$$\Lambda = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (3.4)$$

The matrix Λ says that first node (first line) is equal to next node 1 and says that second node (second line) is also equal to next node 1. We can apply the transformation Λ to the incidence A to make what we want. You can verify that $\Lambda^T A \Lambda = 0$. This means that in the new space where nodes 1 and 2 are connected, there is no more nodes and the incidence is equal to zero. The potential is a property of the border. Making the association between the state variable dq and the potential $d\psi$ (see equation 3.1), we can write:

$$U = \oint_1^1 d\psi = 0 \quad (3.5)$$

while:

$$U = \int_1^2 d\psi = \psi_2 - \psi_1 \quad (3.6)$$

Borders are the limits of the integral. When we close a branch on itself to create a mesh we suppress the borders. This leads to suppress the potential differences.

If we look at ∂U it gives the border of the branch $\partial\psi$. But when a branch is open, no flux can go across it. In real life, there is only one situation where this can appear: in static states, electrostatic for example. In all the other cases, kinetic energy exists and flux exists. It means that only closed circuit can exist. From a set of branches we can construct meshes. The space of meshes will constitute the major space that we will now studied.

3.2 Meshes

Mesh fundamental relation says that the sum of the forces on a closed circulation is zero (Newton's relation). This can be retrieved starting like for mechanics from the energy expression for electrical circuits. For magnetic energy it is given by:

$$E = \frac{1}{2}Li^2$$

by making $d_t\partial_i E$ we obtain:

$$\frac{d}{dt} \frac{\partial E}{\partial i} = \frac{d}{dt} (Li) = U_L \quad (3.7)$$

The potential U_L can be assimilated with a force by the product with the load divided by the distance:

$$F_L = q \frac{U_L}{l}$$

For the potential energy $E' = \frac{1}{2}CV^2$ we make:

$$U_C = \frac{\partial E'}{\partial q} = \frac{q}{C} \quad (3.8)$$

As before:

$$F_C = q \frac{U_C}{l}$$

It remains the last kind of force: the force coming from the dissipation, F_R . This force comes from the dissipation power:

$$W = \frac{1}{2}Ri^2 \Rightarrow \frac{\partial W}{\partial i} = Ri = U_R \quad (3.9)$$

and

$$F_R = q \frac{U_R}{l}$$

Through these various relations and Newton's one we see that:

$$\frac{q}{l} \left\{ \frac{d}{dt} \frac{\partial E}{\partial i} + \frac{\partial E'}{\partial q} + \frac{\partial W}{\partial i} \right\} = 0 \quad (3.10)$$

We retrieve Lagrange's equations. The mesh space is the fundamental space which is established by Lagrange's equations. We may wonder how these equations can be obtained starting from a collection of Kirchhoff's branches? First must be understood that extrinsic sources e comes as right member in the Lagrange's equations:

$$\frac{d}{dt} \frac{\partial E}{\partial i} + \frac{\partial E'}{\partial q} + \frac{\partial W}{\partial i} = e \quad (3.11)$$

After what, each capacitive or dissipative term of this equation can be associated to a partial development of the electric field that obeys to equation 3.2.

We can imagine two situations. A first situation where branches are organized in series. A second situation where the branches are organized in a closed mesh. We suppose to have a collection of N branches. These N branches are characterized by N functions $\mathcal{H}_k + U_k$, $k \in [1, \dots, N]$. To explain the processes, we will consider two branches only. It is easy to extend the mechanism to any number of branches.

3.2.1 Organization in serie

We have two branches, one with the operator \mathcal{H}_1 and one with the operator \mathcal{H}_2 . With these two separate branches we want to construct one branch made of the two previous branches in series. To do that, we create a transformation F that make corresponding the two branches 1 and 2 to a unique one 1'. The matrix of the transformation F is given by (having the original branches as rows and the new branch as column):

$$F = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (3.12)$$

By another side, the collection of the two separate original branches can be synthesized in a single impedance matrix Z , purely diagonal, where the branches are the diagonal components. By making the bilinear product $Z' = F^T Z F$ we transform Z in a matrix Z' where the two branches are gathered. We

compute:

$$Z' = [\mathcal{H}_1 + \mathcal{H}_2] \quad (3.13)$$

and the Kirchhoff's branch 1' has for equation:

$$e_1 + e_2 = U_1 + U_2 + Z' (i) \quad (3.14)$$

The potentials are obtained by UF and the sources by eF .

Note that the new branch has only two borders. We will show why but it is not so easy. First we define the incidence matrix which gives the relations between the nodes and the branches, before the reduction to a single branch:

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad (3.15)$$

Nodes are in columns and branches in rows.

We benefit of this description to explain the mechanism of the mute index. Rather than writing for a sum on an index between a matrix and a vector like:

$$v_x = \sum_y M_{xy} u_y$$

we can use both location up and down to define indices. The index up is associated with rows and the index down with columns. Previous equation can be written:

$$v_x = \sum_y M_x^y u_y$$

But as we can see in this example, the index which is repeated is the index with which we make the sum. Finally, it is not important to detail the sum symbol once we have understood that the repeated index is the index on which the sum is made. Previous equation can be resumed by:

$$v_x = M_x^y u_y$$

For the moment we can stop here this first introduction to tensorial algebra. We will see other properties of this algebra further.

The gathering of the two branches can be translated applying a transformation N to the set of the nodes. This transformation replace the four nodes of the two original branches by the two nodes of the final single branch. Old nodes n are in rows and new nodes n' in columns:

$$N_{n'}^n = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (3.16)$$

$N_{n'}^n$ is the transposed matrix of $N_n^{n'}$. The incidence matrix A_n^b links old nodes with old branches. The matrix $F_{b'}^b$ links old branches with new ones. The matrix product

$$N_n^{n'} A_b^n F_{b'}^b = \mathcal{A}_{b'}^{n'} \quad (3.17)$$

creates an incidence matrix which gives the connection between the new nodes and the new branches.

We see on this simple example how the mute index convention is powerful and helps to find the result of various successive transformations. We can follow the various maps implied in these processes:

$$\begin{aligned}
 N_n^{n'} &: n' \rightarrow n \\
 A_b^n &: n \rightarrow b \\
 F_{b'}^b &: b \rightarrow b' \\
 A' &= N \circ A \circ F : n' \rightarrow b'
 \end{aligned}
 \tag{3.18}$$

The indices send back to corresponding spaces. Tensorial algebra is powerful because it says how datas are managed and it allows to write simply complex equations.

3.2.2 Nodal space

Studying networks through nodes and branches means to work with nodal methods. These method are the most usuals. They include nodal analysis for circuits, nodal finite elements, etc. We are going to demonstrate how to establish the nodal analysis equations, using tensorial algebra.

We start from the Millman's theorem:

$$U = \frac{ey + i}{y} \tag{3.19}$$

By generalizing, this is equivalent to:

$$y^{kn} U_n = e_n y^{mk} + i^k \tag{3.20}$$

The potential differences can be expressed in term of the potentials at the nodes:

$$A_n^q \psi_q = U_n \quad (3.21)$$

We can replace this relation in the equation 3.20:

$$y^{kn} A_n^q \psi_q = e_m y^{mk} + i^k \quad (3.22)$$

now we can multiply each member by A_k^v :

$$A_k^v y^{kn} A_n^q \psi_q = A_k^v e_m y^{mk} + A_k^v i^k \quad (3.23)$$

We call $Y^{vq} = A_k^v y^{kn} A_n^q$ the admittance in the nodes space, $s^v = A_k^v i^k$ the current in the nodes space and $J^v = A_k^v e_m y^{mk}$ the electrical sources of nodes. Finally:

$$Y^{vq} \psi_q = J^v + s^v \quad (3.24)$$

We compute the potential at the nodes ψ_q after the inversion of Y .

Once the potentials at the nodes are known, we can obtain the potentials differences, then the currents, etc. All these last quantities are in the branches space.

3.2.3 The tensorial concept

There are many ways to present tensors. From my opinion I think that there are two fundamental concepts leading to tensors and perhaps being physically their principal reason to be:

1. concept of co-space;

2. concept of invariant.

I'm not sure that my proposals may be completely approved by mathematicians. But I'm sure that they are in practice the expression of the powerful side of tensors.

Co-space

In electricity as in many physics, movement is the natural way of life. As we discuss before, static situations are very abstracts and rarely really applicable in life. Energy needs to change always from one aspect to another. Perhaps this is in relation with the whole movement of the universe. As the universe moves, space-time moves and all our life moves.

So, a first consequence of these movements is that majority of things we can observe have one speed and one direction of movement. As a consequence, it is natural to associate them with vectors. We can recall here some fundamental properties of vectors.

If we consider a set Q of identified points $\{a, b, c, \dots\}$. A vector \mathbf{v} can be seen in a first step as an ordered couple of points. For example (a, d) . Best, the couple of values can be organized in a column matrix:

$$\mathbf{v} = \begin{bmatrix} a \\ d \end{bmatrix} \quad (3.25)$$

The vector \mathbf{v} belongs to the set $Q \times Q$, here in a space of dimension 2. The set of vectors associated with the operation of sum "+" is an abelian group.

The orientation of the vector is described by the order of the numbers in the matrix. By the fact the vector:

$$-\mathbf{v} = \begin{bmatrix} -a \\ -d \end{bmatrix} \quad (3.26)$$

is opposite to the vector \mathbf{v} . We understand that this process can be applied to many sets of the life. Each time we link two quantities in a given order, we create a set of vectors. Two vectors being equal if they have same amplitude and direction. In the life, each time we mind on movement, during travels, when we walk and use the "GPS", etc., we refer to vectors. When we write on a sheet, following specific lines and with respect to rules: we write from left to right or up to down, etc., we refer to vectors. The question may rather be: is there some actions we do that cannot be made in reference with vectors? Vectors are the natural life. For this reason, we call them, once we have chosen their definition in a special case, the "natural space". This space can be of very high dimension. For example if we consider colors of the rainbow. We may create a space where each fundamental color is a direction of this space. In this case we may have seven directions (Isaac Newton has separated the rainbow in seven fundamental colors).

Now, we can create vectors in this colors space by affecting a factor to each color, defining by this action the vector associate with a particular composite color. So, in the space of colors red, orange, yellow, green, blue, indigo and purple, we affect real factors of intensities $(\alpha_r, \alpha_o, \alpha_j, \alpha_v, \alpha_b, \alpha_i, \alpha_t)$ and any vector \mathbf{v}

is declined under the form of the column matrix:

$$\mathbf{v} = \begin{bmatrix} \alpha_r \\ \alpha_o \\ \alpha_j \\ \alpha_v \\ \alpha_b \\ \alpha_i \\ \alpha_t \end{bmatrix} \quad (3.27)$$

We can add vectors, we can multiply them by a number (a real number or a complex one), and as they constitute a group, it exists a null vector, a symmetric for the addition, etc. But if we want to transmit these kind of vectors to a friend by phone, it is difficult. We cannot describe its direction because this direction can be very difficult to explain. If we want to describe the distance there is between two vectors, it is very difficult also. This is all the more difficult that our vectors may be complex, for example if we associate with the colors a phase to say when these colors appear. In this case, the components of the vectors would be complex numbers: $\alpha_x e^{-\beta p}$ (p is the Laplace's operator). What would be very easy is to be able to describe the distance between two vectors using a simple number.

Metric

Once we can define distance, we can locate the position of any points. That's the marvelous property of neighborhood. Neighborhood is the simplest way to define a topology, to describe the space.

By definition, the distance s between two points identified by the two vectors v^x and u^y is given by:

$$s^2 = G_{xy}v^x u^y \quad (3.28)$$

This notion is difficult to understand. Often it is presented taking as an example the cartesian space case and its particular metric. But it masks the more global properties of this notion.

To illustrate the distance we will take as an example a circuit. We consider two circuits purely resistive of values R_{11} and R_{22} . The system of equations that pilots these circuits through two sources e_1 and e_2 is (we will see further how to establish these equations):

$$\begin{cases} e_1 = R_{11}i^1 \\ e_2 = R_{22}i^2 \end{cases} \quad (3.29)$$

The equation $e_x = f(i^y)$ can be seen as a parameterized function of two variables. The directions \mathbf{b}_k are given by the flux derivatives:

$$\begin{cases} \mathbf{b}_1 = \partial_{i^1} e_x(i^y) = [R_{11}, 0] \\ \mathbf{b}_2 = \partial_{i^2} e_x(i^y) = [0, R_{22}] \end{cases} \quad (3.30)$$

Note that the result of the derivatives creates a couple of values in the space of dimension 2. This leads well to vectors: base vectors. With these base vectors we can define a metric. The metric G is defined by:

$$G_{ij} = \langle \mathbf{b}_i, \mathbf{b}_j \rangle \quad (3.31)$$

so:

$$G_{ij} = \begin{bmatrix} R_{11}^2 & 0 \\ 0 & R_{22}^2 \end{bmatrix} \quad (3.32)$$

The operation made to find the metric uses the jacobian matrix $J_b^a = \partial_{i^b} e_a$. Each base vector is in fact a column of J_b^a . From this jacobian matrix we can obtain its transposition Γ_a^b . If we take a look to equation 3.29 $e_a = z_{ab} i^b$ where z_{ab} is the impedance matrix (it is the same matrix as J_b^a) given by:

$$z_{ij} = \begin{bmatrix} R_{11} & 0 \\ 0 & R_{22} \end{bmatrix} \quad (3.33)$$

If we multiply each member by Γ_c^a , we obtain:

$$\Gamma_c^a e_a = \Gamma_c^a z_{ab} i^b$$

but:

$$\Gamma_c^a z_{ab} = G_{cb} \quad (3.34)$$

The system 3.29 can be written $\Gamma_c^a e_a = G_{cb} i^b$. What happens now if we compute $G_{cb} i^c i^b$? We find $(R_{11} i^1)^2 + (R_{22} i^2)^2$ and the distance is given by:

$$s = \sqrt{(e_1)^2 + (e_2)^2} \quad (3.35)$$

The distance s is the classical distance defined by Pythagore's theorem: we are in an euclidian space where s is the hypotenuse and e_1 and e_2 the two orthogonal directions.

Through the concept of distance, we are able now to make a link between vectors and numbers, also named scalars. This can be done thanks to the metric G which is a tensor twice covariant. But even before that, the impedance matrix has made correspondences between the components of the vector i^a and the collection of numbers e_b which are motive forces (electromotive force, magnetomotive force, mechanical force, etc.).

This last discussion is very important. To speak of topology in electromagnetism means to define a metric which describes the electromagnetic space. It's a mathematical description. There are no direct relations between the geometrical proximity and the topological proximity. Some descriptions that are made starting from intuitive and visual separation of zones where electromagnetic exchanges occur are not real mathematical topologies applied on electromagnetism. To define a metric is the incontournable first step to speak of electromagnetic topology.

Referential, Dual space, invariant

If we move a chair from a point A to a point B, this action is independent from the choice of referential used to describe mathematically this movement. Vectors and tensors have their own existence, independently from the matrix which represents them.

The notion of referential is very important. Every time we do something, every time we describe something we should detail before in what referential we consider the problem. We will see that for electromagnetism, the referential is defined by

operators of impedance. Every time we speak of results in electromagnetism, we should specify what are the impedances used to do the measurements or the computation. The notion of referential impacts all what we do in life. When we speak of animals, in general we describe their behaviors in our referential, without knowing in fact what they feel and think. It's the same for human, one thing that can be funny for one person in one cultural referential can be unbearable for another person in another referential. Fortunately, it exists invariants. Invariant remains the same in any referential. The only problem is to find it! I can measure an object using a metric referential and transform my value in another referential to communicate it to a friend. He will be completely able to understand my measurement because, whatever the system to measure the distance, a distance remains a distance. Regardless of the culture, the location, etc., if I walk to the north, I walk to the north.

In relativity¹ for example, the invariant is the interval ds . We have:

$$ds = \sqrt{c^2 dt^2 - dx^2} \quad (3.36)$$

In another referential, time and space are transformed following Lorentz's transformation:

$$\begin{cases} dt' = \gamma (cdt + \beta dx) \\ dx' = \gamma (dx + \beta cdt) \end{cases} \quad (3.37)$$

$$(\beta = \sqrt{1 - v^2/c^2}, \gamma = \beta^{-1}).$$

¹Lifchitz, L. L. E., & Landau, L. (1970). *Théorie des champs*. Edition MIR.

You can easily verify that $c^2 dt^2 - dx^2 = c^2 dt'^2 - dx'^2$. It means that the distance ds doesn't depend on the referential chosen to measure it.

And using a distance, I use a number obtained from a vector. As we have already written it vectors are not so easy to manipulate. This is interesting if I can make a correspondence between a vector and a number, and this for all the vectors of my space. We look for something which is isomorphic to my vectorial space. A map that gives to each vector a number, obtained from the vector after some operation.

In relativity, the "4-vector" \mathbf{v} has a first component linked with the temporal side and a second component of dimension three linked with the spatial side: $\mathbf{v} = (ct, \vec{x})$. A dual vector of \mathbf{v} can be obtained by giving a minus sign to the spatial part: $\mathbf{v}^* = (ct, -\vec{x})$. The scalar product of the vector by its dual leads to the invariant s^2 :

$$s^2 = \mathbf{v} \cdot \mathbf{v}^* = c^2 t^2 - x^2 \quad (3.38)$$

In the electromagnetic case, a number that can be attached with each branch, we have seen it, is the electromotive force. This electromotive force comes from the work of the electric field on the length of the branch. So the operation used to compute this work may be our linear form that transforms a vector in number. Let's try this.

$$\mathbf{j} = \sigma \mathbf{E} \Rightarrow \int_0^\chi \mathbf{dx} \cdot \mathbf{E} = \frac{1}{\sigma S} \chi \cdot i + k = -U \quad (3.39)$$

i is the current on the branch of length χ and k the constant of integration. $-U$ (remember that $U = -\int_x \mathbf{dx} \cdot \mathbf{E}$) is the

potential difference linked with the field work. The constant is determined by:

$$k = R.i + U$$

R is given by $R = (\sigma S)^{-1} \chi$ and S is the section of the flux, σ the conductivity. As the current like the electric field must be created by an external source, the constant k cannot be something else than the electromotive force e .

We obtain once more

$$e = R.i + U$$

A similar linear form can be found for dielectric medium with:

$$\mathbf{E} = \frac{1}{\epsilon S} \int_t dti \Rightarrow U = - \int_0^x \mathbf{dx} \cdot \mathbf{E} = -\frac{1}{C} \int_t dti + k \quad (3.40)$$

C is the capacitance of the medium of section S . Final result for a capacitive branch is:

$$e = U + \frac{1}{C} \int_t dti$$

Back to the graphs

We dispose of a set of oriented branches, each of them wearing a direction of one space. This collection constitutes the components of a global vector of the currents for the branches i^k .

These components are the coordinates of the vector current in a multidimensional space where base vector are defined by the

impedances. The metric is obtained by the operation previously described. For a pure dissipative system, it is equivalent to the square of the impedance.

There is another collection of objects: the collection of electromotive forces. They create a covector, i.e. a dual of the vector i^k : $e_k = \Gamma_k^q e_q$. We have seen that this covector comes from the work of the electric field. As said before, the metric is the only way to go from the vectors to the covectors. In the branches space, the metric is basically made of resistances and capacitances.

Tensorial algebra

Dual vectors are called covectors and identified with an index down: e_k . Vectors are identified with index up: v^k . A metric ζ allows to transform a vector in covector: $e_k = \zeta_{km} v^m$. That's the only way to pass from a vector to a covector. Its inverse makes the inverse operation: $v^m = y^{mk} e_k$. Note that when we inverse a tensor, indices exchange their locations and they go from down to up or up to down. In electromagnetism, the metric is an impedance matrix.

For electromagnetism, the invariant is the power (or normalized energy with $q = 1$). Making the scalar product $e_k i^k$ leads to $(\zeta_{kn} i^n + U_k) i^k = W$. W is the invariant.

When we change of referential, we transform the component of a vector, for example a flux vector \mathbf{f} in other components F^α . To do that we use a matrix. A matrix, for example Λ , has one index up and one index down: $F^\alpha = \Lambda_k^\alpha f^k$.

The operations made using the mute index rule are very ef-

ficient. Once we use this technique, very complex computations becomes easy to do. The study of the establishment of the nodal equation was a first example. Many others will be described all along this book.

A last question that can appears is, what is a tensor? Is there any definition for this object? We have said two fundamentals things:

1. it exists an invariant that we must retrieve in any change of reference frame;
2. a tensor is an object like a vector independent of the matrix that represents it.

(note that in fact, a vector is a tensor of order one. A metric is a tensor of order two, etc.). The global idea is to say that each time that in an equation, an object is suspected to be a tensor, the equation mustn't change of form in a change of referential. That's the first criteria for saying if an object is a tensor. But in fact is it very important? I think that what is very important is to identify properties of particular objects we use, keeping the same name to call them, and to respect the rules of the tensorial algebra.

Among the objects we have described there is the metric G (and we will see the inertia tensor), obtained from operations on a parametrized curve, the impedance operator ζ that we will more clearly define, and its little brothers z or \mathcal{H} defined in the branches space. There is the vector of fluxes v^k (we can use other symbols, for example for currents this will be i or J) and the covector of forces e . There are also some important

matrices: the incidence A , the connectivity \mathcal{C} , rotations Λ , and others we will be able to use. There are finally pseudo-tensors, sometimes called under given properties tensorial densities. ζ can be a tensorial density and Γ the transposed matrix of the jacobian \mathcal{J}^2 .

3.2.4 Organisation in loop

The closed circulation of the electric field sends to a closed circulation of branches. We have seen how to transform a collection of branches to organize them in serie. Here, we present a transformation that leads to an organization in loops called meshes. This transformation is called a connectivity (remember that we have seen the incidence before). We consider two branches b_1 and b_2 separated. We create a mesh m_1 made by the closed circulation made with the branches b_1 and b_2 . Principle of this action is presented figure 3.2.

If we note \mathcal{C} the connectivity, we can determine the change of referential by writing the relations between the old impedances and the new one (here we have only one mesh, so only one impedance in this space). We can report the belonging of each impedance of branches to the impedance of meshes. In this first case we have:

$$R_1(b_1) \in Z(m_1) \quad R_2(b_2) \in Z(m_1) \quad (3.41)$$

²Remember that the jacobian is defined by $\mathcal{J}_{ab} = \partial_b (f_a)$, where f_a is any function.

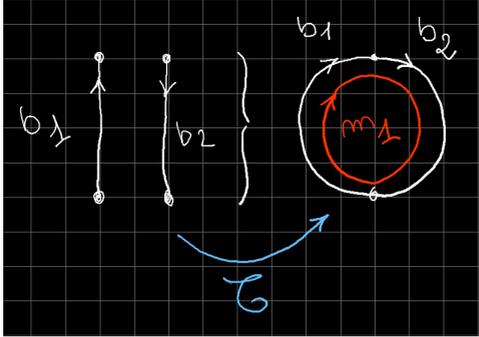


Figure 3.2: Characterization of a branch

From these two relations we can deduce the matrix \mathcal{C} :

$$\mathcal{C} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (3.42)$$

The impedance matrix in the branch space is given by:

$$z = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \quad (3.43)$$

It results from the simple direct sum of all the element of the collection of objects that makes the system. We transform this matrix using the connectivity \mathcal{C} and making a bilinear product:

$$\mathcal{C}^T z \mathcal{C} = C_\alpha^a z_{ab} C_\beta^b = \zeta_{\alpha\beta} \quad (3.44)$$

When we apply this product we find a single real value given by: $\zeta_{\alpha\beta} = R_1 + R_2$.

Let's take another example with two loops coupled. If we consider the figure 3.3. It shows two meshes having a common branch like a frontier: the branch 2. The belonging of each branch to the corresponding meshes leads to the next connectivity matrix:

$$C = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \quad (3.45)$$

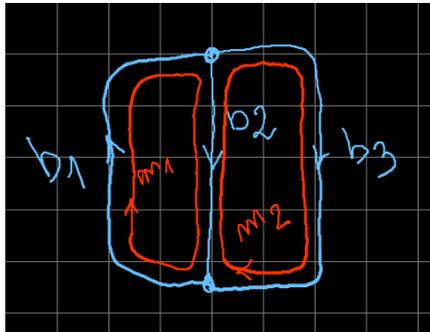


Figure 3.3: Two coupled meshes

The belonging of branch 2 to the two meshes 1 and 2 is not so evident. What means a -1 relation of belonging? Rather than

belonging, we should now speak of change of reference frame. Imagine people walking in the street. They don't know if the street turn around further and makes a loop. They see other people walking for some of them in the same direction in the street, and others walking in the other sens in the street. If we consider a change of referential applied on the speed of particles in the branches compared to the speed of the same particles in the meshes at the same locations, we can easily understand that a minus sign links branch 2 and mesh 2. Connectivity is a change of reference frame.

Transformation applies to forces and fluxes

The change of reference frame has finally been applied using speeds for particles in branches. It is evident that it can be applied to the fluxes in the branches. From now we can call these fluxes: currents, even if they are not electrical ones. Between the currents in the branches i and those of the mesh space J we have the relation:

$$i^k = C_\sigma^k J^\sigma \quad (3.46)$$

We start from this relation to transform Kirchhoff's equation into Kron's one. Now we can use a fundamental mathematical property. As e_k is dual of i^m we have by definition if \check{e}_α are the forces in the meshes space:

$$\check{e}_\alpha = C_\alpha^m e_m \Rightarrow e_m = C_m^\alpha \check{e}_\alpha \quad (3.47)$$

When we go from one space to another and when this second space belongs to a higher order dimension than the first one,

the connectivity which gather simple elements to make a more complex one, is applied necessarily to the lower order space. It says how the simplest element are organized as intersections of the more complex ones. This can surprise somewhere, but let's think of that. It is a systemic problem. It seems evident that starting from little elements we can construct a system. It's what we do every time we play with legos or similar games. But if we make reverse engineering, it's many more easy to identify intersections between volumes to go back after towards the elements that compose the system. And it's possible to work using this approach even to construct a system from little parts. This approach obliges to imagine what is the system before to begin its construction. Thinking to this, in fact it's quite natural! We first imagine what we want to construct, and after we imagine how to organize the parts to reach our objective.

Branches are frontiers between meshes, or participate to the meshes which are the borders of surfaces. Surfaces are themselves borders of volumes. Knowing the properties of a volume, we only need to know its limits to describe its behavior.

The surfaces we are speaking of are abstracted ones: they are parametrized hyper-surfaces. Physical volumes disappear and included inside these abstracted surfaces.

That's why finally the connectivity is applied to the meshes currents to generate the ones of the branches. This first relation translates a relation that goes from the meshes space to the branches space. Associate vector of currents is said natural. But we see that the forces are transformed using the transposed

matrix \mathcal{C}^T and in general the inverse of $\mathcal{C} = \mathcal{C}^{-1}$.³ That's why also, e_k is a covector and not a vector.

One difficult exercise using graphs is to imagine the volumes that are behind. But mathematically we have seen this by looking for the linear forms.

When we look at a resistance, we integrate the field on the height of the branch following:

$$U = - \int_x \mathbf{dx} \cdot \mathbf{E} = \frac{1}{\sigma S} \int_x dx i \quad (3.48)$$

But i/S is the current density J . This is equivalent to write:

$$U = - \int_x \mathbf{dx} \cdot \mathbf{E} = \frac{1}{\sigma} \int_x \mathbf{dx} \cdot \mathbf{J} \quad (3.49)$$

This last expression implies the integral of x . This is a linear form of first order, noted 1-form. For the dielectric, we obtain a similar expression that involves a 1-form:

$$U = - \int_x \mathbf{dx} \cdot \mathbf{E} = \frac{1}{\epsilon} \int_{(x,t)} \mathbf{dx} \cdot \mathbf{J} dt \quad (3.50)$$

We may wonder why we do not work with the current density rather than with the current? Because when we use the connectivity to describe a part of a system, we make equivalence between currents of branches and currents of meshes. This is not possible with the current densities, because they are not the

³The connectivity matrices are Hadamard's ones. So their inverse is equal to their transposition.

same between two connected branches. This would be possible using sections rather than integers in the connectivity matrix. But this gives no added values to compute the models of the systems.

As soon as the branches are connected in meshes, a new physical phenomenon appears that cannot be seen in the branches space. Following Maxwell's equation we have:

$$\check{e} = \oint_c d\mathbf{C} \cdot \mathbf{E} = -f \left(\int_x dx \int_y dy p \mathbf{B} \right) \quad (3.51)$$

(f) is a function. We know that the electromotive force comes from the time variation of the magnetic field. In the previous relation, p is the Laplace's operator. But this variation occurs inside a section. The previous relation is not correct because we have a scalar as result and a vector on the member of the right. How can we do to translate the scalar product of the field with the section?

The more elegant way to make this is to use external product⁴. The expression:

$$\int_x \int_y d\mathbf{x} \wedge d\mathbf{y} \mathbf{n}_z \quad (3.52)$$

creates a normal vector which amplitude is equal to the surface enclosed by the directions x and y . The external product sends a quantity equal to this surface and \mathbf{n}_z is the normalized vector

⁴Troyanov, M. (2009). Cours de géométrie. Presses polytechniques et universitaires romandes.

perpendicular to the surface. Now equation 3.51 can be written by using 3.52:

$$\check{\epsilon} = \oint_c d\mathbf{C} \cdot \mathbf{E} = -p \int_x \int_y d\mathbf{x} \wedge d\mathbf{y} \mathbf{n}_z \cdot \mathbf{B} \quad (3.53)$$

The equation 3.52 defines a linear 2-form.

The electromotive force can be simply defined by: $\check{\epsilon} = -p\phi$. Which means immediately:

$$\phi = \int_x \int_y d\mathbf{x} \wedge d\mathbf{y} \mathbf{n}_z \cdot \mathbf{B} \quad (3.54)$$

This flux belongs to the mesh and cannot be defined in the branches space. But the magnetic flux ϕ can also be written $\phi = \mathcal{L}i$. If the mesh is a perfect loop of radius r , the magnetic field is linked with the current by:

$$\mathbf{B} = \frac{\mu}{2r} \mathbf{J}A$$

for a section of conductor equal to A . By replacement we obtain:

$$\mathcal{L} |\mathbf{J}| A = \frac{\mu A}{2r} \int_x \int_y d\mathbf{x} \wedge d\mathbf{y} \mathbf{n}_z \cdot \mathbf{J} \quad (3.55)$$

This leads to:

$$\mathcal{L} = \frac{\mu}{2r} \int_x \int_y d\mathbf{x} \wedge d\mathbf{y} \quad (3.56)$$

When we search for a relation leading to the force in the branch space, we use 1-forms to define resistance and capacitance. To obtain the inductance, we need 2-forms. The impedances

in the branch space can be components of a vector. But the inductances are components of a tensor of order 2, twice covariant.

Excepted when associated with a component and a branch, any mesh adds an inductance to its impedance operator. This inductance is intrinsic to the mesh and cannot belong to its branches. The same principle is observed with the electromotive forces \check{e} . And we will see that the meshes space has others properties.

More on the duality There is another way to define duality. We start from two branches, one branch is a resistance and the other a capacitor. We make a mesh with these two branches. All along the resistance, an electric field is developed. If l_R is this circulation we have:

$$Ri = \frac{1}{\sigma} \frac{j}{\phi} \phi j$$

j is the current density. Along the capacitor on a length l_C we have:

$$\frac{1}{C} \int_t dt i = \frac{l_C}{\epsilon_0 \phi} \int_t dt j \phi$$

So, for the whole circulation:

$$\oint_c d\vec{c} \cdot \vec{E} = l_R \frac{j}{\sigma} + l_C \frac{1}{\epsilon_0} \int_t dt j = l_R E_R + l_C \frac{q}{\epsilon_0 \phi} \quad (3.57)$$

but:

$$\frac{q}{\phi} = \epsilon_0 E_C$$

finally we confirm that:

$$e = \oint_c d\vec{c} \cdot \vec{E} = l_R E_R + l_C E_C \quad (3.58)$$

We have:

$$e = \oint_c d\vec{c} \cdot \vec{f}(i)$$

for example

$$\vec{f}(i) = \frac{R}{l_R} \vec{i}$$

under this definition, e is a 1-form. But on the other side:

$$e = -\mu \int_{(x,y)} (d\vec{x} \wedge d\vec{y}) \cdot \dot{H}$$

which identifies e as a 2-form. How can we make a link between a 2-form and a 1-form ? Thanks to μ which must be a metric - an Hodge operator⁵. As a consequence, e is the dual of i .

The potential differences disappears

Starting from the collection of branches connected we much reach a given system. This means that applying some connectivity to this collection and its vector should lead to the expression

⁵Bossavit, A. (2001). On the notion of anisotropy of constitutive laws: Some implications of the ‘‘Hodge implies metric’’ result. COMPEL-The international journal for computation and mathematics in electrical and electronic engineering, 20(1), 233-239.

of this system. Let's the N branches be described by the Kirchoff's law:

$$s_k = \mathcal{H}_{km}(i^m) + U_k = z_{km}.i^m + U_k \quad (3.59)$$

where s_k are generators attached with the branches. Using the relation $i^m = \mathcal{C}_\beta^m J^\beta$ we obtain:

$$s_k = \mathcal{H}_{km}(i^m) + U_k = z_{km}.\mathcal{C}_\beta^m J^\beta + U_k \quad (3.60)$$

We can multiply all members on the left by \mathcal{C}_σ^k

$$\mathcal{C}_\sigma^k s_k = \mathcal{C}_\sigma^k z_{km}.\mathcal{C}_\beta^m J^\beta + \mathcal{C}_\sigma^k U_k \quad (3.61)$$

Now $\mathcal{C}_\sigma^k s_k = \check{s}_\sigma$, $\mathcal{C}_\sigma^k z_{km}.\mathcal{C}_\beta^m = \zeta_{\sigma\beta}$ and $\mathcal{C}_\sigma^k U_k = 0$. This last relation, a fundamental one, says that the closed circulation of the differences of potentials is zero. It's a very important property of the meshes space to reduce the number of variables to the unique meshes currents.

The equation of the system is now reduced to:

$$\check{s}_\sigma = \zeta_{\sigma\beta} J^\beta \quad (3.62)$$

This equation is incomplete because it doesn't include the inductances attached with each mesh. Of fact in the mesh space we must add the tensor $\mathcal{L}_{\sigma\beta}$ which includes the self-inductance of each mesh and the mutual inductances between meshes. we must add also the electromotive forces \check{e}_σ induced in the meshes by external fields. The equation becomes:

$$\check{f}_\sigma = \check{s}_\sigma + \check{e}_\sigma = (\zeta_{\sigma\beta} + \mathcal{L}_{\sigma\beta} p) J^\beta \quad (3.63)$$

Specific properties of meshes: the inertia tensor

The tensor $\mathcal{L}_{\sigma\beta}$ can be called inertia tensor. This tensor translates the reaction of the diffracted field in response to the excitation of the incident field. The behavior is the same as the inertia for the mechanical excitation. That's why we can call it in general the inertia tensor.

What is particular with this tensor is that when we compute the current derivative of the function $\check{f}_\sigma(J^\beta)$, the application of this derivation to the inertia part leads to zero. When we do:

$$\frac{\partial}{\partial J^\alpha} (\mathcal{L}_{\sigma\beta} p J^\beta) = \mathcal{L}_{\sigma\beta} p \cdot \delta_\alpha^\beta = 0 \quad (3.64)$$

When we compute the base vectors of the parametrized surface \check{f}_σ , all the components of \mathcal{L} lead to zero. To retrieve the original equation we must add this term to the product of the jacobian by the mesh currents:

$$\left\{ \begin{array}{l} \check{f}_\sigma = \zeta_{\sigma\beta} J^\beta + \mathcal{L}_{\sigma\beta} p J^\beta \\ \Rightarrow \check{f}_\sigma - \mathcal{L}_{\sigma\beta} p J^\beta = \zeta_{\sigma\beta} J^\beta \\ \Rightarrow \Gamma_\alpha^\sigma (\check{f}_\sigma - \mathcal{L}_{\sigma\beta} p J^\beta) = \Gamma_\alpha^\sigma \zeta_{\sigma\beta} J^\beta \\ \Rightarrow T_\alpha = G_{\alpha\beta} J^\beta \end{array} \right. \quad (3.65)$$

The force-inertia term T_α includes the external sources $\Gamma_\alpha^\sigma \check{f}_\sigma$ and the inertia tensor $\Gamma_\alpha^\sigma \mathcal{L}_{\sigma\beta} p J^\beta$. We can take a simple example to illustrate this expression.

We consider two meshes which are resistances (R circuits) coupled through a real function A . The impedance tensor in the mesh space is given by:

$$z = \begin{bmatrix} R_1 + L_1 p & -A \\ -A & R_2 + L_2 p \end{bmatrix} \quad (3.66)$$

We compute the base vectors:

$$\mathbf{b}_1 = \begin{bmatrix} R_1 \\ -A \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} -A \\ R_2 \end{bmatrix} \quad (3.67)$$

The metric is:

$$G = \begin{bmatrix} R_1^2 + A^2 & -A(R_1 + R_2) \\ -A(R_1 + R_2) & R_2^2 + A^2 \end{bmatrix} \quad (3.68)$$

The force-inertia tensor is:

$$T = \begin{bmatrix} R_1 \check{f} - R_1 L_1 p J^1 + A L_2 p J^2 \\ -A \check{f} + A L_1 p J^1 - R_2 L_2 p J^2 \end{bmatrix} \quad (3.69)$$

To simplify this first study, we can consider a weak coupling ($A \rightarrow 0$) with:

$$G = \begin{bmatrix} R_1^2 & 0 \\ 0 & R_2^2 \end{bmatrix} \quad (3.70)$$

$$T = \begin{bmatrix} R_1 \check{f} - R_1 L_1 p J^1 \\ -R_2 L_2 p J^2 \end{bmatrix} \quad (3.71)$$

First equation is:

$$R_1 \check{f} - R_1 L_1 p J^1 = R_1^2 J^1$$

This is equivalent to:

$$\check{f} - L_1 p J^1 = R_1 J^1$$

That's the equation we can obtain using equation 3.63 under the same assumption.

Meshes choice, fundamental Euler-Poincare's relation and Roth's diagram

For those who wouldn't be convinced by the interest of the mesh space, we can compare one case with the nodal approach and recall an interesting technique to extract the eigenvalues.

Figure 3.4 shows a circuit that we will resolve using two different formalisms: Kron's one and the formalism of the nodal method.

In the mesh space it is easy to find the impedance tensor:

$$\zeta = \begin{bmatrix} \frac{1}{C_p} + 2Lp & -Lp \\ -Lp & \frac{3L}{2}p + \frac{1}{\gamma p} \end{bmatrix} \quad (3.72)$$

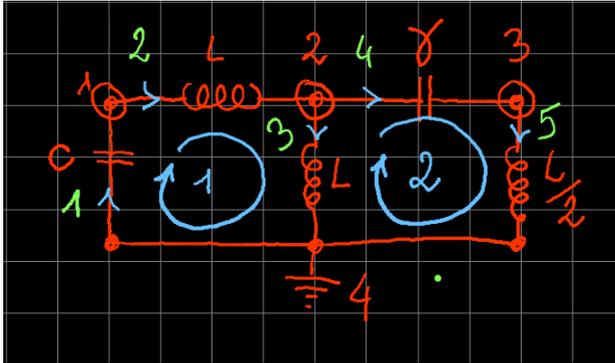


Figure 3.4: A particular circuit

Seeing this matrix we can define the kinetic operator \mathcal{T} :

$$\mathcal{T} = \begin{bmatrix} 2L & -L \\ -L & \frac{3L}{2} \end{bmatrix} \quad (3.73)$$

and the potential operator U :

$$U = \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{\gamma} \end{bmatrix} \quad (3.74)$$

The characteristic equation of the system is given by

$$\mathcal{T}^{-1}U - \omega^2 I = 0$$

This gives:

$$\left(\frac{3}{4LC} - \omega^2\right) \left(\frac{1}{L\gamma} - \omega^2\right) - \frac{L^2}{\gamma C} = 0 \quad (3.75)$$

The roots of this equation are the eigenvalues of the characteristic equation.

In the nodal space, the admittance matrix is given by:

$$Y = \begin{bmatrix} \frac{1}{Lp} + Cp & \frac{1}{Lp} & 0 \\ -\frac{1}{Lp} & \frac{2}{Lp} + \gamma p & -\gamma p \\ 0 & -\gamma p & \gamma p + \frac{2}{Lp} \end{bmatrix} \quad (3.76)$$

This time we have for the energy operators:

$$\mathcal{T} = \begin{bmatrix} \frac{1}{L} & \frac{1}{L} & 0 \\ -\frac{1}{L} & \frac{2}{L} & 0 \\ 0 & 0 & \frac{2}{L} \end{bmatrix} \quad (3.77)$$

and for U :

$$U = \begin{bmatrix} C & 0 & 0 \\ 0 & \gamma & -\gamma \\ 0 & -\gamma & \gamma \end{bmatrix} \quad (3.78)$$

Now, because we are in admittance, we compute $\mathcal{T}U^{-1}$. We see that this expression leads to degenerescence because two terms

of the characteristic equation are equals (even without taking into account the determinant which is already singular). The nodal method gives the values of the potentials. It gives the solutions for the potential covector. This corresponds to the hamiltonian. At the contrary, solving for the mesh currents gives the solution for the current vector which corresponds to the lagrangian.

As said Penrose⁶, it would have been better if we could solve the problems in the co-space (cotangent space), but reality shows that the lagrangian (tangent space) seems to be the single able to solve any problem.

Someone may wonder if similar problem appears with the modified nodal analysis (MNA) method. The answer is yes. The MNA matrix is:

$$\begin{bmatrix} Cp & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma p & -\gamma p & 0 & 0 & 0 \\ 0 & -\gamma p & \gamma p & 0 & 0 & 0 \\ 1 & -1 & 0 & -pL & 0 & 0 \\ 0 & 1 & 0 & 0 & -pL & 0 \\ 0 & 0 & 1 & 0 & 0 & -p\frac{L}{2} \end{bmatrix} \quad (3.79)$$

⁶Penrose, R., & Jorgensen, P. E. (2006). The road to reality: A complete guide to the laws of the universe. *The Mathematical Intelligencer*, 28(3), 59-61.

which can be identified with the matrix of the energies:

$$W = \begin{bmatrix} \mathcal{U} & 0 \\ 0 & \mathcal{T} \end{bmatrix} \quad (3.80)$$

If we try to do $(\mathcal{T}\mathcal{U})^{-1}$, the matrix is singular and it is impossible to obtain its inverse, even to compute the determinant of the minors.

A question can emerge: how to determine the set of meshes knowing that there are many possibilities to chose it? We introduce here an important concept linked with graphs: the spanning tree. The spanning tree is a subgraph that allows to reach all the nodes of a graph. In the tensorial analysis of networks, it was introduced through nodes pair space, but I prefer to introduce it through what I call virtual meshes. It is in deep relation with current sources and the concept of sources of known currents. The known sources of currents are physically quite rares. In général, nature creates forces that create currents. But to add particles directly to a medium is less usual. Anyway, in some cases, it is easier to consider the source like if particles come from elsewhere than to take into account the whole phenomenon. An example is lightning. It is very difficult to take into account the whole phenomenon, with the clouds, the whole circulation of loads over the clouds and the earth, etc. In a first approximation we can consider that the location of the lightning impact has weak influence on the spark-gap of the lightning and so to start from its source of current to study the problem.

Figure 3.5 shows two branches numbered 1 and 2, a current source in clear white and two meshes. The first mesh is linked

with branches 1 and 2. The second mesh is not closed and is in relation with the known current i^3 . This current is in fact the current of this unclosed mesh J^2 . We call this kind of mesh a virtual mesh. If we take a look to the connectivity of this circuit we find:

$$\mathcal{C} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \quad (3.81)$$

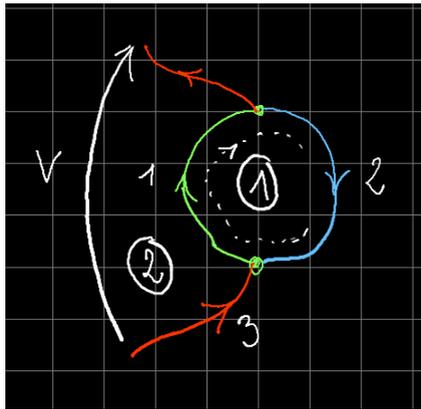


Figure 3.5: Virtual mesh

Note that, using the virtual mesh, we obtain a space of meshes of dimension 2. The same dimension as the branches. If a and b are the impedance operators of the two branches. We

start from a impedance matrix in the branches space coming from the direct sum of all the separate branches as diagonal elements:

$$z = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \quad (3.82)$$

Computing $\zeta = \mathcal{C}^T z \mathcal{C}$ we obtain:

$$\zeta = \begin{bmatrix} a+b & a \\ a & a \end{bmatrix} \quad (3.83)$$

J^2 is a known current. But the unknown variables are J^1 and V , the potentials difference across the virtual mesh. The equations of our system for a covector source $[\check{e}, 0]$ are:

$$\begin{cases} (a+b)J^1 + aJ^2 = \check{e} \\ aJ^1 + aJ^2 = V \end{cases} \quad (3.84)$$

First equation gives us J^1 (as J^2 is known), and the second equation gives us V . As previously we can compute the second geometrization, seeing the system of equations as a parametrized hypersurface. Here this would give us:

$$G = \begin{bmatrix} (a+b)^2 + a^2 & a(a+b) + a^2 \\ a(a+b) + a^2 & 2a^2 \end{bmatrix} \quad (3.85)$$

The spanning tree is the branch 1 which allows to reach the two nodes of the circuit. The nodes-pair current or current source J^2 uses the branch of the spanning tree. That's a rule. All the other branches are branches of closing. Once the spanning tree and its branches are defined, we create the meshes by closing these branches by other ones. This can be an algorithm to find automatically the meshes. But it is not so easy. In fact if we want to construct well conditioned matrix, we must use a different rule when a decoupling structure exists. For example if we have two meshes separate by a capacitor, we may use a unique spanning tree with the first branch and declare two meshes having for common branch the spanning tree. But in that case the matrix can be badly conditioned. It is better to declare two branches with the capacitor as the common branch. If a , b and c are the three impedances of the circuit, b being the capacitor impedance, under the spanning tree construction we obtain:

$$\zeta = \begin{bmatrix} a + b & a \\ a & a + c \end{bmatrix} \quad (3.86)$$

In the second assumption we obtain:

$$\zeta = \begin{bmatrix} a + b & -b \\ -b & b + c \end{bmatrix} \quad (3.87)$$

If b is a very small value, the first solution leads to:

$$\zeta = \begin{bmatrix} a & a \\ a & a + c \end{bmatrix} \quad (3.88)$$

where the impact of b disappears completely. With the second rule we obtain:

$$\zeta = \begin{bmatrix} a & -b \\ -b & c \end{bmatrix} \quad (3.89)$$

where b remains visible. This second solution leads to a better conditioned matrix. The computation in this case will be correct.

What is interesting by using G is that the distance is defined in volts, a normalized energy.

When we look at any graphs, we can number the nodes \mathfrak{N} , (the dimension of the space \mathbb{N}), the branches \mathfrak{B} (the dimension of the space \mathbb{B}) and the meshes \mathfrak{M} (the dimension of the space \mathbb{M}), all grouped on networks (space \mathbb{S} and number \mathfrak{S}). The cords create relations between these objects, but are not parts of the graph itself. Euler-Poincaré's relation says how are linked the spaces \mathbb{N} , \mathbb{B} , \mathbb{M} and \mathbb{S} :

$$\mathfrak{M} - \mathfrak{B} + \mathfrak{N} - \mathfrak{S} = 0 \quad (3.90)$$

It's a fundamental relation when we want to know how many meshes we need to solve a system. An important concept is the one of connected graph. Each time a group of branches are

separate from another group they constitutes a network. The separation means that no branches goes from this group to the other. It doesn't prohibit the fact that cords can exist between the two groups. A connected graph is a graph made of only one network.

Other applications of graphs under TAN formalism

The general approach consists in finding a first natural space, a vectorial space, then to find a dual space to define finally a metric or a pseudo-metric. Once this metric is found, the major part of the work is done. After, we can see if some operations like the second geometrization can add information to the formalism. Let's take an example in economics. Natural space is the activity of people and of firms. It's a vectorial space because this activity has directions (the kind of activity) and can increase or decrease, so being positive or negative. There is a null vector, the lack of activity, and they can be added as leading to zero if an activity is in opposition with the same activity but in regression. We note f^k the vector of activity.

A source s_u that creates this activity is money. Fundings generate economical activity in a classical economical society where money is the material of exchange and the reference to compare many different activities. We hope finally to create a link between activity and funds, something that is similar to $s_u = h_{uk} f^k$. This has meaning: s is in euros and f for example in days of activity. Under this choice, h is the day cost of work. A study was made with this space of configuration to

demonstrate strategies in research funding⁷.

We now present another application in biology.

Use for biology First work consists in defining dual space from original one. It means that some observables are scalar one contrary to some others that can be seen as flux. The fact that some of them are known or not is not the problem. The identification of both kind of variables must be attached to their nature: coming from a vectorial structure or associated with numbers (scalars). If this distinction can be done, the work can begin for defining a metric. We recall the notion of metric and its importance in the approach. Then the formalism⁸ can be used to find new interpretations of the system analyzed. We conclude on future possible works to see if this approach can lead to new results or if this work can conduct to a new modeling in bacteria growth analysis.

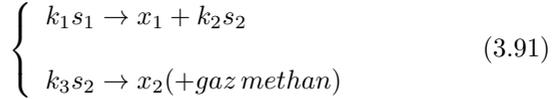
This reflexion was conducted thanks to Prs. Alain Vander Wouwer and Laurent Dewasme from university of Mons, Belgium on the base of their work on bacteria growth⁹.

⁷*Financement de la recherche d'une structure privée*. Olivier Maurice. Res-Systemica, Vol.15, October 2015.

⁸Olivier Maurice, Philippe Durand. Modélisation des systèmes complexes. : Présentation des grands principes de la méthode xTAN. Présentation effectuée lors de la réunion du groupe CESIR de l'AFSCET à l'ESIGELEC, le 19 décembre. 2015. hal-01246740

⁹Dewasme, L., Renard, F., & Wouwer, A. V. (2007, July). Experimental investigations of a robust control strategy applied to cultures of *S. cerevisiae*. In Control Conference (ECC), 2007 European (pp. 4785-4791). IEEE.

inputs We define a biomass concentration x and a substrate s . Reaction scheme leads to the equations:



The application of mass budgets leads to:

$$\begin{cases} \frac{ds_1}{dt} = -k_1 \mu_1 x_1 - D.s_1 + D.S_{in1} \\ \frac{ds_2}{dt} = -k_3 \mu_2 x_2 + k_2 \mu_1 x_1 - D.s_2 + D.S_{in2} \\ \frac{dx_1}{dt} = \mu_1 x_1 - D.x_1 \\ \frac{dx_2}{dt} = \mu_2 x_2 - D.x_2 \end{cases} \quad (3.92)$$

with μ_α kinetic coefficient given by $\mu_\alpha = f(s_\alpha)$. k are the stoichiometric functions and D the dissipation functions (debit per volume. It's a real function).

Usually a vector ξ is defined as $[s_1 \ s_2 \ x_1 \ x_2]^T$. Noting:

$$K = \begin{bmatrix} -k_1 & 0 \\ k_2 & -k_3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (3.93)$$

and $F = [DS_{in1} \ DS_{in2} \ 0 \ 0]^T$. With these definitions, we can resume the whole relations by matrix equation:

$$\frac{d\xi}{dt} = K\phi - D.\xi + F \quad (3.94)$$

Using $\phi = [\mu_1 x_1 \quad \mu_2 x_2]^T$.

Defining natural and dual space It appears clearly that ξ can be used as a vector from a natural space. It represents our unknown variables and its components are in relation with flux. Flux coming from substrate or biomass. But to transform equation 3.94 in an homogeneous equation with ξ appearing, we must replace ϕ by ξ . By defining:

$$\kappa = \begin{bmatrix} 0 & 0 & -k_1\mu_1 & 0 \\ 0 & 0 & k_2\mu_1 & -k_3\mu_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.95)$$

Using κ , equation 3.94 can be written:

$$F = \frac{d}{dt}\xi - \kappa\xi + D.\xi \quad (3.96)$$

Here we clearly see the dual space F appearing which corresponds with a scalar. So we can now fix the mathematical nature of each component and write the equation using tensorial algebra. For that we define the fundamental tensor G by:

$$\begin{bmatrix} \frac{d}{dt}(\cdot) + D_{11} & 0 & -k_1\mu_1 & 0 \\ 0 & \frac{d}{dt}(\cdot) + D_{22} & k_2\mu_1 & -k_3\mu_2 \\ 0 & 0 & \frac{d}{dt}(\cdot) + D_{33} & 0 \\ 0 & 0 & 0 & \frac{d}{dt}(\cdot) + D_{44} \end{bmatrix} \quad (3.97)$$

and now equation 3.96 becomes

$$F_a = G_{ab}\xi^b \quad (3.98)$$

From G to the graph Seeing at the structure of the fundamental tensor 3.97 we can draw a graph where each diagonal element becomes a mesh unity and each extra-diagonal element becomes a cord between two meshes. Starting from these principles, we arrive to the graph presented figure 3.6.

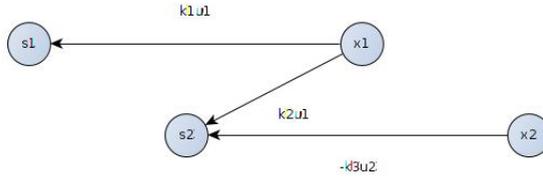


Figure 3.6: Graph coming from G

Analysis Next step to benefit of tensorial writing¹⁰ is to conduct an analysis based on mathematical operations applied to equation 3.98. First tentative can be to derive the source F_a depending on the flux ξ^b :

¹⁰Olivier Maurice, Philippe Durand, Alain Reineix. METHODOLOGY OF NETWORKS ANALYSIS USING XTAN. 2015. hal-01167819

$$\frac{\partial F_a}{\partial \xi^h} = \frac{\partial}{\partial \xi^h} G_{ab} \xi^b + G_{ab} \frac{\partial}{\partial \xi^h} \xi^b \quad (3.99)$$

But

$$\frac{\partial}{\partial \xi^h} \frac{d}{dt} \xi^b = 0, \quad \frac{\partial}{\partial \xi^h} D_{ab} = 0$$

so that:

$$\frac{\partial}{\partial \xi^h} G_{ab} = -\frac{\partial}{\partial \xi^h} \kappa_{ab} - \kappa_{ab} \frac{\partial}{\partial \xi^h} + D_{ab} \frac{\partial}{\partial \xi^h}$$

The partial derivative of flux depending on them leads to Kronecker's operator:

$$\frac{\partial}{\partial \xi^h} \xi^b = \delta_h^b$$

Finally:

$$\frac{\partial F_a}{\partial \xi^h} = -\frac{\partial}{\partial \xi^h} \kappa_{ab} \xi^b - \kappa_{ab} \delta_h^b + D_{ab} \delta_h^b + G_{ab} \delta_h^b \quad (3.100)$$

As μ depends on s_α we can note:

$$\frac{\partial}{\partial \xi^h} \kappa_{ab} = H_{ab,h} \quad (3.101)$$

and equation 3.100 becomes:

$$\frac{\partial F_a}{\partial \xi^h} = -H_{ab,h} \xi^b - \kappa_{ab} \delta_h^b + D_{ab} \delta_h^b + G_{ab} \delta_h^b \quad (3.102)$$

and so for example:

$$\begin{aligned} \frac{\partial F_1}{\partial \xi^1} \xi^1 &= -H_{11,1} (\xi^1)^1 - H_{12,1} \xi^2 \xi^1 + \dots \\ &\dots - \kappa_{11} \xi^1 + D_{11} \xi^1 + G_{11} \xi^1 \end{aligned}$$

As a consequence, it can be shown that

$$F_a - \frac{1}{2} \frac{\partial F_a}{\partial \xi^h} \xi^b = \alpha_{ab} \frac{d\xi^b}{dt} + \beta_{ab} H_{ab,h} \xi^b \xi^h \quad (3.103)$$

where α_{ab} and β_{ab} are coefficients equal to $\pm 1/2$.

Use for automatic We consider a motor system made of one "RL" circuit and an electrical motor¹¹. The modeling is the classical representation presented in automatic courses. If C_m is the motor couple and C_r the resistive couple, K_d the constant of friction, K_e the electromotive constant, K_t the electromagnetic couple and J the inertia moment, we write:

$$\begin{cases} C_m = K_t i \\ K_t i - K_d w - C_r = J \frac{dw}{dt} \end{cases} \quad (3.104)$$

w is the angular speed.

¹¹Olivier Maurice, Philippe Durand. Complex Systems Modeling Developed for Electromagnetic Compatibility Applied to Automatic Problems. 2016. hal-01345514

Knowing these equations and seeing the next diagram (figure 3.7) we obtain the following equation system:

$$\begin{cases} u = (R + Lp) i - K_e w \\ -C_r = -K_t i + (K_d + Jp) w \end{cases} \quad (3.105)$$

with p Laplace's operator.

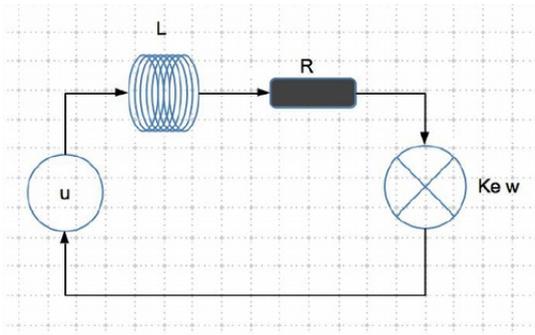


Figure 3.7: Motor control diagram

With some use, the engineer may have drawn the diagram figure 3.8. From our previous discussion, we deduce from this graph the following fundamental tensor:

$$z = \begin{bmatrix} R + Lp & -K_e \\ -K_t & K_d + Jp \end{bmatrix} \quad (3.106)$$

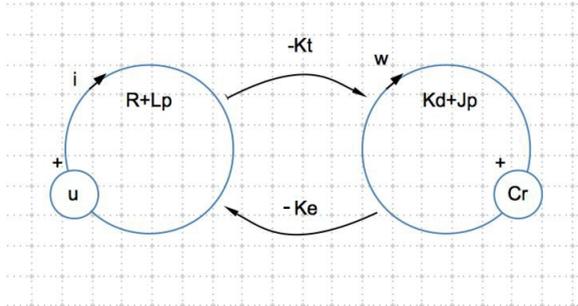


Figure 3.8: xTAN technique diagram for the motor control

associated with the covector $u_k = [u \quad -C_r]$ and the vector $f^m = [i \quad \omega]^T$. It's clear that we immediately obtain equations 3.105 through the tensorial description $u_k = z_{km} f^m$.

Use for cyber-physical systems theoretical analysis If we consider a system of systems, it is represented by a diagonal metric where the interactions between its components point to the nearest neighbor, hence this diagonal structure. By developing needs, extra-diagonal components enrich the basic structure of the metric to fill it drastically. This may include the emergence of exchange networks, "cloud", etc. The filling of this metric is easy to understand since it follows the conventional filling in the design of a system, going from the element to the system or system of systems. In the "cyber" approach, we first create an interconnected network on which we will be able to

rely. A collection of applications will be part of this network to provide different services accessible to the nodes. The means of realizing these services are grafted onto these nodes like so many potential providers. Ultimately, if we can predetermine their properties, we can adapt the network to these consumers before their addition, it is even the object of the software updates.

In the first case, the classical construction of the system starts from a M_{ij} material collection represented by a two-covariant metric tensor. The system S is established by direct sum of the collected components: $S_{ij} = \oplus_M M_{ij}$ then by transformations applied to this set to create the system: $S_{\alpha\beta} = \Lambda_\alpha^i S_{ij} \Lambda_\beta^j$. Communications $\mathcal{C}_{\alpha\beta}$ are then added to allow the hardware to communicate with each other, but these communications are initiated by the hardware and not the network, Apply a connection operation is translated by a Λ transformation:

$$T_{\zeta\sigma} = \Lambda_\zeta^\alpha (S_{\alpha\beta} + \mathcal{C}_{\alpha\beta}) \Lambda_\sigma^\beta \quad (3.107)$$

The system of systems T is here controlled by the starting materials M_{ij} .

In the second approach, we start from the network \mathcal{C}_{ij} . To this network we add equipment that can meet the needs of the network, $\oplus_M M_{ij}$ and connect them:

$$T'_{\alpha\beta} = \Lambda_\alpha^i (\mathcal{C}_{ij} + \oplus_M M_{ij}) \Lambda_\beta^j \quad (3.108)$$

This is the network that drives the hardware that connects to it, the network as a set of connected people. These concepts and differences between "classical" and "software-based" approaches

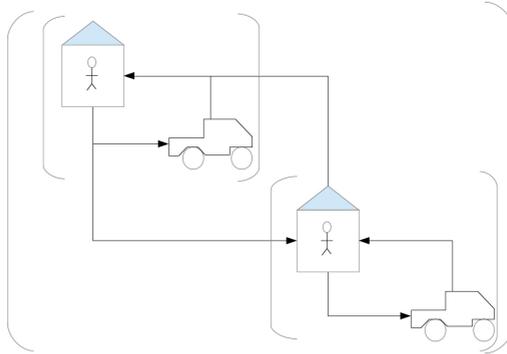


Figure 3.9: Classical organization

are well modeled by topological formalisms. The concepts discussed can be translated by the notion of neighborhood. Each system can be modeled by a matrix of operators. These are the matrices we describe. To illustrate their structures, we can symbolically represent them with small drawings that illustrate the systems modeled within the matrices. The figure 3.9 thus shows a structure organized conventionally with actors each equipped with means and communicating with each other.

In a cyber-physical structure, the players rely on a software architecture of services and will require a need, a demand. The service platform will pass on this request to providers who will make offers to meet demand. The size of the system is in fact superior at least in the number of different types of actors, adding the players of the platform.

Actors have their means and decide on actions that can be carried out using these means or by contacting directly other providers. In a software-based company, requests are sent to a platform that passes on to potential providers who respond to the tender. The returns are then managed by the platform to the customers. Figure 3.10 illustrates the matrix that models this articulation.

On this graph the queries x_i constitute the components of the dual space while the actions y^j are the natural space. Applications classified by category produce a spectrum of need as a frequency spectrum on which the calculation is established. The strings, for example r , transmit and transform the query action to a next entity. In the regular graph, the client (x_1, y^1) directly transmits its need to the providers $(x_2, \dots, x_n; y^2, \dots, y^n)$.

Providers make θ^k offers that are as many possible actions in a multiplayer game where "opponent" is the customer. The latter makes his choices according to his expectation of gain for each offer. This choice, if it is complete, cancels its request (cords e). The whole system of systems can be modeled in the form of the following equations (established directly in the space of the meshes of the graph):

$$\begin{cases} x_i = \omega_{ij}y^j \\ e_u = G_{uv}\theta^v \end{cases} \quad (3.109)$$

or $\theta^\nu = y^\nu$ and if we note:

$$S_\alpha = \begin{bmatrix} x_\alpha \\ e_\alpha \end{bmatrix} \quad \zeta_{\mu\nu} = \begin{bmatrix} \omega_{\mu\nu} & 0 \\ 0 & G_{\mu\nu} \end{bmatrix} \quad \mathcal{Y}^\beta = \begin{bmatrix} y^\beta \\ \theta^\beta \end{bmatrix}$$

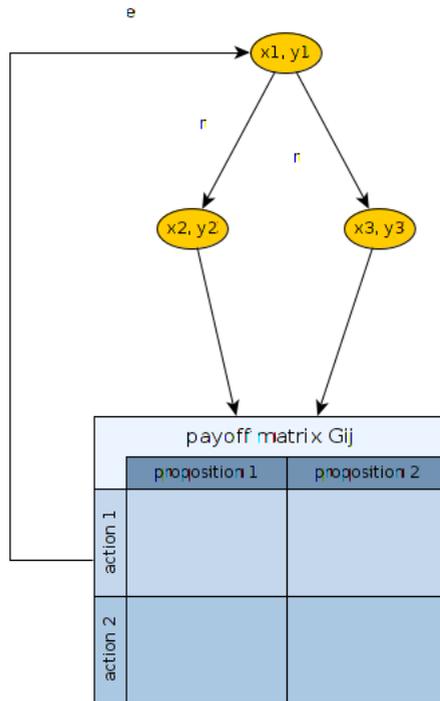


Figure 3.10: Graph for a classical organization

The whole system of system is modeled by the equation:

$$S_\alpha = \zeta_{\alpha\sigma} \mathcal{Y}^\sigma \quad (3.110)$$

For a cyber-physical SoS, a platform (x_2, y^2) intervenes between the service providers and the customer. It supports the query (r_{21}) and seeks solutions. It is this time that it plays with the providers $(x_3, y^3, \dots, x_n, y^n)$ and retaining a proposal returns it to the client (P_{12}) . The platform appears as an added cycle. We find then for the metric of classical organization:

$$\begin{bmatrix} \omega_{11} & G_{12} & G_{13} \\ r & \omega_{22} & 0 \\ r & 0 & \omega_{33} \end{bmatrix} \quad (3.111)$$

Obviously if we had N providers, the matrix would be $N + 1$. For the cyber-physical organization we find as metric always for 3 providers:

$$\begin{bmatrix} \omega_{11} & p_{12} & 0 & 0 \\ r_{21} & \omega_{22} & G_{23} & G_{24} \\ 0 & r & \omega_{33} & 0 \\ 0 & r & 0 & \omega_{44} \end{bmatrix} \quad (3.112)$$

The cyber-physical organization of the SoS adds a software layer. One would think that it takes the client away from the providers. We will show that this is not the case. Figure 3.11 shows the graph associated with the organization with a "cyber" layer.

We can define a base for parametrized surface S_α :

$$\vec{b}_v = \frac{\partial S_\alpha}{\partial \mathcal{Y}^v} = \zeta_{\alpha v}$$

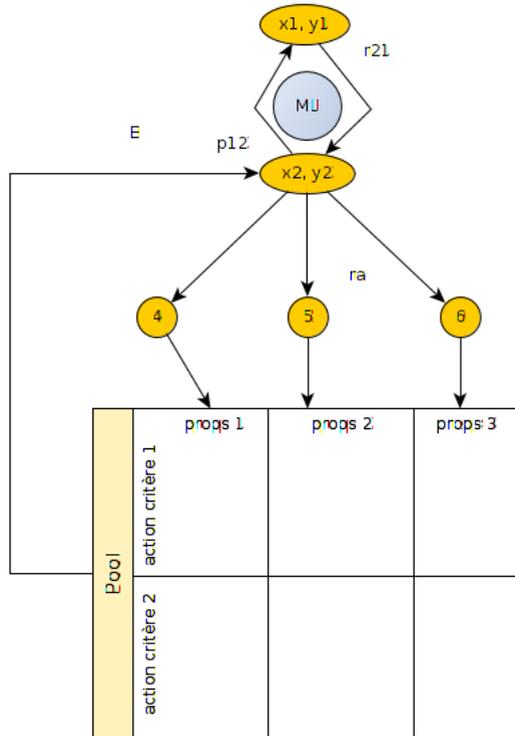


Figure 3.11: Graph for a cyber-physical organization

we can define after a metric $g_{km} = \langle \vec{b}_k, \vec{b}_m \rangle$. In the local organization, that is to say classical, the distance between the client and a provider is defined by (for k providers):

$$d_{1v}^l = \sqrt{g_{1v} \mathcal{Y}^1 \mathcal{Y}^v} = \{2(\omega_{11} G_{1v} + r\omega_{vv}) \mathcal{Y}^1 \mathcal{Y}^v + \dots \\ ((\omega_{11})^2 + kr^2) \mathcal{Y}^1 \mathcal{Y}^1 + ((G_{1v})^2 + (\omega_{vv})^2) \mathcal{Y}^v \mathcal{Y}^v\}^{\frac{1}{2}} \quad (3.113)$$

In the cyber-physiscal system this distance becomes:

$$d_{1v}^g = \sqrt{g_{1v} \mathcal{Y}^1 \mathcal{Y}^v} = \{2(r_{21} G_{2v}) \mathcal{Y}^1 \mathcal{Y}^v + \dots \\ ((\omega_{11})^2 + (r_{21})^2) \mathcal{Y}^1 \mathcal{Y}^1 + ((G_{2v})^2 + (\omega_{vv})^2) \mathcal{Y}^v \mathcal{Y}^v\}^{\frac{1}{2}} \quad (3.114)$$

We want to compare these two distances. If the request is transmitted as is by the platform, we can consider that $r_{21} = r$. Then in factor of \mathcal{Y}^1 we have: $(\omega_{11})^2 + kr^2 > (\omega_{11})^2 + r^2$.

If the platform behaves psychologically in a way close to the client, according to his interests, we can assume $G_{2v} \approx G_{1v}$, then the factor in \mathcal{Y}^v is the same.

The comparison of d_{1v}^l and d_{1v}^g uses the relation $\omega_{11} G_{1v} + r\omega_{vv}$ versus $r_{21} G_{2v}$. Noting $r_{21} = r$ and $G_{2v} = G_{1v} = G$, we try:

$$\omega_{11} G + r\omega_{vv} < rG$$

or:

$$G(\omega_{11} - r) + r\omega_{vv} < 0 \quad (3.115)$$

This implies that $r > \omega_{11}$, but r is issued from ω_{11} . If $x_1 = \omega_{11} \mathcal{Y}^1$ the request wants to report \mathcal{Y}^1 so at the maximum $r =$

ω_{11} . The inequality 3.115 is impossible. Finally:

$$\omega_{11}G + r(\omega_{vv} - G) > 0$$

and

$$d_{1v}^l > d_{1v}^g$$

CQFD.

The platform brings services closer to the customer by adding a step between the client and the service providers because it supports all interconnections with service providers. This is visible in the structure of the metric where the cyber-physical SoS adds a "cyber" layer that cancels direct client-provider interactions.

3.3 Diakoptic

Diakoptic is surely most popular Kron's work. Again today, it provokes many original works in research. The fundamental of diakoptic is to deduce the whole solution of a system using previous results obtained with parts of this system, and without restarting the computation from zero. It's a generalization somewhere of the coupled systems theory.

There are many different ways to speak of diakoptic. I want here to show only one technique, just to explain the approach and the method to conduct it.

Imagine we have a quite complicated system described by its metric ζ and with one branch, i^a which is a frontier of the

system. The equation of the system for the moment is the classical one $u_a = \zeta_{ab} J^b$. And we have a connection between i and J : $i^a = C_a^u J^u$.

To simplify the expose we consider a simple mesh made of two branches of impedances a and b . Its mesh impedance is $a+b$. If \check{e}_1 is a generator on branch 1, its equation is $\check{e}_1 = (a+b)Q^1$, Q^1 is the mesh current. Now a second branch of impedance γ is available. We want to connect it to the branche of impedance b . The equation of the system is now:

$$\begin{cases} \check{e}_1 = (a+b)J^1 - bJ^2 \\ 0 = -bJ^1 + (b+\gamma)J^2 \end{cases} \quad (3.116)$$

But we have previously compute Q^1 and \check{e}_1 remains the same in both systems. By replacing \check{e}_1 with $(a+b)Q^1$, Q^1 being known, we can obtain the new current J^1 using this previous result:

$$J^1 = \left[\frac{(a+b)(b+\gamma)}{(a+b)(b+\gamma) - b^2} \right] Q^1 = \beta Q^1 \quad (3.117)$$

A second equation completes the previous one:

$$J^2 = \left(\frac{b}{b+\gamma} \right) J^1$$

We understand with this couple of equation that if $b = 0$, $Q^1 = J^1$ and $J^2 = 0$, there is no coupling. Another disturbing case is if $b = -\gamma$. In that case, $J^1 = 0$, and as a consequence $J^2 = 0$. But

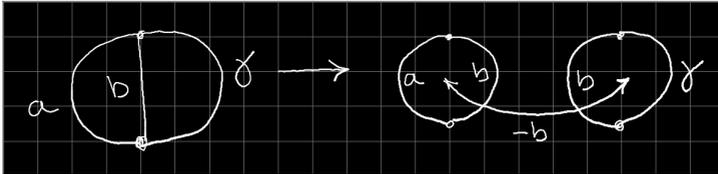


Figure 3.12: Equivalence of two circuits

Q^1 is not equal to zero, it is equal to $\check{e}_1/(a - \gamma)$. It means that this is the connection of the branch γ which leads to suppress the current J^1 .

Now let's take a look to the figure 3.12. It represents two coupled meshes. When we look at the circuit on the left, the metric associated with this circuit is equal to the metric associated with the circuit on the right. We cannot differentiate these two circuits. It is a very important result of topology. It means that every time two meshes are coupled through a common branches, we can obtain the same behavior with two meshes coupled by a function which is identical to the previously shared branch.

3.4 Reality, Graphs & manifolds

A system is finally a set of coupled meshes by cords. All the meshes are diagonal components of the metric and the coupling are carried by the cords. The dimension of the system is the number of meshes.

If we translate the representation of the system in the mesh space, meshes becomes nodes and cords can be seen as branches. Under this new description, the structure of the manifold appears by studying the property of its graph in the mesh space. There is a fundamental link between the graph and the manifold. Here cords are the branches between the nodes which are the meshes. One cycle will correspond to closed interactions between some elements. The shortest way for going from one node to another will indicate the shortest interaction between these two nodes, etc. We have here a simple and efficient technique to analyze our system. We reuse the properties of classical graphs with only nodes and branches to find special properties of the systems described under meshes coupled through cords¹². To make the difference between the branches and the nodes in the basics spaces and the branches and nodes in the mesh and cords spaces, we now call the meshes, vertex and the cords: edges, in this last space. We give first some basic definitions. Note that the complete definition of our manifolds will come after, when we will study the domains. But the equation part of their definition can already be analyzed here.

- The degree of a vertex is the number of edges (so the number of interactions) attached to this vertex;
- a path is the number of edges to go from one vertex to another. This path mustn't use twice the same edge. BE CAREFUL, here, contrarily to the classical graph theory,

¹²*A la découverte des graphes*. Christian Laforest. EDP sciences, 2017

a path of length 0 can exist only if an interaction acts on the same mesh. The figure 3.13 shows such a case;

- A cycle is a closed circulation of edges. It returns to a vertex, coming from the same vertex but without using twice time the same edge;
- A tree is a graph without any cycle.

Now, as we work in a special space where meshes are the vertex, and because the graph we trace are our systems and their associate manifolds, we can directly call these graphs: manifolds \mathcal{M} (remember that a part of the definition will come later).

A manifold \mathcal{M} is defined by the number of vertices V and the number of edges E : $\mathcal{M}(V, E)$.

If $d_{\mathcal{M}}$ is the sum of all the degrees of the vertices of a manifold, this sum is equal to $2\bar{E}$ where \bar{E} is the number of edges of the manifold.

There is no constraints for a manifold having N vertices in term of number of edges. It can have zero edge, meaning that there are no interactions between all its vertices. But to count the number of edges of a manifold versus its interactions, we must apply one rule. Let c being a cord between two vertices u and v . If the interaction c exists only from u to v , this cord is represented by an arrow that goes from u to v . At the contrary, an interaction from v to u is represented by an arrow from v to u . And finally, a symmetric interaction between u and v is represented by a double arrow between u and v . We cannot use two arrows going from u to v . It means that it cannot exist more than one interaction between two vertices, even if

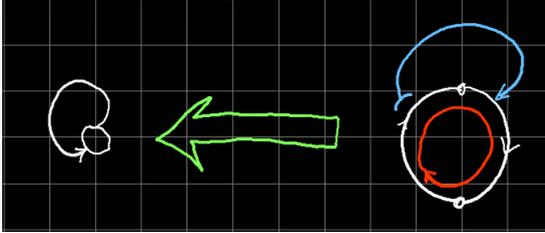


Figure 3.13: A 0 length path (an example of 0 length path is the self-inductance of a loop)

this interaction includes many different equations. We will see with the domains how this kind of segmentation can be applied. Figure 3.14 shows the processus considered.

To analyse manifolds, we need to understand how to play the interactions. There are enclosed in two matrices. The inertia \mathcal{L} and the interaction tensor μ which contains the extra-diagonal components of the tensor ζ or of the metric G . The diagonal components of the inertia \mathcal{L} gives all the zero length paths of the manifold. In the path notion, intervenes the fact that the interaction is oriented or not. To go from one vertex to another, the notion of orientation of the edges is fundamental. We cannot go from the vertex u to the vertex v if the interaction goes only from v to u .

When a manifold has cycles, the number of cycles gives the genus \mathfrak{G} of the manifold. Knowing the dimension of the set of vertices \mathfrak{V} , the dimension of the set of edges \mathfrak{E} , the genus of the

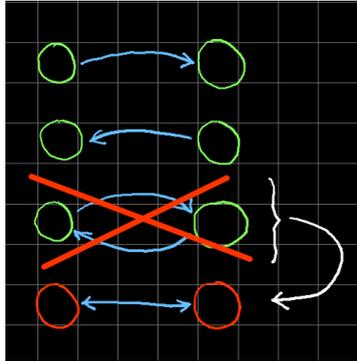


Figure 3.14: Various edges for symmetric or not symmetric interactions

manifold which gives the number of cycles is given by:

$$\mathfrak{G} = \mathfrak{E} - \mathfrak{V} + 1 \quad (3.118)$$

This is another version of the Euler-Poincaré's relation. We call a face a cycle that is not crossed by any edge. It means that if two persons are inside a face, they can move anywhere in this face without touching any edge to be in contact. The relation between the number of edges \mathfrak{E} , the number of vertices \mathfrak{V} and the number of faces \mathfrak{F} is given by Euler's equation:

$$\mathfrak{V} - \mathfrak{E} + \mathfrak{F} = 2 \quad (3.119)$$

\mathfrak{G} and \mathfrak{F} are two fundamental numbers for characterizing the manifold.

Let's study various cases to understand how these concepts play depending on the systems studied and to understand what kind of manifolds are associated with them.

3.4.1 Antennas

A couple of antennas can be modeled using two faces and a bidirectional edge. This structure is a connected manifold of genus 1 as it exists at least one face made by the exterior of the graph limited by its edges. We can verify this because the manifold has 2 vertices, 1 edge and so the number of faces must be equal to 1 to respect Euler's formula.

3.4.2 A shielded link with a pigtail

We imagine a shielded link with the shield connected to the ground. On one side of this link, the connection with the load is perfectly closed. On the other side, a pigtail connects the shield to the load making an interruption of the shield.

An external magnetic field created by a loop illuminates the shielded link. The magnetic field is in interaction with the loop between the shield and the ground. The current on the shield creates an electromotive force through the transfer impedance of the cable into the internal domain of the cable, which constitutes another circuit. But the loop can also interact with the pigtail. The whole makes a system of three meshes, i.e. three vertices in interaction, with three edges. Its genus is equal to 2, 1 face exterior as in all graph and one face created by the three edges. This result has a strong information. If we study a system

starting from the exterior. First vertex represents the exterior: the environment. Next vertex will represents the input of the system envelop. Then others vertices represents the electronics inside the envelop. If the manifold has this structure, its genus is equal to one. Now if the environment is directly in interaction with one of the element inside the envelop, it means that an edge is created from the first vertex to some vertex after the second one, and in this case the genus of our manifold will be equal to two. The higher its genus, the higher the number of interactions between the electronics and the environment.

3.4.3 Networks: graphs with oriented edges (arcs)

We consider now our cords and their orientations. In this case, edges can be called arcs. Now, how can we know where the energy flows or, in other words, can we understand what are the preferred paths for the energy? Figure 3.15 shows a first situation.

The arcs are unidirectionals. The force induced on vertex 2 is given by:

$$e_2 = \alpha f^1 \quad (3.120)$$

It creates a flux of value $f^2 = e_2/\zeta_2$. In the third vertex the force is:

$$e_3 = \beta f^2 = \beta \frac{e_2}{\zeta_2} = \frac{\beta \alpha}{\zeta_2} f^1 \quad (3.121)$$

Finally:

$$f^2 = \frac{\alpha}{\zeta_2} f^1 \quad f^3 = \frac{\beta}{\zeta_3} \frac{\alpha}{\zeta_2} f^1 \quad (3.122)$$

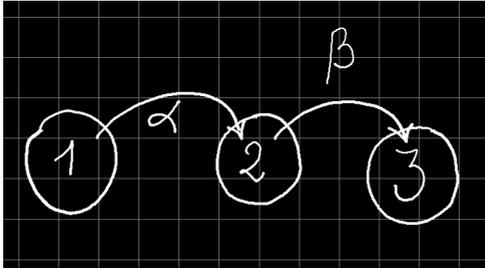


Figure 3.15: Manifold with three vertices coupled

so

$$f^3 = \frac{\beta}{\zeta_3} f^2 \quad (3.123)$$

If $\beta > \zeta_3$, the current in the third vertex is superior to the current in the second vertex. This can be true even if the genus of the manifold is 1.

From now we can speak indifferently of arcs, cords, edges ! Vertices points out the meshes. A graph with oriented cords is called a network. But as this network is associated with a manifold (we will add later the inequalities to have a manifold in the rigorous meaning of the word), we will speak of manifolds rather than networks or graphs, because there is a perfect similarity between the drawn and the system of equations deduced from this drawn.

A manifold where all the vertices are connected through cords is said complete.

3.4.4 On manifolds

The relevance of the tensor calculation approach applied to the objects of the Kron analysis was discussed earlier in the day of differential geometry, mainly by authors like Lynn or Hoffmann or Papin and Kaufmann. It is proposed here to revise this discussion by approaching it with a point of view more closely related to modern concepts of differentiable manifolds. But at first it is hung up to the definition of tensor given by Elie Cartan¹³

Definition of a tensor through Elie Cartan's point of view

We quote the definition proposed by Elie Cartan: we call in general tensor a system of a number defining analytically a geometric (or physical) being so that by a change of Cartesian coordinates, the components of the tensor undergo a linear substitution whose coefficients do not depend on the numerical values of these components but only on the two systems of coordinates (and also on the nature of the tensor).

When we look at the tensor of the impedances in a chosen space, for example the space of the branches. If we change coordinates to express this tensor in the space of the meshes, the tensor undergoes no transformation which is dependent on the

¹³This definition is taken from the remarkable article by A. Rottman, "Gabriel Kron and the formulation of a technique for solving complex systems on the basis of the overall theory of the electrical circuit (electrodiakoptic)."

values of the impedances, but only on the connections between the two spaces of the branches and the meshes.

Consider, for example, a dispersion matrix. The coefficients of the matrix are derived from ratios between incoming waves θ_e and outgoing θ_s . If we add an offset to incoming waves to write them as $\theta_e + \theta_0$, this implies the expression of the dispersion coefficients and the transformation depends on their values. In the spaces and co-spaces defined by Kron, the addition of an offset current to the components of the current vector will have no effect on the values of the impedance tensor.

Manifold of N dimensions and Kron's philosophy

Following Nakahara¹⁴, a sub-manifold is the generalization of our familiar ideas on curves and surfaces to any dimension. A curve in a three-dimensional Euclidean space is parametrized locally by a single number, usually the time $(x(t), y(t), z(t))$. In the case of two parameters, for example time and speed, a surface will be described. More generally, we can define the notion of topological or differentiable manifold which makes no reference to an embedding in the ambient space.

In the case of networks, Kron's idea stems from a first observation: "Circuit representation does not help in understanding and predicting the values taken by currents and tensions". This observation cited by Kron himself contradicts the intuition of the electronic technicians. We will detail its relevance. Admitted, the network is then replaced by a fundamental tensor (and

¹⁴Geometry, Topology and Physics, M. Nakahara, second edition, edition Taylor and Francis. ISBN 0-7503-0606-8

it is the object of this article to discuss its tensor character) and a current vector whose properties are studied. We will come to the definition of this tensor by first detailing the concepts of spaces of configuration, manifolds and systems of coordinates. The final goal is to know if the Kron currents space can be correctly described by an n -manifold. We follow in this the pattern of reasoning led by Penrose¹⁵.

Space of configuration

A configuration space \mathcal{C} is a space whose different points are the different states associated with an object during its evolution. For example, for an ordinary rigid body in the Euclidean space, we have a configuration space with 6 dimensions: three for the position of the body at each moment and three for its direction. In the case of an electromagnetic system of three coupled cells, there is a three-dimensional configuration space: the three intensities as a function of time. The dimensions of the space \mathcal{C} are as many degrees of freedom associated with the body. How does this configuration space \mathcal{C} differ from the continuous space \mathbb{R}^n (seen as a Cartesian product: $\mathbb{R} \times \mathbb{R} \dots \times \mathbb{R}$, each \mathbb{R} describing one of the degrees of freedom)? In the space of the reals, one can pass from any point to any other continuously by translation. Another way to characterize \mathbb{R} is to note that if we follow a loop on the \mathbb{R} values, we can reduce this loop to a point anywhere in \mathbb{R} . In the space \mathcal{C} one can have constraints: to be constrained to move on a surface with presence of holes. These holes prevents

¹⁵Roger Penrose, "A la découverte des lois de l'Univers". Edition Odile Jacob, ISBN 978-2-7381-1840-0

a loop surrounding the hole from reducing this loop to a point. On the other hand, it will be possible locally, in "sub-surfaces" of a complete surface, to carry out this identification of a loop at a point.

This number of "holes" differentiates a simply connected space, like the Euclidean space, from multiply connected spaces with several holes. It is all the interest to study spaces like \mathcal{C} of non-trivial topology and of high dimensions. In the case of the spaces generated by the currents of an electromagnetic network, the dimension can be very high. For an ordinary electronic circuit, there are currently dimensions which exceed several hundreds. In the case of a gas, to take the example given by Penrose, the dimension reaches 60.10^{18} !

Manifolds and system of coordinates

How to study mathematically the structure of an n -manifold associated with a network? A network and its cell description in this approach serve only to establish the vector of the currents in a chosen cell space and to deduce the associated \mathcal{C} configuration in the form of a current vector. We distinguish our various k coordinates under the writing x^k . How can we represent our variety \mathcal{M} ? We imagine it as a collage of different local coordinate systems (or "charts"), each chart being an open region of \mathbb{R}^n . In the bonding procedure, there will be passage functions that will connect the coordinates of a first system as a function of those of a second system. From the point of view of Kron, this arrangement is made in electrical machines. The couplings between windings give, as seen from a particular coil, as much

locally Euclidean space for each angular position of the rotor with respect to the stator. The passage of the expressions of the currents for an angular position to the other is then made, if the variety of a Riemannian structure is provided, by means of the coefficients of the connection of Christoffel. We detail these points later. But we can already say that the justification of the tensor character of the analysis proposed by Kron rests fundamentally on the particular behavior of magnetic couplings in electrical machines. Then, it is by generalization and sometimes abuse of language¹⁶, that this terminology is used for all types of networks.

From a network one can deduce a configuration space and the dimension of the variety that will be generated from this network. The network defines as many axes on which the components of the current vector are projected over time. These axes in all rigor are deduced themselves from the rate of variation of a scalar function ϕ which is associated with the global load of the network. It is a smooth function that passes through all the values covered by the manifold, and its rate of variation on each direction attached to an element of the starting cell topology defines an axis of the space considered for the currents ($d\Phi$ is a 1-form that can be assimilated to an integration of the current $\int_u \vec{du} \cdot \vec{i}$). Thus, if we consider a network with two meshes

¹⁶For, among other things, the metric in tensorial analysis of networks is not necessarily symmetrical. This point was discussed by Papin and Kauffmann in their work "tensor calculus". Abuse is legitimate from the point of view of physical understanding, for a distance in ohms between electromotive forces and currents is well established.

x and y , the current vector ξ of this network can be written as:

$$\vec{\xi} = a \frac{\partial}{\partial x} + b \frac{\partial}{\partial y}.$$

A covector is then defined as a projection on the variations dx^i of the spatial directions (currents) retained. The load variation is connected to a potential difference which is a covector. In the case of the meshes, the covector is a magnetic flux. The element dx^i is a plane perpendicular to the direction x^i . This plane element can also be represented as the outer product of the $(n - 1)$ directions carried by the corresponding independent vectors (chosen by balasubramanian to describe the space of magnetic fluxes).

Consider three loops in interaction. One revolves around the other two. The coupling between these three loops is a function of time and angle. We can then plot a surface of the three mesh currents $k^1(t, \theta)$, $k^2(t, \theta)$, $k^3(t, \theta)$. In this exercise, the impedance tensor is reduced to the inductances of the loops and mutuals between the single movable loop and the two fixed perpendicular loops. The graph presented in figure 3.16 is obtained.

We see that the temporal values of the current draw a torus in the configuration space k^1, k^2, k^3 . We see here the manifold associated with this configuration space \mathcal{C} which is here of genus 1. It is on this manifold that we want to determine the applicable formalisms, of which the tensor is the object of this article.

We can ask first if our manifold \mathcal{M} is a topological space. We can consider the set of points in the space \mathcal{C} as a set X composed

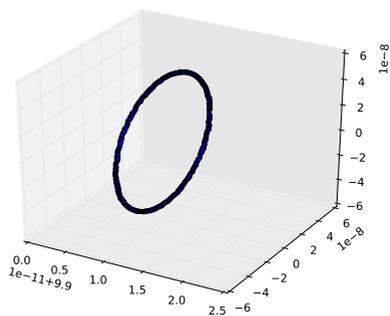


Figure 3.16: Geometrical representation of the three coupled inductances

of subsets defined for each angle value. A \mathcal{T} collection of these subsets is such that the (X, \mathcal{T}) is a topological space, depending on whether:

1. \emptyset and X belong to \mathcal{T} ;
2. any union of subsets of \mathcal{T} belongs to \mathcal{T} ;
3. any finite intersection of subsets of \mathcal{T} belongs to \mathcal{T} .

Our manifold \mathcal{M} is thus constructed on the basis of pairs $(\{k^i(t, \theta)\}, \phi_\theta =]k^i(t)[_\theta)$. The subsets of the subsets make $\mathcal{M}: \cup_\theta \{k^i(t, \theta)\}_\theta = \mathcal{M}$. Each subset created by the open of current values for all values of t but for an angle value therefore returns by a ϕ_θ map at an open interval of \mathbb{R}^N . The transition of a subset $\{k^i(t, \theta)\}_i$ to another $\{k^i(t, \theta)\}_j$ being smooth. It is hereby guaranteed to be differentiable because the angle intervenes in the coupling by trigonometric functions. The manifold is therefore differentiable. In other words, we can always compute expressions of the type:

$$\frac{\partial k^a}{\partial \theta}$$

For a curve in \mathcal{M} obtained for a particular value of θ , we can compute a vector tangent to this curve by derivation: the vector

$$\frac{\partial k^i}{\partial t}$$

To define a vector we need a curve: $k^i(t)|_\theta$ and a $\phi_\theta : \mathcal{M} \rightarrow \mathbb{R}$ function. The rate of increase in the direction t is written as:

$$\frac{d\phi_\theta}{dt}$$

In terms of local coordinates, the derivative becomes:

$$\frac{\partial\phi_\theta}{\partial x^\mu} \frac{dx^\mu}{dt}$$

Where we find that the vector is constructed on a partial derivative of the form:.

$$X^\mu \frac{\partial}{\partial x^\mu}$$

At any point on the curve, we can build a local coordinate system from the tangents $\frac{\partial}{\partial x^\mu}$ and project a vector. In this case, we have seen that we have computed the partial derivatives of our function $\frac{\partial\phi}{\partial k^i}$ at a point (t, θ) particular. These partial derivatives finally refer to the directions other than the direction i on which we derive.

At the same point, a covector can also be constructed from the increments of the directions $k^i : dk^i$. It is naturally found that:

$$\left\langle dx^\nu, \frac{\partial}{\partial x^\mu} \right\rangle = \delta^\nu_\mu$$

The covectors constitute a dual space of the space of the vectors constructed on the rates of increase of the local coordinates.

These definite markers, we can state the notion of tensor. Since a vector or a covector is declined on the directions of increments or variations locally, the tensor is the set of coefficients applied to either vector products or covectors and which returns to a real number. In the case of Kron, the inductance tensor returns a product of covectors projected on the temporal variations of the currents towards the reactive power by writing:

$$P = L_{\mu\nu} \frac{dk^\mu}{dt} \frac{dk^\nu}{dt}$$

The object $L_{\mu\nu}$ is thus here a tensor defined on the local codirections.

Kron's electrical machine and the reasons of its tensoriality

We develop here the argument presented by Lynn in his book "Tensors in electrical engineering"¹⁷

The equation of an electrical machine is here developed as a generalization of the matrix equation attached to a set of fixed coils. The first configuration to be considered is that in which stator axes are connected to the coils of the stator and the rotor axes are connected and rotate with the coils of the rotor. In this case, the inductances and mutual inductances are themselves functions of the rotation angle θ and when the rotor rotates,

¹⁷J.W.Lynn is in the Electrical Engineering Department of the University of Liverpool. This work dates from 1963, published by Adward Arnold, London.

these functions are implicitly functions of time. The equations of a simplest machine are then:

$$v_e = R_{ea}i^a + \frac{d}{dt}(L_{ea}i^a) = R_{ea}i^a + L_{ea}\frac{di^a}{dt} + i^a\frac{\partial L_{ea}}{\partial\theta}\frac{d\theta}{dt}$$

The principle will be to show that the last two terms of this expression can be seen as a general form of temporal derivation of the current and the equation can be rewritten in the form:

$$v_e = R_{ea}i^a + L_{ea}\frac{\delta i^a}{\delta t}$$

To derive this equation, we proceed in two steps. In a first step, we develop the notion of covariant derivative, in a second time, it is applied to Lagrange's equation.

Covariant derivative

A vector \vec{A} can be written depending on its contravariant coordinates: $\vec{A} = A^q\vec{a}_q$. The vector differential $d\vec{A}$ is:

$$d\vec{A} = dA^q\vec{a}_q + A^q d\vec{a}_q \quad (3.124)$$

and also:

$$\frac{\partial\vec{A}}{\partial x^j} = \frac{\partial A^i}{\partial x^j}\vec{a}_i + A^i\frac{\partial\vec{a}_i}{\partial x^j} \quad (3.125)$$

But as we have also: $g_{ij} = \vec{a}_i \cdot \vec{a}_j$, then:

$$\frac{\partial g_{ij}}{\partial x^k} = \frac{\partial\vec{a}_i}{\partial x^k}\vec{a}_j + \frac{\partial\vec{a}_j}{\partial x^k}\vec{a}_i \quad (3.126)$$

with the notation:

$$\frac{\partial \vec{a}_i}{\partial x^j} \vec{a}_k = [ij, k] \quad (3.127)$$

The correct permutation of the indices in 3.126 gives an explicit expression of the coefficients $[ij, k]$ as a function of the metric:

$$[ij, k] = \frac{1}{2} \left(\frac{\partial g_{kj}}{\partial x^i} + \frac{\partial g_{ki}}{\partial x^j} + \frac{\partial g_{ij}}{\partial x^k} \right)$$

Coefficients $[ij, k]$ are called Christoffel's coefficients of first kind. We can then write:

$$\left\{ \begin{matrix} m \\ ij \end{matrix} \right\} = g^{km} [ij, k]$$

Multiplying each member by \vec{a}^k :

$$\frac{\partial \vec{a}_i}{\partial x^j} \vec{a}_k \vec{a}^k = [ij, k] \vec{a}^k$$

but $\vec{a}^k = g^{km} \vec{a}_m$ so:

$$\frac{\partial \vec{a}_i}{\partial x^j} = [ij, k] g^{km} \vec{a}_m$$

Note that the balance of the indices corresponds well on each side. We can then rewrite first equation in the form:

$$\frac{\partial \vec{A}}{\partial x^j} = \frac{\partial A^i}{\partial x^j} \vec{a}_i + \left\{ \begin{matrix} m \\ ij \end{matrix} \right\} A^i \vec{a}_m$$

By organizing the indices we obtain:

$$\frac{\partial \vec{A}}{\partial x^j} = \frac{\delta A^m}{\delta x^j} \vec{a}_m \quad (3.128)$$

with:

$$\frac{\delta A^m}{\delta x^j} = \left[\frac{\partial A^m}{\partial x^j} + \left\{ \begin{matrix} m \\ ij \end{matrix} \right\} A^i \right]$$

We will now show that the metric tensor refers to an "inductance tensor".

Lagrangian

In a dynamic system, the kinetic energy is represented by the formula:

$$T = \frac{1}{2} m v^2 \quad (3.129)$$

An invariant measurement of the distance is given by: $l^2 = g_{ab} dx^a dx^b$ ¹⁸

And so:

$$\left(\frac{dl}{dt} \right)^2 = v^2 = g_{ab} \dot{x}^a \dot{x}^b$$

and finally:

$$T = \frac{1}{2} m g_{ab} \dot{x}^a \dot{x}^b \quad (3.130)$$

¹⁸This result is not so trivial. l is given by $\sqrt{g_{ab} dx^a dx^b}$. After a derivation and using the square of the result we obtain the previous result. We must recall that under the holonomic assumption g doesn't depend on time or speeds, but depends on the state variables x^i .

Purely kinetic Lagrange's equations and without dissipation are given by:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}^c} \right) - \frac{\partial T}{\partial x^c} = f_c \quad (3.131)$$

With replacing T by its expression and after derivation we obtain:

$$\frac{\partial T}{\partial \dot{x}^c} = \frac{1}{2} m g_{cb} \dot{x}^b + \frac{1}{2} m g_{ca} \dot{x}^a \quad (3.132)$$

$$\frac{\partial T}{\partial x^c} = \frac{1}{2} m \frac{\partial g_{ab}}{\partial x^c} \dot{x}^a \dot{x}^b \quad (3.133)$$

Using $d_t g = d_x g \cdot d_t x$ we obtain:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}^c} \right) = \frac{1}{2} m \left(\frac{\partial g_{cb}}{\partial x^a} + \frac{\partial g_{ac}}{\partial x^b} \right) \dot{x}^a \dot{x}^b + m g_{cb} \ddot{x}^b \quad (3.134)$$

and finally using previous results we obtain:

$$f_c = \frac{1}{2} m \left(\frac{\partial g_{cb}}{\partial x^a} + \frac{\partial g_{ac}}{\partial x^b} - \frac{\partial g_{ab}}{\partial x^c} \right) \dot{x}^a \dot{x}^b + m g_{cb} \ddot{x}^b \quad (3.135)$$

In synthesis:

$$f_c = m \{ [ab, c] \dot{x}^a \dot{x}^b + g_{cb} \ddot{x}^b \} \quad (3.136)$$

In a purely electrokinetic system, state variable is the load and kinetic energy is given by the development:

$$T = \frac{1}{2} L_{\mu\nu} \dot{i}^\mu \dot{i}^\nu \quad (3.137)$$

The analogy with the generic writing is immediate and was at the origin of Kron's idea to use tensorial analysis to describe the physical behavior of the electrical machines and in general of all electrodynamic system. The equation of the electrical machine comes naturally from our results and is:

$$v_c = R_{ca}i^a + L_{ca} \frac{di^a}{dt} + [ab, c] i^a i^b \quad (3.138)$$

3.4.5 Seconde geometrization

Major method uses to solve electrical networks is the “modified nodal analysis”. But exists the direct method, the nodal analysis and the Kron's method. These methods can be separated in two groups: methods making homogeneous mathematical objects and methods that does not. After a short recall of differential geometry, we show that the second kind of methods cannot lead to interesting form through the differential geometry while for the first kind, the Kron's method leads to a new and efficient representation of networks. This continues Kron's idea: once equations are established, the network graphs are no more useful and doesn't help to understand the current and voltages evolution. Following this approach, we take a look to a more theoretical representation of the networks, using classical differential geometry rigorously¹⁹. The mathematical tools developed here can be used for any applications using networks and graphs. It is for electromagnetic circuits for sure, but also for social, mechanical, thermal, chemical, etc., uses.

¹⁹This thoughts were conducted with the help of Philippe DURAND, mathematician at CNAM university, Paris

3.4.6 Basics in differential geometry

We are interested in parametrized surfaces under the differential geometry formalism (DGF). We consider a surface S , $S \in \mathbb{R}^3$. A map $\phi : \Omega \rightarrow \mathbb{R}^3$ makes a relation between the domain $\Omega \in \mathbb{R}^2$ with $\phi(\Omega) \subset S$ and the vectorial space \mathbb{R}^3 . We write u_1, u_2 the curvilinear coordinates on the parametrized surface S . The parametrization of the surface S can be made by the map ϕ given by the definition:

$$\phi(u_1, u_2) = (x(u_1, u_2), y(u_1, u_2), z(u_1, u_2)) \quad (3.139)$$

With ϕ we can define two vectors \mathbf{b}_1 and \mathbf{b}_2 as a base of the vectorial tangent plan TpS on the point $p = \phi(u_1, u_2)$:

$$\mathbf{b}_1 = \frac{\partial \phi}{\partial u_1}, \quad \mathbf{b}_2 = \frac{\partial \phi}{\partial u_2} \quad (3.140)$$

The third one, normal to TpS makes a mobile referential. We define: $\mathbf{n} = (\mathbf{b}_1 \times \mathbf{b}_2) \|\mathbf{b}_1 \times \mathbf{b}_2\|^{-1}$

From both vectors \mathbf{b}_1 and \mathbf{b}_2 we can construct the Gram's matrix G of these two vectors which is called the metric tensor of the surface S in the parametrization ϕ [1]. By definition,

$$G_{ij} = \langle \mathbf{b}_i, \mathbf{b}_j \rangle \quad (3.141)$$

3.4.7 Basic circuit

To better explain our approaches we use a simple circuit made of three branches. Whatever the real components of the circuit

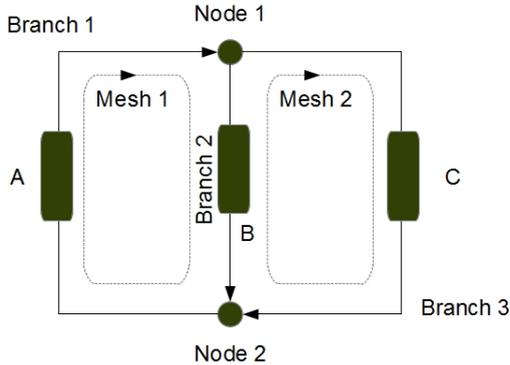


Figure 3.17: Graph considered

considered, the graph presented figure 3.17 remains the same. This single network (R) has two nodes (N), three branches (B) and the number of meshes (M) is given by: $M = B - N + R$ [2]. In this case, the number of meshes is 2.

3.4.8 Direct method of resolution

Kirchhoff's currents law (KCL) and Kirchhoff's voltages law (KVL) lead to next equations, A, B and C being impedance and branch currents having same numbering as the branches themselves.

$$\begin{cases} i_1 = i_2 + i_3 \\ Ai_1 + Bi_2 = 0 \\ -Bi_2 + Ci_3 = 0 \end{cases} \quad (3.142)$$

These equations can be synthesized under the form $\sum_q Z_{lq} i^q = E_l$ with E_l the source vector. This gives:

$$Z_{lq} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & B & 0 \\ 0 & -B & C \end{bmatrix} \quad (3.143)$$

In order to make appearing two parameters, we can define $E_l = 0$, $\forall l$ and take i_1 as source to write:

$$\begin{cases} i_1 = i_2 + i_3 \\ Ai_1 = -Bi_2 \\ 0i_1 = -Bi_2 + Ci_3 \end{cases} \Rightarrow \begin{cases} (1+A)i_1 = (1-B)i_2 + i_3 \\ Ai_1 = -2Bi_2 + Ci_3 \end{cases} \quad (3.144)$$

Then we can define $x(i_2, i_3) = (1+A)i_1$ and $y(i_1, i_2) = Ai_1$. After what we can define $z(i_2, i_3) = i_1$. From these definitions is obtained:

$$\begin{cases} \mathbf{b}_1 = \frac{\partial \phi}{\partial i_2} = (1-B, -2B, 1) \\ \mathbf{b}_2 = \frac{\partial \phi}{\partial i_3} = (1, C, 1) \end{cases} \quad (3.145)$$

From these two vectors we obtain the metric $G_{ij} = \langle \mathbf{b}_i, \mathbf{b}_j \rangle$:

$$G_{ij} = \begin{bmatrix} 5B^2 - 2B + 1 & 2 - B(1 + 2C) \\ 2 - B(1 + 2C) & 2 + C^2 \end{bmatrix} \quad (3.146)$$

To reach this result and obtain such a metric, we have made various operations. But starting from an inhomogeneous system, it was possible to define parametrized functions using the first relation and mixing it with the two others. This implies that the relation $i_1 = i_2 + i_3$ must be understood as $1i_1 = 1i_2 + 1i_3$ where impedances of one ohm are arbitrarily added to the KCL. Now, looking at the result, we see that its dimension is not homogeneous too. It involves ohms, square of ohms, etc.

In conclusion, the direct method is not an adequate one to analyze the circuit variables in a geometrical way.

3.4.9 Nodal analysis

Another way to solve Kirchhoff's equations is called "nodal analysis" [3]. Seeing a Kirchhoff's branch (shown figure 3.18), we can imagine a network made of many of these elementary circuits.

Each node of this network is associated with a potential v_n . The voltage differences between nodes v_b are linked with the nodes potential through the incidence matrix A : $v_b = A^T v_n = v - v_s$. On another side $i = -i_s = Yv$ where Y is the admittance function. Combining these various relations we write: $AYv = -Ai_s \Rightarrow AY A^T v_n = -AYv_s - Ai_s$. Defining $AY A^T = Y_n$, Y_n

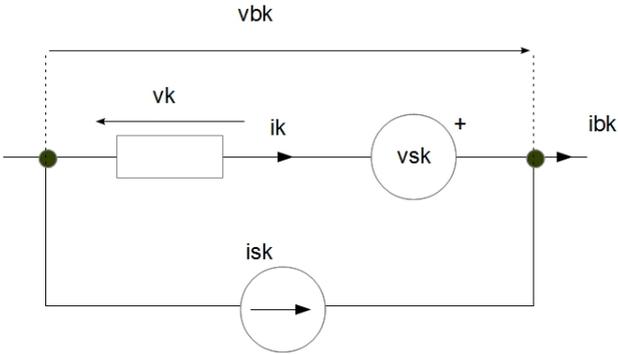


Figure 3.18: Kirchhoff's branch

being the admittance matrix in the nodes space. If Y_n^{-1} exists, we obtain finally:

$$v_n = -Y_n^{-1}AYv_s - Y_n^{-1}Ai_s \tag{3.147}$$

For a circuit where $i_s = 0$, remains $v_n = -Y_n^{-1}AYv_s$.

Looking to the circuit figure 3.17, we define the incidence matrix A :

$$A^T = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} \tag{3.148}$$

We obtain for Y_n :

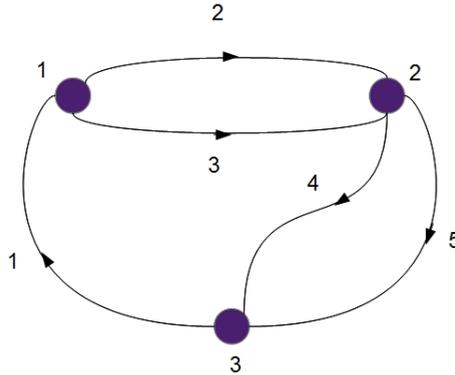


Figure 3.19: A circuit of three nodes

$$Y_n = \begin{bmatrix} \frac{1}{A} + \frac{1}{B} + \frac{1}{C} & -\left(\frac{1}{A} + \frac{1}{B} + \frac{1}{C}\right) \\ -\left(\frac{1}{A} + \frac{1}{B} + \frac{1}{C}\right) & \frac{1}{A} + \frac{1}{B} + \frac{1}{C} \end{bmatrix} \quad (3.149)$$

with:

$$AYv_s = \begin{bmatrix} \frac{E}{A} \\ -\frac{E}{A} \end{bmatrix} \quad (3.150)$$

The system obtained is over dimensioned and we have to fix $v_2 = 0$. Then $v_1 = E/(\alpha A)$ with $\alpha = A^{-1} + B^{-1} + C^{-1}$. In this case we see that it's impossible to define a parametrized surface. We look to another circuit made of three nodes (figure 3.19).

In this case the incidence matrix is :

$$A^T = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \quad (3.151)$$

If a_i is the impedance of edge i and $b_i = (a_i)^{-1}$, we obtain two equations defining the potentials v_1 and v_2 (for one source E on the edge 1):

$$\begin{cases} v_1 = \frac{E}{a_1} (b_3 + b_2 + b_1) + \frac{E}{a_1^2} \\ v_2 = -\frac{E}{a_1} (b_3 + b_2) - \frac{E}{a_1} (b_5 + b_4) \end{cases} \quad (3.152)$$

Seeing to these equations, no parameters appear clearly to define a parametrized surface. But we can suppose that the parameters may be the sources. In this case, we must define two different sources, for example E_1 on edge 1 and E_2 on edge 2. In this case the system becomes:

$$\begin{cases} v_1 = (b_3 + b_2 + b_1) (b_1 E_1 - b_2 E_2) - \dots \\ \dots - (b_3 + b_2) b_2 E_2 + (b_1)^2 E_1 \\ v_2 = -(b_3 + b_2) (b_1 E_1 - b_2 E_2) + \dots \\ \dots + (b_2 + b_3 + b_4 + b_5) b_2 E_2 + (b_4 + b_5) b_1 E_1 \end{cases} \quad (3.153)$$

Now with this new system we can define a parametrized surface with the two parameters E_1 and E_2 . Taking $x(E_1, E_2) = v_1$

and $y(E_1, E_2) = v_2$, we need a definition for $z(E_1, E_2)$. Anyway, let's take a look to the two first derivatives:

$$\begin{cases} \frac{\partial v_1}{\partial E_1} = b_1 (b_1 + b_2 + b_3) + b_1^2 \\ \frac{\partial v_2}{\partial E_1} = b_1 (b_2 + b_3 + b_4 + b_5) \end{cases} \quad (3.154)$$

We see as a consequence that the vectors \mathbf{b}_1 and \mathbf{b}_2 will depend on square of the admittance. It means that the metric will be associated with the fourth power of the admittance. This doesn't lead to any understandable notion.

Once more, the configuration space obtained under a nodal approach does not lead by a simple way to a readable metric.

3.4.10 Kron's method

The Kron's method consists in solving the circuit in the mesh space, using his formalism. Using the circuit described figure 3.17, we can construct through Kron's formalism [2] the impedance matrix in the mesh space given by:

$$Z = \begin{bmatrix} A + B & -B \\ -B & B + C \end{bmatrix} \quad (3.155)$$

Staying without current source (no use of the spanning tree) but only with a mesh source on mesh one, the circuit is now represented by the system of equations: $E_k = \sum_q Z_{kq} i_q$, $k \in \{1, 2\}$. To complete these two equations we can define a transfer function through $V_3 = B i_2$. The parametrized surface can be

link with the function $\phi(E_1, E_2, V_3)$, each function depending on i_1, i_2 . The base vectors can be defined by:

$$\begin{cases} \mathbf{b}_1 = (A + B, -B, 0) \\ \mathbf{b}_2 = (-B, B + C, B) \end{cases} \quad (3.156)$$

With this, the metric G_{ij} is given by:

$$G = \begin{bmatrix} A^2 + 2B^2 + 2AB & -(2B^2 + AB + BC) \\ -(2B^2 + AB + BC) & 3B^2 + C^2 + 2BC \end{bmatrix} \quad (3.157)$$

This time, the metric is given in ohm square. This allows a direct analogy with the canonical expression $ds^2 = G_{ij}x^i x^j$ saying that as G is in Ω^2 , the vector components being currents (flux in general) implies that the coordinates x^i are in Ampere. By the fact, ds^2 is in volt square. So the elementary distance is in volt, which seems very physical for electromagnetism. Kron's method leads to an available geometrization of the current space.

3.4.11 Deeper in space description with the Kron's mesh one

Having found an adequate description of graphs to project the currents in a geometrical one, we take a look to the conditions that should be set in order to create some curvature in the current space. In a first discussion we see that current depending impedance are the condition to create curvature. In a second

discussion, we consider the minimum graph in a complete space to define a parametrized surface and the conditions to have curvature or the general links between Kron's description and the geometrical one.

Basic principles

The two vectors \mathbf{b}_1 and \mathbf{b}_2 allow to define a normal vector to the mobile tangential surface TpS . It's defined by:

$$\begin{cases} \mathbf{n} = \frac{\mathbf{b}_1 \times \mathbf{b}_2}{|\mathbf{b}_1 \times \mathbf{b}_2|} \\ \mathbf{n} = \frac{(-B^2, B(A+B), (A+B)(B+C)-B^2)}{B^4 + [B(A+B)]^2 + [(A+B)(B+C)-B^2]^2} \end{cases} \quad (3.158)$$

With \mathbf{n} we can compute the second fundamental form. First step is to compute the vectors b_{qk} with:

$$\mathbf{b}_{qk} = \frac{\partial \mathbf{b}_q}{\partial i_k} \quad (3.159)$$

but until \mathbf{b}_1 or \mathbf{b}_2 depends on i_1 or i_2 , all \mathbf{b}_{qk} are equal to zero. It means that only circuits using the Kron's formalism for the mesh space where the currents are involved in the impedance function can create a second fundamental form and as a consequence, a curvature of the space.

Current depending impedance functions

If the impedance depends on the currents, something like $z = \alpha i$, the product $zi = \alpha i^2$ has for derivative $2\alpha i$, and so, the \mathbf{b}_{11}

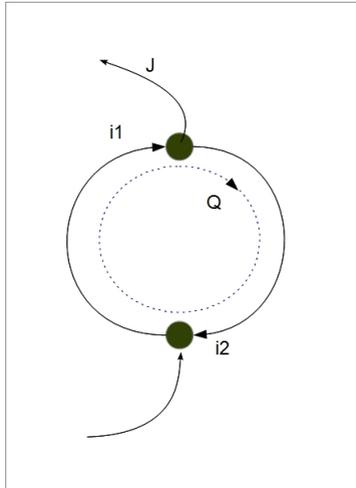


Figure 3.20: Nodes pair current

vector may have for value α which is different from zero. To generalize the approach we start from a simple graph made of one mesh but with two sources: one electromotive force on the mesh and one current source applied on the two nodes of the mesh. By this graph, we study a complete space as KRON consider it [2]. Figure 3.20 shows this circuit and both mesh and nodes pair currents.

Complete space and parametrized surface

The connexion between edges 1 and 2 and the mesh and nodes pair currents Q and J is defined by the direction chosen for the currents. It gives the connection:

$$\mathcal{C} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \quad (3.160)$$

The direct sum of the edge impedances gives the matrix:

$$Z = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \quad (3.161)$$

The transformation $\mathcal{C}^T Z \mathcal{C}$ define the complete space impedance matrix U :

$$U = \begin{bmatrix} a+b & a \\ a & a \end{bmatrix} \quad (3.162)$$

This allows to define two of the coordinates functions of ϕ :

$$\begin{cases} x(Q, J) = e = (a+b)Q + aJ \\ y(Q, J) = V = aQ + aJ \end{cases} \quad (3.163)$$

It remains to define $z(Q, J)$. We can define a general function $z(Q, J) = f(Q, J)$.

Metric analysis

basis vectors are given by:

$$\begin{cases} \mathbf{b}_1 = \left(a + b, a, \frac{\partial f}{\partial Q} \right) \\ \mathbf{b}_2 = \left(a, a, \frac{\partial f}{\partial J} \right) \end{cases} \quad (3.164)$$

The metric is immediately given by:

$$G = \begin{bmatrix} (a+b)^2 + a^2 + \left(\frac{\partial f}{\partial Q} \right)^2 & a(a+b) + a^2 + \frac{\partial f}{\partial Q} \frac{\partial f}{\partial J} \\ a(a+b) + a^2 + \frac{\partial f}{\partial Q} \frac{\partial f}{\partial J} & 2a^2 + \left(\frac{\partial f}{\partial J} \right)^2 \end{bmatrix} \quad (3.165)$$

From its construction, the metric is always symmetric. This even if the impedance matrix is not. With the two basis vectors we can define the normal vector \mathbf{n} :

$$\begin{cases} \alpha = \left(a \left[\frac{\partial f}{\partial J} - \frac{\partial f}{\partial Q} \right], \frac{\partial f}{\partial Q} a - (a+b) \frac{\partial f}{\partial J}, b^2 \right) \\ \mathbf{n} = \frac{\alpha}{\|\alpha\|} \end{cases} \quad (3.166)$$

Curvature analysis

First step is to study the variation of the basis vectors depending on the parameters. We compute $b_{qk} = \partial_k \mathbf{b}_q$. this gives :

$$\left\{ \begin{array}{l} \mathbf{b}_{11} = \left(0, 0, \frac{\partial^2 f}{\partial Q^2} \right) \\ \mathbf{b}_{12} = \left(0, 0, \frac{\partial^2 f}{\partial Q \partial J} \right) \\ \mathbf{b}_{21} = \left(0, 0, \frac{\partial^2 f}{\partial Q \partial J} \right) \\ \mathbf{b}_{22} = \left(0, 0, \frac{\partial^2 f}{\partial J^2} \right) \end{array} \right. \quad (3.167)$$

Due to the \mathbf{b}_{qk} components, the second fundamental form h is defined by:

$$h_{qk} = \langle \mathbf{b}_{qk}, \mathbf{n} \rangle = b^2 \sqrt{\langle \mathbf{b}_{qk}, \mathbf{b}_{qk} \rangle} \quad (3.168)$$

\mathbf{b}_{qk} vectors indicate how basis vectors depends on parameters Q and J . If \mathbf{b}_{12} is equal to zero, it means that \mathbf{b}_1 does not depends on J . It's the case when \mathbf{b}_1 runs on a plan and doesn't change in direction or amplitude whatever J values. When the vector changes it can mean that for example, the surface is turning and the mobile tangential plan TpS has to turn too. But this doesn't imply curvature. \mathbf{b}_{qk} vectors indicate the fact that \mathbf{b}_q ones change when Q or J change. But to control if the angles between the basis vectors are no more 90 degrees, we have to look at the scalar product between the normal vector to the plan and its fundamental base. That's the purpose of the object h . In our case, this is determined by the square of impedance b and the second order partial derivatives of the function f used to define $z(Q, J)$ in the parametrized surface ϕ .

When we have to compute the variation of any vector \mathbf{A} , for example $\partial_Q \mathbf{A}$ this involves both variation of vectors which implies basic ones and the variation of space. If \mathbf{A} is a natural vector developed on \mathbf{b}_q :

$$\mathbf{A} = A^q \mathbf{b}_q \quad (3.169)$$

When Q changes, the mobile tangential TpS changes too and :

$$\frac{\partial \mathbf{A}}{\partial Q} = \frac{\partial}{\partial Q} (A^k \mathbf{b}_k) = \mathbf{b}_k \frac{\partial}{\partial Q} A^k + A^k \frac{\partial}{\partial Q} \mathbf{b}_k \quad (3.170)$$

With previous computations, this leads to:

$$\frac{\partial \mathbf{A}}{\partial Q} = \mathbf{b}_k \frac{\partial}{\partial Q} A^k + A^k \mathbf{b}_{kq} \quad (3.171)$$

This relation stills difficult to use. It would be easier if only basis vectors appear, making a link to the known TpS surface on any point (Q, J) . Fortunately, \mathbf{b}_{qk} can be written depending on \mathbf{b}_q through the Christoffel's symbols[1]:

$$\mathbf{b}_{qk} = \Gamma_{qk}^1 \mathbf{b}_1 + \Gamma_{qk}^2 \mathbf{b}_2 + h_{qk} \quad (3.172)$$

This time, the partial derivative of \mathbf{A} is completely defined in the base vectors of TpS at any point. Christoffel's symbols give the information on the curvature of the parametrized surface directly linked with the impedance dependencies on currents.

Dimensions \mathbf{b}_k are impedances. It means that \mathbf{A} can be voltages and its components A^k amperes. We have seen that the distance is fundamentally in volt. To compute Christoffel's symbols we create a dual base c^k for TpS . This base makes basis vectors as admittances and the corresponding covectors in ampere with coordinates in volts. We find the fundamental relation giving $\mathbf{b}_k \mathbf{c}^q = \delta_k^q$ and the fact that the invariant is the power.

3.4.12 Remarkable manifolds

If we can keep the structure of a manifold simply by moving its vertices and changing the value of its cords, the new manifold is said to be isomorphic to the first one. We can by this technique create many manifolds that are isomorphic to a canonical manifold. We want here to identify some of these canonical manifolds.

Coupled lines

Two coupled lines are represented by two couple of two vertices. A simple line consists in two vertices linked with a non symmetrical cord²⁰

The figure 3.21 shows the structure on the left. We can wonder if it is possible to avoid the cords to cross each other?

²⁰Alain Reineix, Olivier Maurice. Progrès récents dans la modélisation CEM de câblages électriques de systèmes complexes. CCT-CEM: Progrès récents dans la modélisation CEM de câblages électriques de systèmes complexes, Mar 2017, Toulouse, France. hal-01495620

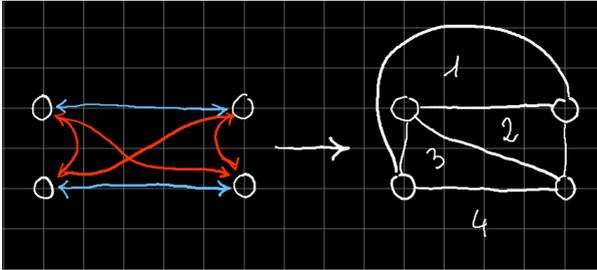


Figure 3.21: A 4 faces manifold

If it is possible, the manifold is said to be planar. The drawn on the right shows that it is possible to do that by moving one of the cord. In that case, four faces appear and Euler's relation for planar manifolds can be verified. The number of vertices \mathfrak{V} is 4, the number of cord \mathfrak{E} is 6 and the number of faces \mathfrak{F} , 4. We verify $4-6+4=2$.

What's the number of cycles? The number of cycles is given by $\mathfrak{E} - \mathfrak{V} + 1 = 3$. The number of cycles is the number of faces minus one. This comes from the fact that no cycles exist at the exterior of the network. If we connect two manifolds like the one presented figure 3.21, and we add two edges between the two opposites vertices in diagonal. If we draw the network of the figure 3.21 on a sphere, it appears clearly that no edges cross each other. If we do the same with the combination of two networks, adding two edges on the external diagonal, it becomes impossible. Except if we make an edge going inside the sphere

to reach the opposite vertex. in that case the face created by these two edges is not in the same plan that all the others.

Planar networks are interesting for EMC, because when the interactions can be located on a sphere without any cross talks, it means that less interactions appear in the system. They can be easily managed due to the fact that we can draw them on a sheet without needing a 3D representation. A class of solids respect this condition. They are called Platon's solids.

Starting from Euler's equation:

$$\mathfrak{V} + \mathfrak{f} - \mathfrak{E} = 2 \quad (3.173)$$

Platon's solids have a remarkable property: all the faces have the same number n of edges, and as each edge is common to two faces, we have:

$$2\mathfrak{E} = n\mathfrak{f} \quad (3.174)$$

There are others informations:

1. the same number p of edges start from each vertices;
2. any edge touches two vertices.

We obtain with these informations:

$$2\mathfrak{E} = p\mathfrak{V} \quad (3.175)$$

With Euler's formula, we can write:

$$2\frac{\mathfrak{E}}{p} + 2\frac{\mathfrak{E}}{n} - \mathfrak{E} = 2$$

By multiplying each term by np we obtain:

$$\mathfrak{E}(2n + 2p - np) = 2np$$

As the number of edges can be only positive, we can write:

$$2n + 2p - np > 0$$

or

$$np - 2n - 2p + 4 < 4$$

Finally:

$$(n - 2)(p - 2) < 4 \quad (3.176)$$

If we impose that a face has at least 3 edges, five cases respect the assumptions:

- $n = 3, p = 3$: tetraedra (pyramide);
- $n = 3, p = 4$: octaedra;
- $n = 3, p = 5$: icosaedra;
- $n = 4, p = 3$: hexaedra (cube);
- $n = 5, p = 3$: dodecaedra

Each of these figures defined by the Greek can be drawn on a sheet in 2D.

In a complete system which includes the undesired interactions between vertices what can be very interesting is to be able to evaluate the various paths of the energy from one source to a target.

When we study a reduced system, i.e. starting from a generator and going to some loads without losses, we always forget one particular path. This path is the energy given to the generator and the losses going into the environment. This adds a mesh, so a vertex that is not taken into account in Euler-Poincaré's formula :

$$\mathfrak{V} + \mathfrak{M} - \mathfrak{E} = \mathfrak{R} \quad (3.177)$$

As in the case of a single system, $\mathfrak{R} = 1$ we obtain:

$$\mathfrak{V} + \mathfrak{M} - \mathfrak{E} = 1 \quad (3.178)$$

Looking at Euler's formula:

$$\mathfrak{V} + \mathfrak{f} - \mathfrak{E} = 2 \quad (3.179)$$

\mathfrak{f} being the number of faces, we see that we obtain the same result but increased of 1. This added face, corresponding to an added mesh is the external mesh made of the energy given to the generator and the losses radiated in the universe. As the energy of the generator comes from the universe also, the loop is closed between this input of energy and the output of the radiated losses.

This confirms Feynman's reflexion when he said that the entropy of a light in a room was balanced by the information that the light gives on the room. It is the same for us here, the entropy of the radiated losses is balanced by the concentrated and organized energy given to the generator. The figure 3.22 illustrates this mechanism in the case of two vertices. A first vertex gives energy to a second one which dissipates it. This circuit is of genus 1, while the number of meshes is 0.

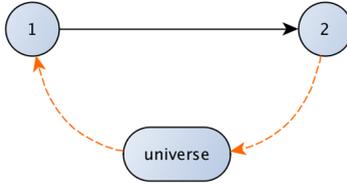


Figure 3.22: Mesh linked with the universe

Let's studying a circuit presented figure 3.24 where three vertices are in interactions. A functional interaction goes through vertices 2 and 3 through cords $-G$ and $-Mp$, starting from vertex 1. But an unintentional interaction $-\gamma$ exists directly between vertices 1 and 3. A feedback loop is represented by the interaction $-\alpha$.

Just seeing the graph we can establish the impedance operator tensor ζ of the manifold:

$$\zeta = \begin{bmatrix} \zeta_{11} & 0 & -\alpha \\ -Mp & \zeta_{22} & 0 \\ -\gamma & -G & \zeta_{33} \end{bmatrix} \quad (3.180)$$

γ being an unintentional interaction, we can wonder what is its influence compared to the command coming from the path 1-2-3? To do that, we want to compute the metric g_{31} . By

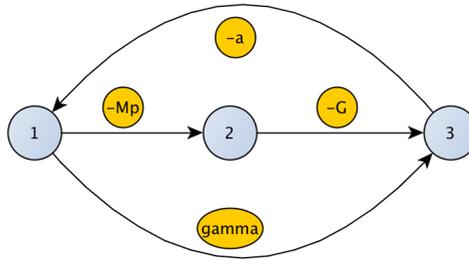


Figure 3.23: A circuit with feedback loop

definition,

$$g_{31} = \frac{x_3}{y^1}$$

x^k being the covector of sources and y^m the vector of fluxes.

We compute:

$$x_3 = -Gy^2 - \gamma y^1 = -\frac{G}{\zeta_{22}}x_2 - \gamma y^1 \Rightarrow g_{31} = \frac{GMp}{\zeta_{22}} - \gamma$$

it shows that:

$$g_{31} = \frac{\zeta_{32}\zeta_{21}}{\zeta_{22}} - \zeta_{31} \quad (3.181)$$

The ratio r of wanted interactions on unwanted interactions is given by:

$$r = \frac{\zeta_{32}\zeta_{21}}{\zeta_{22}\zeta_{31}} \quad (3.182)$$

It's clear that to go from vertex 1 to vertex 3 there are multiple possible paths. Note that if we don't make appear x_1 , the feedback loop does not intervene in the result. Let's test another topology (another manifold).

A first case is the same network as the one presented figure 3.24. We consider the case with $\alpha = 0$ and all the interactions are symmetric (i.e. they act on both sides. For example the interaction $-Mp$ acts on vertices 1 and 2). In that case the impedance operator is given by (we replace the interactions functions by three symbolic functions a, b, c):

$$\zeta = \begin{bmatrix} \zeta_{11} & a & c \\ a & \zeta_{22} & b \\ c & b & \zeta_{33} \end{bmatrix} \quad (3.183)$$

c is the undesired interaction. If we compute the metric between vertex 3 and source 1 we find:

$$x_3 = cy^1 + by^2$$

but:

$$y^2 = \frac{1}{\zeta_{22}} (ay^1 - by^3)$$

by replacement:

$$x_3 \left(1 + \frac{b^2}{\zeta_{22}\zeta_{33}} \right) = cy^1 + \frac{ab}{\zeta_{22}} y^1$$

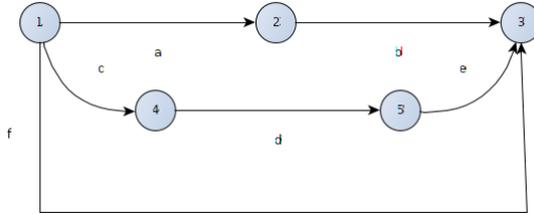


Figure 3.24: A network with three paths to vertex 3

Finally:

$$g_{31} = \frac{x_3}{y^1} = \frac{c\zeta_{22}\zeta_{33} + ab\zeta_{33}}{\zeta_{22}\zeta_{33} + b^2} \quad (3.184)$$

In many cases, we will verify $b^2 \ll \zeta_{22}\zeta_{33}$.

Under this assumption:

$$g_{31} \approx c + \frac{ab}{\zeta_{22}} \quad (3.185)$$

The ratio is that time given by:

$$r = \frac{1}{\zeta_{22}} \frac{ab}{c} \quad (3.186)$$

Reusing the previous case where all the interactions were asymmetric, we can think that the interaction between vertices 1 and 3 is given by:

$$g_{31} = f + \frac{ab}{\zeta_{22}} + \frac{cde}{\zeta_{44}\zeta_{55}} \quad (3.187)$$

It's easy to verify that the metric effectively has this expression. So, we can generalize the equation of a path when all the interactions are asymmetrical relations:

$$g_{ab} = \zeta_{ab} + \sum_k \frac{\zeta_{ak}\zeta_{kb}}{\zeta_{kk}} + \sum_{m,n} \frac{\zeta_{am}\zeta_{mn}\zeta_{nb}}{\zeta_{mm}\zeta_{nn}} + \dots \quad (3.188)$$

A path of order N evolves N vertices between the source vertex and the target one.

There is no direct relation between the importance of a path and the energy that it gives to the target vertex. Another point is that previous equations can be written if the impedance operators can be reduced to Laplace's functions. Without what the expressions are more complicated because we cannot divide by a so simple way.

If we limit the study to the path of order one with intermediate vertices m , the expression for g_{ab} in case of symmetrical interactions becomes:

$$g_{ab} = \sum_m \left\{ \zeta_{ab} \frac{\zeta_{bb}\zeta_{mm}}{\zeta_{bb}\zeta_{mm} + (\zeta_{bm})^2} + \frac{\zeta_{bm}\zeta_{ma}\zeta_{bb}}{\zeta_{bb}\zeta_{mm} + (\zeta_{bm})^2} \right\} \quad (3.189)$$

Filters

We use the structure presented figure 3.12. Branches a and γ are resistances R_1 and R_2 , in series with an inductance L_2 . The

branch b is a capacitor C . The impedance tensor of this network is:

$$\zeta = \begin{bmatrix} R_1 + \frac{1}{Cp} & -\frac{1}{Cp} \\ -\frac{1}{Cp} & R_2 + \frac{1}{Cp} + L_2p \end{bmatrix} \quad (3.190)$$

We add a last stage including a transformer of gain G with supplies a load $R_3 + L_3p$ which is a third vertex. The tensor becomes:

$$\zeta = \begin{bmatrix} R_1 + \frac{1}{Cp} & -\frac{1}{Cp} & 0 \\ -\frac{1}{Cp} & R_2 + \frac{1}{Cp} + L_2p & -Gp \\ 0 & -Gp & R_3 + L_3p \end{bmatrix} \quad (3.191)$$

Now an undesired interaction exists between first and last vertex, γ . The metric is finally:

$$\zeta = \begin{bmatrix} R_1 + \frac{1}{Cp} & -\frac{1}{Cp} & 0 \\ -\frac{1}{Cp} & R_2 + \frac{1}{Cp} + L_2p & -Gp \\ -\gamma & -Gp & R_3 + L_3p \end{bmatrix} \quad (3.192)$$

By noting:

$$D = (R_3 + L_3p) \left(R_2 + \frac{1}{Cp} + L_2p \right) + G^2p^2 \quad (3.193)$$

The distance between the load and the source is dependent of:

$$g_{31} = \frac{1}{D} \frac{G}{C} (R_3 + L_3 p) - \frac{\gamma}{D} (D - G^2 p^2) \quad (3.194)$$

which can be written:

$$g_{31} = \frac{G}{DC} \zeta_{33} - \gamma \left(1 - \frac{G^2 p^2}{D} \right) \quad (3.195)$$

We can search the extrema of the distance depending on the coupling γ :

$$\frac{\partial g_{31}}{\partial \gamma} = 0 \Rightarrow (R_3 + L_3 p) \left(R_2 + \frac{1}{Cp} + L_2 p \right) = 0 \quad (3.196)$$

For the two roots:

$$p_i = \frac{-R_2 C \pm \sqrt{R_2^2 C^2 - 4L_2 C}}{2L_2 C}$$

g_{31} doesn't depend on the parasitic coupling γ . In other cases, if

$$\gamma > \left| \frac{G \zeta_{33}}{C (D - G^2 p^2)} \right|$$

The parasitic coupling becomes the major reason of the flux measured on vertex 3.

How to interpret this distance? The metric g_{31} is not sufficient by itself. Imagine you want to go from the point A to the point B . You know the distance AB but it remains to choice how to cover this distance and to pass the interfaces between the

location where you are and the beginning of the path. In electromagnetic compatibility as in mechanics, depending on the receiver - the end of the trip - it exists always a probability to reach this receiver. Its sensitivity is not a fixed value, but an average one with its own distribution around the average value. For example if x is the location of the receiver, the repartition function $\mathcal{P}(x)$ says what is the probability for x to be inferior to χ :

$$\mathcal{P}(\chi) = \int_0^{\chi} P(x < \chi) dx \quad (3.197)$$

The emitter has a similar repartition function \mathcal{A} that says how it emits around its average level of emission:

$$\mathcal{A}(Y) = \int_0^Y P(y < Y) dy \quad (3.198)$$

From this definition we can obtain the complementary one $\bar{\mathcal{A}}$ that says what is the probability that the emitter radiates beyond to a given value:

$$\bar{\mathcal{A}}(Y) = 1 - \int_0^Y P(y < Y) dy \quad (3.199)$$

The effective distance of interaction between the emitter and the receiver is then defined by the integral of the intersection between the two functions $\bar{\mathcal{A}}(y)$ and $\mathcal{P}(x)$ at the end of the path. The level of intersection d between vertices 1 and 3 is:

$$d(1, 3) = \frac{1}{\nu - \mu} \int_{\mu}^{\nu} dx, y \bar{\mathcal{A}}[g_{31}(y^1)] \cdot \mathcal{P} \quad (3.200)$$

with:

$$\bar{\mathcal{A}}[g_{31}(y^1)] = 1 - \int_0^Y P(g_{31}(y^1) < Y) dy \quad (3.201)$$

distance d is normalized. In electromagnetic compatibility we try to find the distance d equal to zero between emitters undesired and receivers. If the distance is not low enough (seeing that the meaning is inverse to the one used with geometrical distances), there are two ways to decrease it:

- by increasing the segregation between the vertices. i.e. to make them less neighbors;
- by increasing the filters that can protect the receiver.

In the first case we make prevention. In the second case we make protection. These are the two fundamental approaches of safety and we may trace a curve giving the limits for each couple of solution in prevention and protection. We look at this approach later.

Cavities on a single mode

A cavity is a resonator when it works on a single mode. This resonator can be modeled using a simple *RLC* circuit. The values of the components are obtained starting from the electromagnetic energy stored in the cavity. For a given distribution function of the electric field $E_x f(y, z)$ we compute the energy

stored which must be equal to CV^2 :

$$CV^2 = X \int_{y,z} dydz \epsilon_0 E_x^2 f^2(y, z) \quad (3.202)$$

The relation for the inductance links the magnetic energy stored and Li^2 .

Once the *RLC* circuit is determined, it gives one frequency of the mode defined by the distribution of the field considered in the computation of the components. What happens if the cavity is filled? We can imagine a track of a PCB, loaded by two resistances R_L . This track is located at known coordinates in the volume of the cavity. If the magnetic field at this location is $B_{y,z}$, the EM force induced in the track of length Y and height h is $hYpB_{y,z}\vec{u}_y\vec{u}_x \times \vec{u}_B$. The magnetic field is linked with the current in the resonator using also the relation in energy:

$$Li^2 = \frac{X}{\mu} \int_{y,z} dydz B^2 \text{Sin}^2 \left(n\pi \frac{y}{Y} \right) \text{Cos}^2 \left(q\pi \frac{z}{Z} \right) \Rightarrow B = 2\sqrt{\mu L}i$$

A cord can be added between the resonator circuit and the circuit of the track of value:

$$\frac{e}{i} = hXYp2\sqrt{\mu L}\vec{u}_y\vec{u}_x \times \vec{u}_B \quad (3.203)$$

The fact that the two circuits are coupled will automatically change the mode frequency and its amplitude. If all the couplings of the objects inside the cavity with the mode are computed, the effect of filling the cavity is automatically taken into account in the mode modification.

Chapter 4

Domains for multi physics

Any material has different behaviors when it is submitted to various environments. For example a solid can become liquid if the temperature is high enough. Its capacity to conduct electricity depends on its state: solid or liquid. For each of these state, it exists a law that says how the material conducts electricity. But when the temperature goes from low to high, the material changes progressively from solid to liquid. And there is a moment where it is partly solid and partly liquid. So we need a mechanism that allows to change of law depending on parameters and allows to mix laws when the material is in transition state. To do that we define domains.

4.1 Principles

We have seen that the system can be defined using impedance operators and their tensor ζ . In general any part of a system is characterized by a given law like $e_1 = \zeta_{11} \cdot f^1$. This law makes a link between the effort e_1 and the flux f^1 .

Now we said that depending on parameters, this equation could become: $e_1 = \zeta'_{11} f^1$. But even more, if the parameter is changing of value, the law may be a composite one like: $e_1 = \alpha \zeta_{11} f^1 + \beta \zeta'_{11} f^1$. α and β are weights associated with each states.

The function arctangent evolves from $-\pi/2$ to $+\pi/2$. So the function D_{x_0} :

$$D_{x_0} = \frac{1}{\pi} \left[\frac{\pi}{2} + \arctan(x - x_0) \right] \quad (4.1)$$

evolves from 0 to 1. The value is 1 when $x > x_0$ and 0 before. The complementary \bar{D}_{x_0} evolves from 1 to 0. We can define a domain \mathcal{D} by:

$$\mathcal{D}_{[x_0, x_1]} = D_{x_0} - D_{x_1} \quad (4.2)$$

(note that others functions than the arctangent function can be used. The logistic function for example). We can respect:

$$\sum_i \mathcal{D}_{[x_i, x_{i+1}]} = 1 \quad (4.3)$$

We consider two laws attached to two values of an operator v . First law corresponds to the values of v in the interval $[v_1, v_2]$, and the second law corresponds to the values of v that belong

to $[v_2, v_3]$. These two laws are grouped in a single impedance operator defined by:

$$\zeta = \overset{v}{\mathcal{D}}_{[v_1, v_2]} \zeta_1 + \overset{v}{\mathcal{D}}_{[v_2, v_3]} \zeta_2 \quad (4.4)$$

This complicated operator can be applied to a flux, and the active law depending on the parameter v will automatically change during the computation.

Theoretically, the domain must cover the all real domain of the parameters values. If they belong to \mathbb{R} , we should cover \mathbb{R} . But in practice, the studies are focused on part of \mathbb{R} ; see physically, the interval $[-\infty, 0[$ can be meaningless (for example in temperature). So it's clear that the intervals will be reduced to values of interest, the only constraints being that all the parameters must be defined for the same global interval in the same problem.

4.2 Modeling a diode

A diode is a component that has two states: one where it behaves like an open circuit and one where it behaves like a short-circuit.

For this component we study a first domain, $v \in \mathbb{R}^-$ for the voltage across the diode for which the diode is an open circuit, and a second domain for the voltage $v \in \mathbb{R}^+$ where the diode is a short-circuit. The curve linked with the physical component is given figure 4.1. This curve could be more complicated, but for the moment we want to limit the number of parameter to one.

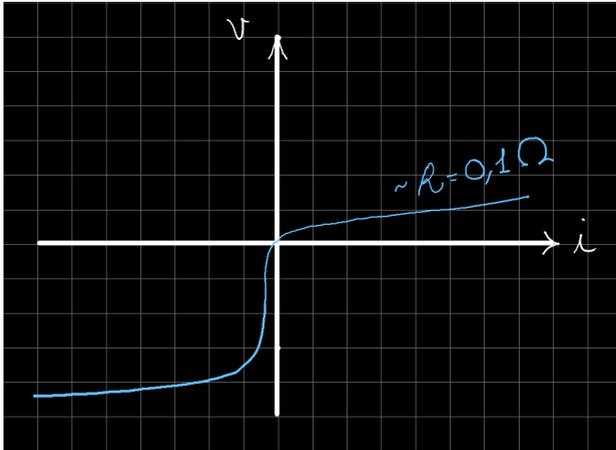


Figure 4.1: Characteristic curve of a diode

If v belongs to \mathbb{R}^+ , the diode behaves like a resistor of value $0,1\Omega$. If v belongs to $]0, -v_b]$ the diode behaves like a resistor of $100k\Omega$. And if v is inferior to $-v_0$ by negative values, it behaves like a resistor of $0,1\Omega$ in series with a force of v_0 volts. So we have three domains for v and two domains for the force. We can study this component through a single vertex including a source e_1 , its own impedance R_1 , an inductance L_1 and the diode. The impedance operator is:

$$\zeta = R_1 + L_1p + z_{diode} \tag{4.5}$$

4.3 Modeling a two parameters object

If two parameters are involved, a tab must be inform to describe what is the law attached with each intersection of domains. For example if we consider a component that depends on the voltage across it and the current flowing inside it.

The first work consists in defining the laws associated with each assumption of domains. If there are two parameters, four laws must be define. But sometimes, the same law can be used for more than one domain. Next tab shows the kind of information which has to be completed.

	$[-v_L, v_0]$	$[v_0, v_L]$
$[-i_L, i_0]$	R_H	$R_c + Lp$
$[i_0, i_L]$	R_i	$R_c + Lp$

At each intersection of cases, the product of domains select the corresponding laws:

	$[-v_L, v_0]$	$[v_0, v_L]$
$[-i_L, i_0]$	$\overset{v}{\mathcal{D}}_1 \overset{i}{\mathcal{D}}_1$	$\overset{v}{\mathcal{D}}_2 \overset{i}{\mathcal{D}}_1$
$[i_0, i_L]$	$\overset{v}{\mathcal{D}}_1 \overset{i}{\mathcal{D}}_2$	$\overset{v}{\mathcal{D}}_2 \overset{i}{\mathcal{D}}_2$

Once this tab available, all the work is done. The impedance operator that represents the real object is completely defined. It can be enclosed in the global operator $H(\zeta)$ of the problem (H represents the electronic part of ζ which is complete by the metric \mathcal{L}). This technique is very physical, more than using "if" instructions. There are no break in the evolution of the system. The component can also be in a transition phase where two states coexists. These states of transition correspond to the intersection between two domains where two laws intervene in the metric between the flux and the source. The technique with the domains allows in fact to represent any kind of behaviors observed on the real objects.

4.4 Power chopper modeling

The power choppers are nowadays widely used in systems to embed efficient power supplies. Their main disadvantage concerns electromagnetic compatibility (EMC). Due to the need of decreasing thermal emissions, the switches are often commanded with fast rise times. This creates wide frequency band of emissions in EMC. So, there is a real need to be able to prevent such emissions having models available. The purpose of this article is to submit some modeling techniques focused on power choppers.

These models are enclosed in the tensorial analysis of networks formalism ¹.

4.5 Basic R-L circuit with switch

Before to treat of complicated power choppers, the first step consists in looking for very simple circuits like RC or RL circuits. This kind of test with the simplest circuits should be made every time validating the switches models.

The tensorial analysis of networks (TAN) works with impedance operators. In a RC circuit we have four components:

1. the generator;
2. the resistance R ;
3. the capacitor C ;
4. the switch Q which is a complicated function.

Through an external command u that drives the switch Q , this switch behaves like an open or a short circuit. The circuit can be realized using a closed circulation of the components. By the fact, it is made of one mesh which impedance operator is:

$$\zeta = R + Q + \frac{1}{Cp} \quad (4.6)$$

¹Leman, S., Demoulin, B., Maurice, O., Cauterman, M., & Hoffmann, P. (2009). Use of the circuit approach to solve large EMC problems. *Comptes Rendus de Physique*, 10(1), 70-82.

p is Laplace's operator. The switch Q is driven by an external source. It is a solid switch like an IGBT transistor. The driver is a command law u . When u is high, the switch is closed and when u is low, the switch is opened. To model this behavior, we use domains. These domains select a law for the switch depending on some external parameters, here the command law. The domain is noticed $\overset{u}{\mathcal{D}}_i$. This means that if the parameter u belongs to the interval i , the law behind \mathcal{D} is selected. Here our operator seems like:

$$Q = \overset{u}{\mathcal{D}}_{[-1,0]} R_H + \overset{u}{\mathcal{D}}_{[0^+,1]} R_b \quad (4.7)$$

R_H is the high value of resistance of the transistor channel when it is blocked and R_b is its low value resistance when it is closed. So if u belongs to the interval of voltages $[-1, 0]$ the transistor behaves like a resistor of value R_H , and when u belongs to the interval of voltage $[0^+, 1]$ the transistor behaves like a resistance of value R_b . Let's take $R_H = 100 \text{ k}\Omega$ and $R_b = 1 \Omega$. When the command goes from one state to the other, an intermediate state can exist. For example if $u = 0,1 \text{ V}$ it is at the limit of both intervals. Physically there are no senses to say that the transistor changes of states in an infinitely thin interval of command. We want that an intersection exists where the two states can exist simultaneously. The function of domain \mathcal{D} cannot be abrupt and must have smooth borders.

4.5.1 Possible domain functions

Various functions have been tested: logistic functions, arctangent functions, erf functions. They have all their advantages and disadvantages. Here we use the erf function that can be easily programmed in Python with the package *math*. The function evolves for a parameter x that belong to $[0, 1]$. We take a look to the function:

$$\text{erf} [\alpha(x - x_0)] \quad (4.8)$$

x_0 is a delay. Figure 4.2 shows the various allures of the erf function changing the coefficient α from 10 to 100. Now if we

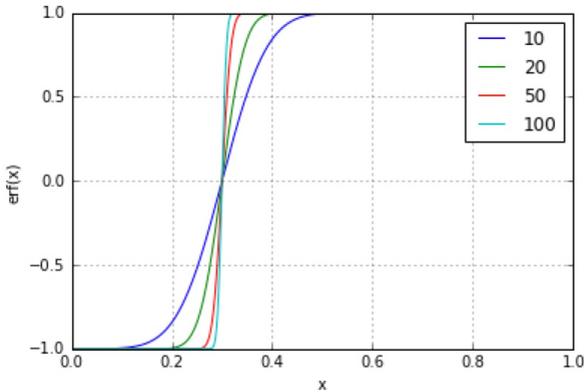


Figure 4.2: Alpha dependences

retire from this function the same one but delayed, this will generate a window function. Next figure 4.3 shows two windows made by adding positive and negative erf functions. A factor 1/2 is applied to normalize the amplitudes. The positive window that is traced is:

$$\mathcal{D}_{[0.01,0.4]}^u = \frac{1}{2} (\operatorname{erf} [100(x - 0,01)] - \operatorname{erf} [100(x - 0,4)]) \quad (4.9)$$

x going from -0,5 to +0,5.

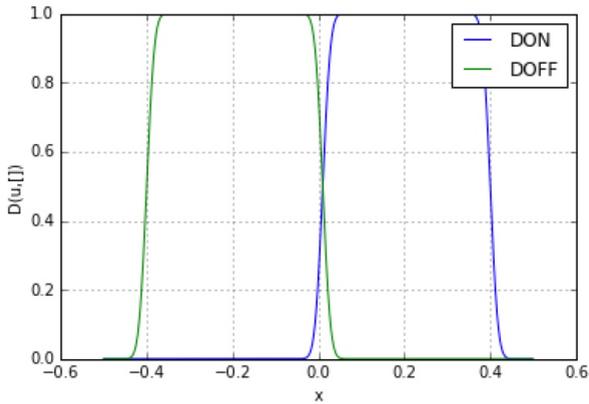


Figure 4.3: D domain

Once the domain functions are defined, the impedance operator becomes easy to define. The only condition is to respect

the intervals used. It means that here, u must evolve between $-0,5$ V and $+0,5$ V.

4.5.2 Seeing the impedance operator changing

Keeping the same domains, we can create a law to test the switch. In our case we can directly use a sinusoidal waveform as law. It evolves between $-0,5$ V and $0,5$ V. Figure 4.4 shows our command law that we will apply to the transistor modeling.

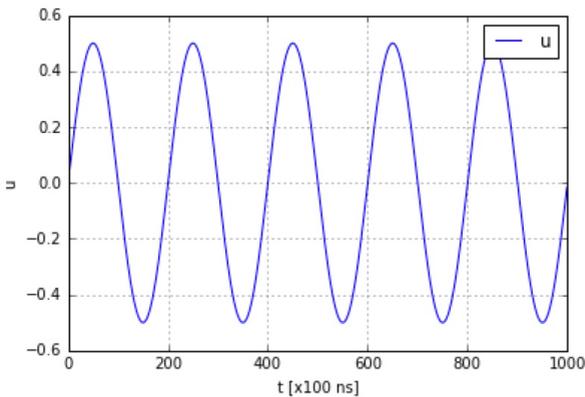


Figure 4.4: Command law

In fact we can set the amplitude of the command depending on the switch speed we want. For an amplitude of $0,1$ V, the

evolution of $Q(t)$ is shown figure 4.5 (the impedance amplitude of Q is divide by 10^5 in order to see both Q and u on the same graph).

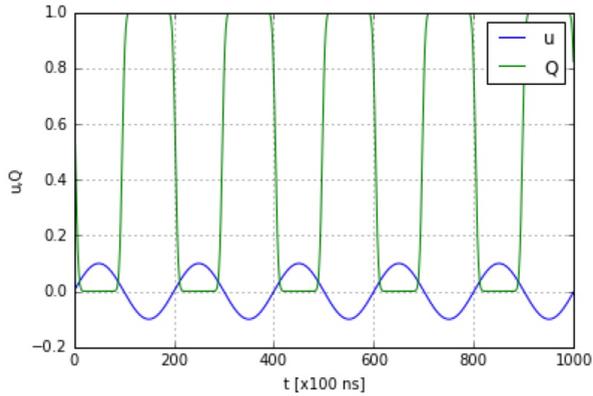
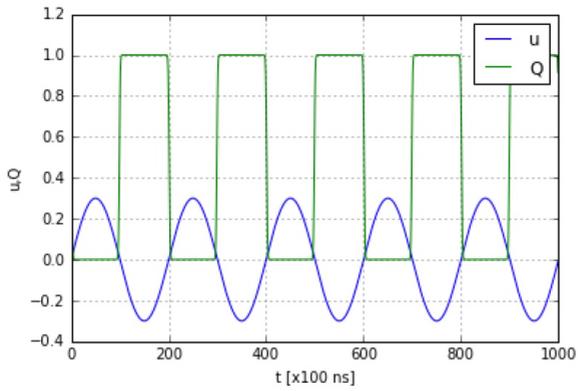


Figure 4.5: Switch response versus u

With a higher level of command, the profile of $Q(t)$ changes as shown figure 4.6. This level is adequate with what we want to do.

4.5.3 Driving a small motor

The previous switch in series with a generator of 10 V can drive the inductance $L = 10 \mu H$ of a motor. As we are interesting

Figure 4.6: Switch response versus u for 0,3 Vp

in real systems, this inductances is in parallel with a resistance that $RL = 1 \Omega$ represents losses or dissipations. The source impedance is $R_0 = 10 \Omega$. The figure 4.7 shows the circuit considered.

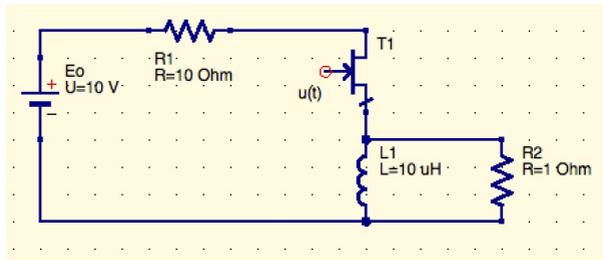


Figure 4.7: Simple RL circuit

The impedance operator of this circuit in the mesh space is given by:

$$\zeta = \begin{bmatrix} R_0 + Q(u) + Lp & -Lp \\ -Lp & Lp + R_L \end{bmatrix} \quad (4.10)$$

The figure 4.8 shows the result with the two mesh currents (we see that the mesh current K^1 is positive and negative. The circuit may also be seen as a DC-AC converter).

But for thermal reasons, a radiator is installed on the transistor. This radiator presents a parasitic capacitor $C = 1 \text{ nF}$ with

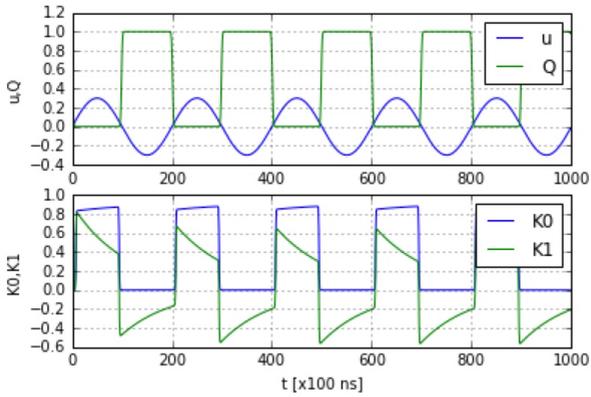


Figure 4.8: Simple RL circuit currents

the structure. The figure 4.9 shows the new graph considered, giving also the three meshes involved in the computation.

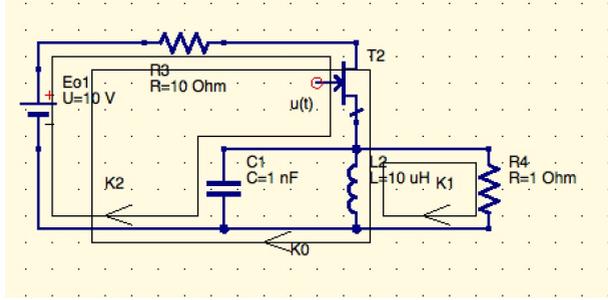


Figure 4.9: Adding parasitic capacitor

The inductance of the connections was negligible compared to the inductance of the load in the mesh 0, but for this new added mesh with the capacitor, we must consider it. So we add for this mesh an inductance L_0 of 100 nH. We add also the resistance of the big inductance $r_{rL} = 10^{-3} \Omega$. The new impedance operator is now:

$$\zeta = \begin{bmatrix} R_0 + Q(u) + Lp + r_{rL} & -Lp - r_{rL} & R_0 + Q(u) \\ -Lp - r_{rL} & Lp + R_L + r_{rL} & 0 \\ R_0 + Q(u) & 0 & \frac{1}{Cp} + R_0 + Q(u) + L_0p \end{bmatrix} \quad (4.11)$$

When we compute this case, we obtain the curves shown figure 4.10. On the curves, we clearly see the oscillations on the mesh 2.

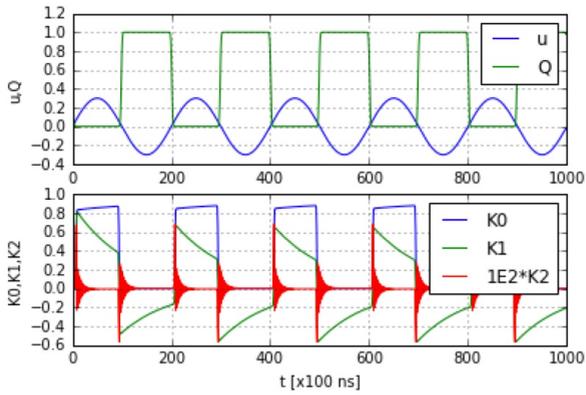


Figure 4.10: Current curves

The parasitic capacitor creates very fast impulse in the common ground. Its amplitude is lower than the main current (near 1 ampere), but it covers a frequency band of 20 MHz. The main current covers 100 kHz. We understand that the high frequency emission of the power chopper will be here due to the parasitic capacitor. Even if the current is one hundred time less than the main one, it is sufficient to go over the limit in radiated or conducted emissions.

But before to study a complete and representative structure, we wonder how the harmonic study may give information on the R-L circuit equipped with a switch? i.e., may have we seen the same behaviors if we take a look to the harmonic modeling?

4.6 Harmonic modeling

When we look at the figure 4.8, we see that the current K^1 makes a little time to be stabilized around 0. During a transition phase, its amplitude evolves to reach this continuous behavior. It is clear that the modeling in the frequency domain does not take into account this transition phase.

4.6.1 RL circuit

If we consider in a first step the simple RL circuit, without the capacitor. Seen from the current K^0 , and neglecting the residual current when the transistor is off, we may retrieve the same current using a pulse generator of duration $\tau = 10 \mu s$? The idea is to replace the continuous power supply in series with the

switch by a single generator of pulse giving the same duration of energy, $10 \mu s$ with the same amplitude ($A = 10 V$) and self resistance (10Ω). Laplace's transform of such an idealized pulse is well known:

$$e(p) = A_f \left(\frac{1}{p} - \frac{1}{p} e^{-\tau p} \right) \quad (4.12)$$

The principle consists in taking the Thèvenin equivalent model for the generator seen across the transistor and inductance branch. Basically, the fundamental frequency is given by $1/T$ where T is the signal period ($20 \mu s$). But equation 4.12 gives the spectrum for a continuous signal while we want to compute it using sampling technique. The spectrum energy density is f_0 time less if we have a single spectrum ray at the frequency f_0 than for a continuous spectrum starting from 0 to f_0 (under the assumption that the spectrum is flat). So we have to multiply the sampling spectrum by a factor $1/T$ to retrieve the good amplitude. As the first zero in the spectrum is given by $1/\tau$, the number of rays should be given by T/τ . How could we set all these parameters using a sampling spectrum? A good criterion is Parseval's relation. Whatever the representation, the power of the signal must remain the same. The amplitude value \hat{e} of the signal obtained from its spectrum (root mean square value: *rms*) is given by:

$$\hat{e} = \sqrt{\sum_n A_n^2} \quad (4.13)$$

where A_n are the Fourier's coefficients.

A first action is to consider the low frequencies impact on the *rms* value is to decrease the fundamental frequency using a

scale factor f_m and

$$f_0 = \frac{1}{f_m T} \quad (4.14)$$

Then we can increase the number of points to go higher in frequency using a factor f_c . Finally the number of points computed is:

$$N = f_m f_c \frac{T}{\tau} \quad (4.15)$$

and the spectrum becomes:

$$\hat{e}(np_0) = \frac{A_f}{T} \left(\frac{1}{p} - \frac{1}{np_0} e^{-\tau np_0} \right) \quad (4.16)$$

A_f as f_m are seted to retrieve 7.56.

Figure 4.11 shows the result obtained for $f_m = 100$, $f_c = 100$, $T = 20\mu s$, $\tau = 10\mu s$ giving $N = 20000$. The values given are the *rms* values computed for e , K^0 and K^1 , to be compared with the temporal signals figure 4.8.

What is very interesting is to see that just looking to their spectrums, we can conclude that the temporal waveform of the current K^0 is similar to the temporal waveform of e , while K^1 has no continuous component, which means that it is symmetric to the 0 volt. The absence of DC value implies the decreasing of amplitude of this signal with time (the suppression of the DC value means a derivative of the original signal), with a time constant of around $30\mu s$ (the cutoff frequency of Bode's equivalent diagram).

This good result suggests that we may use the same approach to study the problem with the capacitor?

At : (9.96988863176+0j)

K0 : (0.857430802774+0j)

K1 : (0.673762374656+0j)

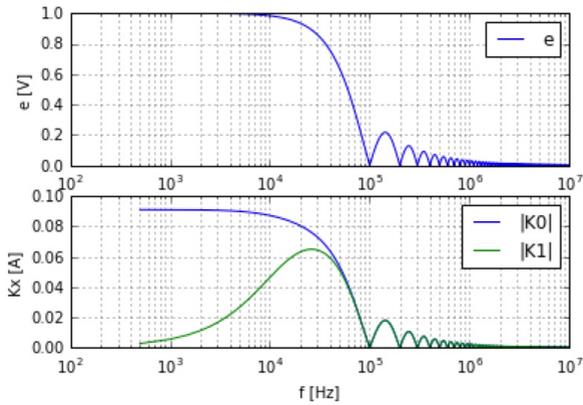


Figure 4.11: Results for RL circuit in harmonic domain

4.6.2 RLC circuit

Figure 4.12 shows the results obtained following the same development, but with this time the added mesh enclosing the parasitic capacitor. The amplitude of K^2 is multiplied by 1000 in order to be visible by comparison to K^0 and K^1 . We see that its frequency spectrum is many more higher than for the other currents.

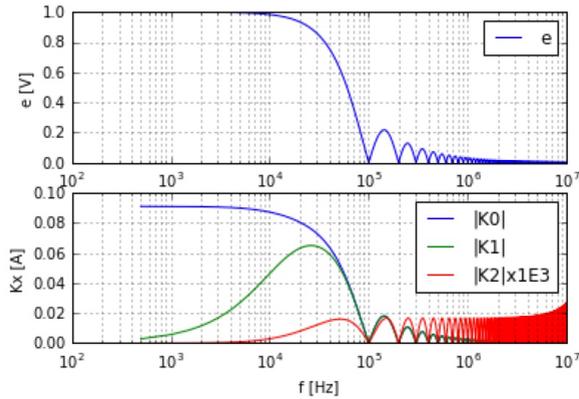


Figure 4.12: Results for RLC circuit in harmonic domain

But we have a problem: the frequency band is not wide enough to show how evolves the spectrum in high frequencies. We have to increase the factor f_c to 1000. With this new set,

we obtain the curve shown figure 4.13.

At : (9.97446078152+0j)
 K0 : (0.857800649095+0j)
 K1 : (0.674087604434+0j)
 K2 : (0.0124008413444+0.0124008413444j)

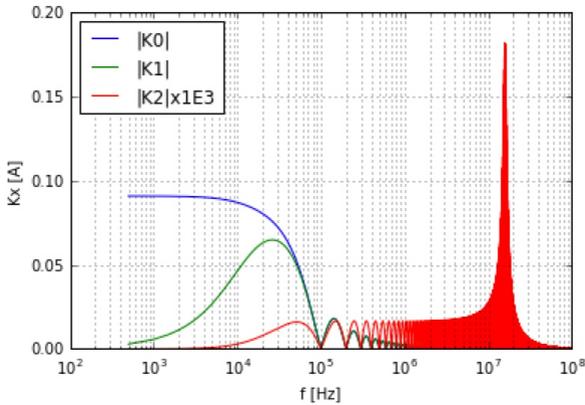


Figure 4.13: Results for RLC, $f_c = 1000$

This time we see the whole spectrum of the parasitic signal. The computation time on a classical computer (not a computation server) is a little higher (around 20 seconds compare to one second before!) but remains acceptable. This new result

shows clearly a frequency of an absorbed oscillation at 18 MHz. Even if we decrease the fundamental frequency, this resonance appears. Here the harmonic study can be many more accurate than the temporal one. We can confirm that the *rms* values remain corrects. It's a good indicator to evaluate the quality of the simulation. The difficulties in the time domain come from the lack of dynamic. We cover as predicted the frequency band of 20 MHz, but the temporal waveform didn't show this level of detail about the absorbing oscillation. The main frequency was evaluated, but it is clearly defined here. Anyway, both temporal and frequency domains lead to the same results.

In this last exercise, our impedance operator is similar to 4.11 but with $Q(u) = 0$. The source covector T is equal to:

$$T = [\hat{e}(np_0) \quad 0 \quad \hat{e}(np_0)] \quad (4.17)$$

and we solve the equation

$$T_\mu = \zeta_{\mu\nu} K^\nu \quad (4.18)$$

4.7 A simple two arms power chopper

A simple power chopper with two arms is represented figure 4.14. Each arm allows to make the current flowing in a chosen direction in the inductive load. By using one time the first arm then the second arm, it gives the possibility to make a motor running one way then the other way.

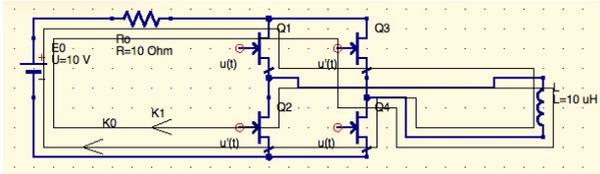


Figure 4.14: Power chopper with two arms

4.7.1 RL circuit

Trusting in our previous results we can simulate the behavior of this structure using a generator with the same Laplace's definition as before. It should give us the solution for the first pair of transistors Q_1, Q_2 . Another interesting point is that seeing the topology, we understand that the two meshes have only two elements in common: the load and the generator. But from the same topology we calculate that we need 3 meshes: the number of branches is 6, the number of nodes is 4, so: $M = 6 - 4 + 1 = 3$. These three meshes can be easily seen: $(R_0 - Q_1 - Q_2)$, $(Q_1 - Q_3 - L)$, $(Q_2 - Q_4 - L)$. These meshes lead to badly conditioned matrices, because they all include a transistor in a blocked mode.

When we understand the working of this circuit, we understand that for reasons of security, both arm cannot works simultaneously with the other. Because we don't want at any time that two transistors like Q_1 and Q_2 for example may short-circuited the power supply. So the two circuits work on two

separate time intervals 1 and 2. If we accept the idea that when the transistors are off, they have an infinite resistance, it results from this reasoning that the two circuits work separately. How may we translate this?

When constructing the meshes, we define the connectivity matrix that gives the links between the branches and the meshes². If we note R the whole resistance including the source resistance R_0 and the conducting transistor junctions R_j , the branch space includes three elements: two resistances R for each mode of working (i.e. for each conducting pair of transistors) and the inductance L . The first branch R wears a generator $\hat{e}(np_0)$. The second branch R wears a generator $-\hat{e}(np_0)e^{-\tau T/2P}$. The delay describing the time separation there is between the two modes, each being represented by a pulse in frequency of duration τ and period T . The connectivity matrix must say that in a first step, branch 2 doesn't exist, i.e. it doesn't participate to the mesh describing the circuit, and in a second step that's the branch 1 which is suppressed. To do that we use a gate function π which is a domain:

$$\pi_1 = \overset{u}{\mathcal{D}}_{[0,T/2]} \quad \pi_2 = \overset{u}{\mathcal{D}}_{[T/2,T]} \quad (4.19)$$

²The technique presented here was first evoked by François COSTA during my HDR defense

With these functions, the connectivity can now be defined:

$$\mathcal{C} = \begin{bmatrix} \pi_1 & 0 \\ 0 & \pi_2 \\ 1 & -1 \end{bmatrix} \quad (4.20)$$

With this connectivity we compute the bilinear product $\mathcal{C}^T \zeta \mathcal{C} = g$ which leads to

$$g = \begin{bmatrix} \pi_1^2 R + Lp & -Lp \\ -Lp & \pi_2^2 R + Lp \end{bmatrix} \quad (4.21)$$

First interesting thing is that the two circuits are cross talked through L , what we have already said. Knowing $\pi^2 = \pi$ and calculating: $W = \mathcal{C}^T e$ where W and e are the source covectors we obtain

$$W = [\pi_1 \hat{e}(np_0) \quad -\pi_2 \hat{e}(np_0) e^{-\tau T/2p}] \quad (4.22)$$

Finally our circuit becomes equivalent to the couple of circuits shown figure 4.15.

We see that in the impedance operator expression, there are no Christoffel's symbols coming from some time derivatives of the connectivity. That's why in that case, the use of a time dependent connectivity remains quite simple. To ensure that the spectrum do not shows particular effects coming from the singular aspect of the unique pulse represented in the source, it

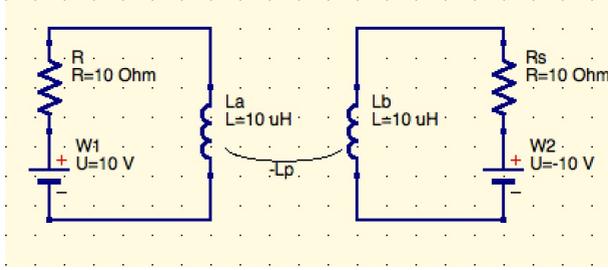


Figure 4.15: Power chopper in a RF representation

can be interesting to reproduce the periodicity of the process, writing and using:

$$\hat{e}'(np_0) = \hat{e}(np_0) (1 + e^{-Tp} + e^{-2Tp} + e^{-3Tp} + \dots) \quad (4.23)$$

We compute the result this time for an inductance of $500 \mu H$ which is more representative of this kind of circuit. We go until four times the period for the sources and this gives the curves presented figure 4.16.

To obtain the good correspondence with the peak value, the amplitude was reduced to $0,1 \text{ V}$ for A_f , with $f_m = 10^4$ and $f_c = 10$.

4.7.2 RLC circuit

The load has a parasitic capacitor with the structure $C_m = 100 \text{ nF}$, as the generator $C_c = 1 \text{ nF}$. The figure 4.17 shows the

At : (9.66247714837+0j)

Ats : (9.66247714837+0j)

K1 : (0.696800746149+0j)

K2 : (0.736637053628+0j)

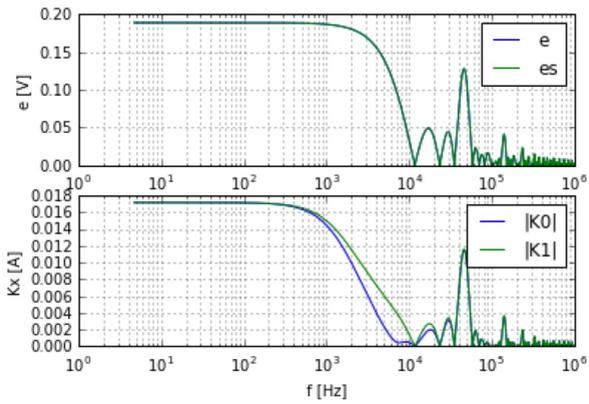


Figure 4.16: Power chopper in 2 coupled circuits

graph for one pair of transistors. We see that the parasitic couplings add a mesh that includes the parasitic capacitors and the common ground. To make in evidence the effect of this new path, we need to consider the impedance r in parallel to the ground, i.e. the resistance of one transistor (eventually the resistance of the wires, etc.).

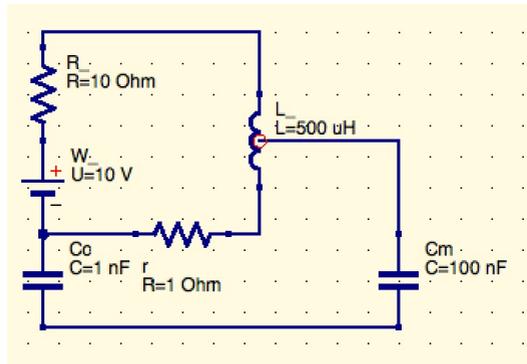


Figure 4.17: Parasitic path

The impedance operator of this half circuit is given by:

$$\zeta = \begin{bmatrix} R + Lp + r & r - Mp - \frac{L}{2}p \\ r - Mp - \frac{L}{2}p & \frac{L}{2}p + r + \frac{1}{C_m p} + \frac{1}{C_c p} \end{bmatrix} \quad (4.24)$$

A mutual inductance M can exist between the two loops. The

inductance of the loop between the low stage transistor and the ground can be estimated considering the wire as a line up to the ground. The characteristic impedance of this line is:

$$z_c = 60 \ln \left(4 \frac{h}{\phi} \right) \quad (4.25)$$

where h is the height from the wire axe to the ground and ϕ the wire diameter. Knowing z_c we obtain: $L = z_c x/v$, x being the length of the wire. Same estimation can be made to compute the inductance of the line going from the power chopper to the load. The mutual inductance between the two loops is very important and is a major parameter in the result. We can estimate it saying that the current i running on the common wire between the two loops creates a magnetic field in the capacitors loop simply given by:

$$B = \mu \frac{i}{2\pi y} \quad (4.26)$$

So, for a length x and a height h , the mutual is obtained saying:

$$e = -SpB = -px \int_y dy \mu \frac{i}{2\pi y} \Rightarrow \frac{e}{i} = M = \frac{\mu}{2\pi} x \ln \left(\frac{2h}{\phi} \right) \quad (4.27)$$

The evaluation of this mutual inductance is important because the losses are weak in this loop when the transistor is ON (around 1Ω or less). Depending on the value of M we can have a oscillator in or under the critical working (signal decreasing between one period and the next one) given by:

$$\pi R \sqrt{\frac{C}{L - M}} \quad (4.28)$$

Typically, M is ten times lower than L . In our case for one meter of length we find $L = 1,5 \mu H$ for the inductance of the line and $M = 921 nH$ for the mutual inductance. The curves given by the calculations are presented figure 4.18 ($f_m = 1000$, $f_c = 100$).

At : (58.8955645803+0j)
 K0 : (2.52934194807+0j)
 K1 : (0.106127400832+0j)
 Vrc : (44.412734227+0j)

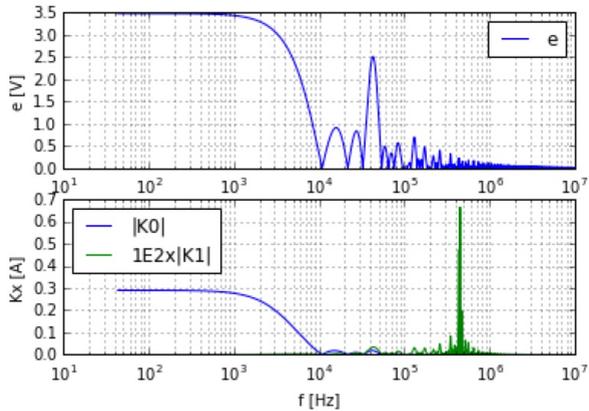


Figure 4.18: Current in the ground

We see that the estimation of the voltage developed across the low stage transistor (V_{rc}) is very high due to the quality factor of the resonant ground loop. We must be careful that our assumption to make the correspondence with the *rms* value are not really respected here, because the signal has a straight band. Anyway it gives an indicator on the risk. We can try to decrease this voltage by increasing the value of C_c , adding a real capacitor between the box of the power chopper and the structure. Let's try using $0,1 \mu F$. We obtain the curves figure 4.19. The *rms* value decreases to 33 V rather than 44. The resonance frequency decreases as waited around 45 kHz rather than 450 kHz.

Probably a solution is elsewhere to avoid these high level currents flowing into the common ground (they will create high level of radiated emissions, as common impedance coupling). Using a shielded cable may be one of these solutions, thinking that it is not possible to change the capacitor C_m .

4.7.3 RLC circuit with two arms

We keep the same approach as before with this time the two arms involved. Figure 4.20 shows the circuit considered.

Note that this new case is very easy to implement. You just have to make the direct summation of the previous impedance operator $\zeta \oplus \zeta$ then to add the coupling between meshes 1 and 3. A first result with $C_c = 1 nF$ is given figure 4.21. We see that the level of voltage across the low stage transistor becomes this time very high, increased by the coupling between the two arms. If we change the value of C_c to 100 nF, the decreasing of

At : (58.8955645803+0j)
K0 : (2.52096486491+0j)
K1 : (0.669290732043+0j)
Vrc : (33.4322540063+0j)

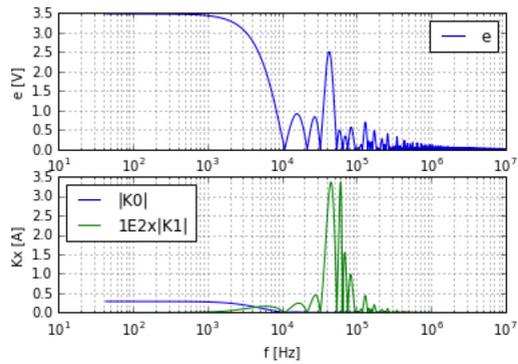


Figure 4.19: Current in the ground with $C_c = 100 \text{ nF}$

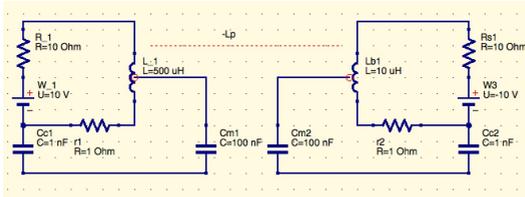


Figure 4.20: Circuit with 2 arms and capacitors

voltage is this time evident, being around ten time lower.

4.8 Theoretical analysis

The force of the tensorial analysis of networks formalism comes from the compact aspect of the systems of equations it gives even for complicated problems. But another force for which it is perhaps the unique method to reach effectively this capacity, is that it offers the possibility to study theoretically the problem. Looking to the system 4.24, we can study theoretically its behavior, depending on the values of C_c (remember that we cannot chose C_m).

Simplifying this system we write:

$$\zeta = \begin{bmatrix} \sigma + 2Hp & -Hp \\ -Hp & r + Hp + \frac{1}{\gamma p} \end{bmatrix} \quad (4.29)$$

A_t : (58.8955645803+0j)
 K_0 : (3.73162341375+0j)
 K_1 : (0.0902465658015+0j)
 K_2 : (3.96614176892+0j)
 K_3 : (0.0902241001854+0j)
 V_{rc} : (224.420908928+0j)

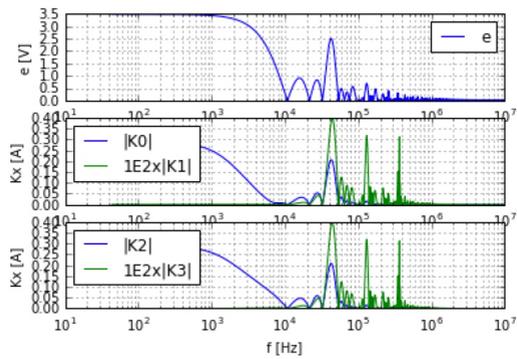


Figure 4.21: Circuit with 2 arms and $C_c = 1 \text{ nF}$

At : (58.8955645803+0j)
 K0 : (1.93987424706+0j)
 K1 : (0.611835171388+0j)
 K2 : (2.30882481892+0j)
 K3 : (0.594600813801+0j)
 Vrc : (23.0018828933+0j)

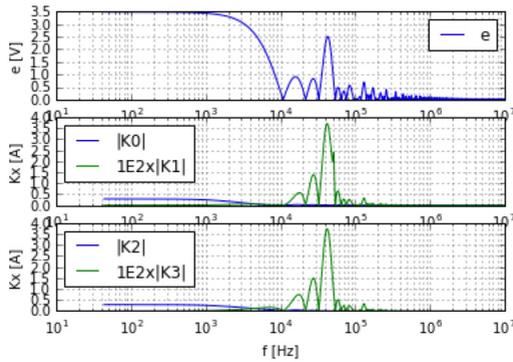


Figure 4.22: Circuit with 2 arms and $C_c = 100 \text{ nF}$

The determinant is:

$$\Delta = (\sigma + 2Hp) \left(r + Hp + \frac{1}{\gamma p} \right) - H^2 p^2 \quad (4.30)$$

Taking into account that r and σ are low values, we simplify the determinant to:

$$\Delta \approx 2H \left(\frac{1}{\gamma} + Hp^2 \right) \quad (4.31)$$

We can calculate the voltage $|1/(\gamma p)K^1|$:

$$\left| \frac{1}{\gamma p} K^1 \right| = \frac{1}{2} \frac{1}{|1 - \gamma H \omega^2|} \quad (4.32)$$

The cutoff frequency of Bode's diagram for this function is $\omega_0 = \sqrt{\gamma H}$. The higher γ , the lower ω_0 . So, it is interesting to increase γ in order to decrease ω_0 then to benefit of a narrower frequency band. If the frequency band is narrower, the energy associated with this frequency band will be lower too, and as a consequence, the *rms* value will be lower also. But this solution has for impact to accept that a parasitic current flows into the ground. A better solution may be to keep this current enclosed in a shielded cable making the link between the power chopper and the motor.

4.9 Multiphysics

It must be clear that the models I submit are perhaps not pertinent. I obviously don't pretend having competencies in other

jobs than the EMC. But the principles and approaches proposed to treat multi physic problems are correct, even if the models are not.

Multi-physics means that various manifold are summed, one for the electromagnetism, one for mechanics, one for temperature, etc. Coupling functions are not homogeneous and report a temperature into a law of a resistance, and so on. The report can be achieved directly into a law or as the value of a parameter attached with a domain. When many physics are implied, the use of domains can be a simplest way to solve some modelings.

For illustrating these mechanisms, we take a look to transducers, then we imagine a patch antenna that is submitted to a surface that can see its own temperature increasing and its mechanics submitted to vibrations. This can be typically the case of a patch antenna located near to a motor on a car, and mounted on the structure of the bonnet. The work is organized in three steps:

1. to determine the graph for each physic;
2. to extend each sub-manifold including the parameters of the other physics;
3. to add the function of coupling between the physics.

4.9.1 Transducers

A transducer is a mechanism that makes the transformation of electrical energy to mechanical energy (motor mode) or from mechanical energy to electrical energy (dynamo mode).

It is a RLC circuit connected to a coil. This coil is immersed in a static magnetic field, and linked to a mass and a spring. The figure 4.23 shows this mechanism.

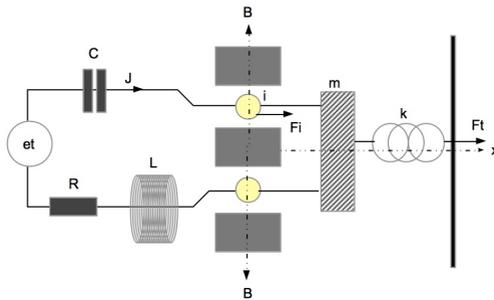


Figure 4.23: Transducer

Principles of this mechanism are well introduced by Gabilard³ And I start here from his description.

If there is a current i in the coil, with the presence of the magnetic field \mathbf{B} , Laplace's force appears given by:

$$\mathbf{F}_i = i\mathbf{l} \times \mathbf{B} \quad (4.33)$$

³R.Gabillard, *Vibrations et phénomènes de propagation*. Dunod university editor, 1969 Paris, France.

An EM force is created given by:

$$f_e = \frac{1}{q} \int_x dx \mathbf{F}_i = \mathbf{l} \times \mathbf{B} \frac{d}{dt} \int_x dx = -l \mathbf{B} \times \mathbf{v} \quad (4.34)$$

avec $|\mathbf{v}| = \dot{x}$. A force F_t is applied on the mass m through the spring k . The signe of F_i depends on the coil orientation. We chose it in order to have F_i in the positive direction of x . Then:

$$m \frac{d}{dt} v = F_i + F_t - kx - \alpha v \quad (4.35)$$

α being the losses coefficient.

For the circuit of the generator we obtain in the mesh space:

$$L \frac{d}{dt} J + RJ + \frac{1}{C} \int_t dt J = e_t + f_e \quad (4.36)$$

Finally:

$$\begin{cases} (mp + \alpha) v + \frac{k}{p} v - \phi J = F_t \\ (Lp + R) J + \frac{1}{Cp} J + \phi v = e_t \end{cases} \quad (4.37)$$

with $\phi = lB$ the linear density of magnetic flux. Noting:

$$x^\alpha = \begin{bmatrix} J \\ v \end{bmatrix} \quad \pi_\beta = [e_t \quad F_t] \quad (4.38)$$

and

$$\zeta = \begin{bmatrix} Lp + R + \frac{1}{Cp} & \phi \\ -\phi & mp + \alpha + \frac{k}{p} \end{bmatrix} \quad (4.39)$$

we want to solve $\pi_\beta = \zeta_{\beta\alpha}x^\alpha$. Due to the negative sign of ζ_{21} , opposite to the sign of ζ_{12} , the coupling is said to be *gyroscopic*. If the sign is the same for both ζ_{21} and ζ_{12} the coupling is said to be *parametric*. The difference is important. If the coupling is gyroscopic, D the diagonal elements of ζ and C the couplings, the determinant will have the form $D^2 + C^2$. In the parametric case, it is $D^2 - C^2$. The first form doesn't have poles. It is a fundamental difference in the behavior of the two kinds of systems and linked manifolds.

Motor mode

In that case, $F_t = 0$. We can look for the impedance across the input of the mechanism:

$$Z_E = \frac{e_t}{J} = R + Lp + \frac{1}{Cp} + \phi^2 \frac{\alpha - j \left(m\omega - \frac{k}{\omega} \right)}{\left[\alpha^2 + \left(m\omega - \frac{k}{\omega} \right)^2 \right]} \quad (4.40)$$

$\Delta = \left[\alpha^2 + \left(m\omega - \frac{k}{\omega} \right)^2 \right]$ being the determinant of ζ .

Writing:

$$Z_E = (R + R_m) + j \left(L\omega - \frac{1}{C\omega} - \chi_m \right) \quad (4.41)$$

we obtain:

$$\begin{cases} R_m = \frac{\alpha\phi^2}{\left[\alpha^2 + \left(m\omega - \frac{k}{\omega}\right)^2\right]} \\ \chi_m = \frac{\left(m\omega - \frac{k}{\omega}\right)\phi^2}{\left[\alpha^2 + \left(m\omega - \frac{k}{\omega}\right)^2\right]} \end{cases} \quad (4.42)$$

If we trace (R_m, χ_m) depending on the frequency, we obtain a circle named Kennelly's circle. It is a circle of diameter ϕ^2/α with a point noticed $\Omega_0 = (\phi^2/\alpha, 0)$.

The electrical power given to the machine is $W_E = (R + R_m)J^2$, while the mechanical power is $W_m = R_mJ^2$. The yield ρ of the mechanism is given by:

$$\rho = \frac{W_m}{W_E} = \frac{R_m}{R + R_m} \quad (4.43)$$

Except for the trivial but not physical solution $R = 0$, the yield is maximum when R_m is maximum, i.e. on the point Ω_0 of Kennelly's circle.

Dynamo mode

In that case $e_t = 0$ and a load is used to extract the electrical energy generated by the mechanical forces, R_0 . So we have to replace R by $R + R_0$. We obtain under this configuration :

$$J = -\frac{1}{\Delta}\phi F_t \quad (4.44)$$

We look at the transfer function F_t/J . Noting $\sigma = R + R_0$

and

$$\Omega = \left(\frac{L\omega}{\sigma} - \frac{1}{\sigma C\omega} \right)$$

we obtain:

$$\frac{F_t}{J} = -\frac{\sigma}{\phi} \left\{ \left[\alpha - m\omega\Omega + \frac{k\Omega}{\omega} + \frac{\phi^2}{\sigma} \right] + j \left[m\omega + \alpha\Omega - \frac{k}{\omega} \right] \right\} \quad (4.45)$$

which can be written:

$$\frac{F_t}{J} = -\frac{\sigma}{\phi} \{ [\alpha + \alpha_0] + j [m\omega + M] \} \quad (4.46)$$

Now if $L\omega/\sigma = (\sigma C\omega)^{-1}$ and $k = m\omega^2$, $\Omega = 0$ and:

$$\left| \frac{F_t}{J} \right| = \alpha \frac{\sigma}{\phi} + \phi \quad (4.47)$$

If the losses α are weak enough,

$$\left| \frac{F_t}{J} \right| \approx \phi \quad (4.48)$$

Now we can look at transducers through the seconde geometrization process. Starting from 4.39, we obtain:

$$\mathbf{b}_1 = \begin{bmatrix} R + \frac{1}{C_p} \\ -\phi \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} \phi \\ \alpha + \frac{k}{p} \end{bmatrix} \quad (4.49)$$

So:

$$G = \begin{bmatrix} \left(R + \frac{1}{Cp}\right)^2 + \phi^2 & \phi \left[R + \frac{1}{Cp} - \alpha - \frac{k}{p}\right] \\ \phi \left[R + \frac{1}{Cp} - \alpha - \frac{k}{p}\right] & \left(\alpha + \frac{k}{p}\right)^2 + \phi^2 \end{bmatrix} \quad (4.50)$$

We can remark that under the second geometrization process there are no more reasons to distinguish parametric or gyroscopic couplings. The transposed jacobian is equal to:

$$\Gamma = \begin{bmatrix} R + Lp + \frac{1}{Cp} & -\phi \\ \phi & mp + \alpha + \frac{k}{p} \end{bmatrix} \quad (4.51)$$

Now computing the distance $ds = \sqrt{G_{12}Jv} = \sqrt{G_{12}x^1x^2}$ we find:

$$ds = \sqrt{\phi \left[R + \frac{1}{Cp} - \alpha - \frac{k}{p}\right] Jv} \quad (4.52)$$

This distance can be equal to zero if $R = \alpha$, $k = C^{-1}$.

Computing $\Gamma\zeta$ we find that $\pi = \zeta x \Leftrightarrow \Gamma\pi - \mathcal{I}x = Gx$, with:

$$\mathcal{I} = \begin{bmatrix} L^2p^2 + 2Lp \left(R + \frac{1}{Cp}\right) & \phi p(L - m) \\ \phi p(L - m) & m^2p^2 + 2mp \left(\alpha + \frac{k}{p}\right) \end{bmatrix} \quad (4.53)$$

With $R = \alpha$, $k = C^{-1}$, G becomes purely diagonal G_d . If we set an electrical frequency $\omega_e = \sqrt{LC}^{-1}$ equal to the mechanical

frequency $\omega_m = \sqrt{k/L}$, we obtain with $m \gg L$:

$$\mathcal{I} = \begin{bmatrix} R^2 & -\phi pm \\ -\phi pm & \alpha^2 \end{bmatrix} \quad (4.54)$$

Noting $A = R+1/Cp = \alpha+k/p$, the problem becomes described by the equation:

$$\begin{cases} Re_t - \phi F_t = (A^2 + R^2 + \phi^2) J + \phi pmv \\ \phi e_t + \alpha F_t = \phi pmJ + (A^2 + \alpha^2 + \phi^2) v \end{cases} \quad (4.55)$$

If $mp\phi^2 \ll R(A^2 + R^2 + \phi^2)$ with $F_t = 0$ this leads to:

$$\frac{e_t}{J} \approx R + \frac{\phi^2}{R} \quad (4.56)$$

which is our previous result.

4.9.2 Patch antenna in electromagnetism

The patch antenna is a resonator. It can be realized like the drawn figure 4.24.

If x is one direction of the patch antenna, the magnetic field can exist inside the structure through the modes $B_0 \sin(n\pi x/X)$. If S is the total surface of the antenna and h the constant height to the structure, its capacitor can be approximated by $C = \epsilon_0 S/h$. The inductance L is $\mu_0 hX/\sqrt{S}$. The EM force induced on the antenna is $e = -hXpB_0$. A load R_L receives the

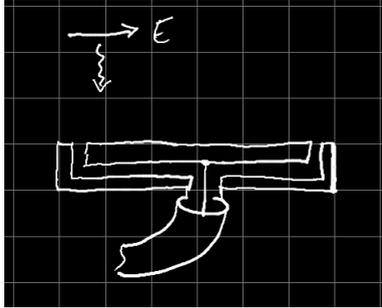


Figure 4.24: Patch antenna

energy captured by the antenna. We can add the far source of field antenna that illuminate the patch antenna by a mesh made of two matched resistances and a Green function obtained from Friis's relation:

$$G = \frac{h}{x} \sqrt{30\mathcal{G}R_a}$$

R_a being the radiation resistance of the emitting antenna and \mathcal{G} its gain. x is the distance between the two antennas and h the effective height of the patch antenna.

The operator of this electromagnetic system is:

$$\zeta_E = \begin{bmatrix} 2R_a & -G \\ -G & R + \frac{1}{Cp} + Lp \end{bmatrix} \quad (4.57)$$

The source is simply $u_E = [U \ 0]$.

4.9.3 Patch antenna from the mechanical point of view

A very interesting work on the equivalence between electrical network and mechanical one was done by G.W. Van Santen [4]. Classical equivalence was not so simple to use, except in simple cases. Van Santen in 1957 submits a new equivalence that allows to make directly the equivalence, even on the structure itself of the network.

For this equivalence:

- the flux f is the "force" p in $[kg]$;
- the load in coulomb becomes the impulsion q in $[kg][s]^{-1}$;
- the EM force (source) becomes the speed at the extremities of the system, \dot{x} , $[cm][s]^{-1}$;
- the inductance L becomes the inverse of the elasticity \bar{k} , $[cm][kg]^{-1}$;
- the resistance R is the inverse of the dissipation \bar{r} , $[cm][kg s]^{11}$;
- the capacitor is the mass m in $[kg s^2][cm]^{-1}$;
- the impedance Z is linked with its inverse \bar{z} , $cm][kg s^2]^{-1}$;
- the power remains the power $p \cdot \dot{x}$, $[kg cm][s]^{-1}$.

With this convention, all impedance expression keeps its form. For example $jL\omega$ becomes $j\bar{k}\omega$ in mechanics, etc. So, under this convention, we construct the graph following the same

approach than for electrical circuits. The impulsion that excites the plate, corresponding to an electromotive force in electromagnetism, becomes a source of speed in mechanics. The extremities of the plate may have create capacitors with the structure. They are transformed in mass for the mechanical graph. It remains the plate itself that acts like a spring: the model is an inductance in series with a resistance that translates the dissipations. Finally we obtain the graph presented figure 4.25.

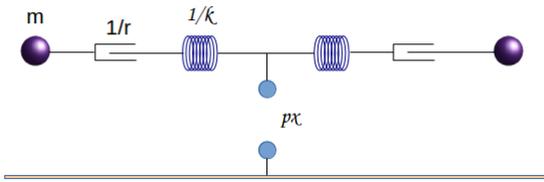


Figure 4.25: Graph of the mechanical part

We may add a dissipation component \bar{r}_0 in series with the source of impulsion to take into account the losses in the transmission of the force to the plate of the patch antenna. We conclude that the operator for this circuit is:

$$\zeta_M = \begin{bmatrix} \bar{r}_0 + \bar{k}p + \bar{r} + \frac{1}{mp} & \bar{r}_0 \\ \bar{r}_0 & \bar{r}_0 + \bar{k}p + \bar{r} + \frac{1}{mp} \end{bmatrix} \quad (4.58)$$

The source of the mechanical system is the acceleration communicated to the system \dot{x} : $u_M = \begin{bmatrix} px & px \end{bmatrix}$.

4.9.4 Temperature

The equivalence between electromagnetism and temperature follows the correspondances:

- $P \leftrightarrow i$;
- $\Delta T \leftrightarrow \Delta U$;
- $mc \leftrightarrow C$.

c is the specific heat. mc is the massique heat. ΔT is the difference of temperatures and P the power. The difference of temperature is equal to the thermal resistance multiplied by the power: $\Delta T = R_{th}P$.

k being the thermal conductivity, S the section of a piece of material and l the length of this piece, the power is given by:

$$P = k \frac{S}{l} \Delta T \quad (4.59)$$

Which conducts to:

$$R_{th} = \frac{1}{k} \frac{l}{S} \quad (4.60)$$

The massique heat is defined by:

$$\Delta T = \frac{1}{mc} \int_t dt P \quad (4.61)$$

or:

$$mc \Delta T = \int_t dt \frac{dW}{dt} = \Delta W \Rightarrow mc = \frac{\Delta W}{\Delta T}$$

The thermal capacity mc is the heat that must be communicated to the body for increasing its temperature of one degree centigrade. The structure reaches a temperature ΔT_s . The temperature is transmitted through the gap of air to the plate which has a given inertia. The thin contact used for the electrical part is neglected.

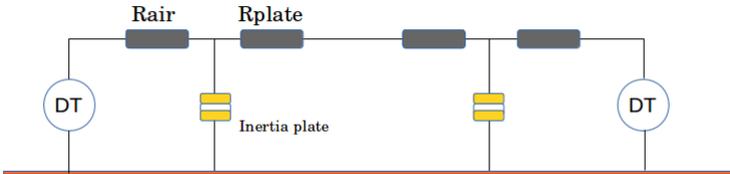


Figure 4.26: Temperature graph for the patch antenna

Figure 4.26 represents the exchange in temperature between the structure and the antenna. The impedance operator for the temperature is:

$$\zeta_T = \begin{bmatrix} R_{air} + \frac{1}{mcp} & -\frac{1}{mcp} & 0 \\ -\frac{1}{mcp} & 2R_{plate} + \frac{2}{mcp} & \frac{1}{mcp} \\ 0 & \frac{1}{mcp} & \frac{1}{mcp} + R_{air} \end{bmatrix} \quad (4.62)$$

The source is: $u_T = [\Delta T \quad 0 \quad \Delta T]$.

4.9.5 Direct sum of the three physics

Once the operator are defined in each physics, first action is to realize the direct sum of all these operators. In our case this means to compute $\zeta = \zeta_E \oplus \zeta_M \oplus \zeta_T$. Same operation is made for the sources: $u = u_E \oplus u_M \oplus u_T$.

For the electromagnetic part, we have seen that the metric is represented by the inductances operator. Probably moreover we should add an inductance L_0 to the mesh of the emitting antenna. In that case, the metric for the electromagnetic part is:

$$\mathcal{L}_E = \begin{bmatrix} L_0 & 0 \\ 0 & L \end{bmatrix} \quad (4.63)$$

But what can it be for the mechanical physics? Reading the correspondences of the new equivalence, it should be the inverse of the elasticity.

Does that make sense? The elasticity gives the relation between the displacement and the force: $\mathbf{F}_x = k \cdot \mathbf{x}$. So $\mathbf{x} = \bar{k} \cdot \mathbf{F}_x$. The inverse of the elasticity \bar{k} leads to:

$$e = \frac{dx}{dt} = \bar{k} \frac{d}{dt} F_x = \bar{k} \frac{d}{dt} f$$

This is completely coherent and \bar{k} appears in this convention as a metric. A last information is that the bilinear product $1/2\bar{k}f^1f^2$ gives the product of the distance by the force xf^2 which is a work, a potential energy.

It remains to define the metric for the temperature. But no component seems to be comparable with a metric.

We must accept that, in the meaning we have given before, it doesn't exist any metric for the physics of the temperature (we will see after that other choices can be made). For all, another observable may have play the role of metric. But the impedance operator is made of only resistance and massique heat and doesn't include time derivative of a variable. By the fact, it doesn't generate with these choices any metric in Kron's meaning.

4.9.6 Adding non homogeneous coupling operators

The vibrations will deform the plate of the patch antenna while the temperature will increase the value of the resistance. In this illustration, there are no other couplings involved between the various physics. But one is symmetric. The temperature increase the resistance, and the power developed in the resistance participates to the radiation of heat.

The first coupling operator links mechanics with the capacitor value of the patch antenna (major effect) and the second operator of coupling links the value of the resistance of the patch antenna with the temperature of the structure.

The resistance depends on the temperature through the law:

$$R = R_0 \left[1 + \left(\frac{T}{T_s} \right)^2 \right].$$

The vibrations change the value of the capacitor, changing the height h of the structure.

The system manages three kinds of fluxes: the currents J , the impulsions q and the power P .

The link between the distance defining the capacitor and the impulsion is:

$$h = \frac{1}{m} \int_t dt q^1$$

as the problem is symmetric, the link with q^1 doesn't need to be completed. The system of equations of the multi physic problem is:

$$\left\{ \begin{array}{l} 2R_a J^1 - GJ^2 - U = 0 \\ -GJ^1 + \left\{ R \left[1 + \left(\frac{T}{T_s} \right)^2 \right] + Lp \right\} J^2 + \frac{q^1}{p^2 \epsilon_0 S m} J^2 = 0 \\ \left(\bar{r}_0 + \bar{k}p + \frac{1}{mp} \right) q^1 + \bar{r}_0 q^2 - px = 0 \\ \bar{r}_0 q^1 + \left(\bar{r}_0 + \bar{k}p + \bar{r} + \frac{1}{mp} \right) q^2 - px = 0 \\ \left(R_{air} + \frac{1}{mcp} \right) P^1 - \frac{1}{mcp} P^2 - \Delta T = 0 \\ -\frac{1}{mcp} P^1 + \left(2R_{plate} + \frac{2}{mcp} \right) P^2 + \frac{1}{mcp} P^3 \\ \frac{1}{mcp} P^2 + \left(\frac{1}{mcp} + R_{air} \right) P^3 - \Delta T = 0 \end{array} \right. \quad (4.64)$$

These systems are often complex to solve because they imply complex functions. Here we have two unknowns multiplied (here

a function of q^1 multiplied by J^2). Classical technique to solve the system is to use Newton's method. Using the symmetry of the thermic network, the vector of the unknowns x^k is given by:

$$x^k = \begin{bmatrix} J^1 \\ J^2 \\ q^1 \\ q^2 \\ P \end{bmatrix} \quad (4.65)$$

The sources being enclosed in the covector u_m .

The system of equations 4.64 can be represented by the vector of functions:

$$\psi = [\psi_\eta (x^k, u_m)] \quad (4.66)$$

The system 4.64 can be synthesized under the form $\psi = 0$. We will solve the system by a succession of approximations. Let's make the assumption that we know the the approximation of order p : $x^k(p)$. The exact solution can be written: $x^k = x^k(p) + \epsilon(p)$, $\epsilon(p)$ being the error between the approximation and the real value. By replacing x^k by its approximation in the vector of function, we obtain: $\psi (x^k(p) + \epsilon(p)) = 0$ (remember that it's always possible to replace the sources by equivalent fluxes that belong to the spanning tree). If the distance between the real value and its approximation is sufficiently thin, we can make the assumption that the function can be derived in this intervalle. Using this property we can write:

$$\psi (x^k(p) + \epsilon(p)) = \psi (x^k(p)) + \psi' (x^k(p)) \epsilon(p) = 0 \quad (4.67)$$

Once developed, this relation leads to the system:

$$\begin{cases} \psi_1(x^k(p)) + \frac{\partial \psi_1}{\partial x^1}(x^k(p)) \epsilon_1(p) + \frac{\partial \psi_1}{\partial x^2}(x^k(p)) \epsilon_2(p) + \dots \\ \psi_2(x^k(p)) + \frac{\partial \psi_2}{\partial x^1}(x^k(p)) \epsilon_1(p) + \frac{\partial \psi_2}{\partial x^2}(x^k(p)) \epsilon_2(p) + \dots \\ \dots \end{cases} \quad (4.68)$$

In fact we see that the derivation of ψ is given by the jacobian matrix $\omega(x)$:

$$\omega(x) = \begin{bmatrix} \frac{\partial \psi_1}{\partial x^1} & \frac{\partial \psi_1}{\partial x^2} & \dots & \frac{\partial \psi_1}{\partial x^n} \\ \frac{\partial \psi_2}{\partial x^1} & \frac{\partial \psi_2}{\partial x^2} & \dots & \frac{\partial \psi_2}{\partial x^n} \\ \dots & \frac{\partial \psi_n}{\partial x^1} & \frac{\partial \psi_n}{\partial x^2} & \dots & \frac{\partial \psi_n}{\partial x^n} \end{bmatrix} \quad (4.69)$$

The equation 4.67 can be written under the form:

$$\psi(x^k(p)) + \omega(x^k(p)) \epsilon(p) = 0 \quad (4.70)$$

which gives:

$$\epsilon(p) = -[\omega(x^k(p))]^{-1} \psi(x^k(p)) \quad (4.71)$$

Therefore

$$x^k(p+1) = x^k(p) - [\omega(x^k(p))]^{-1} \psi(x^k(p)), \quad p = 0, 1, 2, \dots \quad (4.72)$$

Note that once the first approximation given, next approximation at each time step is given by the previous value obtained at the previous time step.

The original condition and first approximation can be obtained starting from the rest state (ambient temperature, etc.).

4.9.7 Using domains

The previous approach has the disadvantage to lead to complex systems to solve. Domains may allow to treat the problem by adding complexity in the beginning of the project but leading to a more simple system to solve. The idea is to approximate some of the variables to make them constant on domains. This reduces the order of the system, by suppressing some product of variables. It's a kind of discretization (but remember that the intersections of the domains is not zero). For example if we take a look to the figure 4.27. It's a mechanism where a weighty loop is maintained up to a magnet thanks to the magnetic field emitted by the magnet. The loop has inside a battery that creates a DC current in the loop.

The weight of the loop and the magnetic field give the forces of Newton's law:

$$F_y = -mgy_l + 2\pi riB_r$$

r is the radius of the loop. This is equal to the acceleration of the loop following y :

$$-mgy_l + 2\pi riB_r = m \frac{d^2}{dt^2} y_l \quad (4.73)$$

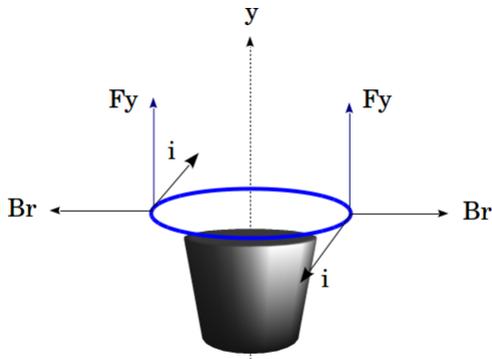


Figure 4.27: Mechanism with a magnet

y_l is the location of the loop on the axis y . A second equation says simply that the current in the loop is created by the battery:

$$e = Ri \quad (4.74)$$

The magnetic field B_r can be approximated constant on defined intervals. We write:

$$B_r(y) = \overset{1}{\mathcal{D}}_y B_1 + \overset{2}{\mathcal{D}}_y B_2$$

We use this development rather than writing a law giving $B_r(y_l)$. With:

$$z_{12} = 2\pi r \left(\overset{1}{\mathcal{D}}_y B_1 + \overset{2}{\mathcal{D}}_y B_2 \right) \quad (4.75)$$

the system becomes:

$$\begin{cases} -m \left(g + \frac{d^2}{dt^2} \right) y_l + z_{12} i = 0 \\ Ri = e \end{cases} \quad (4.76)$$

Finally in that case the system to solve is:

$$\begin{bmatrix} -m \left(g + \frac{d^2}{dt^2} \right) & z_{12} \\ 0 & R \end{bmatrix} \begin{bmatrix} y_l \\ i \end{bmatrix} = \begin{bmatrix} 0 \\ e \end{bmatrix} \quad (4.77)$$

This system can be solved without using Newton's method.

Now if we take a look to the impedance operator ζ in 4.77, we can compute the basis vectors of the corresponding surface:

$$\mathbf{b}_1 = \begin{bmatrix} -mg - 2\pi r i \dot{B}(y) \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 2\pi r B \\ R \end{bmatrix} \quad (4.78)$$

knowing that $d/dy(md^2/dt^2y) = 0$.

Seeing these vectors we understand that a condition to accept the approximation 4.75 is that $\ddot{B}(y) = 0$. Or, in other words, $\Gamma_{\alpha\beta,1} = 0$. Effectively, if we write the system under the form:

$$e_\mu + m_{\mu\nu} \frac{d^2 K^\nu}{dt^2} = H_{\mu\nu} K^\nu$$

The hamiltonian H is given by:

$$H = \begin{bmatrix} -mg & 2\pi r B \\ 0 & R \end{bmatrix} \quad (4.79)$$

The solutions are obtain using:

$$K^\nu = (H_{\mu\nu})^{-1} \cdot T_\mu \quad (4.80)$$

with:

$$T_\mu = e_\mu + m_{\mu\nu} \frac{d^2 K^\nu}{dt^2}$$

$m_{\mu\nu}$ begin the inertia:

$$m = \begin{bmatrix} m & 0 \\ 0 & 0 \end{bmatrix} \quad (4.81)$$

In fact we want to compare:

$$\begin{bmatrix} -mg & 2\pi r f(y) \\ 0 & R \end{bmatrix}^{-1}$$

and:

$$\begin{bmatrix} -mg & 2\pi r \sum_{\alpha} \overset{B}{\mathcal{D}}_{\alpha} \bar{B} \\ 0 & R \end{bmatrix}^{-1}$$

where $f(y)$ is a continuous function of y and \bar{B} a locally constant value of B . For the solution, only K^1 is impacted by the error coming from the approximation. After inverting the matrices we obtain in the first case:

$$K^1)_1 = 2\pi r f(y)e + mR \frac{d^2 y}{dt^2}$$

and in the second case:

$$K^1)_2 = 2\pi r \left(\sum_{\alpha} \overset{B}{\mathcal{D}}_{\alpha} \bar{B} \right) e + mR \frac{d^2 y}{dt^2}$$

so the error is associated with

$$\epsilon(\%) = \left| \frac{K^1)_1 - K^1)_2}{K^1)_1} \right|$$

giving:

$$\epsilon(\%) = \left| \frac{f(y) - \sum_{\alpha} \overset{B}{\mathcal{D}}_{\alpha} \bar{B}}{f(y)} \right| \quad (4.82)$$

The error can be reduced without difficulties if the vectors \mathbf{b}_x moves continuously on the surface, without curvatures. This implies $\Gamma_{xy,z} = 0$.

4.10 The lagrangian approach for multiphysic

We consider a parabolic elliptic of equation:

$$z = \frac{a}{2}x^2 + \frac{b}{2}y^2$$

The axis z being vertical. We imagine a ball of mass m , rolling freely inside the surface. It is submitted to its weight and to the corresponding Newton's force:

$$f = -mg$$

Its potential energy will be

$$U = mgz$$

By replacement we obtain:

$$U = \frac{k_x}{2}x^2 + \frac{k_y}{2}y^2 \quad (4.83)$$

with $k_x = mga$ and $k_y = mgb$.

Another information is the kinetic energy of the ball. If the parabolic elliptic is enough plate, we can make the assumption that:

$$\left(1 + \frac{dz^2}{dx^2 + dy^2}\right) \approx 1$$

so

$$T \approx \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) \quad (4.84)$$

4.10. THE LAGRANGIAN APPROACH FOR MULTIPHYSIC201

We can now write the lagrangian:

$$L = T - U = \frac{1}{2}m (\dot{x}^2 + \dot{y}^2) - \frac{k_x}{2}x^2 - \frac{k_y}{2}y^2 \quad (4.85)$$

if v^q is the vector of the variables ($v^1 = x, v^2 = y$), the system of equations:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{v}^q} - \frac{\partial L}{\partial v^q} = 0 \quad (4.86)$$

leads to a system of independent equations in x and y . Now we want to change of reference frame and to write the system in a new coordinates system (χ, γ) with:

$$\begin{cases} x = \chi \cos\theta - \gamma \sin\theta \\ y = \chi \sin\theta + \gamma \cos\theta \end{cases} \quad (4.87)$$

If we replace these new expressions of x and y in 4.83 and 4.84 we obtain:

$$\begin{cases} U = \frac{1}{2}k_\chi \chi^2 + \frac{1}{2}k_\gamma \gamma^2 + C_{\chi\gamma} \\ T = \frac{1}{2}m (\dot{\chi}^2 + \dot{\gamma}^2) \end{cases} \quad (4.88)$$

The kinetic energy remains unchanged in this transformation, but the potential energy see a new term added $C_{\chi\gamma}$ (here for example $C_{\chi\gamma} = (k_y - k_x) \sin\theta \cos\theta$). The lagrangian becomes:

$$\begin{cases} m\ddot{\chi} + k_\chi \chi + C_\gamma = 0 \\ m\ddot{\gamma} + k_\gamma \gamma + C_\chi = 0 \end{cases} \quad (4.89)$$

We see that in the new reference frame, the equations are no more independents. In the system of coordinates where the coupling disappear, these coordinates are called "eigenvalues".

4.10.1 Functional space

Lagrange's equations can be written under a formalism of functional space. Having a base of functions (it can be the locations of a mobile or other functions of parameters), we can define the "bra" or covector by (we illustrate these notions in a space of dimension 2):

$$x_k = (x| = (x_1 \quad x_2) \quad (4.90)$$

using Dirac's writing. The vector ("ket") is given by

$$|x) = x^n = \begin{pmatrix} x^1 \\ x^2 \end{pmatrix} \quad (4.91)$$

Now we define an operator \mathcal{U} which transforms the ket x^k in a new vector of components k_1x^1, k_2x^2 . This operator is defined by its matrix:

$$\mathcal{U} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \quad (4.92)$$

If we use the definition:

$$\text{if } x^k = \begin{pmatrix} x^1 \\ x^2 \end{pmatrix}, \Rightarrow x_k = (x_1 = (x^1)^* \quad x_2 = (x^2)^*) \quad (4.93)$$

we can verify that

$$(x|\mathcal{U}|x) = 2U \quad (4.94)$$

4.10. THE LAGRANGIAN APPROACH FOR MULTIPHYSIC203

For the kinetic energy we compute:

$$(\dot{x} | \mathcal{T} | \dot{x}) = 2T \quad (4.95)$$

with:

$$\mathcal{T} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \quad (4.96)$$

Now if we imagine a coil moving in order to cut the Earth magnetic field. The potential energy is mgz . The kinetic energy is for the mechanical part $1/2m(v_x)^2$ and for the electrical part $1/2Li^2$. We can define a space of configuration with the variables (z, v_x, i) . Applying Lagrange's equation we find for three sources $f_x(t), f_z, e(t)$:

$$\begin{cases} m \frac{dv_x}{dt} = f_x(t) \\ mg = f_z \\ L \frac{di}{dt} = e(t) \end{cases} \quad (4.97)$$

We can obtain the same result with a different approach. Once the natural "primal" vector is defined:

$$x^k = \begin{bmatrix} v^x \\ \dot{z} \\ i \end{bmatrix} \quad (4.98)$$

From this vector we can define a dual covector starting from

the invariant P with:

$$\begin{cases} f_x v^x = P \\ \dot{z} \cdot f_z = P \\ e \cdot i = P \end{cases} \quad (4.99)$$

Noting $T^u = [f_x \quad f_z \quad e]$, the system is obtained writing:

$$T_\mu = \zeta_{\mu\nu} x^\nu \quad (4.100)$$

with:

$$\zeta = \begin{bmatrix} mp & 0 & 0 \\ 0 & mp & 0 \\ 0 & 0 & Lp \end{bmatrix} \quad (4.101)$$

When various physics are interlaced, the situation is more complicated. If we go back on the example of the resistance. If its expression is $R(1 + \alpha T)i$. Once developed, this expression gives:

$$u = Ri + R\alpha Ti$$

If there are two vectors, a first used for the currents i^k and a second one for the gradient of temperature T , the tensorial product $i \otimes T$ creates a new vector that includes all the possible product i, iT, T . As the dual space is a space of numbers, it can always be reduced to an unique covector u_m . The equation of the system is in this case given by:

$$e_\beta = \zeta_{\beta\gamma\sigma} (i^\gamma \otimes T^\sigma) \quad (4.102)$$

what we notice

$$e_\beta = \zeta_{\beta\gamma\sigma} i^\gamma T^\sigma \tag{4.103}$$

In a first step, the tensor ζ gives the solution of the system, but we have seen that it is not easy to solve. It needs often to call for Newton's method due to its non linear properties. To reduce the complexity we can create a new vector, made of the arranged tensorial product with $k^\omega \rightarrow i^\gamma T^\sigma$. With this substitution:

$$e_\beta = \zeta_{\beta\omega} k^\omega \tag{4.104}$$

Now if we apply a second geometrization process, we will define a metric G which is in fact of dimension three $G_{\beta\gamma\sigma}$, and if there are Christoffel's symbols, we will write them in a first step $\Gamma_{\beta\alpha,\omega}$ which correspond to $\Gamma_{\beta\alpha,\gamma\sigma}$.

In each case the process is the following. Having $t^{\alpha\beta} = i^\alpha \otimes u^\beta$ where i and u are any vectors, we create the vector s defined by:

$$s^{[\sigma]} = \{C_{\alpha\beta}^\sigma(\sigma) t^{\alpha\beta}\}^T \tag{4.105}$$

The matrix C has only one component different to zero: it is the component of the row number (σ) and first column. Each product between C and t gives a row vector. Once transposed, this vector constitutes the series of components $[\sigma]$ of s . To include the cases where only components of i or u appear, we have to add a first component "1" in both vectors i and u . The component s^0 can be defined in this technique having the value "1". It represents eventual constants in the system, independent from any variables. We understand here that each vector attached to various values of the temperature (for example) may be seen

as a simple flux modified by a different operator which depends on the temperature. The term $R\alpha Ti$ can be seen as a modified impedance operator $R\alpha T$ which depends on the temperature applied to i , or a constant impedance operator $R\alpha$ applied to the component of the tensorial product of T and i . The first solution is written:

$$\overset{T}{\mathcal{D}}_x z(i) \quad (4.106)$$

with $\overset{T}{\mathcal{D}}_1 z(i) = R$ and $\overset{T}{\mathcal{D}}_2 z(i) = R(1 + \alpha T)$. the second solution is:

$$R(Ti)^1 + R\alpha(Ti)^2 \quad (4.107)$$

The first solution is the best one, because:

1. the temperature is continuous and cannot easily be attached to a vector definition;
2. it reduces the dimension of the problem and helps to its solving, cutting it in sub-parts where parameters can be approximated as constants.

Domains are one key point to solve multi physic problems. But both techniques can be useful, for example in problems where both speed and currents are involved: $i^a v_b$. But this kind of situation is quite easy to solve, because often there are in relation with machines ou induced EM force on mobile conductors, and Grassmann's algebra. For each current, three speeds are available. Knowing $\vec{u}_i \wedge \vec{u}_v = A_{iv}$, we can create the vector

$$s^\gamma \rightarrow i^k \otimes v^n / s^{[\gamma]} = A_{iv} \left\{ C_{kn}^{\gamma iv}(\sigma) i^k \otimes v^n \right\}^T \quad (4.108)$$

After what, one impedance operator, sets depending on parameters through domains, applies transformations to s to make a covector f . The general equation of this kind of system is given by:

$$f_\mu = \overset{T}{\mathcal{D}}_x \zeta_{\mu\nu} (s^\nu) \quad (4.109)$$

4.11 Some more ideas on manifolds and domains

4.11.1 Primitive manifold changing

We consider an impedance defined with domains. This definition is linked with a curve giving for all possible values or one variable x^k the result e_m through $e_m = \sum_n \Pi_m \mathcal{D}_n^m z_{mk}(n) x^k$, and this for one set of values for the parameters (x^k being a generalized flux - i.e. currents in the electrical case). If the parameter values change, the impedance function changes and the previous curve becomes a set of curve, i.e., a surface. This surface can be seen as a manifold issued from a moving curve. As an example, we make a program defining first the windows of each domains, with two domains. In that case, we consider only one parameter to make the illustration simplest. Figure 4.28 shows the result obtained for the domain windows.

Once the domains defined, we can use them to create a parametrized impedance in two laws.

Let's take for example next impedance law:

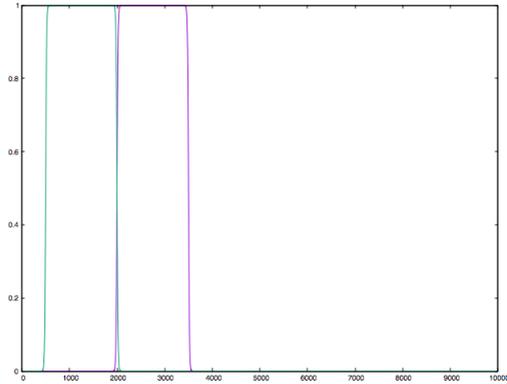


Figure 4.28: Domain windows

$$Z(i, R, \alpha) = \mathcal{D}_1^i R i_1 + \mathcal{D}_2^i R (1 + \alpha i_1) i_1$$

Once defined this law, we can give i_1 all possible values in one context, for a given resistance R and for various values of the parameter α .

Previous equation was drawn and gives the figure 4.29. First domain creates only one curve as it doesn't depend on any parameter. At the contrary, second domain creates many curves that make the surface seen on the figure.

It's clear that depending on domain values, the manifold associated with the law can change radically of appearance.

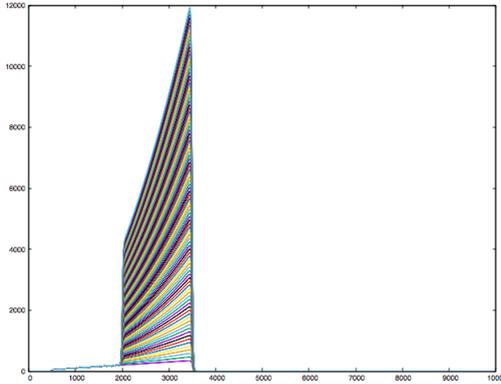


Figure 4.29: Curves for various parameter values

4.11.2 Connecting primitive manifolds

Various of previous like manifolds can be glued in order to create more complex ones [9]. The only constraints is to make in accordance the domains. If we imagine two laws of definitions:

$$Z_1 = \mathcal{D}_1 Z_1^1 + \mathcal{D}_2 Z_1^2 \quad Z_2 = \mathcal{D}'_1 Z_2^1 + \mathcal{D}'_2 Z_2^2$$

If we want to make the direct summation of the two previous primitives, it gives:

$$Z = \mathcal{D}_1 Z_1^1 + \mathcal{D}_2 Z_1^2 + \mathcal{D}'_1 Z_2^1 + \mathcal{D}'_2 Z_2^2$$

It's clear that domains should be equal to point out same values of parameters coming from the environment conditions.

Another possibility exists when domains complete each other. For example: $\mathcal{D}'_1 + \mathcal{D}_{1|} = \mathcal{D}''_1$. Note that one of the domain limit must be open (private of the last value) in order to connect it continuously with the other one. We may write: $\mathcal{D}''_1 = \mathcal{D}_1 \setminus [+ \mathcal{D}'_1$. In both cases, direct summation becomes possible and has meaning. If one of the impedance law doesn't depends on limits, the direct summation results from the classical addition of the two original laws.

4.11.3 Impact on the second geometrization process

When impedance is written using such laws defined on domains (restricted ones, which should be always the case for real object), what is the consequence on the second geometrization process? In other words, how the metric will be expressed when these domains come in square power?

We consider the system represented by the impedance matrix:

$$Z = \begin{bmatrix} \mathcal{D}_1 A + \mathcal{D}_2 B & C \\ C & D \end{bmatrix} \quad (4.110)$$

Note that even if a current threshold intervene in \mathcal{D} , it is simplest to choose another current than the current multiplied by this domain. In this case, when computing the derivation to extract base vectors, the result is directly the impedance operator itself. So, previous matrix leads to the jacobian:

$$J = \begin{bmatrix} \mathcal{D}_1 A + \mathcal{D}_2 B & C \\ C & D \end{bmatrix} \quad (4.111)$$

under the hypothesis that $A \dots D$ are all pure reals.
The metric coming from this jacobian is:

$$G = \begin{bmatrix} (\mathcal{D}_1 A + \mathcal{D}_2 B)^2 + C^2 & (\mathcal{D}_1 A + \mathcal{D}_2 B)C + CD \\ (\mathcal{D}_1 A + \mathcal{D}_2 B)C + CD & C^2 + D^2 \end{bmatrix} \quad (4.112)$$

Due to its decomposition in domains, the first term (without C^2) can be developed in:

$$(\mathcal{D}_1 A)^2 + (\mathcal{D}_2 B)^2 + 2(\mathcal{D}_1 A \cdot \mathcal{D}_2 B)$$

Now in case of perfect differentiation in the domain, i.e. $\mathcal{D}_1 \cdot \mathcal{D}_2 = \delta_{12} = 0$, the square is reduced to the summation of the squares of each impedance domain. But this case is rarely a physical one. More, it often leads to instability in numerical computations. So, if we use a common interval to both domains \mathcal{D}_1 and \mathcal{D}_2 as previously, the product $2(\mathcal{D}_1 \cdot \mathcal{D}_2) AB$ is different from zero. This term leads to a power P_δ linked with the transition phases when the material change of characteristics under environment parameters. If K and Q are the two mesh currents, we have for example:

$$P_\delta = 2(\mathcal{D}_1 \cdot \mathcal{D}_2) AB.K^2$$

So that the metric can be finally written: $G + G_\delta$, where G_δ integrates all the phase changes of the manifold. Note that

these transition states belong to each primitive manifold and also to shared energy (extra-diagonal element $G_{ij}, i \neq j$) if any impedance law is constructed under the domain formalism. Noting

$$\mathcal{D}_{ij} = \mathcal{D}_i \cdot \mathcal{D}_j \quad (4.113)$$

some interactions can disappear or not depending on the domain intersections, giving the metric a new meaning: it gives a deep description of the distance dependance versus the environment parameters. To solve quite easily the problems, it's clear that it's better when these parameters are not directly the flux themselves.

4.11.4 Curvature

In one domain, the law can integrate a function that's depends on one current. For example: $Z(\mathcal{D}) = \mathcal{D}_1 R + \mathcal{D}_2 R.K$. In that case, first base vector could be (in a three dimensions space):

$$\mathbf{b}_1 = \begin{bmatrix} \mathcal{D}_1 R + \mathcal{D}_2 R.K \\ b_2 \\ b_3 \end{bmatrix} \quad (4.114)$$

where b_2 and b_3 are functions without currents.

Due to the component dependance with current, $\mathbf{b}_{11} = \partial_K \mathbf{b}_1$ exists. It means that Christoffel's coefficients $\Gamma_{11,i}$ can exist. What does it means ? It means that for some environment conditions, the base vector \mathbf{b}_1 change with current K value. Its components on the local TpS tangent plan \mathbf{b}_i are given by

Christoffel's coefficient [11]. In other words, from one location to another $d\mathbf{b}_1 = \Gamma_{11,i}\mathbf{b}_i dK^i$.

If Christoffel's coefficients exist, this is a testimony that environment influence deeply the object behavior. The dependance of space with currents translates the fact that energy looks for a technique to compensate a kind of saturation in energy distribution. It translates also hysteresis phenomenons [10] in the system behavior. Remanent processes are involved and modify space characteristics that creates base vector evolution depending on the currents. In a general consideration, these effects always exist for any system. Linearities are approximations for limited domain definition of the impedances. It corresponds to limited definition of the manifolds.

4.12 Synthesis on multiphysic

In this discussion we consider the state variables as the positions or the speeds.

We have seen that various possibilities exist to describe thermal interactions as mechanical ones. I submit here a proposal of synthesis that can be used anytime, and I give some justifications on these choices. For electronics, it's clear, we have seen the choices from the beginning of this book. For thermal this is not so evident. Temperature don't seem to be vectorial. But we can work using the gradient of temperature, θ . This gradient can be the basic space or natural one. Then the dual space is naturally the power communicated to the circuits or materials. The electrical power is naturally a scalar, so it seems

perfectly coherent to use it as the dual space. For mechanics the choice seems more evident: the speed is the natural space and the force seen as a normalized work is the dual one. This leads to the table:

	natural	dual	metric	sources
electronics	J^k	e_μ	$\zeta_{\mu k}$	e_μ
thermal	θ^α	ω_β	$t_{\beta\alpha}$	ω_β
mechanics	v^k	f_σ	$g_{\sigma k}$	f_σ

For mechanics, some examples of operators are the elasticity of a spring: k , with $f = kx \Rightarrow f = k \int_t dtv$ or the losses per friction: $-\lambda v$. In thermal there is the thermal capacity m which is the inertia:

$$\omega - m \frac{d\theta}{dt} = 0$$

and the thermal conductivity σ given by: $\omega = \sigma\theta$. Note that often in thermal, the complementary choice is made, i.e. that the gradient of temperature is considered as the dual and the power as the flux. The advantage of the other choice is that the inertia appears similar to the inertia in electronics. In mechanics, the inertia is simply the mass of inertia mdv/dt , but we have seen that other choices can be made also, using the elasticity as inertia. Both choices are usable. Depending of each situation, the engineer will be able to use one option or the other, to make his equations as understandable as possible.

But the two choices are not completely equivalent. When we take the force as flux, it leads to an invariant which is in relation with the potential energy. Choosing the speed as flux, this leads to an invariant in relation with the kinetic energy. I think that

in fact this last choice is in general the best one, giving to the equations the property of a lagrangian, because we can write it under the form (T is the kinetic energy, G the metric):

$$\frac{d}{dt} \frac{\partial T}{\partial v} - G \int_t dt \frac{\partial T}{\partial v} = 0$$

We may write the system equations using the potential energy U :

$$\frac{v}{2G} \frac{dU}{dt} - U = 0$$

Under this form, we use the inverse of the metric while in the previous writing, the metric is directly used. Using directly the metric means that the solutions are in the natural space. While by using the inverse of the metric means to solve the equations in the dual space. Solving the problem in the natural space appears here through the fact that operators of integration belongs to the equations. In the dual space, simple differential equations translate the problem. We may think that this form is better, conducting to a simplest form of the equations. Integro-differential equations are harder to solve. However the integration allows to take into account all the past of the system, while the derivatives take into account only the previous state. If a system behavior is based on a far past, the dual space expression can be degenerate, while the natural space keeps in mind this story, eventually in limit condition, and will be more pertinent.

Any problem should group the three kind of equations. In any problem there are notions of mechanics, electronics and

thermal. How these three physics are coupled? The power in thermal comes from the electronics. The induced thermal force comes from the electrical current and we can define a coupling operator: ω/J . Between mechanics and electronics, the coupling is assured by the cut off magnetic flux: $e = vxB$. The friction creates heat and the power of heat is transmitted to the thermal network. The three physics are coupled by these mechanisms.

Chapter 5

Cords

Kron was used to use lumped elements to translate wave propagation. For a line, the model of the telegraphists with a succession of RLC cells for example gives a good but heavy model for a line. By incorporating Branin's model in Kron's formalism, we open a new opportunity, a powerful opportunity to translate guided waves. But it was possible to extend this process of transport of an energy from one graph to another, each graphs being connected. I have decided to call this process, a cord (after a first period where I called this, a chord). Mathematically this kind of link between connected graphs can be seen as a functor. Anyway I want here to explore the marvelous horizon that was open using cords in Kron's formalism¹

¹the most resulted article on this subject is probably "Maurice, O., Reineix, A., Durand, P., & Dubois, F. (2014). Kron's method and cell

5.1 What is a cord?

Basic principle of a cord c is to link a component of a covector d with a component of a vector f :

$$d = c(f) \quad (5.1)$$

In case of a simple and linear factor, the cord becomes $d = c.f$. One unusual function for a cord is for example a simple real value R . This is the case with common mode impedance in electromagnetic compatibility for example.

A cord can wear information. It is a medium for behavioral modeling. A cord can be created between two branches or two meshes.

5.2 Cord, symmetry and branins

Usually a cord exists for both directions between the two networks. It means that if ζ_{21} exists for example, ζ_{12} should exist also. We can consider for example two loops in interaction. We know that the electromotive force (EM force) induced on one loop (the second loop e_2), coming from the current of the other loop (first loop i^1) is given by:

$$e_2 = -\dot{\phi}_2 = -\frac{1}{\mathcal{R}} \frac{di^1}{dt} \quad (5.2)$$

complexes for magnetomotive and electromotive forces. arXiv preprint arXiv:1412.0187." The oldest article being perhaps "Olivier Maurice. Theoretical Application of the Tensorial Analysis of Network for EMC at the System Level. 2007. hal-00166215"

with the reluctance \mathcal{R} :

$$\mathcal{R} = \frac{1}{\mu} \frac{l}{S}$$

but the reluctance remains the same even if we take a look to the opposite coupling:

$$e_1 = -\dot{\phi}_1 = -\frac{1}{\mathcal{R}} \frac{di^2}{dt} \quad (5.3)$$

The relation is perfectly symmetric because $\mathcal{R} = \mathcal{R}_{12} = \mathcal{R}_{21}$. The reluctance makes part of the coupled system. In fact the reluctance is directly equal to a magnetic or mutual inductance cord. The metric is given by:

$$\mathcal{L} = \begin{bmatrix} L_1 & -\mathcal{R} \\ -\mathcal{R} & L_2 \end{bmatrix} \quad (5.4)$$

The determinant of the metric is given by: $\Delta = L_1 L_2 - \mathcal{R}^2$. The currents are given by:

$$\begin{bmatrix} J^1 \\ J^2 \end{bmatrix} = \frac{1}{p\Delta} \begin{bmatrix} L_2 & \mathcal{R} \\ \mathcal{R} & L_1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \quad (5.5)$$

The maximum for the current J^2 is obtained for $\mathcal{R} = k\sqrt{L_1 L_2}$. If $L_1 = L_2$ and $e_1 = e_2$ we find:

$$\frac{J^1}{J^2} = 1 \quad (5.6)$$

If the metric is not symmetric, the same calculus leads to:

$$\frac{J^1}{J^2} = \frac{1}{1+k} \quad (5.7)$$

But k is the coupling coefficient. If this coefficient is high, near 1, it implies that the coupling is strong. If the coupling is strong, the metric must be symmetric to take into account the back impact of the coupled current on the source. If both loops are excited equally, the currents are equal also and this regardless the value of k . But if the coupling is not symmetric, the current in the first loop tends to be half the current in the second loop, which is not correct because under our assumptions, all the conditions are equals. At the contrary, if the coupling coefficient is low, near 0, both currents tends to e/Lp and the first relation is respected. But the second relation becomes true, verifying for the two currents that the value reached is e/Lp .

If there is a long time of propagation of the coupling interaction between the two meshes, the situation is different.

5.3 Branin's model

Branin's model for guided waves can be used for many process of propagation in fact. Figure 5.1 represents a graph that shows the principle of this model.

The model is made of two meshes. Between these two meshes, a cord creates a coupling. This coupling function includes a time delay τ . The source has a self impedance Z_c which is equal to

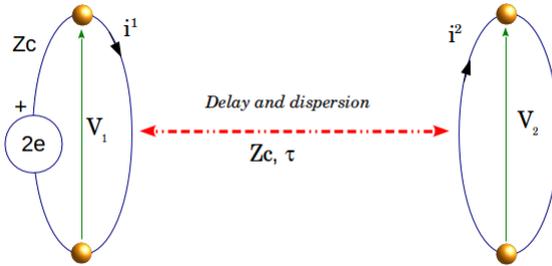


Figure 5.1: Branin's model

the medium impedance. The transported energy can be represented by a little circuit including the EM force $2e$ and the self impedance Z_c . When we take a look to the voltage V_1 transmitted to the medium we can write:

$$V_1 = 2e_1 - Z_c i^1 \quad (5.8)$$

In this equation, $e_1 = e$ and i^1 is the current of the first mesh. This implies:

$$2e_1 = (V_1 + Z_c i^1) \quad (5.9)$$

As the voltage arrives at the second mesh with the delay τ we find finally:

$$2e_1 = (V_1 + Z_c i^1) e^{-\tau p} \quad (5.10)$$

Same reasoning applied to the second mesh leads to:

$$2e_2 = (V_2 - Z_c i^2) e^{-\tau p} \quad (5.11)$$

and this with the convention for the current directions shown on the figure. Equations 5.10 and 5.11 are Branin's equations. They can be applied each time a process of propagation is implied in a physical problem. The function in factor that takes in charge the transmission, $exp(-\tau p)$ can be very different and is in general a Green's function. We note it now γ . So, Branin's equations becomes:

$$\begin{cases} 2e_1 = \gamma_{12} (V_2 - Z_c i^2) \\ 2e_2 = \gamma_{21} (V_1 + Z_c i^1) \end{cases} \quad (5.12)$$

The figure 2 shows a symbolic representation of the propagation of energy emitted by the source $2e_1$.

The wave propagating can be seen as a branch. When the wave reaches its target, the branch is connected to the load and communicates its energy to it. If we look at the hamiltonian before the wave packet is emitted, it is simply equal to:

$$H = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \quad (5.13)$$

Once the energy is received by the second mesh, the coupling interaction can be measured and considered. H becomes equal to:

$$H = \begin{bmatrix} R_1 & 0 \\ \frac{1}{i} \gamma_{21} (V_1 + Z_c i^1) & R_2 \end{bmatrix} \quad (5.14)$$

If all the energy is not dissipated in the load, a part of this energy is transmitted to the emitter. After one duration of propagation, the hamiltonian is:

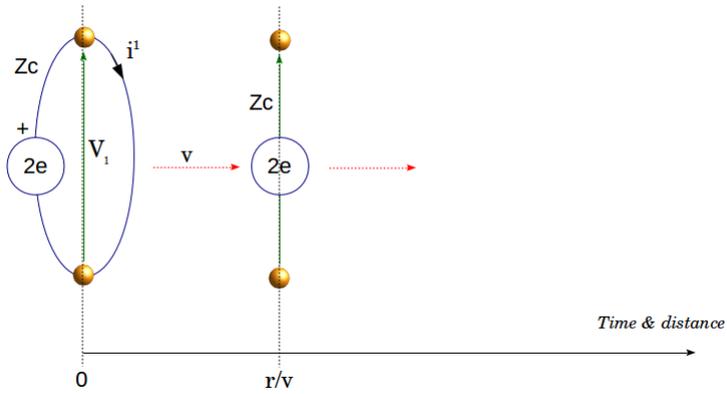


Figure 5.2: Propagation of energy

$$H = \begin{bmatrix} R_1 & \frac{1}{i^2} \gamma_{12} (V_2 - Z_c i^2) \\ \frac{1}{i^1} \gamma_{21} (V_1 + Z_c i^1) & R_2 \end{bmatrix} \quad (5.15)$$

This hamiltonian is not symmetric. Why?

If the medium is symmetric in its behavior, $\gamma_{12} = \gamma_{21}$. If the system is matched, i.e. if all the transmitted energy is dissipated by the loads, in that case $R_1 = R_2 = Z_c$ and $(V_2 - Z_c i^2) = (V_1 + Z_c i^1) = 0$. No waves are returned back to the emitter. But until the wave reaches the second mesh, it cannot know the conditions of matching. So the interaction has a given expression depending on the first mesh and the medium of propagation. The backward energy depends on the target mesh, not of the first one. It seems logical that the expressions of the two interactions are not obliged to be equals.

Knowing \mathcal{L} and U the part of the hamiltonian that includes the capacitance, we can compute the characteristic equation:

$$|\mathcal{L}^{-1}U - \omega^2 I| = 0 \quad (5.16)$$

I being the identity matrix. If we consider that U is purely diagonal, if \mathcal{L} is also purely diagonal, the eigenvalues are directly given by:

$$\omega = \sqrt{\mathcal{L}^{-1}U}$$

If the metric is symmetric but not purely diagonal, we obtain the eigenvalues from:

$$\Pi_k (d [\mathcal{L}^{-1}U]_k - \omega^2) - \Pi_k \bar{d} [\mathcal{L}^{-1}U]_k = 0 \quad (5.17)$$

$d[]$ pointing out the diagonal components of the matrices and $\bar{d}[]$ the extra-diagonal components of the matrices.

Now what happens if one of the matrices is not symmetric? For example:

$$\mathcal{L} = \begin{bmatrix} L_1 & 0 \\ -\gamma & L_2 \end{bmatrix}$$

and:

$$U = \begin{bmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \end{bmatrix}$$

We obtain:

$$|\mathcal{L}^{-1}U - \omega^2 I| = \left| \left[\begin{array}{cc} \frac{1}{L_1 C_1} - \omega^2 & 0 \\ -\frac{\gamma}{C_1 L_1 L_2} & \frac{1}{L_2 C_2} - \omega^2 \end{array} \right] \right| = 0$$

The characteristic equation is:

$$\left(\frac{1}{L_1 C_1} - \omega^2 \right) \left(\frac{1}{L_2 C_2} - \omega^2 \right) = 0$$

A special problem appears when the matrices are not symmetric, more particularly the metric, or at least when the matrices don't have components on all extra-diagonal locations: the eigenvalues don't take into account these elements. But this verdict is in fact completely logical. If the symmetry is lost on the metric, this means that a weak coupling interaction is modeled. If the coupling is low, there are no impacts of each mesh to another. If this assumption is true, the eigenvalues are not dependent on the coupling.

Symmetry means interdependencies and strong coupling paths. When the interactions are far and delayed, the coupling never

exist simultaneously. It means that the metric is never symmetric rigorously. So eigenvalues depend only of the diagonal components.

One good criterion to detect reciprocal influences in coupling process is to measure the deviations in frequencies on the eigenvalues when the object are separated or coupled.

In electromagnetism, cords are first of all made of fields. Using Coulomb's gauge, the electric far field is simply $p\mathbf{A}_x$ and the magnetic field is $\partial_{[y}\mathbf{A}_{z]}$ ².

The sources are loads and currents. They can be synthesized in the 4-vector $I^k = (qc, idx)$. This 4-vector can be easily obtained from the voltage across the capacitors for q and the currents across branches for i . (qc, idx) comes from EM forces e and these EM forces are created by the field. Under far field assumption, we have:

$$e = -p \int_h d\mathbf{h} \cdot \mathbf{A} = -ph_v I^v, v \neq 0 \quad (5.18)$$

The first component of the 4-vector I^0 generates near field interaction, but is not involved in the far field process. Many quite complex things coming from Maxwell's theory can be retrieved using an equivalent schematic. But in that exercise we mustn't forget the radiative part of the energy. The characteristic impedance of a line becomes the radiative resistance for the free field emission. And in fact, both can be seen as the real energy radiated in one case in free field, and in the other case in a guided waves process.

²We use here Penrose's notation. The bracket means that we alternate the variables with a minor sign: this gives $\partial_y \mathbf{A}_z - \partial_z \mathbf{A}_y$.

The power transported by the field is simply given by:

$$ds = -p^2 \frac{\mathbf{A} \cdot \mathbf{A}}{\mu c} \quad (5.19)$$

If we imagine an isotropic antenna, it radiates in all the space on a surface of $4\pi r^2$ value at the distance r from the source. The power flux of the field is given by:

$$s = \int_S d\mathbf{S} \cdot \eta s = p^2 \frac{4\pi r^2}{\mu c} \mathbf{A} \cdot \mathbf{A} \quad (5.20)$$

η is Poynting's vector. But this flux must be equal to the power transmitted to the antenna for the radiation: $R_a(I^1)^2$, I^1 being the current in the radiation resistance R_a .

From this relation we obtain the intensity of the electric field at a distance r :

$$|p\mathbf{A}| = \frac{1}{r} \sqrt{\frac{\mu c R_a}{4\pi}} I^1 \quad (5.21)$$

The EM force e_2 induced in a far circuit is obtained from:

$$e_2 = -p\mathbf{h}_E \cdot \mathbf{A} = -\frac{h_E}{r} \sqrt{\frac{\mu c R_a}{4\pi}} I^1 e^{-\mathbf{k} \cdot \mathbf{r}} \text{Sin}(\theta [\vec{u}_h, \vec{u}_r]) \quad (5.22)$$

From this relation we can define the propagator:

$$\gamma = \frac{e^{-\mathbf{k} \cdot \mathbf{r}}}{r} \quad (5.23)$$

and a coupling coefficient \mathcal{C} :

$$\mathcal{C} = -h_E \text{Sin}(\theta [\vec{u}_h, \vec{u}_r]) \quad (5.24)$$

the \vec{u}_χ being the unitary vectors aligned on the light line and aligned on the direction of reception of the receiving antenna (be careful, \mathbf{k} is a complex quantity).

At the emission, the circuit concerned by this first computation comes from a topology given by a $L - R_a$ circuit. The part of active power devoted to the radiation is made equivalent to a Joule effect of losses $R_a(I^1)^2$. I^1 is also the mesh current. The source circuit is shown on the left, figure 5.3 with $i^0 = I^1$.

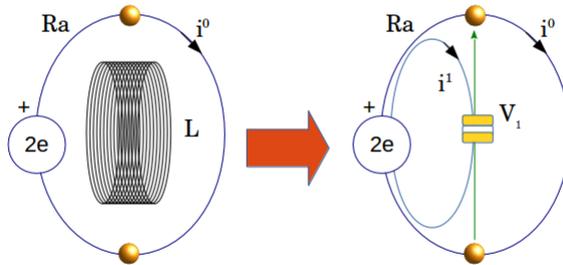


Figure 5.3: Source circuits

The radiation resistance R_a is located in series with the generator of self impedance R_0 . The field is created by the current coming from this generator, inside some conductive part. As the circulation of the currents can take many different forms, the radiation diagram which results from these circulations are also very varied. A function describes the volume of the space

where the antenna radiates. It increases the field intensity and if G is this function, we obtain finally:

$$e_2 = -\gamma \mathcal{C} G \zeta_b I^1 \quad (5.25)$$

with

$$\zeta_b = \sqrt{\frac{\mu c R_a}{4\pi}}$$

The function G includes the polarization of the light and its description of repartition around the light line r .

But the field can also come from a dipole, i.e. a couple of loads, one positive and one negative, oscillating around a middle point. This loads are represented by the plates of a capacitor across which the voltage V_1 is developed. Taking into account both fields coming from i and V_1 gives the expression of the radiation in a Branin's form. Note that often when using Branin's model, one of the problem to solve is to determine the expressions of the voltages V_n . Once these voltages are known, impedance operators of coupling appear and the theoretical expression of the problem is also known.

We have $V_1 = \Delta q/C$. If we look only at the far field created by this dipole, we find in any book on antennas³:

$$E_\theta = p \sin(\theta) \frac{e^{-\mathbf{k} \cdot \mathbf{r}}}{r} \frac{h}{4\pi\epsilon_0 c^2} I^2 \quad (5.26)$$

h is the height of the dipole, θ the height angle. Once more we

³For example, see "Antennas"; John D.Kraus and Ronald J.Marhefka, Mc Graw Hill edition 2002, NY

can define the functions γ , G and ζ_E :

$$\left\{ \begin{array}{l} \gamma = \frac{e^{-\mathbf{k} \cdot \mathbf{r}}}{r} \\ G = \sqrt{2} \text{Sin}(\theta) \\ \zeta_e = p \frac{h}{4\pi\epsilon_0 c^2} \end{array} \right.$$

With the same coupling function, we finally obtain for our antenna:

$$e_2 = -\gamma (pCG [\zeta_e I^2 + \zeta_b I^1]) \quad (5.27)$$

which is a Branin's model here for antennas in far field. The radiation resistances are defined by the integral of the power flux of the field all over the radiation surface. At the equator, we integrate the elementary surface $r d\theta d\alpha$ on 2π , θ being the height and α the azimuth. Up to the equator, the perimeter decreases as the radius becomes $r \text{Sin}(\theta)$. The integral is so given by:

$$P = \int_0^\pi d\theta \int_0^{2\pi} d\alpha r \text{Sin}(\theta) p^2 \frac{A^2(\theta, \alpha)}{\mu c} \quad (5.28)$$

Having P , R_a is defined by:

$$R_a = 2 \frac{P}{I_0^2} \quad (5.29)$$

I_0 is the power supply current of the antenna.

When we take a look to the source circuit figure 5.3, we can

write the impedance operator:

$$\zeta = \begin{bmatrix} R_0 + R_a + Lp & R_0 \\ R_0 & R_0 + R'_a + \frac{1}{Cp} \end{bmatrix} \quad (5.30)$$

We suppose here that the geometrical dimensions of the antenna are many much smaller than the wavelength. R_a and R'_a are the radiation resistances associated with each kind of field (field coming from the parts I^0 and I^k of the 4-vector current). The total power involved is:

$$w = \zeta_{ab} i^a i^b$$

$$w = (R_0 + R_a + Lp) i^1 i^1 + 2R_0 i^1 i^2 + \left(R_0 + R'_a + \frac{1}{Cp} \right) i^2 i^2 \quad (5.31)$$

In this power we recognize the power dissipated by the generator in its real impedance $R_0 (i^1 + i^2)^2$; the power stored in the inductances and capacitors $Lp(i^1)^2 + (Cp)^{-1}(i^2)^2$ and the power radiates w in the environment $R_a(i^1)^2 + R'_a(i^2)^2$.

5.4 Quantum field

If the field intensity decreases, we known that it cannot go below $h\nu$. Maxwell's equations are completely equivalent to Telegraph's equations with $V = E.x$ and $i = H.y$ for a wave propagating in the z direction with height x and width y . It means that a simple lumped element line should be able to represent

the propagation of a photon. This line is a succession of LC cells. Figure 5.4 represents this mechanism. Note that the coupling phenomenon when we take the equivalent topology to the classical line topology, with coupled but independent cells goes only in one direction. This is very important. It tells in what direction the photon propagates.

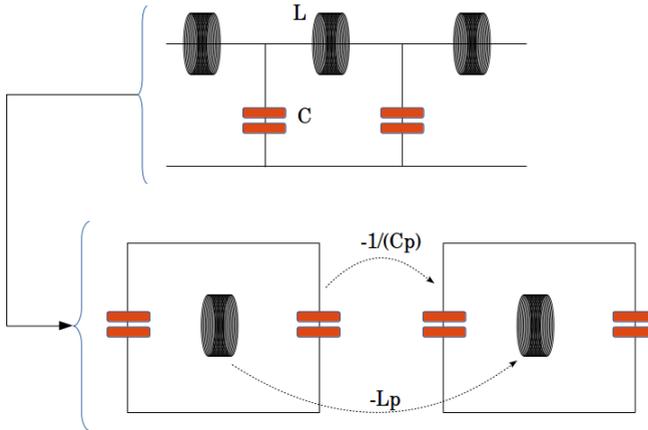


Figure 5.4: Quantum field propagation

If you compute the eigenvalues, it is possible because as the propagation goes only one direction, the determinant of the met-

ric is not zero, we have:

$$\mathcal{L} = p \begin{bmatrix} L & 0 \\ -L & L \end{bmatrix} \quad (5.32)$$

and

$$H = \frac{1}{p} \begin{bmatrix} \frac{1}{C} & -\frac{1}{C} \\ -\frac{1}{C} & \frac{1}{C} \end{bmatrix} \quad (5.33)$$

Making

$$|\mathcal{L}^{-1}H - \omega^2| = 0$$

we find naturally $\omega = \pm 1/\sqrt{LC}$. The stored kinetic energy is $\lambda/(2c)L(i)^2$ which must be equal to $h\nu$. We deduce from these relations:

$$i = \sqrt{\frac{2hc\nu}{\lambda L}}$$

as $L = \mu c/c = \mu$ per meter or $\mu\lambda/2$ for one photon, we can compute the value of current that must be created at the origin, knowing that there are no losses in the propagation. If $\lambda = 100 \text{ nm}$, the current is around $3 \cdot 10^{-14}$ (it means that with 3 pA we have only 100 photons). That's not so small! If the value is not itself a good reason to think that the world is a quantum world, what can be the criterion?

The criterion is the way the field is distributed in the space. The field coming from the Joule's effect respects entropy and it has no specific organization, being lost in the space. While under other conditions, the field can be distributed within modes, respecting the laws that governs the stationary waves. When the field is stationary, the field is a quantum field. Under this

assumption, how the free field emitted by anything can be a quantum field? How a photon can exist? Once a field of very short wavelength is inside the matter, it can found many borders in order to create modes, to exist through modes and to be recognize as a photon. If a field of 100 nm wavelength is in a box, the time for a round trip is $7,3 \cdot 10^{-16}$ s. It is impossible to measure! But from the time we introduce probes to try to measure the field, we disturb the environment with metallic structures and we create modes. And if two antennas are separated with χ meters, they constitute metallic limit conditions for the field and whatever the distance, a modal field will be created between the two antennas, on the direct line that joins them. Does it mean that the photon exists only when we look at the field? Or when it has the time to establish a stationary wave?

The photon can propagate at the infinite without losses if the equivalent line is lossless. The characteristic impedance of the line is the one of the empty space: $\mu_0 c$. But this impedance appears because the line is infinite. Between two limits that are matched, the field is distributed in $\cos(2\pi E/ht)$. It is not a stationary wave, but simply the distribution of the field due to its time function. But if the photon exists only when we observe it, how can we explain the constant value of the field at the floor of $h\nu$? The quantification is spatial. The energy tries to use the maximum of space that is reachable to it. We can accept the assumption that at very low levels of energy, a minimum, must be reached to excite the empty space which is $h\nu$. It was Planck's demonstration, nothing more but nothing less. As the line is a lossless one, this floor energy can go ahead

continuously until some observations creates stationary waves. The presence of a single metallic part is sufficient to organize the field in such a mode.

The field is distributed in the space following modes. We will see that in some cases, it is possible to see these modes. But at radio-frequencies, there is a lack of coherence to visualize the modes. Anyway, there are somewhere no problems! In any case the unique problem is to compute the field, and after, if modes appear, it is possible to imagine photons under the quantification of the field that the modes represent.

The probability that the photon uses one way rather than another respects Feynman's rules and the concept of way in the Feynman's meaning. The field is the fundamental form of energy transmission and the photon explains the minimum step of energy that can be exchange between the matter and the space. And we must never forget that this minimum is very low, somewhere often immeasurable ($h = 6,610^{-34} [J][s]$).

5.5 Waveguides

We consider a waveguide of width Y , height X and length Z . The general approach to determine the dispersion is to make the assumption that the field propagates for example under the mode

$$E_x(y, z) = E_{0x} \text{Sin} \left(n\pi \frac{y}{Y} \right) e^{-jkz}$$

In the volume of the waveguide, the field must respect Helmholtz's equation:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + \left(\frac{\omega}{c}\right)^2 E_x = 0 \quad (5.34)$$

by replacement we obtain the dispersion:

$$-\left(\frac{n\pi}{Y}\right)^2 E_x - k^2 E_x + \left(\frac{\omega}{c}\right)^2 E_x = 0$$

so:

$$k_n = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{n\pi}{Y}\right)^2} \quad (5.35)$$

This dispersion k for the mode n , k_n , is associated with the modes $TE_{xy} = TE_{0n}$.

Regardless of the geometry, the approach to determine the dispersion can always be the same. To represent a waveguide with Branin's model, we need a dispersion (that's done!) and a characteristic impedance η_c . The characteristic impedance in a line is obtained computing the ratio V/i . We can try to compute it using the same formula:

$$V = - \int_x E_x dx = -X E_x$$

and

$$i = \int_y dy B_y$$

This last relation needs to know the magnetic field. Maxwell's equation says us that:

$$\frac{\partial \mathbf{E}_x}{\partial z} = -p\mu \frac{\mathbf{B}_y}{c} \quad (5.36)$$

if we look to the B_y component of the magnetic field. We have so:

$$B_y = -\frac{c}{\mu p} (-jk) E_{0x} \text{Sin} \left(n\pi \frac{y}{Y} \right) e^{-jkz}$$

We can know the wave impedance η_0 , knowing that $B = \mu H$:

$$\eta_0 = \mu \left| \frac{\mathbf{E}_x}{\mathbf{B}_y} \right| = \mu \frac{\omega}{k_n} \quad (5.37)$$

Now we can compute i_z

$$i_z = H_y Y$$

and V_x :

$$V_x = E_x X$$

and

$$\eta_c = \frac{V_x}{i_z} = \mu \frac{X}{Y} \frac{\omega}{k_n} \quad (5.38)$$

This impedance can be considered as the characteristic impedance of the waveguide, and gives us the material to model the waveguide similarly to a line. Now if we consider the B_z component of the magnetic field. It concerns the field propagation in the y direction of the waveguide. We obtain first:

$$B_z = \frac{pY}{n\pi c^2} E_{0x} \text{Cos} \left(n\pi \frac{y}{Y} \right) e^{-jkz}$$

and in this case:

$$i = -\frac{pY^2}{n^2\pi^2 c^2} E_x$$

which gives:

$$\eta_c = \frac{Xn^2\pi^2c^2}{pY^2} \quad (5.39)$$

For the first mode where $2Y = \lambda$, the characteristic impedance becomes

$$\eta_c = \frac{X\pi c}{Y}$$

This value is not infinite but is very high, comparable to an open-circuit. The problem of our method is that it computes the characteristic impedance on one point at the input of the waveguide. Let's try another method to compute this value, very important for us: considering the transmitted power to the waveguide, the same reasoning we made for antennas with the radiation resistance.

In this new approach we consider the characteristic impedance Z_c as an impedance to verify that $|Z_c i_0^2|$ is equal to the power transmitted by the waveguide. This power is:

$$|Z_c i_0^2| = \iint_S ds \left(\frac{E^2}{\eta} + c^2 \frac{B^2}{\eta} \right) \quad (5.40)$$

The power transmitted by the waveguide is the same under the two fields. We can compute the power transmitted by the electric field:

$$\int_y dy \int_x dx E^2 = 2XE_{0x}^2 e^{-2\frac{z}{c}p} \int_y dy \left(\frac{1 - \text{Cos}(2n\pi y/Y)}{2} \right) \quad (5.41)$$

using the fact that: $V = E.X$ we obtain:

$$Z_c = \eta \frac{X}{Y} \quad (5.42)$$

with $\eta = \eta_0 \mu \omega (k_n)^{-1}$. We obtain the same result as previously.

With both equations 5.35 and 5.42, we can modeled the waveguide like a line with a Branin's model. If we connect a generator e_1 of self impedance $\eta_g \neq \eta XY^{-1} = z_0$ at the input of the waveguide, the voltage V_1 is equal to $e_1 - \eta_g i^1$. As V_2 is equal to $R_L i^2$, R_L being the load connected at the output of the waveguide, we obtain by replacement:

$$\zeta_{21} = (z_0 - \eta_g) e^{-jkz} \quad \zeta_{12} = (R_L - z_0) e^{-jkz} \quad (5.43)$$

but what is also important is that a delayed generator is also reported on the output of the waveguide: $e_1 e^{-jkz}$. This generator is the only source of signal on the output if the waveguide is matched on all its inputs. Effectively, in that case the coupling impedance $(z_0 - \eta_g)$ and $(R_L - z_0)$ are equal to zero because $R_L = z_0$ and $\eta_g = z_0$.

5.6 Chain matrices

When the frequency increases, the diagram of emissions can be reduced. If we consider an antenna like a collection of elementary dipoles of source i_s , separated by a distance θ , each of them radiates and at a point M of the space, we can compute

the intensity i_M :

$$i_M = \alpha i_s \int_{\theta} d\theta \exp\left(-\frac{1}{2}\left(\frac{p^2 r^2}{c^2} + \frac{p^2}{c^2} n^2 r^2 \theta^2\right)\right) \frac{1}{R}$$

$R = \sqrt{r^2 + n^2 \theta^2}$. Noting $p = j\omega_0$ and $\omega_0 d\theta = d\Omega_0$, we obtain ($\bar{r} = r/c$):

$$i_M = i_s \frac{e^{-\bar{r}p}}{\omega_0} \int_{\Omega} d\Omega_0 \frac{e^{-jn\Omega_0\bar{r}}}{R}$$

But this is the inverse Fourier's transformation of the function R^{-1} . The higher the harmonic Ω_0 , the thinner the pencil of radiation.

We have seen that the field inside waveguide propagates through modes. Is it possible to understand now the modes of a free field? We take a look to lasers. Lasers emit light at very high frequencies and it will be perhaps easier to understand how light behaves with them.

To model the way followed by the light, we use matrices called "ABCD" matrices. They make a link between the distances d to the optical axe and angles θ with the axe at the input and output of an optical structure. We may write:

$$\begin{bmatrix} d_2 \\ \theta^2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} d_1 \\ \theta^1 \end{bmatrix} \quad (5.44)$$

A succession of different ways is modeled by the product of the chain matrices.

Once the chain matrix is known, we can transform it in an

impedance matrix using:

$$Z = \frac{1}{C} \begin{bmatrix} A & \det(ABCD) \\ 1 & D \end{bmatrix} \quad (5.45)$$

with $\det(ABCD) = AD - BC$. The impedance operator make the link between the covector d_k and the vector θ^q . Chain matrices can be a good solution to obtain the expression of the Green's function. Anyway, the field A must respect the wave equation:

$$\nabla^2 A + \frac{\omega^2}{c^2} A = 0 \quad (5.46)$$

A solution of this equation should be:

$$A = \psi(x, y, z) e^{-jkz} \quad (5.47)$$

Replacing 5.47 in 5.46 gives:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - k^2 \psi + \frac{\omega^2}{c^2} \psi = 0 \quad (5.48)$$

neglecting the derivatives of ψ depending on z , under the assumption that ψ varies slowly with z . The function:

$$\psi = \exp\left(j \frac{q\lambda}{2\sqrt{2}} x^2 + j \frac{n\lambda}{2\sqrt{2}} y^2\right) e^{j(\omega t - kz)} \quad (5.49)$$

leads to the dispersion:

$$k = \sqrt{\frac{\omega^2}{c^2} - \left(\frac{q\lambda}{2}\right)^2 - \left(\frac{n\lambda}{2}\right)^2} \quad (5.50)$$

The dispersion says that the light can propagate using various modes of characteristics q, n . With this expression we have put in evidence modes and a spatial quantification of the field. This modes and their patterns can be seen with a laser on a target where impacts appear in each direction and with a number given by the mode degree.

For a given mode of propagation, we can study how the chain matrix can work. We give here a simple example to illustrate how the use of the chain matrices can help to construct the impedance operator.

Knowing a source circuit, we can compute the field emitted by this circuit. Coming from the voltage developed at the input of the radiating channel, we can obtain the electric field:

$$\mathbf{h} \cdot \mathbf{E} = V_1 \quad (5.51)$$

The same identity can be made between the current and the magnetic field:

$$\mathbf{B} = \mu_0 \frac{i^1}{2\pi r} \quad (5.52)$$

A matrix S can be created to link the primary variables (V_1, i^1) with the field (\mathbf{E}, \mathbf{B}):

$$S = \begin{bmatrix} h^{-1} & 0 \\ 0 & \mu_0(2\pi r)^{-1} \end{bmatrix} \quad (5.53)$$

Once created, the field can propagate thanks to a Green's oper-

ator G :

$$G = \begin{bmatrix} e^{-\frac{r}{c}p} & 0 \\ 0 & e^{-\frac{r}{c}p} \end{bmatrix} \quad (5.54)$$

and finally the field is transformed through S' in another location in potential and current:

$$S' = \begin{bmatrix} -h' & -Ap \\ p\epsilon_0 A' & \frac{1}{\mu_0} A_2 \nabla \times \end{bmatrix} \quad (5.55)$$

$A, A', A_2, r, h, h', \dots$ are geometrical parameters.

The whole set of operations made is:

$$\begin{bmatrix} V_2 \\ i_2 \end{bmatrix} = S'GS \begin{bmatrix} V_1 \\ i_1 \end{bmatrix} \quad (5.56)$$

The global chain matrix is obtained making:

$$T = S'GS = \begin{bmatrix} -h'e^{-\frac{r}{c}p}h^{-1} & -Ape^{-\frac{r}{c}p}\mu_0(2\pi r)^{-1} \\ p\epsilon_0 A'e^{-\frac{r}{c}p}h^{-1} & e^{-\frac{r}{c}p}(2\pi r)^{-1}A_2 \nabla \times \end{bmatrix} \quad (5.57)$$

The impedance operator is deduced from this chain matrix using definition 5.45:

$$\zeta = \frac{1}{p\epsilon_0 A'e^{-\frac{r}{c}p}h^{-1}} \begin{bmatrix} -h'e^{-\frac{r}{c}p}h^{-1} & \Delta \\ 1 & e^{-\frac{r}{c}p}(2\pi r)^{-1}A_2 \nabla \times \end{bmatrix} \quad (5.58)$$

and

$$\Delta = (Ap\mu_0p\epsilon_0A'h^{-1} - h'h^{-1}e^{-\frac{r}{c}p}A_2\nabla\times) \frac{e^{-2\frac{r}{c}p}}{2\pi r}$$

The same technique can be used to model lines, waveguide, etc.⁴

5.7 Cords with domains

Interactions themselves can change depending on parameters. So, various kind of interactions of cords can exist between two meshes, each cord being activated once in function of the parameters values. A logical gate for example can be modeled using this technique. A source e creates a current in a first mesh. If the impedance of the mesh is one ohm, the current is equal to the voltage. If the voltage exceeds a defined threshold, a cord is activated and creates a source on a second gate which transmits the information. The cord is the operator $\overset{v}{D}_1$. If the current i^1 exceeds the threshold, the cord is activated and $\overset{v}{D}_1 = 1$. A generator of 1 volt feeds the second mesh of one ohm also and the output is equal to 1. It's an identity gate. If we define the cord $-\overset{v}{D}_1$, and if we give to the second mesh a source of 1 volt, the activation of the cord will create a generator in opposition with the source of the second mesh, and this time the output is zero. In that case, the system works like a NOT gate.

⁴Pr Alain Reineix has conducted advanced works on the use of chain matrices for the modelling of cavities. It has given each time very efficient results.

5.8 Breaking cords

If we consider a Branin's structure of source impedance R_0 , load R_L and characteristics (z_c, τ) . This branin links two connex circuits without any branches between them. We want to study the first circuit without taking care of the second circuit. Seen from the input of the line we can write:

$$\begin{cases} V_1 = V_i (1 + \sigma e^{-2\tau p}) \\ J^1 = J_i (1 - \sigma e^{-2\tau p}) \end{cases} \quad (5.59)$$

This allows to say that seen from the first circuit, the presence of the line loaded by R_L is equivalent to the impedance:

$$z_R = \frac{V_1}{J^1} = z_c \left(\frac{1 + \sigma e^{-2\tau p}}{1 - \sigma e^{-2\tau p}} \right) \quad (5.60)$$

with $V_i/J_i = z_c$ and

$$\sigma = \frac{R_L - z_c}{R_L + z_c}$$

When the conditions are gathered to compute an equivalent backward impedance, this technique allows to cut the cord and forget all the part of the original circuit connected through the line. It shows also that the two meshes linked by a cord can be synthesized in an unique mesh. At low frequencies this possibility is fundamental. It translates the fact that Green's function faints when the frequencies decrease. If we take a short-circuited line ($R_L = 0$): at low frequencies it's a simple inductance. This

is simply and well described by the backward impedance expression. Taking $R_L \rightarrow 0$ we obtain:

$$z_R = z_c \frac{R_L + jz_c tg(2\tau p)}{z_c + jR_L tg(2\tau p)} = jz_c tg(2\tau p)$$

which has the dimension of an inductance.

5.9 Cord & game theory

Cyber Physical Systems involve machines, numerical networks and humans. Humans act depending on three datas:

1. their past experience (and the past can go very far in the past!);
2. the current state of their environment;
3. their hope in a future the more or less far.

That's all how works a gamer playing football: it uses his training and the previous trajectory of the ball, detect the environment (where are the other gamers, etc.), and play the ball depending on what he wants to do with this ball in a near future. This interaction is a cord. A particular cord but anyway, a cord. How may we define this cord in a simplest way (everyone will be able to treat of more complex cases starting from this basic case)? We consider an output J^1 of a machine (this machine can embed intelligence or not). A user can drive a command e_2 depending on his choice, this choice being driven by

J^1 (for example, it can be any current $J^{(n)}$ as any force $e_{(m)}$). These commands make a covector c_y . For each couple of values (J^1, c_y) the user wins a defined gain ω with a bayesian probability $P_y(c_y | J^1, \omega_1^y)$. A first matrix (*payoff* matrix) ω_1^y says how many wins the player for each of his choices and for the output J^1 . The hope of earnings h_{11} of the player is an operator applied to J^1 and defined by:

$$h_{11} \cdot J^1 = \omega_1^y P_y(c_y | J^1, \omega_1^y) f(J^1) \quad (5.61)$$

The command e_2 is determined knowing h_{11} by a transfer function Q with:

$$e_2 = Q_2^1 h_{11} \cdot J^1 \quad (5.62)$$

The whole process behaves like an impedance operator ζ_{21} a little special perhaps, but involved in the whole chain of definition of a system, enclosing the human factor in its working.

Note that the probability for the player to choice c_1 or c_2 or etc., depends on his psychological profile. If we want to compute some behavior without any psychological interpretation, we can do it using the "rational player": the player who make his choices in order to win the maximum. But for a human or an animal, to win a maximum can have various sides. For them, death can be the best choice if it allows them to save their Childs. But from a pure payoff matrix, this choice can be the worst case. Sometimes it's possible to consider the rational cases as the more pertinents. Sometimes not. Each situation must be considered separately. This complexity is logical and in relation with the complexity of the animal behaviors. Anyway, the advantages of computers is to be able to make as many

assumptions as necessary to explore all the possibilities and get out the risks when some of these assumptions may provoke risks.

Let's take an illustration.

We imagine an electronic which measure the height to the ground from a plane. It's an antenna sending a pulse in the direction of the ground. After some time, this pulse is reflected and sent back to the antenna. The time duration between the two pulses is an image of the distance to the ground. A pilot can read this information and drive the altitude of the plane. To do that, he activates a command with two buttons, one to get higher and one to get lower. Each button commands a motor which lifts up or down the plane acting on the wings.

The antenna is a simple radiation resistance with the generator matched. The reflected pulse is a generator applied on the radiation resistance. If V is the level of potential transmitted to the antenna, $E_0 = V/h_a$ is the initial electric field radiates to the ground, h_a being the effective height of the antenna. The reflected field has for equation $E_r = E_0 e^{-2H/cp} e^{-2\alpha H}$ where H is the height to the ground, c the speed of the light and α the losses in propagation. The mesh that represents the antenna can be powered by two generators $h_a E_0$ or $h_a E_r$. In parallel a circuit is powered by -1 volt when the field is emitted and 1 volt when a field is received between two incident pulses sending. This 1 volt of power on 1 ohm generates a current J^2 of 1 ampere which is the reference readed by the pilot (we suppose it activates a time counter in relation with the height). The other circuit is a simple mesh where the load is the resistance of the motor activating some part of the wings. One button gives 100 volts to the motor and the other button gives -100 volts to the

motor. The figure 5.5 illustrates the system. It shows that one cord goes through a human to activate a force (V_{up} or V_{down}) from the information associates with a current (J^2).

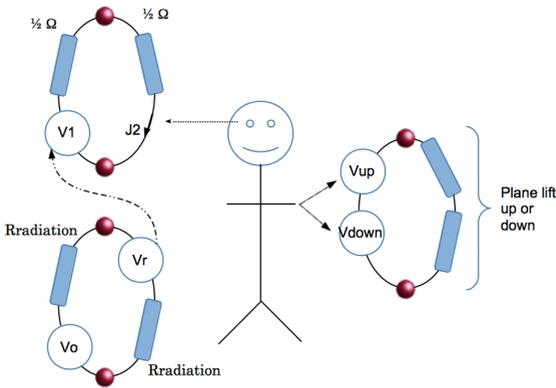


Figure 5.5: System with a pilot

The plane being to an altitude H_0 we make the assumption that if the plane goes up, it will consume more petrol and if it goes down, it increases the risk to touch some obstacle but consumes less than if it goes up. In the first case, its gain is -10 and in the second case -35 (we don't care the exact signification of these gains!). The changes in the force translating the incident of backward field on the antenna is taken in charge by a gamma matrix (see the paragraph "sources and gamma matrices" for

that), γ that can take two forms depending on time:

$$\gamma(t \in [0, T]) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \gamma(t > T) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (5.63)$$

the source covector being for this circuit:

$$e = [V_0 \quad V_r]$$

The payoff matrix is defined by⁵:

action	J^2
Vdown	$J^2 = -1, \omega_2^d = -35$
Vup	$J^2 = 1, \omega_2^u = -10$

The pilot wants to go down in order to consume less if there are no indications of near ground (γ doesn't change before T), or going up if a reflexion is detected soon (γ changes before T). Depending on the psychological profile of the pilot, the probability that he decides to go up or down is different. We make the assumption that the pilot is a prudent person, but he suffers of the pressure of his company to consume less. This situation is complicated to evaluate. Now imagine that the altimeter is disturbed. A parasitic emission modifies the state of the electronic and a parasitic operator $\tilde{\gamma}$ defined by:

$$\tilde{\gamma} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5.64)$$

⁵I have simplified the payoff matrix here, just to illustrate the mechanism without going too much in the details

is added to γ . We see here that the problem becomes different. The pilot don't see any obstacle. If he trusts the electronic, he goes up, if he trust his vision he goes down. We suppose that he trusts in general the electronic, this defining his psychological profile as t .

We make the assumption that ($V_{up} \rightarrow V_u, V_{down} \rightarrow V_d$):

$$\begin{cases} P(V_u | J^2 = 1, t) = 99\% \\ P(V_d | J^2 = -1, t) = 1\% \end{cases}$$

in that case we define $\Rightarrow h_{22} \cdot J^2 = (-10.0, 99 + -35.0, 01) |J^2| = -10, 25 |J^2|$. The function Q_3^2 can be for example

$$Q_3^2 = (-1) \left[\mathcal{D}_{[-10, -30]}^{h_{22} J^2} \beta_u + \mathcal{D}_{[-30, -40]}^{h_{22} J^2} \beta_d \right] \delta_{t, \tau} \quad (5.65)$$

β_u and β_d are coefficients that leads to V_u and V_d . The operator $\delta_{t, \tau}$ is here to translate the time that the pilot takes to make his decision, depending on his profile t and acting on the time as a delay τ .

If we define the coupling between the meshes 1 and 2 with $e_2 = V_1$ and:

$$\zeta_{21} = \mathcal{D}_{<0}^{J^2} G + \mathcal{D}_{>0}^{J^2} (-G') \quad (5.66)$$

the impedance operator of our system becomes:

$$\zeta = \begin{bmatrix} 2R_{ray} & 0 & 0 \\ \zeta_{21} & 1 & 0 \\ 0 & Q_3^2 h_{22} |\bullet| & 2R_m \end{bmatrix} \quad (5.67)$$

(the bullet shows that the operator is applied to the current, the current taking place where the bullet is located).

R_m is the load for the motors. The sources are:

$$w = \begin{bmatrix} (\gamma(t) + \tilde{\gamma}(t)) e \\ 0 \\ 0 \end{bmatrix} \quad (5.68)$$

and the system is modeled as usual by the generic equation

$$\omega_\mu = \zeta_{\mu\nu} J^\nu \quad (5.69)$$

With cords defined on game theory, this allows engineers to enclose human factor in the system modeling.

Chapter 6

Christoffel's symbols

6.1 Introduction

Despite all the problems that can be elegantly solved using Kron's formalism, some engineers still do not want to recognize the qualities of this formalism. But there is one domain where, whatever they say, it is impossible to cover it using other techniques: the domain of relativity. Electrical machines is one special application of this capacity, incorporating the accelerated movement between the stator and the rotor. Electrical machine is one of the most interesting domain to abord using Kron's formalism. That's why we begin by this thematic before to look at other problems involving Christoffel's symbols.

6.2 Basic principle of electrical machines

Electrical machines work using the magnetic force. When we look at second Maxwell's equation:

$$\frac{\vec{\eta}}{S} \oint_c d\vec{c} \cdot \vec{B} = \mu \vec{j} \quad (6.1)$$

If the magnetic field is integrated over a length l_c and $\vec{j} = F/S\vec{\eta}$ this gives:

$$l_c \cdot B = \mu S j \Rightarrow \frac{l_c}{\mu} B = F \Rightarrow \frac{l_c}{\mu S} \phi_B = F \quad (6.2)$$

F is called the magnetomotive force (MM force) and is equal to the product of the current and the number of turns of wire browsed by this current. ϕ_B is the magnetic flux. Making the similarity with Ohm's law we define the reluctance:

$$R = \frac{l_c}{\mu S} \Rightarrow F = R\phi_B \quad (6.3)$$

The magnetic force comes from the electric one. The magnetic force is a relativistic transformation of the electric force. But the relation is reversible and we may consider that the magnetic force is the primitive one and the electric force derives from the magnetic force. But this second assumption makes some problems, principally about the electrostatic force that cannot be obtained coming from the magneto-static force. Anyway, it is very interesting to accept the idea of magnetic load. This load has only one particular property: it cannot exist alone, and any

positive magnetic load must be paired with a negative magnetic load.

A magnetic load b applies a magnetic force following:

$$\vec{f}_B = -b\vec{B} \quad (6.4)$$

The magnetic field emitted at a distance r by a load b' is given by Biot & Savart's law:

$$\vec{B} = \frac{\mu}{4\pi r^2} b' \vec{u}_r \quad (6.5)$$

giving:

$$\vec{f}_B = -\frac{\mu}{4\pi r^2} bb' \vec{u}_r \quad (6.6)$$

the minus sign indicates that opposite magnetic loads attract them each other and loads of same sign repel each other. We can take two very long magnets to make the experience, in order to isolate one load from the other on each magnet. Figure 6.1 illustrates this experience.

The magnetic field is defined in tesla ($[T]$). The magnetic load or "mass" in ampere meter ($[A][m]$).

If we apply a magnetic field on a bar of magnet, it creates a magnetic torque as shown figure 6.2. The force applied by a magnetic field created for example by a MM force F coming from a solenoid on the magnetic mass at the extremities of the magnet of length $2l$ (we also call the magnetic mass "pole", with the north pole equivalent to the positive mass and the south pole equivalent to the negative mass), appears through the magnetic torque C of value:

$$C = mlB\sin(\theta) = f_B l \sin(\theta) \quad (6.7)$$

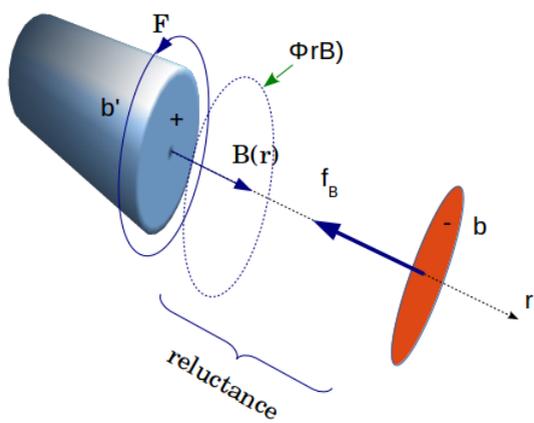


Figure 6.1: Magnetic force

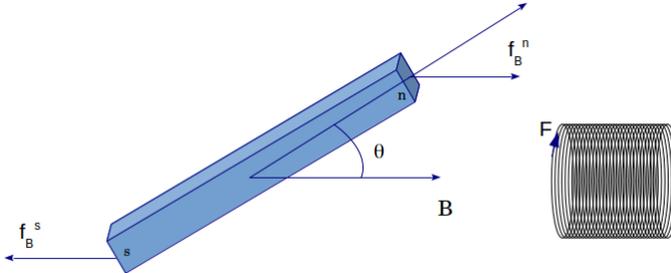


Figure 6.2: Magnetic torque

Now we begin to be able to imagine how to construct an electrical machine. But it involves a solenoid. First task consists so to be able to construct and control a solenoid. And the task is not so easy that it may appear. It becomes really difficult if we want also to model this solenoid in high frequencies for the electromagnetic compatibility for example.

A solenoid behaves like a magnet. It has two poles. To create a performant magnetic flux inside the machines, it is necessary to have a good overcome of the solenoid making. The solenoids inside the electrical machines use steel support in order to increase the magnetic permeability μ . A first consequence to transmit the magnetic force from an emitter to a receiver, is that a strong permeability allows to control the propagation of the flux. It creates a low reluctance, indicating by there that

the flux will use in first the way going through the steel rather than going through the air.

First work is to compute the value of the inductance of a single loop. Figure 6.3 shows a loop located around a piece of steel.

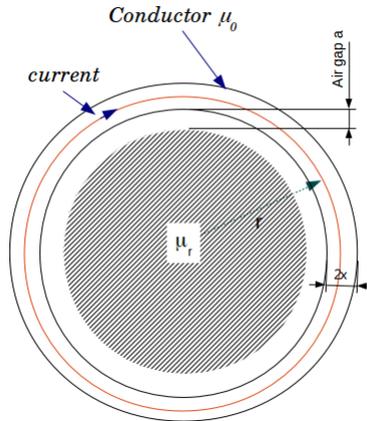


Figure 6.3: Loop for machine

If no material is inside the loop, we can compute the magnetic field coming from both sides of a line passing through the

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center of the loop. We have:

$$dB(y) = \frac{\mu_0 i r d\theta}{4\pi} \left[\frac{1}{y^2} + \frac{1}{(D-y)^2} \right] \quad (6.8)$$

with $D = 2r - 2x$. If we trace this function, we see that it is near to be constant except very near to the borders of the loop. This typical value is given by the integral:

$$B = \int_0^{2\pi} d\theta r \frac{\mu_0 i}{4\pi r^2} = \frac{\mu_0 i}{2r} \quad (6.9)$$

This gives us the MM force. Now we imagine a second loop. It can be located up to the first one, with a diameter near to be the same of the first loop. Or it can be located around the first loop. Let's study the first assumption.

The reluctance from one loop to the other is made of two reluctances in parallel. A reluctance concerns the tube of material (steel) and the other concerns the gap of air. The EM force induced in the second loop which defines the mutual inductance, is given by:

$$\dot{\phi}_B = - \left(\mu \frac{(r-a)^2}{l_C} + \mu_0 \frac{2\pi r a}{l_C} \right) \dot{F} \Rightarrow M = \left(\mu \frac{(r-a)^2}{l_C} + \mu_0 \frac{2\pi r a}{l_C} \right) \quad (6.10)$$

l_C is the height between the two loops, from center to center.

But a particular effect changes a little these expressions: the skin effect. On the separate loop, from equation 6.9, we can obtain the inductance in free space:

$$L = \frac{\mu_0 \pi r}{2}$$

But at low frequencies, a part of the magnetic flux uses the metallic medium of the loop. Its surface is $2\pi r\delta$, δ being the skin depth:

$$\delta = \frac{1}{\sqrt{\pi\nu\mu\sigma}} \quad (6.11)$$

ν is the frequency and σ the conductivity. The inductance can be estimated, in presence of the magnetic material (steel) by:

$$L = \frac{\mu_0\pi}{2x} \left[\mu_r (r - x - a)x + 2 \left(r - x - \frac{a}{2} \right) a + \frac{2r}{\sqrt{\pi\nu\mu\sigma}} \right] \quad (6.12)$$

The frequency increasing, the material blocks the penetration of the magnetic field by the same phenomenon which is the skin effect. It means that the reluctance, and as a consequence the mutual inductance change in frequency. The mutual becomes:

$$M = \frac{\mu_0}{l_C} \left[2\pi r a + \mu_r \left\{ (r - a)^2 + \frac{2\pi(r - a)}{\sqrt{\pi\nu\mu\sigma}} \right\} \right] \quad (6.13)$$

Note that the relative permeability is a function of the form $\mu_r [e^{-\beta\nu} - j(1 - e^{-\beta\nu})]$.

We can study a first and well winding solenoid, made of three loops. This first simple example will help us to complete the model of the solenoid with the parasitic capacitors. The distance minimum between two loops or between a loop and the kernel of steel is the depth of the isolating. To the inductance and mutual inductance, we must add the capacitor between two successive loops. The capacitor between one loop and the kernel

of steel is:

$$C_{lk} = \epsilon_0 \frac{4\pi^2 r}{\ln \left(\frac{2h}{d} + \sqrt{\left(\frac{2h}{d}\right)^2 - 1} \right)} \quad (6.14)$$

and the capacitor between two loops is:

$$C_{ll} = \epsilon_0 \frac{2\pi^2 r}{\ln \left(\frac{D}{d} + \sqrt{\left(\frac{D}{d}\right)^2 - 1} \right)} \quad (6.15)$$

D is the distance between two loops, d the diameter of the wire, h the distance between one loop and the kernel of steel. For three loops, the graph is given figure 6.4. Each wire making a loop having its own resistance:

$$R = \frac{1}{\sigma} \frac{2\pi r}{\pi x^2} + \frac{1}{\sigma} \frac{2\pi r}{2\pi x} \sqrt{\pi \nu \mu \sigma} \quad (6.16)$$

This graph should conduct to the expression of the impedance of a solenoid made of three loops.

It remains to connect the three loops between them. We can represent the loops on a plane to show clearly their connections. Figure 6.5 shows this presentation.

The capacitor g exists between the open extremities of one loop. The structure is quite complex and needs to establish the connections between the branches and the meshes to define the mesh properties. We hope to find some periodicity that could be reuse for a higher number of loops. We define six meshes from θ to E . The first mesh θ groups the three loops and the circuit of the generator. Noting $M_{n,n+2}$ the mutual inductance between

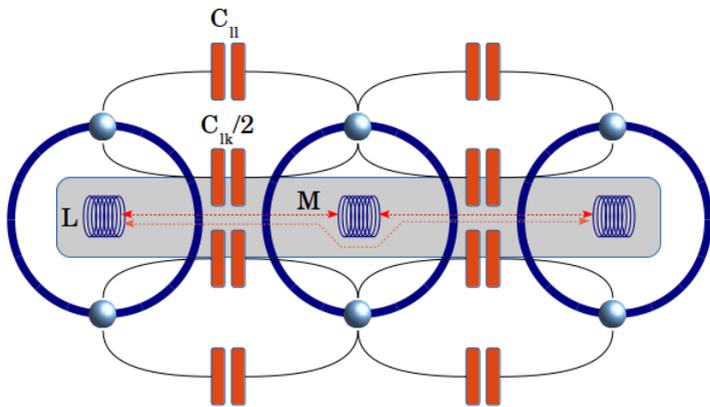


Figure 6.4: Three loops coupled

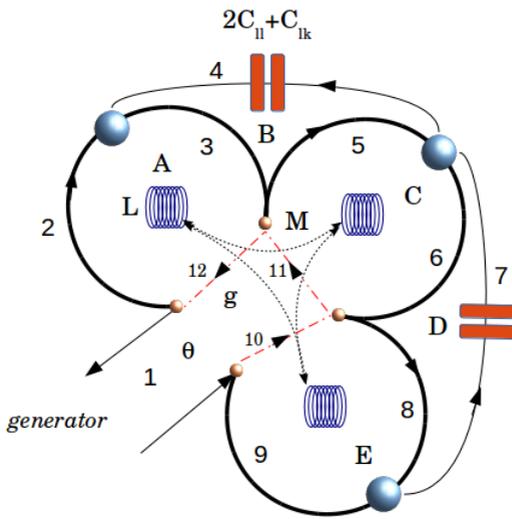


Figure 6.5: Three loops coupled and connected

two loops that are nearby and $M_{n,m}$ the mutual inductance between two loops that are not nearby and $C = 2C_{ll} + C_{lk}$ we find the operator ζ given by:

$$\zeta = \begin{bmatrix} R_0 + 3R + 3Lp & R & R & R & R & R \\ R & Lp + R + \frac{1}{gp} & \frac{R}{2} & -M_{ACP} & 0 & -M_{AEP} \\ R & \frac{R}{2} & \frac{1}{Cp} + R & \frac{R}{2} & 0 & 0 \\ R & -M_{ACP} & \frac{R}{2} & R + Lp + \frac{1}{gp} & \frac{R}{2} & -M_{CEP} \\ R & 0 & 0 & \frac{R}{2} & \frac{1}{Cp} + R & \frac{R}{2} \\ R & -M_{AEP} & 0 & -M_{CEP} & \frac{R}{2} & Lp + R + \frac{1}{gp} \end{bmatrix} \quad (6.17)$$

We look at the input impedance of the solenoid. It is given by:

$$z_E = \frac{u_1 - R_0 J^1}{J^1}$$

Seeing the structure of the lagrangian ζ , we see that if R tends to zero, the input impedance is $z_E = R + 3Lp$. In general $J^1 = y^{1x} u_x$. Note that the impedance we look for here, is the impedance of the differential mode. The figure 6.6 shows the impedance module computed for typical geometrical values. We see that the added element like the capacitors doesn't change, in this frequency band, the global inductive behavior of the solenoid.

From the electromagnetic compatibility point of view, the problem comes from the common mode impedance. The topology of this impedance is not similar to the differential mode impedance. This comes from the fact that the capacitors to the steel kernel are in series in the differential mode and in parallel in the common mode.

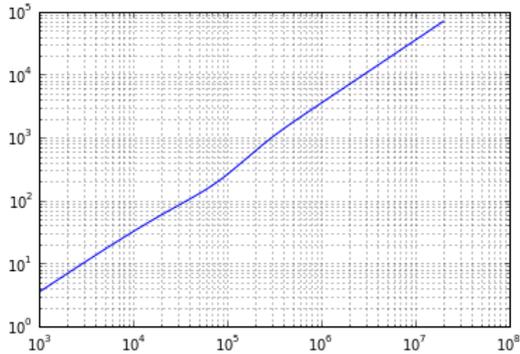


Figure 6.6: Differential mode impedance

The figure 6.7 shows the topology considered in common mode measurement. Note that we have change a little the organization of the capacitors. But this should be of second order. First remarkable fact is that in common mode, the capacitors between the loops and the structure are of first influence.

If we consider the added mesh for the measurement as the first one, we obtain the following impedance operator for this circuit:

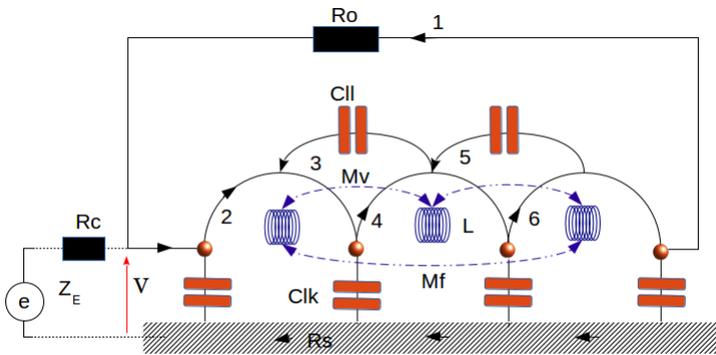


Figure 6.7: Circuit for common mode measurement

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$$\zeta = \begin{bmatrix} R_c + L_c p + C_{lk} p & 0 & -\frac{1}{R} C_{lk} p & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_0 + 3R + 3Lp & \frac{2}{C_{lk} p} + R + R_s + Lp & 0 & 0 & R & 0 & 0 & R \\ 0 & 0 & \frac{R}{2} & \frac{1}{C_{lk} p} + R & 0 & \frac{R}{2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_{lk} p} - M_{vp} & -\frac{1}{C_{lk} p} - M_{vp} & 0 & -\frac{1}{C_{lk} p} - M_{vp} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_{lk} p} - M_{vp} & -\frac{1}{C_{lk} p} - M_{vp} & 0 & -\frac{1}{C_{lk} p} - M_{vp} & 0 & 0 & 0 \\ 0 & 0 & \frac{R}{2} & \frac{R}{2} & 0 & \frac{R}{2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_{lk} p} - M_{vp} & -\frac{1}{C_{lk} p} - M_{vp} & 0 & -\frac{1}{C_{lk} p} - M_{vp} & 0 & 0 & 0 \\ 0 & 0 & \frac{R}{2} & \frac{R}{2} & 0 & \frac{R}{2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_{lk} p} - M_{vp} & -\frac{1}{C_{lk} p} - M_{vp} & 0 & -\frac{1}{C_{lk} p} - M_{vp} & 0 & 0 & 0 \end{bmatrix} \quad (6.18)$$

Figure 6.8 shows the impedance measurement in common mode.

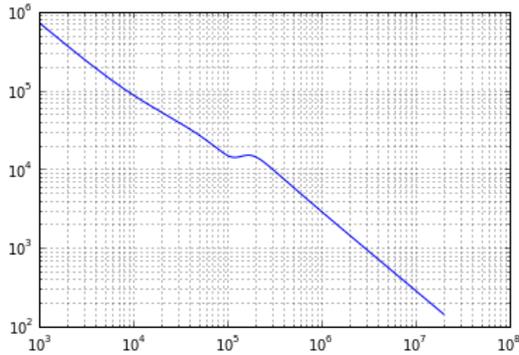


Figure 6.8: Input impedance for common mode measurement

The local increase of impedance around 200 kHz comes from the impedance of the kernel of steel and its complex and real properties. Its conductivity, even low and its permeability - a high one - conducts to this kind of behavior. In this computation, the impedance of the kernel is given by:

$$R_s = \frac{1}{\sigma_K} \frac{l_C}{0.5\pi r^2} + \frac{1}{\sigma_K} \frac{l_C}{2\pi r} \sqrt{\pi\nu\mu_0\mu_r(\nu)\sigma_K}$$

Using the same method we can model solenoid with higher number of turns. For example the figure 6.9 shows a solenoid made of two layers where three new loops are added and located

on top of the three previous ones. We understand that depending on how the layers are made, various capacitors and mutual inductances are distributed all over the whole solenoid.

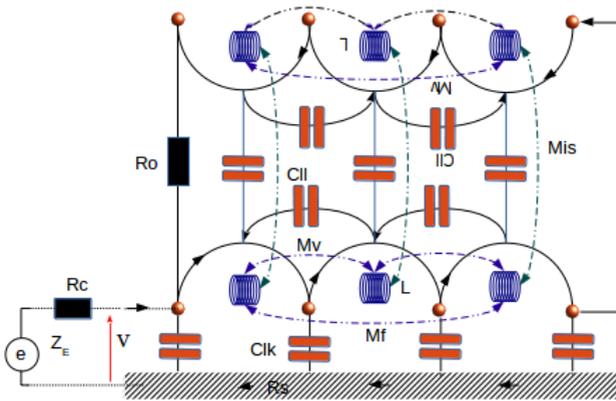


Figure 6.9: Solenoid with two layers

More complex fitting out can be imagined. They can call for statistic approaches if the location of the wires are not determined in an accurately defined way.

Once we know how to make the coils, we have to imagine the mechanical process that will use the magnetic force in order to create movement. First, let's come back on this force and its mechanism.

Intrinsically, the magnetic force comes from the current. It doesn't exist magnetic loads (masses) and the magnetic force coming from magnet is generated by the electrons and their spins.

The magnetic force is given by:

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad (6.19)$$

as $q\vec{v} = x\vec{i}$ by replacement we obtain:

$$\vec{F}_B = x\vec{i} \times \vec{B} \quad (6.20)$$

Note that I have chosen here to take the current as the vector. It can be obtained through $\vec{i} = i\vec{u}_x$, \vec{u}_x being the normalized vector in the direction of x that carries the current. As the magnetic mass cannot exist without appearing as a pair, this leads to the fundamental result that the divergence of the magnetic field is zero.

The magnetic force can be well understood looking at the figure 6.10. A loop of current i is equivalent with a magnet with its two poles north and south. But if this current is immersed in a magnetic field, two forces are applied on the wire associated with the two components of the field, following the two directions x and y .

The forces on x are in opposition from one side of the loop to the other and lead to a zero force in this direction. But the force on y applies a torque. If only By exists, there are no forces applied on the loop but an electromotive force is induced inside the loop. If there is only the magnetic field in the direction x , the torque is applied and the loop tends to turn around its

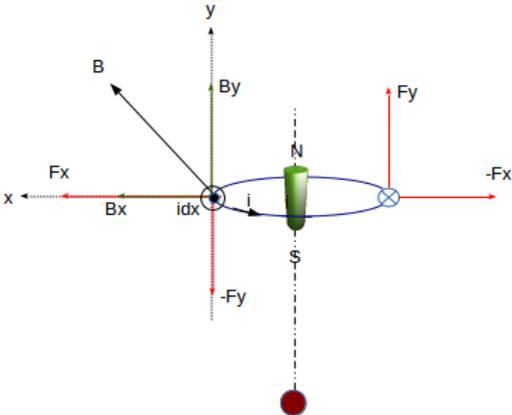


Figure 6.10: Forces on a loop

central axis. Once this process is understood, we can discern two kinds of coupling: a first coupling of mutual inductance M between loops associated with the y component and a second coupling of cut off flux G associated with the x component of the field. To these inductances is added the self inductance of each coil L . The figure 6.11 represents all these couplings which define the whole metric of the system.

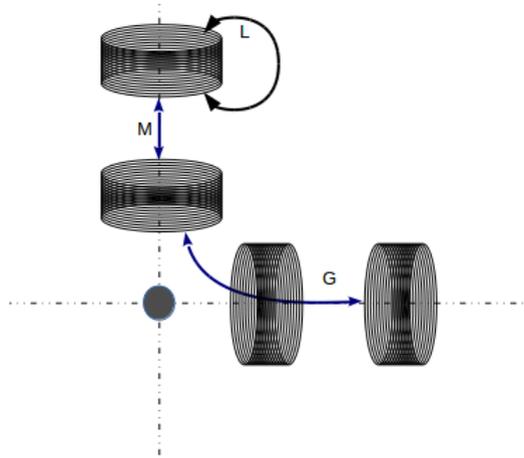


Figure 6.11: Various metrics in the coupled loops

How acts the cut off flux? The force is linked with qvB . It means that its work is given by $qvBl$. But this work is also qe

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where e is an EM force. Finally the EM force is vIB . As B is a function of i , the cut off flux coupling mechanism creates an EM force in relation with the equation $G\Omega i$, Ω being the angular speed of the rotor of the machine. By another side we have seen that the cut off flux was responsible for the torque applied on the rotor. The mechanical equation of the machine is given by:

$$\Gamma = K \frac{d\Omega}{dt} + \rho\Omega + Gii' \quad (6.21)$$

K being the inertia of the machine, ρ the dissipation due to the friction of the axis of the rotor and i and i' the currents in stator and rotor coils. With the assumption that the machine works in constant regime and that there are no frictions, the torque can be simply estimated by:

$$\Gamma = Gii'$$

In general, we have:

$$\Gamma = G_{\alpha\beta} i^\alpha i^\beta \quad (6.22)$$

This comes from the fact that the torque is a power, that must be equal to the electrical power delivered to the coil, ei .

A generalization of the previous relations appears for any number of coils:

$$\begin{cases} e_\alpha = r_{\alpha\beta} i^\beta + \mathcal{L}_{\alpha\beta} \frac{di^\beta}{dt} + G_{\alpha\beta} \Omega i^\beta \\ \Gamma = K \frac{d\Omega}{dt} + \rho\Omega + i^\alpha G_{\alpha\beta} i^\beta \end{cases} \quad (6.23)$$

but these relations are defined for a static rotor. If the rotor moves, the situation is different.

We consider the machine made of four coils represented figure 6.12.

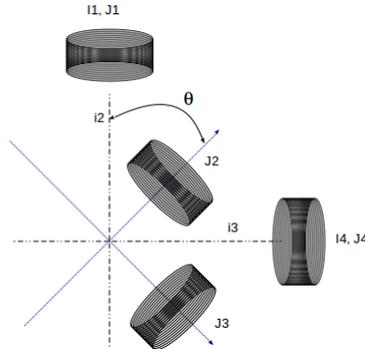


Figure 6.12: Mobile axis for the rotor

The relations written between the coils in a static location can be transformed to take into account the changing position of the coils when the rotor turns. To do that we write the relations between the currents in the imagined static coils and the currents in the mobile coils. As we can understand it seeing the figure 6.12, the relations are:

$$\begin{cases} i^1 = J^1 \\ i^2 = J^2 \cos\theta - J^3 \sin\theta \\ i^3 = J^2 \sin\theta + J^3 \cos\theta \\ i^4 = J^4 \end{cases} \quad (6.24)$$

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These relations leads to the connectivity matrix:

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \text{Cos}\theta(t) & -\text{Sin}\theta(t) & 0 \\ 0 & \text{Sin}\theta(t) & \text{Cos}\theta(t) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6.25)$$

We can replace the currents i by the currents J in the equation of the machine given equation 6.23, using $i^k = \Lambda_m^k J^m$:

$$e_\alpha = r_{\alpha\beta} \Lambda_\sigma^\beta J^\sigma + \mathcal{L}_{\alpha\beta} \frac{d\Lambda_\sigma^\beta J^\sigma}{dt} + G_{\alpha\beta} \Omega \Lambda_\sigma^\beta J^\sigma \quad (6.26)$$

and to end the bilinear product:

$$\Lambda_\gamma^\alpha e_\alpha = \Lambda_\gamma^\alpha r_{\alpha\beta} \Lambda_\sigma^\beta J^\sigma + \Lambda_\gamma^\alpha \mathcal{L}_{\alpha\beta} \frac{d\Lambda_\sigma^\beta J^\sigma}{dt} + \Lambda_\gamma^\alpha G_{\alpha\beta} \Omega \Lambda_\sigma^\beta J^\sigma \quad (6.27)$$

Let's compute the second term.

$$\frac{d\Lambda_\sigma^\beta J^\sigma}{dt} = \Lambda_\sigma^\beta \frac{dJ^\sigma}{dt} + \frac{d\Lambda_\sigma^\beta}{dt} J^\sigma$$

but:

$$\frac{d\Lambda_\sigma^\beta}{dt} = \frac{d\Lambda_\sigma^\beta}{d\theta} \frac{d\theta}{dt} = \frac{d\Lambda_\sigma^\beta}{d\theta} \Omega$$

Noting with a breve the quantities in the mobile axis reference frame and:

$$\Lambda_\gamma^\alpha \mathcal{L}_{\alpha\beta} \frac{d\Lambda_\sigma^\beta}{dt} = \Gamma_{\gamma\sigma,\theta} \quad (6.28)$$

we obtain:

$$\check{e}_\gamma = \check{r}_{\gamma\sigma} J^\sigma + \Gamma_{\gamma\sigma,\theta} \Omega J^\sigma + \check{\mathcal{L}}_{\gamma\sigma} \frac{dJ^\sigma}{dt} + \check{G}_{\gamma\sigma} \Omega J^\sigma \quad (6.29)$$

We see that Γ can also be seen as the derivation of $\check{\mathcal{L}}$ versus θ :

$$\Gamma_{\gamma\sigma,\theta} = \frac{1}{2} \frac{\partial}{\partial \theta} \check{\mathcal{L}}_{\gamma\theta} \quad (6.30)$$

The coefficients Γ are in relation with Christoffel's coefficients.

Christoffel's coefficients are clearly defined in differential geometry. We imagine a system characterized by the operator:

$$\zeta = \begin{bmatrix} L_{11}p + z_{11} & -M_{12}p & 0 \\ -M_{21}p & L_{22}p + z_{22} & -M_{23}p \\ 0 & -M_{32}p & L_{33}p + z_{33} \end{bmatrix} \quad (6.31)$$

From $e_\alpha = \zeta_{\alpha\sigma} J^\sigma$ we can extract the base vectors of the hypersurface $e_\alpha(\zeta, J)$. To do that we have to compute:

$$\mathbf{b}_x = \frac{\partial e}{\partial J^1} \rightarrow \frac{\partial e_\nu}{\partial J^\sigma} = \mathcal{J}_{\nu\sigma} \quad (6.32)$$

We imagine that the mutual inductance M depends on the current (it can be a ferromagnetic material). We have to compute for example $\partial_t(M_{21}J^1)$ (in fact it means that rather than $Mp(\cdot)$ we have $p(M(\cdot))$). We write:

$$\frac{d}{dt}(M_{21}J^1) = M_{21} \frac{dJ^1}{dt} + J^1 \frac{d}{dt} M_{21}$$

$$\frac{d}{dt}(M_{21}J^1) = M_{21} \dot{J}^1 + J^1 \frac{\partial M_{21}}{\partial J^1} \frac{dJ^1}{dt}$$

$$\frac{d}{dt}(M_{21}J^1) = (M_{21} + J^1 \Gamma_{21,1}) \dot{J}^1$$

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The ζ operator is in the time domain:

$$\zeta = \begin{bmatrix} L_{11} \frac{d}{dt} + z_{11} & - (M_{12} + J^2 \Gamma_{12,2}) \frac{d}{dt} & 0 \\ - (M_{21} + J^1 \Gamma_{21,1}) \frac{d}{dt} & L_{22} \frac{d}{dt} + z_{22} & - (M_{23} + J^3 \Gamma_{23,3}) \frac{d}{dt} \\ 0 & - (M_{32} + J^2 \Gamma_{32,2}) \frac{d}{dt} & L_{33} \frac{d}{dt} + z_{33} \end{bmatrix} \quad (6.33)$$

Note that we have simplified the analysis, making the assumption that the mutual inductance depends only of the current for which it is in factor (in general that's not a correct assumption).

Now we have to compute for example $\partial_{J^1} (M_{21} + J^1 \Gamma_{21,1}) \dot{J}^1$ to determine one component of the vector \mathbf{b}_1 , and:

$$\begin{aligned} \frac{\partial}{\partial J^1} (M_{21} + J^1 \Gamma_{21,1}) \dot{J}^1 = \\ \Gamma_{21,1} \dot{J}^1 + \Gamma_{21,1} \dot{J}^1 + J^1 \frac{\partial \Gamma_{21,1}}{\partial J^1} \dot{J}^1 \end{aligned}$$

The last term is zero. Finally:

$$\frac{\partial}{\partial J^1} (M_{21} + \Gamma_{21,1}) \dot{J}^1 = 2\Gamma_{21,1} \dot{J}^1 + J^1 \frac{\partial \Gamma_{21,1}}{\partial J^1} \dot{J}^1 \quad (6.34)$$

We can now compute the basis vectors:

$$\begin{aligned}
 \mathbf{b}_1 &= \begin{bmatrix} z_{11} \\ \left(J^1 \frac{\partial \Gamma_{21,1}}{\partial J^1} + 2\Gamma_{21,1} \right) \dot{J}^1 \\ 0 \end{bmatrix} \\
 \mathbf{b}_2 &= \begin{bmatrix} \left(J^2 \frac{\partial \Gamma_{12,2}}{\partial J^2} + 2\Gamma_{12,2} \right) \dot{J}^2 \\ z_{22} \\ \left(J^2 \frac{\partial \Gamma_{32,2}}{\partial J^2} + 2\Gamma_{32,2} \right) \dot{J}^2 \end{bmatrix} \\
 \mathbf{b}_3 &= \begin{bmatrix} 0 \\ \left(J^3 \frac{\partial \Gamma_{23,3}}{\partial J^3} + 2\Gamma_{23,3} \right) \dot{J}^3 \\ z_{33} \end{bmatrix}
 \end{aligned} \tag{6.35}$$

From these basis vectors we define the metric G . But we can already see something remarkable: the basis vectors depends on the time derivative of the currents. It means that from one location to another, the basis of the tangent space changes.

$$\left\{ \begin{array}{l}
 G_{11} = (z_{11})^2 + \left[\left(J^1 \frac{\partial \Gamma_{21,1}}{\partial J^1} + 2\Gamma_{21,1} \right) j^1 \right]^2 \\
 G_{12} = G_{21} = z_{11} \left(J^2 \frac{\partial \Gamma_{12,2}}{\partial J^2} + 2\Gamma_{12,2} \right) j^2 + z_{22} \left(J^1 \frac{\partial \Gamma_{21,1}}{\partial J^1} + 2\Gamma_{21,1} \right) j^1 \\
 G_{22} = \left[\left(J^2 \frac{\partial \Gamma_{12,2}}{\partial J^2} + 2\Gamma_{12,2} \right) j^2 \right]^2 + (z_{22})^2 \\
 G_{23} = G_{32} = z_{22} \left(J^3 \frac{\partial \Gamma_{23,3}}{\partial J^3} + 2\Gamma_{23,3} \right) j^3 + z_{33} \left(J^2 \frac{\partial \Gamma_{32,2}}{\partial J^2} + 2\Gamma_{32,2} \right) j^2 \\
 G_{13} = G_{31} = \left(J^1 \frac{\partial \Gamma_{21,1}}{\partial J^1} + 2\Gamma_{21,1} \right) j^1 \left(J^3 \frac{\partial \Gamma_{23,3}}{\partial J^3} + 2\Gamma_{23,3} \right) j^3 \\
 G_{33} = \left[\left(J^3 \frac{\partial \Gamma_{23,3}}{\partial J^3} + 2\Gamma_{23,3} \right) j^3 \right]^2 + (z_{33})^2
 \end{array} \right. \quad (6.36)$$

Starting from the equation $e_\nu = \zeta_{\nu\sigma} J^\sigma$ we can transform it in:

$$e_\nu - \mathcal{L}_{\nu\sigma} j^\sigma = H_{\nu\sigma} J^\sigma \quad (6.37)$$

but we see that:

$$H_{\nu\sigma} = \mathcal{J}_{\nu\sigma} - \left(J^\alpha \frac{\partial \Gamma_{\nu\sigma,\alpha}}{\partial J^\alpha} + \Gamma_{\nu\sigma,\alpha} \right) j^\alpha$$

with $\mathcal{V}_\mu^\nu = (\mathcal{J}_{\nu\mu})^T$, we obtain:

$$\mathcal{V}_\mu^\nu \left(e_\nu - \mathcal{L}_{\nu\sigma} j^\sigma \right) = \mathcal{V}_\mu^\nu \mathcal{J}_{\nu\sigma} J^\sigma - \mathcal{V}_\mu^\nu \left(J^\alpha \frac{\partial \Gamma_{\nu\sigma,\alpha}}{\partial J^\alpha} + \Gamma_{\nu\sigma,\alpha} \right) j^\alpha J^\sigma \quad (6.38)$$

which gives:

$$T_\mu = (G_{\mu\sigma} - \mathcal{V}_\mu^\nu \Gamma_{\nu\sigma,\alpha} p J^\alpha) J^\sigma - \mathcal{V}_\mu^\nu J^\alpha \frac{\partial \Gamma_{\nu\sigma,\alpha}}{\partial J^\alpha} p J^\alpha J^\sigma \quad (6.39)$$

Now we can take a look to the terms

$$[\nu\alpha, \sigma] = \left\langle \frac{\partial \mathbf{b}_\nu}{\partial J^\alpha}, \mathbf{b}_\sigma \right\rangle$$

The object $[\nu\alpha, \sigma]$ are the coefficients of Christoffel of the first species. If we make the assumption that the coefficients $\Gamma_{xy,z}$ do not depend on the currents at second order, we have

$$\frac{\partial}{\partial J^\alpha} \frac{\partial \Gamma_{\nu\sigma,\beta}}{\partial J^\beta} = 0$$

Under this assumption, we obtain for example:

$$[11, 2] = z_{22} \frac{\partial \Gamma_{21,1}}{\partial J^1} j^1 \quad (6.40)$$

or in general:

$$[\mu\nu, \sigma] = z_{\sigma\sigma} \frac{\partial \Gamma_{\mu\sigma}}{\partial J^\nu} j^\sigma \quad (6.41)$$

and equation 6.39 may becomes:

$$T_\mu = (G_{\mu\sigma} - [\mu\sigma, \nu] p J^\nu) J^\sigma \quad (6.42)$$

The term

$$\mathcal{V}_\mu^\nu \frac{\partial \Gamma_{\nu\sigma,\alpha}}{\partial J^\alpha} \frac{d}{dt}$$

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can be seen as an added inductance due to the dependence of the medium to the flux intensity. It comes in addition with \mathcal{L} . The figure 6.13 shows the curbature of the trajectory of the current for a simple RL circuit when the inductance depends on the current intensity. The inductance increasing, the time rise tends to be longer.

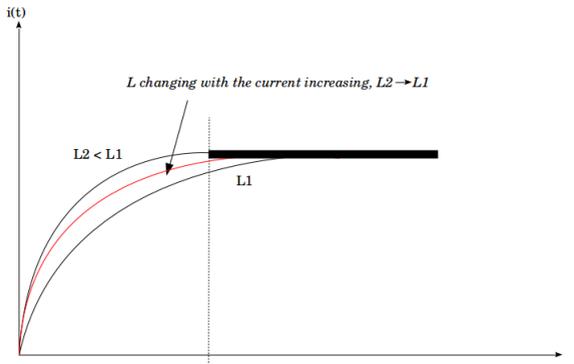


Figure 6.13: Curved space

The curvature of the space appears because at different points of currents, the local time L/R is not the same.

6.3 Curvilinear coordinates: q or J ?

When we study the hypersurface e_γ , we make the derivation

$$\frac{\partial e_\gamma}{\partial x^\beta}$$

In mechanics, x^β is the location of the mobile. The equivalence between mechanics and electronics is often used to replace x^β by q^β in the lagrangian. We may associate q^β with the curvilinear coordinates in the geometrization process of electronics. We have seen that this is possible and leads to the definition of mesh loads.

Anyway it is possible also to take J^β for state variables rather than q^β . The choice has an impact on the tangent plan definition and on the jacobian. In the case of the inductances, we calculate:

$$\frac{\partial}{\partial q^\beta} (L_{\mu\nu} p J^\nu) \quad (6.43)$$

or

$$\frac{\partial}{\partial J^\beta} (L_{\mu\nu} p J^\nu) \quad (6.44)$$

The second case leads to zero for the second member. That's why the metric must be complete by the inertia tensor to retrieve the lagrangian. The second member of the first choice gives:

$$L_{\mu\nu} p \frac{\partial}{\partial q^\beta} J^\nu = L_{\mu\nu} p \frac{\partial}{\partial q^\beta} \frac{dq^\nu}{dt} \quad (6.45)$$

which gives also zero. The difference comes from the terms like $R_{\mu\nu} J^\nu$. Looking for the basic vectors of the hypersurface

we find:

$$\frac{\partial}{\partial q^\beta} R_{\mu\nu} J^\nu = R_{\mu\nu} \frac{\partial}{\partial q^\beta} \frac{dq^\nu}{dt} = 0 \quad (6.46)$$

The fact that the dissipation leads also to zero shows that the good choice is to use the fluxes as curvilinear coordinates. From this choice it seems logical to define new Christoffel's symbols:

$$\Gamma_{\mu\nu,\sigma} = \langle \mathbf{b}_{\mu\nu}, \mathbf{b}_\sigma \rangle \quad (6.47)$$

with:

$$\mathbf{b}_{\mu\nu} = \frac{\partial \mathbf{b}_\mu}{\partial J^\nu}$$

In mechanics we may have defined

$$\mathbf{b}_{\mu\nu} = \frac{\partial \mathbf{b}_\mu}{\partial x^\nu}$$

and the equivalence with the loads may invite to write:

$$\mathbf{b}_{\mu\nu} = \frac{\partial \mathbf{b}_\mu}{\partial q^\nu}$$

but we have seen that it is not the unique choice. If we take the speed in mechanics to define the natural space, Newton's law makes appear immediately the fundamental tensor and the dual space:

$$f_\alpha = m_{\alpha\sigma} p v^\sigma \quad (6.48)$$

The invariant in this case is $f_\nu v^\nu = w$, the elementary power of all the particle that belong to the system studied. As the power is also the invariant of Kron's formalism, we can wonder if it

may be a general invariant for all physics? It is equivalent to consider:

$$w = \frac{1}{2} \frac{d}{dt} (m_{\mu\nu} v^\mu v^\nu) = \frac{1}{2} \Gamma_{\mu\nu,\sigma} \gamma^\sigma v^\mu v^\nu + m_{\mu\nu} v^\mu \gamma^\nu \quad (6.49)$$

with $\gamma = \dot{v}$. We obtain:

$$w = v^\mu \left(\frac{1}{2} \Gamma_{\mu\nu,\sigma} \gamma^\sigma v^\nu + m_{\mu\nu} \gamma^\nu \right) \quad (6.50)$$

giving:

$$f_\mu = m_{\mu\nu} \gamma^\nu + \frac{1}{2} \Gamma_{\mu\nu,\sigma} \gamma^\sigma v^\nu \quad (6.51)$$

If the mass depends on the speed (relativistic effect), the Christoffel's coefficients appear with:

$$\Omega_{\mu\sigma} = \frac{1}{2} \Gamma_{\mu\nu,\sigma} v^\nu \quad (6.52)$$

This inertia is added to the cartesian one $m_{\mu\nu}$ making that for the same force, the speed will be slower than usual.

Chapter 7

Metrics

7.1 Metrics

We have seen that various kinds of metrics can be defined:

1. the rigorous metric G defined through the second geometrical process;
2. the metric \mathcal{L} issued from the kinetic energy and inductance operators;
3. the "pseudo-metric" ζ issued directly from Kron's equation¹.

¹We have $\zeta = H + \mathcal{L}$

In any case, these objects are fundamental and give solutions to complex problems. We explore here these various behaviors and discuss of the use of these various "metrics".

7.2 The point of view of automaticians

Automatics identify inputs, outputs and state variables on systems. Often inputs are noted u and state variables x . Outputs are noticed y . Under the tensorial analysis of networks formalism, the state variables are the fluxes $\rightarrow x^k$ and the inputs are the forces u_m . The outputs are only particular state variables. The system is represented by the operator H_{mk} that links the state variables and the inputs. The inputs can always be represented by nodes pair fluxes, meaning that u_m is also a particular state variables, a known one x^k . The system is often written using the metric \mathcal{L} through:

$$\mathcal{L}_{\alpha\beta} p \frac{dx^\beta}{dt} = H_{\alpha\beta} (x^\beta) + \gamma_\alpha^\sigma u_\sigma \quad (7.1)$$

Implicitly, each state variable belongs to a known interval of values $I(x^k)$.

7.3 Observability

The most interesting thinking of automatics is the discussion around the concept of observability. The state variable $x^{(k)}$ is said observable if a given function y^k readable allows to know

the value of $x^{(k)}$ at a given instant t . The system is defined by:

$$\begin{cases} \mathcal{L}_{\alpha\beta} p \frac{dx^\beta}{dt} = H_{\alpha\beta}(x^\beta) + \gamma_\alpha^\sigma u_\sigma \\ y^q = A_n^q x^n + Y^{qa} u_a \end{cases} \quad (7.2)$$

As the matrix A determines whether or not there is a link between each state variable and the possible observations, the observed state variable $x^{(k)}$ can point out all the vector x^k . The same question and mechanism can be applied to the covector u_m . In fact we can conclude on the observability saying that any state variable x or input u can be known if the corresponding observable y is known.

This implies that the state variables are themselves known. A first consequence is that the system can be inverted in order to compute the state variable.

7.4 Controlability

The commands of the system are enclosed in the terms u_m . Other commands comes from the fluxes - i.e. some states variables - through coupling processes.

The observables are linked with the states variables through $y^k = C_n^k x^n$. But we can replace x^n by $Y^{nq} u_q$. So:

$$y^k = C_n^k Y^{nq} u_q = D^{kq} u_q \quad (7.3)$$

The commands u_q are observable if the object D^{kq} can be defined. That's a first condition to say if the commands are observable.

Now a criterion to say if a command u is effective, is to verify if a change in the value of a chosen input, let's say u_1 change the observable, let's say y^3 as desired. It leads to study $\partial_{u_1} y^3$.

Linear case

If $\zeta_{\alpha\beta}$ is a linear operator, we can write:

$$\frac{\partial y^3}{\partial u_1} = C_\alpha^3 \frac{\partial x^\alpha}{\partial u_1} = C_\alpha^3 Y^{\alpha 1} \quad (7.4)$$

Y being the inverse of the pseudo-metric ζ . But to be able to make this equality, it implies that Y is a linear operator coming from the linear relation $u_1 = \zeta_{1\alpha} x^\alpha$.

Non linear operator

Under that assumption we can use the form:

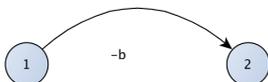
$$\mathcal{L}_{\mu\alpha} \frac{dx^\alpha}{dt} = H_{\mu\alpha}(x^\alpha, u_1)$$

The relation between the observables and the state variables can be chosen invertible: $y^3 = C_\alpha^3 x^\alpha \Rightarrow x^\alpha = C_3^\alpha y^3$. Then:

$$\mathcal{L}_{\mu\alpha} \frac{d}{dt} C_3^\alpha y^3 = \mathcal{L}_{\mu 3} \frac{dy^3}{dt} \quad (7.5)$$

but:

$$\frac{dy^3}{dt} = \frac{y_+^3 - y^3}{\delta_t}$$

Figure 7.1: An asymmetric graph for \mathcal{L}

By replacement we obtain:

$$y_+^3 = \frac{\delta_t}{\mathcal{L}_{\mu 3}} H_{\mu\alpha}(x^\alpha, u_1) + y^3 \quad (7.6)$$

we can create $\mathcal{A}_\alpha^3() = \frac{\delta_t}{\mathcal{L}_{\mu 3}} H_{\mu\alpha}()$.

Now we can say that a command is observable if,

$$u_1 \rightarrow u_1 + \delta u_1 \Rightarrow \mathcal{A}_\alpha^3(x^\alpha, u_1) \neq \mathcal{A}_\alpha^3(x^\alpha, u_1 + \delta u_1) \quad (7.7)$$

We see in that construction that the symmetric property of the metric \mathcal{L} is fundamental. Let's take an example. When we take a look to the figure 7.1, the coupling function linked to \mathcal{L} is not symmetric. A mutual inductance exists from the vertex 1 to the vertex 2, but not in the other direction.

By noting²:

$$\mathcal{L} = \begin{bmatrix} a & 0 \\ -b & c \end{bmatrix} \quad H = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \quad (7.8)$$

² \mathcal{L} can be seen as a metric, H the hamiltonian and ζ the lagrangian

The source covector is given by: $u_m = [u_1 \quad u_2]$. The tensorial equation of the system is given by:

$$\mathcal{L}_{\mu\nu} p y^\nu = u_\mu - H_{\mu\nu} y^\nu \quad (7.9)$$

In this development, it is clear that $y^k = \delta_\alpha^k x^\alpha$, δ being here the unity matrix. The tensorial equation leads to the system of equation:

$$\begin{cases} a p y^1 = -r_1 y^1 + u_1 \\ -b p y^1 + c p y^2 = -r_2 y^2 + u_2 \end{cases} \quad (7.10)$$

where

$$\frac{\partial y^1}{\partial u_2} = 0$$

In that example, the command u_2 has no effect on the observable y^1 .

The concept of observation is intimately linked with the real system. Studying abstractly a system, the fact that a state variable can be measured or not is not a problem. The problem appears with the notion of measurement. But it is very important because in complicated system, the fact that it is possible to access to a given variable is not evident.

It is very interesting also because it introduces the concept of uncertainties. Who speaks of measurement must speak of uncertainties. Another question occurs: is it possible to have a state variable that influence the observable but that cannot be known or not with the desired accuracy? Two major notions are in deep relation with the studying of a system:

1. the uncertainties on the variables measurements;
2. the influences of the variables on the measured result.

7.5 Studying the metric

Finally the observable as the state variables are completely defined if the metric is completely controlled. Uncertainties concern the source and the metric. The influence of a state variable comes from the paths between this state variable and the observable of interest. These paths are defined by the metric. Let's try to detail a little more these mechanisms.

7.5.1 Influence of coupling paths

Finding the influence of some vertex on a result extracted from another vertex is often quite complicated. Even if it is not evident to obtain the theoretical expression of the influence of one element on another, this kind of study gives at least the variables from which depends the result.

We consider the case of two differential lines, inside a shielded cable. We want to understand the coupling mechanism between these two lines. As we are in low frequencies, we do not take into account the propagation, and the wires of the lines are replaced by simples contacts.

The figure 7.2 shows the situation we consider.

The graph has six nodes and ten branches. It requires five meshes.

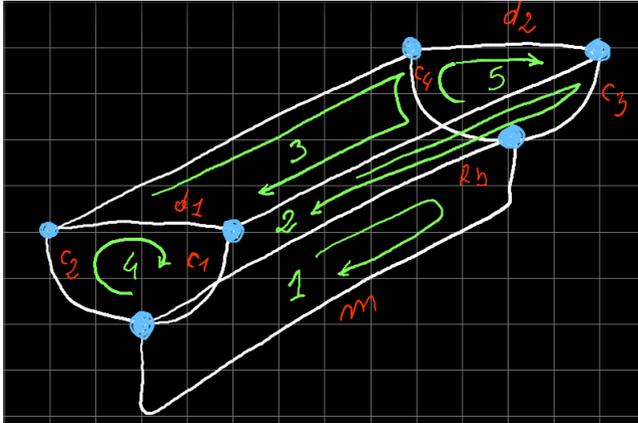


Figure 7.2: Two differential lines coupled

If we identify the meshes in this circuit, we find the five meshes. The first one is the external domain, between the ground plane and the shield of the cable. The second and third meshes are constructed between the wires and the shield. Finally the two last meshes are made with the differential impedances at each extremity of the line. Each time a branch is shared between two meshes it can be replaced by a cord of same impedance operator, with a minus sign. The circuit presented figure 7.2 is equivalent to the graph of vertices and cords presented figure 7.3.

We recognize the five meshes that are in this new representation the five vertices. The number of faces is 4 and the graph is

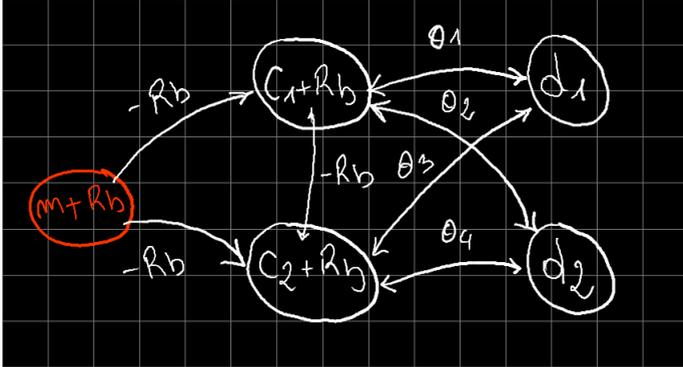


Figure 7.3: Vertices graph of problem 7.2

planar. You can try to change one of the cords linking the common mode with the differential mode in order to avoid any cross talks between cords. What is remarkable is that finally we don't need any mesh for the differential mode. It is completely defined through the definitions of the common modes. This is rigorously demonstrated by Euler's formula saying us the number of faces is 4. A second important fact is that as all the energy is guided, there are no coupling mechanisms between the two differential meshes. i.e. there is no cord between vertices d_1 and d_2 .

The direction of the transfer impedance coupling coming from the shields of the cables Z_T goes from the external domain $m + R_b$ to the internal one: the common mode $c_x + R_b$.

If there are two such lines coupled, and if the source is lo-

cated on one differential vertex, for the case of the line that has the source, the transfer impedance coupling goes from inside to outside. And for the other line, the transfer impedance goes like in figure 7.3 from outside to inside. Between the two lines, a mutual inductance creates an interaction between the two external domains. Finally the problem of two differential coupled lines is represented by the graph given figure 7.4. The numbered of the vertices is indicated on the right. The source (force) is located on vertex d_1 while the observable is located on vertex d'_2 .

With $e_1 = m + R_b$ the impedance operator H for a single line is given by:

$$H = \begin{bmatrix} e_1 & 0 & 0 & 0 & 0 \\ -Z_T & c_1 & R_b & -\theta_1 & -\theta_2 \\ -Z_T & R_b & c_2 & -\theta_3 & -\theta_4 \\ 0 & -\theta_1 & -\theta_2 & d_1 & 0 \\ 0 & -\theta_3 & -\theta_4 & 0 & d_2 \end{bmatrix} \quad (7.11)$$

To obtain such an operator, the differential loads enclose the half capacitor of the line. It means that for the low frequency model of the line, we use a C-L-C model. The metric is given by all self inductances of the vertices and the only mutual inductance of the circuit is between the two external domains e_1 and e'_1 .

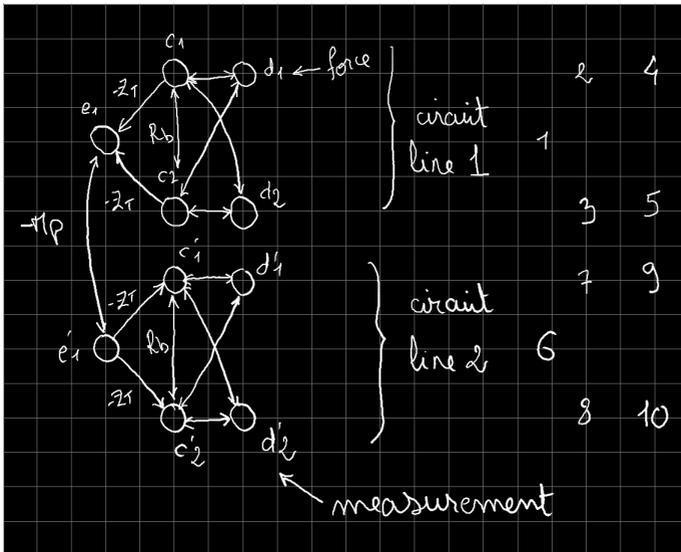


Figure 7.4: Problem of two differential lines coupled

Covariant and natural approaches

For determining the paths between two vertices we can use two different approaches. The first one we call the covariant approach consists in computing the force induced in the target x_k depending on a flux in another vertex somewhere J^m . The natural approach uses the inverse. It gives the flux induced in a vertex depending on a force somewhere in another vertex. The first method establishes:

$$u_k = \zeta_{km} x^m \quad (7.12)$$

The second approach uses:

$$x^m = Y^{mk} u_k \quad (7.13)$$

The first approach has the advantage to lead directly to the invariant:

$$u_k x^k = \zeta_{km} x^m x^k \quad (7.14)$$

Both techniques should lead to the same result anyway. Imagine a system defined by the simple hamiltonian:

$$H = \begin{bmatrix} z_1 & 0 \\ G & z_2 \end{bmatrix} \quad (7.15)$$

We have directly: $u_2 = Gx^1$. And also: $x^2 = Y^{21}u_1$.

But:

$$Y^{21} = \frac{G}{z_1 z_2}$$

Writing

$$x^2 = \frac{G}{z_1 z_2} u_1 \Rightarrow z_2 x^2 = G \frac{u_1}{z_1} \Rightarrow u_2 = G x^1$$

the two expressions are equivalents.

Paths described by the hamiltonian

The matrix H can be considered as an hamiltonian because the lagrangian operator ζ is obtained making $\zeta = \mathcal{L} - H$, giving the classical relation $H = \mathcal{L} - \zeta$.

When we look at the hamiltonian H if we focus on the paths to go from c_2 to e_1 for example, we find two paths: $c_2 - c_1 - e_1$ or directly $c_2 - e_1$. These paths can be seen on the graph figure 7.4. The sum of all the possible paths leads to the whole distance between the target and the source. Let's consider our case with the source in d_1 and the target on d'_2 . A first equation is given with the relation between the induced force u_{10} and the emf u_6 :

$$u_{10} = (\zeta_{10,7} Y^{76} + \zeta_{10,8} Y^{86}) u_6$$

This relation can be obtained simply looking at the graph.

We accept that the coupling mutual inductance M acts only in one direction. So: $u_6 = -M p x^1$.

Then u_1 can be linked with u_4 to complete the whole transfer function.

$$u_1 = -Z_T (x^2 + x^3) = -Z_T (Y^{24} + Y^{34}) u_4$$

Finally:

$$u_{10} = M p Z_T (\zeta_{10,7} Y^{76} + \zeta_{10,8} Y^{86}) (Y^{24} + Y^{34}) u_4 \quad (7.16)$$

We can always replace any force u by a nodes pair current χ that gives the same value. It is just necessary to locate this current on the spanning tree corresponding to the branch where the force is applied. We define: $u_4 = \gamma\chi^4$. With this technique, it is possible to give the definition of the path from χ^4 to u_{10} :

$$g_{10,4} = \gamma MpZ_T (\zeta_{10,7}Y^{76} + \zeta_{10,8}Y^{86}) (Y^{24} + Y^{34}) \quad (7.17)$$

It remains to obtain the Y^{ab} terms. In that case we can compute:

$$\left\{ \begin{array}{l} A = (-R_b (d_1 d_2 R_b - \theta_1 \theta_3 d_2 - \theta_2 \theta_4 d_1)) \\ B = \theta_1 ((-\theta_3 d_2 R_b) + \theta_1 d_2 c_2 - \theta_4 (\theta_1 \theta_4 - \theta_2 \theta_3)) \\ C = \theta_1 (\theta_4 d_1 R_b - \theta_2 d_1 c_2 - \theta_3 (\theta_1 \theta_4 - \theta_2 \theta_3)) \\ D = c_1 (d_1 d_2 c_2 - (\theta_3)^2 d_2 - (\theta_4)^2 d_1) \end{array} \right.$$

giving: $\Delta = e_1 (A - B + C + D)$. Δ' is similar to Δ where you replace d by d' , c by c' etc.

We can now give the expressions of the Y 's:

$$\left\{ \begin{array}{l} Y^{76} = \frac{1}{\Delta'} [(-d'_1 d'_2 R_b Z_T) + (d'_1 d'_2 c'_2 - (\theta_3)^2 d'_2 - (\theta_4)^2 d'_1) Z_T + \dots \\ \dots + \theta'_1 \theta'_3 d'_2 Z_T + \theta'_1 \theta'_4 d'_1 Z_T] \\ Y^{86} = \frac{1}{\Delta'} [(- (d'_1 d'_2 R_b - \theta_1 \theta_3 d'_2 - \theta_1 \theta_4 d'_1) Z_T) + \dots \\ \dots + d'_1 d'_2 c'_1 Z_T - (\theta'_1)^2 d'_2 Z_T - (\theta_1)^2 d'_1 Z_T] \\ Y^{24} = \frac{e_1}{\Delta} [(-\theta_3 d_2 R_b) + \theta_1 (d_2 c_2 - (\theta_4)^2) + \theta_1 \theta_3 d_2] \\ Y^{34} = \frac{e_1}{\Delta} [-\theta_1 (d_2 R_b - \theta_3 \theta_4) - \theta_3 d_2 c_1 + (\theta_2)^2 \theta_3] \end{array} \right. \quad (7.18)$$

We can see that despite of the apparent simplicity of this problem, the components of the cords and vertices are completely imbricated and it is already difficult to discern the influence of each component of the hamiltonian. Anyway, the fact to understand this is also a result.

To analyse the result a method can consist in forcing to zero some interactions (cords) and to explore the impact on the result. This can help to detect the influence of variables, but under the assumption that there are no combine effects.

But in all the products that intervene in the expression, some of them leads to very little values. Practically, they don't have any influence due to the fact that they can be neglected in the result. The products like $\theta_1 \theta_3$ or $R_b Z_T$ etc. gives very weak values. In general the products of interactions give very weak values. This is not always true. For example when the cords are

Branin's ones, representing interactions of guided waves, this assumption becomes false. But in a transformer, the square of the mutual inductance gives very low values.

When we take a look on the graph 7.4, we can presume of the importance of the retroactions between vertices. If this is possible, it becomes many more simple to determine the path to go from one vertex to another. Going from 4 to 1 can be made through:

- 4-2-1;
- 4-2-3-1;
- 4-3-2-1;
- 4-3-1.

Four paths. Then there is only one path to go from 1 to 6 through the coupling $-Mp$. Then it remains to go from 6 to 10 using:

- 6-8-10;
- 6-7-10;
- 6-7-8-10;
- 6-8-7-10.

If we consider the path $4 - 2 - 1$. We can write with the weak coupling assumption:

$$u_1 = -Z_T x^2, \quad u_2 = \theta_1 x^4$$

but:

$$x^2 = \frac{u_2}{c_1}$$

so:

$$u_1 = -\frac{Z_T}{c_1}\theta_1 x^4 \Rightarrow g_{14}^{(4,2,1)} = -\frac{Z_T}{c_1}\theta_1$$

For the path 4 – 3 – 2 – 1 we may find:

$$g_{14}^{(4,3,2,1)} = -\frac{Z_T R_b \theta_3}{c_1 c_2}$$

while:

$$g_{14}^{(4,2,3,1)} = -\frac{Z_T R_b \theta_1}{c_1 c_2}$$

and:

$$g_{14}^{(4,3,1)} = -\frac{Z_T}{c_1}\theta_3$$

and the whole interaction is defined by:

$$g_{14} = -\frac{Z_T}{c_1} \left[(\theta_1 + \theta_3) \left(1 + \frac{R_b}{c_2} \right) \right] \quad (7.19)$$

This relation must be completed by the metric. The self operator of each vertex is increased with their self inductances: $c_1 \rightarrow c_1 + L_1 p$, $c_2 \rightarrow c_2 + L_2 p$.

With these approximations we can understand easily the influence of the components that intervene in the interaction. To cover the whole problem with the two lines, we just have to

duplicate the previous interaction and to add the mutual inductance to obtain:

$$g_{10,4} = Mp \frac{Z_T Z'_T}{c_1 c'_1} \left[(\theta_1 + \theta_3) \left(1 + \frac{R_b}{c_2} \right) \right] \left[(\theta'_1 + \theta'_3) \left(1 + \frac{R'_b}{c'_2} \right) \right] \quad (7.20)$$

if we verify: $R_b/c_2 = R'_b/c'_2 \ll 1$, and $Z_T = Z'_T$, $c_1 = c'_1$ the relation becomes

$$g_{10,4} = Mp \left(\frac{Z_T}{c_1} \right)^2 [(\theta_1 + \theta_3) (\theta'_1 + \theta'_3)] \quad (7.21)$$

The mutual inductance depends on the distance between the two lines. While the transfer impedance of the shielded cables depends on the characteristics of the braids of the shielded cables. For decreasing the coupling path between the vertices 4 and 10, we have two solutions, knowing that we cannot chose the electronic components c_i and θ_j, θ'_k :

1. we can increase the distance in order to decrease the mutual inductance;
2. we can decrease the transfer impedance value.

The first solution plays on the location of the source and receiver, but doesn't change anything in the components of the system. Acting on this parameter we act on its prevention. We can report the consequence of various strategies on a diagram with two axes: one axe where we change locations (we try to separate sources and victims) and one axe where we change protections. Often separation concerns the metric and protections

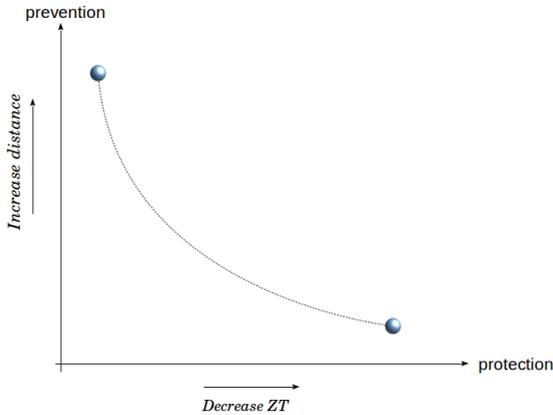


Figure 7.5: Protection-prevention diagram

acts on the hamiltonian. This is all the more true that knowing we can include Green's functions in the metric.

Figure 7.5 represents this kind of diagram. When increasing the protection we can accept to decrease the separation and vice versa. Then for each couple of solutions (prevention, protection) we can trace a curve giving the limit over which the system is safe. All points higher than the curve gives solutions where the system is not disturbed. For lower values, the system can be disturbed. How to determine these couples?

7.5.2 Repartition function and its complementary function

The previous limit is defined by the fact that the studied function is not disturbed by the source. But how to define accurately this notion of disturbance? We take another example to illustrate the problem and see how uncertainties can be taken or not into account like the influent elements.

The curve given figure 7.5 represents somewhere the foundation of the EMC. It says for a system when its configurations can lead to dysfunctions. After What, playing on the locations of some materials or changing filters, adding shielded cables permit to make the points of working that are in the dangerous zone of this diagram to go back in the safe one. It synthesizes all the EMC job and its objectives.

But to be sure that the choices will lead to stable and safe conception of a system, the engineer must enclose uncertainties in the quantities he uses. As we will see, when these uncertainties concern basic components like resistances, it is quite easy to take them into account. But when they concern complicated behaviors like diodes, it can be easier to take them into account through some macromodels that include the variation of major parameters involved in the EMC response of the electronic.

The new system we imagine is made of two antennas, a load and a spark gap.

Figure 7.6 gives the graph of this system.

The manifold associated with this system is quite simple to establish. The generator being matched on mesh 1, the first vertex is $2R_a$ where R_a is the radiation resistance of the antenna.



Figure 7.6: System studied

The receiver uses the same antenna, so: $H_{22} = R_a + R_L$. The spark gap is located near to the load to limit the voltage across it: $H_{33} = R_L + Z_E$. Z_E is an impedance operator defined on domain as seen previously. But in order to make this analysis simplest we just define:

$$Z_E = \overset{v}{\mathcal{D}}_1 R_H + \overset{v}{\mathcal{D}}_2 R_s \quad (7.22)$$

if the voltage across R_L is low enough, the spark gap behaves like a high value resistance R_H and if the voltage across R_L is high enough, the spark gap behaves like a short circuit R_s .

The voltage across the load R_L is a connection to the EM force induced on the receiver: $V_\gamma = C_\gamma^\sigma u_\sigma$. This can be written:

$$V_\gamma = C_\gamma^\sigma \zeta_{\sigma\beta} x^\beta$$

if we note $\mathcal{A}_{\gamma\beta} = C_{\gamma}^{\sigma} \zeta_{\sigma\beta}$ we obtain:

$$V_{\gamma} = \mathcal{A}_{\gamma\beta} x^{\beta} \quad (7.23)$$

To characterize the emitter we need to be able to define the probability that the amplitude may be higher than a given value V_{γ}^0 : $P_E(\mathcal{A}_{\gamma\beta} x^{\beta} > V_{\gamma}^0)$. To compare easily the level projected by the source on the target with the target threshold we define the complementary function of repartition for the source:

$$\bar{P}_E(\mathcal{A}_{\gamma\beta} x^{\beta} < V_{\gamma}^0) = 1 - P_E(\mathcal{A}_{\gamma\beta} x^{\beta} > V_{\gamma}^0) \quad (7.24)$$

The function \bar{P} has the profile presented figure 7.7.

The falling part of the curve has a profile that depends on the original density of probability that characterize the function. Often we don't know the density of probability of the emitted amplitude depending on the level. The unknown is translated in a density extremely spreaded. At the opposite a density like Dirac's function means a perfect known level of threshold. But the reality is quite different. In general we do know only one value as threshold. Without being able to make any more test, we are obliged to deduced the repartition function from this single value. The repartition function is in this case a step function. Is this really a difficulty? What we want to know is if the projected level is equal or superior to the threshold level? In any case the repartition function that are often approximated by gaussian density has a maximum which is difficult to define. Rather than using these classical approaches, we can use the single known value and adding to it a typical dispersion for similar technologies. What we want to determine is the distance d

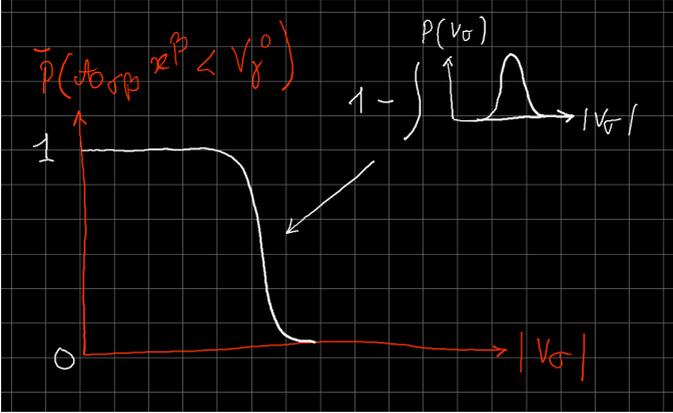


Figure 7.7: Function of repartition

between both repartition functions as shown figure 7.8. With all the uncertainties in a whole chain, the global distance that will be used can accept the unknowledge on the density function of the receiver.

If we are able to determine some couples of solutions prevention-protection we can construct the diagram PP (prevention-protection). With this diagram, the level of risk of the system is directly visible seeing the location of the point of functioning of the system on this diagram. These couples of curves are often of same distance, because the engineers try to keep the margin and risk at the same level also. Figure 7.9 illustrates this methodology.

The distance is given by the metric $\mathcal{A}_{\gamma\beta}$. When we want

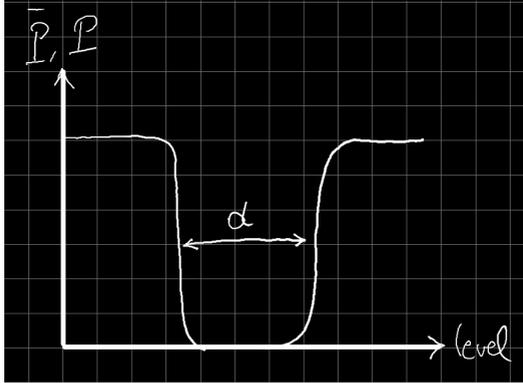


Figure 7.8: Distance between threshold

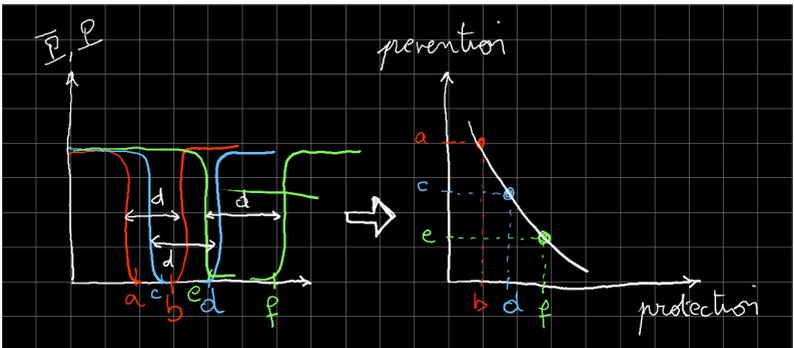


Figure 7.9: Construction of the PP diagram

to play on the physical distance between the emitter and the receiver we will change the components of the metric side of the lagrangian \mathcal{L} and move the prevention level. When we change the electronic, we change the components of H and move the protection level. As in general the source is not reachable, we just can increase the distance to the source. So changing the distance is often in relation with the emitter, while changing the electronic is more in relation with the receiver adding filters or shielded cables in its structure.

The distance d is determined by the modulus of the difference between the voltage brought back through the metric and the threshold of the function considered:

$$d_\alpha = |\mathcal{A}_{\alpha\beta}x^\beta - V_\alpha| \quad (7.25)$$

This distance is itself a covector, as the equation tells us. But on the PP diagram we want to synthesis all the distance extracted from the identified observables of a system. But each signal V_α is associated with functions more or less importants. So, the risk for the system must take differently into account these signals and their critical aspects. Starting from feared events we define critical signals and less critical signals. The distance must take into account these weightings to compute the number that represents it globally. Finally for a couple of solutions in protection and prevention, we can say that the system is not failing if the distance d is respected and has for definition:

$$d = \mathcal{W}^\alpha |\mathcal{A}_{\alpha\beta}x^\beta - V_\alpha| \quad (7.26)$$

To understand the difficulties associated with this kind of ex-

ercise, we study our example. We try a contravariant approach.

The hamiltonian of the problem is:

$$H = \begin{bmatrix} 2R_a & 0 & 0 \\ -G & R_a + R_L & -R_L \\ 0 & -R_L & R_L + Z_E \end{bmatrix} \quad (7.27)$$

The source vector is given by: $u_1 = -R_0 J^s$, all the other components are equal to zero. We look to the voltage on the output load: $V_s = R_L J^2$. The dispersion on the resistances is easy to write:

$$R_a \rightarrow R_a + \delta R_a \quad R_L \rightarrow R_L + \delta R_L$$

Green's function is defined by:

$$G = \frac{h}{r} \sqrt{30\mathcal{G}R_a} e^{-\frac{r}{c}p} (1 + \text{Cos}\theta)^2$$

h is the effective height of the antenna, r the distance to the antenna; θ is the angle under which the antenna is seen by the receiver.

What is the expression of δG ?

$$G = \left(\frac{h + \delta h}{r + \delta r} \right) \sqrt{30(\mathcal{G} + \delta\mathcal{G})(R_a + \delta R_a)} e^{-\frac{1}{c}(r + \delta r)p} (1 + \text{Cos}(\theta + \delta\theta))^2$$

All the dispersions δx are positive or negative. For the first term:

$$\left(\frac{h + \delta h}{r + \delta r} \right) = \frac{(h + \delta h)(r - \delta r)}{r^2 - \delta r^2} \approx \frac{hr - h\delta r + r\delta h}{r^2}$$

As $r \gg h$ we obtain:

$$\left(\frac{h + \delta h}{r + \delta r}\right) \approx \frac{h + \delta h}{r}$$

For the root:

$$30(\mathcal{G} + \delta\mathcal{G})(R_a + \delta R_a) \approx [\mathcal{G}(R_a + \delta R_a) + R_a \delta\mathcal{G}]$$

We make the assumption that all the resistance have the same dispersion: $\delta c = \delta R_a = \delta R_L$, and:

$$\sqrt{30(\mathcal{G}R_a + (\mathcal{G} + R_a)\delta c)} = \sqrt{30\mathcal{G}R_a} \sqrt{1 + \frac{(\mathcal{G} + R_a)}{\mathcal{G}R_a} \delta c}$$

Using a first order development, we obtain:

$$\sqrt{30\mathcal{G}R_a} \left(1 + \frac{\mathcal{G} + R_a}{2\mathcal{G}R_a} \delta c\right)$$

For the delay we have:

$$e^{-\frac{(r+\delta r)}{c}p} \approx e^{-\frac{r}{c}p} \left(1 - \frac{\delta r}{c}p\right)$$

For the radiation diagram:

$$\text{Cos}(\theta + \delta\theta) \approx \text{Cos}\theta \left(1 - \frac{\delta\theta}{2}\right)$$

The uncertainties on Green's function is finally given by:

$$\begin{aligned} \delta G = & \sqrt{30\mathcal{G}R_a} \left\{ -\frac{h}{r} \delta\theta (1 + \text{Cos}\theta) \text{Cos}\theta e^{-\frac{r}{c}p} + \dots \right. \\ & \left. \dots + (1 + \text{Cos}\theta)^2 e^{-\frac{r}{c}p} \frac{h}{r} \left(\frac{\mathcal{G} + R_a}{2\mathcal{G}R_a} \right) \delta c + (1 + \text{Cos}\theta)^2 e^{-\frac{r}{c}p} \left(\frac{\delta h}{r^2} \right) \right\} \end{aligned} \quad (7.28)$$

It remains to determine δZ_E to know δH . The domains used in the equations 7.22 depend on the breakdown voltage that is defined with some dispersion. We have:

$$\begin{cases} \overset{v}{\mathcal{D}}_1 = [1 - \frac{1}{\pi} (\frac{\pi}{2} + \text{arctg}(V - V_b))] \\ \overset{v}{\mathcal{D}}_2 = \frac{1}{\pi} (\frac{\pi}{2} + \text{arctg}(V - V_b)) \end{cases}$$

So:

$$\frac{\partial \overset{v}{\mathcal{D}}_x}{\partial V_b} = \mp \frac{1}{\pi} \left(\frac{1}{1 \pm (V - V_b)^2} \right)$$

which gives:

$$\delta \overset{v}{\mathcal{D}}_x = \mp \frac{1}{\pi} \left(\frac{1}{1 \pm (V - V_b)^2} \right) \delta V_b$$

and so:

$$\delta Z_E = \delta \overset{v}{\mathcal{D}}_1 R_H + \delta \overset{v}{\mathcal{D}}_2 R_s$$

This last element allows to define δH .

The equation $\mathcal{L}_{ab} \dot{x}^b = u_a - H_{ab} x^b$ conducts to the dispersion on the result:

$$\delta \chi^b = \frac{\epsilon}{\mathcal{L}_{ab}} (\delta u_a - \delta H_{ab} x^b) \quad (7.29)$$

ϵ is the step time and χ^b the vector of the solution at the next time for the initial conditions x^b .

Studying the risk to disturb the system, we look at the value of the induced flux χ^b coming from the emitter u_a . We can determine the maximum of the emitter given by $u_a + \delta u_a$. This

moves the curve of the source on the right, like on figure 7.9. The risk is given by the distance $\mathcal{W}_b |\chi^b - \chi_s^b|$ where χ_s^b are the thresholds. This distance becomes minimum if χ^b is increased of the value $\delta\chi^b$. This value is given by the relation 7.29. You have noted that we have not search for a dispersion on \mathcal{L} ? This permits to study the distance $\mathcal{W}_b |\chi^b - \chi_s^b|$ keeping the minimum for a chosen value of the sample ϵ . Once the point is obtained on the axe of protection, tests can be made for various values of \mathcal{L} to reach the limit and define the couple (\mathcal{L}, H) in the PP diagram. Same operation can be made for other assumptions on H .

We have seen that it is not so trivial to find the dispersions, even for a very simple system as the one we use to illustrate this exercise. More, in practice we often didn't have the informations on the dispersion values δx . We can obtain maximum of diversion on data sheets for example, and use experiences and past projects to give a value to the maxima. It remains, for a given path between one emitter and one receiver g_{er} or H_{er} or ..., to determine the conditions for which it is a maximum. This is not also so simple because its maximum is not always obtained taking the maximum of all the dispersions. It always need to study the function of the path, using each time it is possible the weak coupling assumption that leads to more simple expressions.

Paths in low frequencies

Low frequencies mean that all the propagation functions of the form $e^{-\tau p}$ can be reduced to 1 (τ is the delay).

We consider a network made of four meshes. Each mesh has

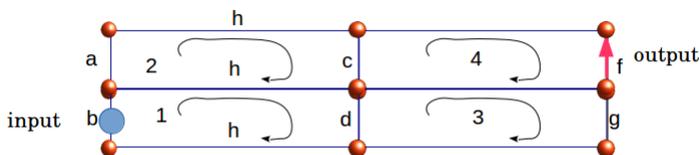


Figure 7.10: Network of four meshes cross talked

two frontiers with its neighbors. The figure 7.10 represents this illustration.

We see six impedance operators from a to h (a, b, c, d, f, g, h). The operator of the first mesh is $2h + b + d$. The operator of the second mesh is $2h + a + c$; etc. We can already give the hamiltonian for the diagonal element of this manifold:

$$H = \begin{bmatrix} 2h + b + d & 0 & 0 & 0 \\ 0 & 2h + a + c & 0 & 0 \\ 0 & 0 & 2h + d + g & 0 \\ 0 & 0 & 0 & 2h + c + f \end{bmatrix} \quad (7.30)$$

Now, just looking to the graph, we can complete H with extra-diagonal elements. They are the shared branches between meshes. From this principle we deduce the completed expression

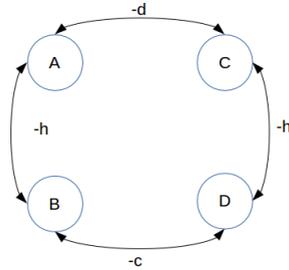


Figure 7.11: Graph on the vertices space

of H :

$$H = \begin{bmatrix} 2h + b + d & -h & -d & 0 \\ -h & 2h + a + c & 0 & -c \\ -d & 0 & 2h + d + g & -h \\ 0 & -c & -h & 2h + c + f \end{bmatrix} \quad (7.31)$$

Using the similarity between shared branch and cord (see paragraph on diakoptic), this topology can also be represented by the graph of vertices and cords presented figure 7.11.

Operator A, B, C, D are the diagonal components of H (for example $A = 2h + b + d$). A source is located on mesh 1 on load b and the output is observed on branch f of mesh 4. We

wonder how the energy can go from the source b to the target f . To understand easily this mechanism we can construct another representation of the topology. It is equivalent to spread the network which is a planar one (it has four faces plus the exterior: its genus is 5). We can associate each vertices to a segment of a circle, and each frontier which is a cord to a radius. A little circle shows the location of the source and a little straight line that indicates the receiver. The figure 7.12 shows this new representation.

On this figure and under this representation, we easily understand that to go from A to D we have to cross the borders $-d$ and h . By the other way the transfer function is hc . All the interactions of the network can be include in a single circle. As the cords considered here are branches, the topology has five faces. But finally, the number of faces is not so interesting to understand how the system works. As there are four segments, the dimension of the manifold is four. The drawn gives also another information. If we want to isolate the source, we can decrease the values of d and h . **The system can be called circular and denoted $\mathcal{S}^{1/2}$** as the interactions can be completely projected on a circle (two circles making a sphere).

Each quarter of the circle is a vertex. But for the moment, this representation is valid only in low frequencies. At least inductances must be added to each vertices. This can include self inductance of borders. As a consequence, this will decrease the quality of the isolation. If we increase again the frequencies, radiated mutual inductance will appear which can add a new direct path from vertex A to vertex C. This new cord can be seen as a new frontier between A and D (imagine that two loops

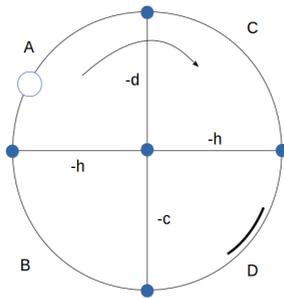


Figure 7.12: Representation in a single circle

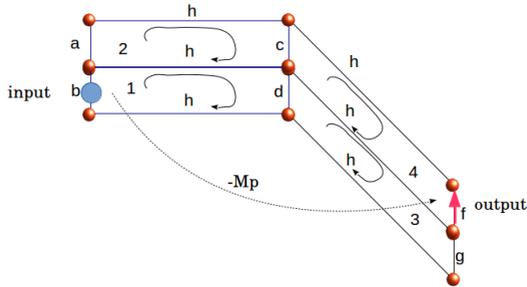


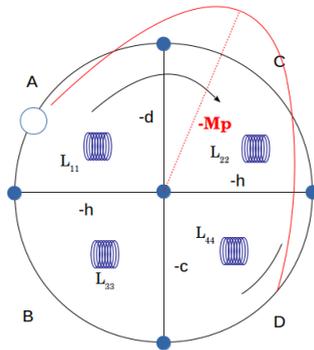
Figure 7.13: Mutual interaction between A and D

in interaction exists at these extremities). Is it possible to add this cord inside the circle?

The figure 7.13 shows this added coupling. As it is a frontier, we may want to add a radius to the previous circle. But it is not possible without making a new vertex incorporated between the others, and no vertices are created using the mutual inductance coupling.

The only solution is to add the new radius on a new half circle with a circulation going from A to D. The figure 7.14 represents this mechanism. The system in this case is $\mathcal{S}^{3/4}$.

The mutual inductance doesn't change the expression of H (equation 7.31) but the metric part \mathcal{L} of the system which be-

Figure 7.14: Adding a half circle for M

comes:

$$\mathcal{L} = \begin{bmatrix} L_{11} & 0 & 0 & -Mp \\ 0 & L_{22} & 0 & 0 \\ 0 & 0 & L_{33} & 0 \\ -Mp & 0 & 0 & L_{44} \end{bmatrix} \quad (7.32)$$

Seeing this new representation, it is easier to find solutions in order to increase the isolation of a vertex. Clearly in our imagined problem, decreasing the value of d and h is no more sufficient, it is also necessary to suppress the M interaction. Now finding the influence of each path is a question of computation, using the distance definition given equation 7.26.

A first conclusion on the systems analysis

To represent the interactions inside a system is a complicated task. Starting from the real system, we describe it as a collection of subpart or sub-manifolds and by a process of direct sum and adding of interactions, we construct the complete manifold associated with the whole system. From this definition, we make a projection of the lagrangian into a graph made of vertices and edges as meshes and cords. In a second step we transform this representation of the manifold in circles defining \mathcal{S} systems or manifolds. Then, solutions can be studied to decrease some paths between emitters and receivers. The distance between them is defined in volts, as demonstrated through the second geometrization process.

Increasing the frequencies and \mathcal{S}^n systems analysis

We continue to discuss with our first case. If the frequencies increase, the extremities of each line will be separate and no longer linked by a non propagative model of wire. Each line will be represented by a branin. The dimension of the manifold increases and include eight meshes. Between the extremities of the line we have a cord and between lines, another cord defined by Vabre's relations³. But we don't care here of Vabre's formula. The figure 7.15 shows the transformation between the graph and its projection on a circle as we do previously.

³Alain Reineix, Olivier Maurice. Progrès récents dans la modélisation CEM de câblages électriques de systèmes complexes. CCT-CEM, Mar 2017, Toulouse, France. hal-01495620

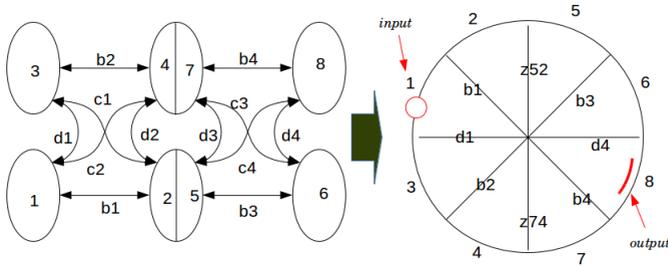


Figure 7.15: High frequency projection on the circle

We have construct the circle starting from the major path between the input and the output. This leads to the circle as it is presented on the figure 7.15. But other paths exist, the paths using the far-end cross talk between the lines. And if we trace them, this gives the first drawn figure 7.16. We understand that we cannot trace these interactions without crossing them if we keep them either on the same circle, or even on a new half circle. But using a new complete circle, it is possible to create paths without any cutting the paths. A first circle is drawn taking into account the interactions $c2$ and $c4$ and a second circle is drawn taking into account the interactions $c1$ and $c3$. The figure 7.17 shows the first circle. The whole set of three circles describes completely the system. Note that all half circles are completed by some vertices in order to wear the total interactions from

the source 1 to the target 8. Having three circles in final, the system is $\mathcal{S}^{3/2}$. Two circles giving a sphere (\mathcal{S}^1) and with one circle more, this leads to $\mathcal{S}^{3/2}$.

The higher the order of \mathcal{S} , the higher the difficulty to isolate the target from the source. For finding a solution in order to avoid disturbances on the function 8 coming from emissions of 1, it's necessary to reduce the problem if possible. First operation is to apply the "weak coupling approximation". Under the "weak coupling approximation" (WCA) we accept the idea that the reaction of the receiver of energy doesn't impact the behavior of the emitter. In other word we may say that the coupling function has an impedance operator that is not changed depending on the receiver. This is false when near field coupling arrives in the process but globally true elsewhere. This is a consequence of the diffusion process. Often the emitter communicates its energy to many paths, and the receivers make the same. Of fact the part of energy that comes back to the emitter, coming from one receiver is neglectable. In near field process, both emitter and receiver become a unique object inside which energy is exchanged. The diffusion in that case is out of purpose and both emitter and receiver influence the whole system realized starting from their common participation.

How is it possible to verify the WCA? For an emitter u and a receiver J (natural written opposite to the covariant one) linked through a coupling function y , if the coupling is a WCA one:

$$J(u) = J(u) \pm \epsilon, \forall J \Rightarrow \frac{\Delta J(u)}{\Delta J} < \epsilon = f(y^2) \quad (7.33)$$

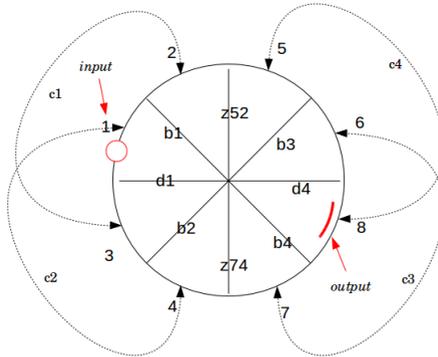


Figure 7.16: Adding cords from far end cross talk

ϵ being a residual change in the current of the emitter.

It is important to understand that this doesn't mean that a current on the receiver $J(u)$ cannot change the current of the emitter J . Because, at the inverse, it can exist a WCA from u seen as an emitter to J seen as a receiver. But it means that the back energy resent to the emitter and coming from itself is neglectable. This can be true even for non symmetrical couplings.

If we have $J = yu$. If z is the impedance operator of J , we have also $v = J/z$. Now we can write $\Delta J(u) = y'v$, so by

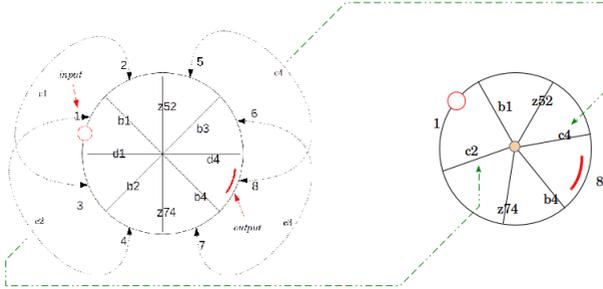


Figure 7.17: Two circles more for other paths

replacement this gives $J(u) = y'J/z$ and finally:

$$\Delta J(u) = \frac{1}{z} y' y u$$

Our criterion becomes:

$$\frac{1}{z} y' y u < \epsilon \quad (7.34)$$

Note that the difficulty comes from the fact that the criterion depends on the impedance operator of the receiver. A way to measure this effect is to measure the impedance seen from the input of the emitter. If the current is influenced by a potential receiver, the impedance will change also, because it means that the generator see another operator on the input. And if the impedance operator is modified by the proximity of another

one that belongs to the receiver, we understand easily that this modification depends on the impedance operator of the receiver.

There is a domain where this problem is clearly identified: the antennas. As long as the environment doesn't influence an antenna, its radiation impedance remains unchanged. If an object becomes to be too much near, the radiation impedance changes and the new antenna is constituted by including both the original antenna and the object.

Under the WCA we can triangularized our matrix H . In low frequencies H becomes:

$$H = \begin{bmatrix} 2h + b + d & 0 & 0 & 0 \\ -h & 2h + a + c & 0 & 0 \\ -d & 0 & 2h + d + g & 0 \\ 0 & -c & -h & 2h + c + f \end{bmatrix} \quad (7.35)$$

and the same for \mathcal{L} :

$$\mathcal{L} = \begin{bmatrix} L_{11} & 0 & 0 & 0 \\ 0 & L_{22} & 0 & 0 \\ 0 & 0 & L_{33} & 0 \\ -Mp & 0 & 0 & L_{44} \end{bmatrix} \quad (7.36)$$

With the assumption of WCA, each path becomes easy to establish. Starting from a current on A (covariant approach) on

vertex B we can write:

$$e_B = -hJ^A$$

passing through a first border. Now $J^B = e_B/(2h + a + c)$ and $e_D = -cJ^B$. So:

$$J^D = \frac{hc}{(2h + c + f)(2h + a + c)} J^A$$

This first transfer function ft associates with this first path is defined by:

$$ft = 20 \log \left(\frac{J^D}{J^A} \right) \quad (7.37)$$

It remains to compute the other paths... The transfer function results from the product of the border functions on the numerator and the product of the impedance operator on the denominator (with WCA and linear assumptions). Bode's diagram is easy to trace as solutions to reduce the impact of various paths.

[First reduction process] A question may be: how to do with strong interactions? There is only one possible response: **a set of branches leading to a network where strong interactions exist must be considered as a single vertex.** The associate manifold is said to be unseparable. On the circle projection, this can be include by making abstraction of the detail inside a quarter. The attention must be focused on the borders for which the WCA can be applied. This constitutes the **first reduction process.**

[Second reduction process] The second reduction process consists in decreasing as far as possible the order of \mathcal{S} . Our previous system being $\mathcal{S}^{3/2}$, an ideal situation may be to reach $\mathcal{S}^{1/4}$ (**segregation principle**)! To do that, it seems simplest to begin by the paths of lower orders, i.e. where the number of borders is the lower. There is a thin relation between the number of borders and the neighborhood. This conducts to define a neighborhood distance:

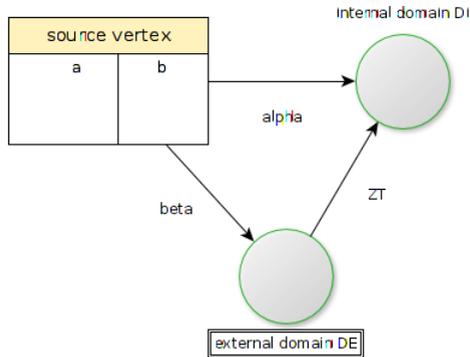
Definition:

The neighborhood distance \hat{d} is the number of borders to cross going from an emitter to a receiver.

The strategy used to decreased the coupling can be to increase the geometrical distance: i.e. playing on \mathcal{L} , or playing on the filters and impedances defined in the operators of H . We find the projections on the diagram 7.5 with displacements along the prevention axis or the protection axis.

Let's take an example. We consider a circuit including two meshes separated by a capacitive filter. A part of this circuit is coupled with a pigtail of a receiver equipped with a shielded cable. Another path is between the same part emitting of the first circuit and the external domain of the receiver. Then through the transfer impedance of the shield, it creates an EM force in the internal domain. The vertex graph of the problem is presented figure 7.18 where we show voluntarily the detail of the two meshes for the first circuit.

The representation of this graph into circles is given figure 7.19. The first reduction was applied, reducing the two meshes of the first circuit into a single part of the circle. Two paths are available to go from b to the internal domain D_i . The order

Figure 7.18: A $S^{1/2}$ system

of the system is $\mathcal{S}^{1/2}$. The first neighborhood of the target is the coupling through the pigtail α . By suppressing the coupling through the pigtail, we suppress one path and remains only the coupling through the transfer impedance of the cable. The system becomes $\mathcal{S}^{1/4}$, last dimension before \mathcal{S}^0 . The neighborhood distance is $\hat{d} = 2$. The transfer function is given by:

$$J^{D_i} = -\frac{1}{Cp} \frac{\beta Z_T}{b D_E D_i} J^a$$

that shows clearly that decreasing Z_T will increase the protection of J^{D_i} .

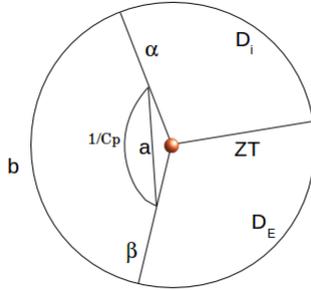


Figure 7.19: Circle representation

In this result, increasing Z_T increase the protection of the receiver. The other parameter is the coupling coefficient β which has the dimension of a mutual inductance. If the geometrical distance between the emitter and the shielded cable increases, this will decrease the EM force induced on the external domain, and, as a consequence, decrease the level induced on the target. For an identical level of protection, we can maintain the product βZ_T constant, or near a given value. β is part of the metric \mathcal{L} while the transfer impedance function is part of the structure H . We can construct a PP diagram (see figure 7.5) to trace the limit curve of safety. Depending on the number of layers for the shield and depending on the distance we obtain three states that give the limit of the PP curve. Figure 7.20 illustrates the process.

Now if for some reason of system layout we need to reduce the distance between the emitter and the receiver, this operation can locate the point of working under the limit, like shown figure 7.21. In that case there are two solutions:

- looking for the nearest point on the limit, which will imply a compromise on both distance and protection;
- to keep the distance desired and looking for the protection involved by this change: here a three layers shielded cable.

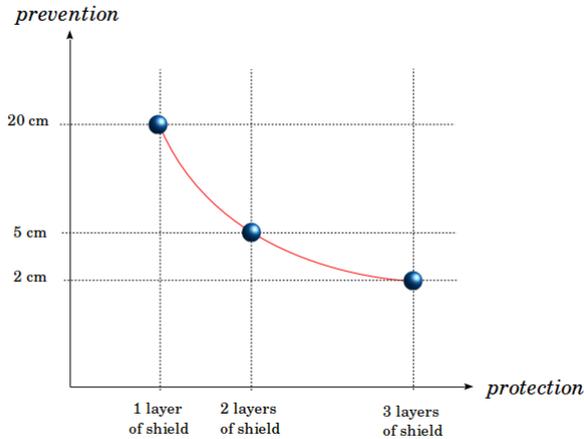


Figure 7.20: Limit curve in the PP diagram

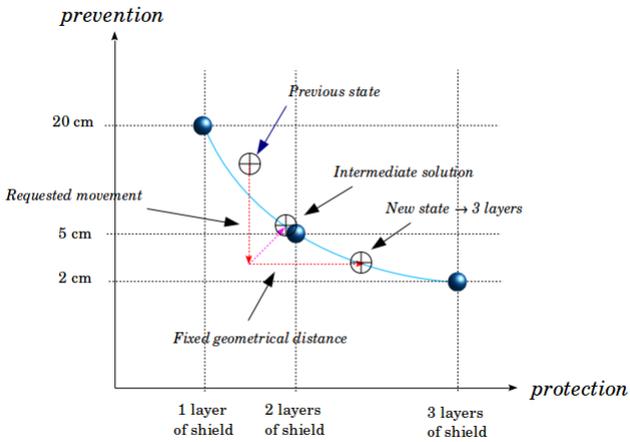


Figure 7.21: Finding solution after a new layout

For analyzing the evolution of the risk of failure in a system consists in following the trajectory of its working point in the PP diagram. Each changing in this film accompanies the movement of the system depending on the phases of its mission and the evolution of its environment. The threshold U_s is the input data from the data sheet giving the level of sensibility for the device. It can be computed for out-band behavior using research works and other sources of information. This value has a given uncertainty and lack of knowledge. For this reason a margin must be taken to decrease this value and to guarantee that the electronic won't be disturbed. Now, around this new threshold, the whole uncertainty including those of the structure, components, locations, etc., should not reach the previous initial value obtained from the data sheet. Figure 7.22 illustrates this mechanism.

To remain under the level fixed, gain in the protection cannot go linearly from one value to another. We see that adding a braid on a cable will give one step 20 dB at least of isolation. That's often the case whatever the constraints and this shows the mechanism in a system conception and the limit in an optimization process.

[What can be done when emitter and receiver are in strong coupling interaction?] Same process must be followed but the engineer should remember that under strong coupling assumption, both emitter and receiver will be impacted by any change in the coupling function.

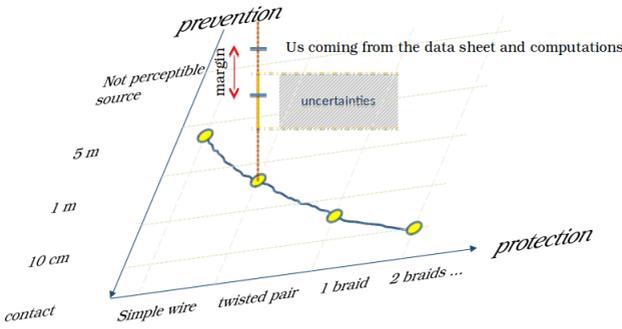


Figure 7.22: Uncertainty around the threshold

7.5.3 Margin management

Once the border determined, the cycle from the emission limits defined to the immunity limits requirements can be made. For a threshold V_s of a component, the choice of architecture leads to a maximum of emissions for all sources M_E . On this level we take a margin to define the immunity limit $I_L = M_E + margin$. This constraint generates a level across the component V_g . Logically, $V_g = V_s + margin$. If no disturbances is observed during the test, it confirms that the margin is reached. The figure 7.23 explains the process.

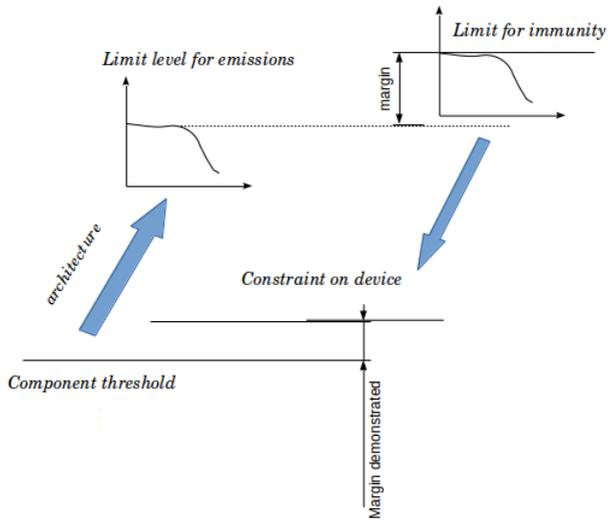


Figure 7.23: Margin process

7.5.4 System life

During its life or its mission, a system changes in its characteristics and can encounter more or less external emitters.

When the system evolves, the story can be drawn on a musical partition. At each step, the metric and the hamiltonian of the system can change like the external sources radiating into the direction of the system. But the relative location of the system and of the sources can change also.

The source vector (using a nodes pair vector) evolves depending on the system movement or evolution. The components of the vector source change depending on time, and the way they are seen by the system is also changing with time. This changes can be taken into account using a matrix that I have called a gamma matrix.

Figure 7.24 shows how the system life can be seen compared to a music sheet.

Once the system chains are conceived, the system can appear like a robot, able to make various actions in response to commands. Depending on its "autonomy", this capacity goes more or less far. For example we can imagine all the versions we have between two robots, a first one being a classical car. I use it to go from Paris to Rouen. It helps me to make this travel with different secure functions, but I have to drive this car anyway! And a second robot which ask me the destination, and make alone all the rest to transport me to this destination.

Under this kind of system, what evolves principally during the mission is the environment. In parallel to the previous music sheet, we can add another one to see how the environment

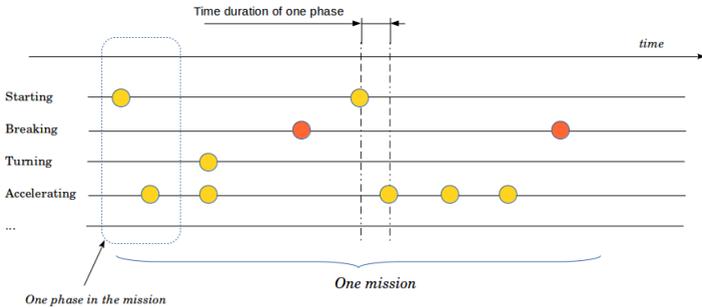


Figure 7.24: System life, mission and phases

evolves during the same mission. Each line takes in charge one source that belongs to the environment. So the problem consists in having a mathematical technique that changes the source covector following its real changing in the mission. Note that the synchronizations (C_n) between the existence of a source in the environment and the electronic state of the system obliges to study the problem through a stochastic process. Each electronic state as phase of mission can be identified with color in correspondance with their critical role in the mission. The same for the constraints that can be more or less dangerous. More, the time duration of each event can be drawn on the music sheet. But a difficulty is that for example, between the time duration of a breaking phase for a vehicle and the duration of lightning

field, the difference of scales is enormous. Figure 7.25 illustrates this approach.

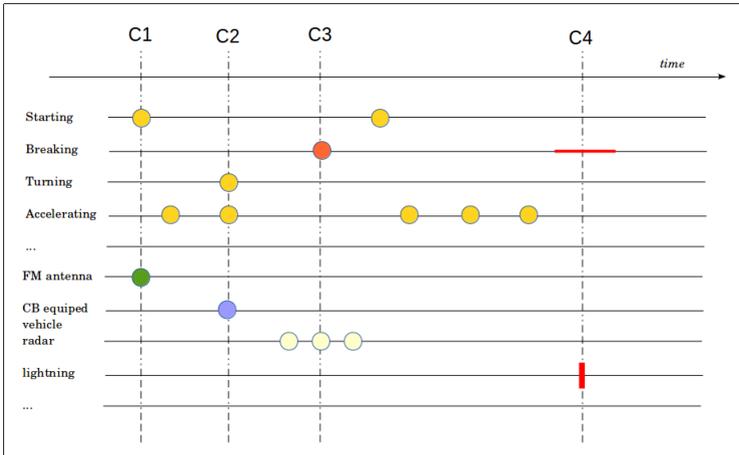


Figure 7.25: System life and environment

The system must be cut in subparts to be studied. It is impossible for the human mind to encompass the whole system. Doctors need to understand each subparts of a human body to make a diagnostic answering to a list of symptoms. The system is made of chains. One chain to distribute the energy. One chain to distribute the intelligence (numeric chain and microprocessors). One chain to manage the perception. One chain to manage the actions (motors, etc.). And so on... Each chain is made of signals exchanged between electronic equipments involved in

this chain. Each equipment is made of electronic components. All of this mechanisms are supported in a mechanical structure or in various mechanical structures. Strategies of exoskeleton or endoskeleton can be used. It's the same for the conception of the system. Two approaches can be followed: bottom-up and up-down. Sometimes, both approaches are used. There are no rules. In final, chains must be defined and incorporated in a chosen structure. What we may say is that the more the system incorporates electronic, the more the bottom-up approach should be used. This because the electronic can adapt the structure to answer to various sollicitations, even mechanical ones. In that case, the mechanical structure can be defined only once the electronic functions are defined themselves.

In any cases, the objectives of the system is to make some missions, to answer for requirements of the customer.

Each chain q is associated with an impedance operator $\overset{q}{\zeta}$. Each of this operator is created using a direct sum of the operators of the equipments $\overset{e}{\zeta}$. Each operator of equipment is constructed from the direct sum of the components operators: $\overset{e}{\zeta} = \oplus_c \overset{c}{\zeta}$. But rather than speaking of operators, we should speak of manifolds. And finally, the system S appears as a direct sum of manifolds:

$$S = \oplus_q \left[\oplus_e \left(\oplus_c \overset{c}{\zeta} \right) \right] \quad (7.38)$$

and $\oplus_c \overset{c}{\zeta} = \overset{e}{\zeta}$, and $\overset{q}{\zeta} = \oplus_e \overset{e}{\zeta}$. But we known that these direct sums must be completed by coupling operators, including the

metric \mathcal{L} .

This has a deep physical meaning. Constructing a system supposes that you group the system in the same space, under the same metric. If not, it becomes impossible to join its pieces or to establish a dialog between its microprocessors. To connect both metrics of the separated systems, the influence of each metric must be transferred to the other. That's the purpose of the mutual inductance. Once more we may discuss of the weak coupling assumption meaning that a system part is weakly coupled to the other part of the system. This means that the coupling acts only one way without back effect on the emitter. But at the system level, this doesn't mean that the added part has no influence on the system. The assumption of weak coupling can concern one physic, and not the whole system mechanisms. For example, adding a microprocessor can change the energy consumption at the margin but decide of the whole system behavior.

7.5.5 Sources and gamma matrices

The sources of the environment can appear many times in different locations, or be seen by various parts of the system because of its movements, or again appear only one time in a particular moment. Knowing Markov's processes, a similar matrix can help to represent this kind of changing in the environment. We just need to replace the source covector e by a modified source covector γe depending of some rhythm that helps to follow the system life.

When we need to study a system, a permanent problem ap-

pears: is it possible to model the system in the frequency domain or is it possible to model it only in the time domain? Why this question? Because they are physical phenomenons that can be easily modeled in the frequency domain, while others can be easily modeled in the time domain.

Skin effect, modes in a cavity, radiations are some of the physical phenomenons that are easier to model in the frequency domain. At the opposite, non linear behaviors like diodes are easier to model in the time domain. In fact, we can consider two facts very importants in this problem:

1. non linear behaviors often depends on low frequency signals and polarisation signals;
2. the first difference between a periodic temporal form and a single temporal form is that in the first case, the spectrum is made of raies (the amplitude is also modified).

A graph in low frequencies decides of the state of the non linear components. The high frequency components are modified depending on these states. The low frequency spectrum is determined making the signal periodical even if this is not the case in the real world. During the computation, a temporal loop makes the non linearities changing and inside this temporal loop, a frequency loop takes in charge the computation of the high frequency signals. This idea uses two important results:

1. when a signal is called "little signal" it has laws defined on only one domain;

2. the signals that makes the polarization of the nonlinear components evolves slowly in a way that allows to neglect the majority of the capacitors and inductances of low values. This simplifies the circuit and the expression of its impedance operator. This is always interesting in the temporal domain.

In a first step we make the signal periodic: $e(t + T) = e(t)$. This allows to find accurately the low frequency spectrum. Each part of $e(t)$ can be retrieved making a moving average computation on the signal. If f_h is the typical central frequency of the high frequency part of the signal and f_b its low frequency envelop, if $f_b \ll f_h$, the average of the signal on a multiple of the duration $1/f_h$ tends to suppress the high frequency signal without impacting the low frequency one.

Once the computation is made, the amplitude found on each high frequency component decides of its amplitude in the temporal sum. To obtain the time domain signal, it's just necessary to add these various sinusoidal components to the low frequency envelop. This kind of strategy is not so easy to write. After having verified that the high frequency signal cannot develop the level responsible for the changes in the domains, we determine two circuits:

1. the low frequency circuit solved in the time domain with the influence of the non linear components. Its source is $q(t)$;
2. the high frequency circuit solved in the harmonic domain of sources $L_h(f)$.

at each time step, we solve the low frequency circuit using time domain finite difference or Newton's method. For each of these steps, we solve the whole frequency domain for the high frequency circuit. The amplitude of the harmonics are memorized then added to the complete solution:

$$s(t) = \zeta[q(t) + \sum_f L_h(f) \text{Cos}(2\pi ft)]$$

One advantage of this decomposition is to take into account changes in the high frequency part, mostly if the high frequency signal comes from a different source from the low frequency one.

We write the signal $u(t)$ with the sum $q(t) + h(t)$ where $q(t)$ is the slowly varying signal with the spectrum $L_q(f)$ and $h(t)$ the high frequency signal with the spectrum $L_h(f)$.

A symbol with a hat points out the spectrum of the signal identified by the symbol ($e(t) \rightarrow \hat{e}(f)$).

If we are in small signal conditions, the change in this voltage or in the corresponding current should not influence the impedance operator. The small signal condition can be written:

$$\forall \hat{e}_\nu, \hat{i}^\alpha \in L_h(f), \frac{\partial \zeta_{\nu\sigma}}{\partial i^\alpha} = 0 \quad (7.39)$$

This can be said with the condition: $\Gamma_{\nu\sigma,\alpha} = 0$. Once more the curvature plays a very important role. We see that as often, working in a flat space simplifies enormously the analysis.

We imagine a signal defined by:

$$u(t) = \alpha \text{Cos} \left(2\pi \frac{t}{T_B} \right) + A \text{Cos} \left(2\pi \frac{t}{T_0} \right) = v(t) + b(t)$$

We wonder how we may treat both signals high and low frequencies which means to be able to identify them in a first step? We make the assumption that the non linear behavior has a cutoff frequency f_c and period T_c . We have $T_B \gg T_c$ and $T_0 \ll T_c$. We want to compute a floating average defined by:

$$\langle u \rangle (t) = \frac{1}{T_0} \int_t^{t+T_0} dtu(t) \quad (7.40)$$

This leads to:

$$\langle u \rangle (t) = \frac{\alpha}{T_0} \int_t^{t+T_0} dt \text{Cos} \left(2\pi \frac{t}{T_B} \right) + \frac{A}{T_0} \int_t^{t+T_0} dt \text{Cos} \left(2\pi \frac{t}{T_0} \right)$$

We obtain:

$$\begin{aligned} & \frac{\alpha T_B}{2\pi T_0} \left[\text{Sin} \left(2\pi \frac{t+T_0}{T_B} \right) - \text{Sin} \left(2\pi \frac{t}{T_B} \right) \right] + \dots \\ & \dots + \frac{AT_0}{2\pi T_0} \left[\text{Sin} \left(2\pi \frac{t+T_0}{T_0} \right) - \text{Sin} \left(2\pi \frac{t}{T_0} \right) \right] \end{aligned}$$

We look at the second term:

$$\begin{aligned} \dots &= \frac{A}{2\pi} \left[\left\{ \text{Sin} \left(2\pi \frac{t}{T_0} \right) \text{Cos} 2\pi + \text{Cos} \left(2\pi \frac{t}{T_0} \right) \text{Sin} 2\pi \right\} - \text{Sin} \left(2\pi \frac{t}{T_0} \right) \right] \\ \dots &= 0 \end{aligned}$$

The first term gives:

$$\begin{aligned} \dots &= \frac{\alpha T_B}{2\pi T_0} \left[\left\{ \text{Sin} \left(2\pi \frac{t}{T_B} \right) \text{Cos} \left(2\pi \frac{T_0}{T_B} \right) + \text{Cos} \left(2\pi \frac{t}{T_B} \right) \text{Sin} \left(2\pi \frac{T_0}{T_B} \right) \right\} \right. \\ & \left. \dots - \text{Sin} \left(2\pi \frac{t}{T_B} \right) \right] \end{aligned}$$

But:

$$\frac{T_0}{T_B} = \epsilon \rightarrow 0$$

so:

$$\dots = \frac{\alpha T_B}{2\pi T_0} \left[\text{Cos} \left(2\pi \frac{t}{T_0} \right) \text{Sin} \left(2\pi \frac{T_0}{T_B} \right) \right]$$

and:

$$\text{Sin} \left(2\pi \frac{T_0}{T_B} \right) \rightarrow 2\pi \frac{T_0}{T_B}$$

Finally:

$$\langle u \rangle (t) = \alpha \text{Cos} \left(2\pi \frac{t}{T_B} \right) = v(t)$$

the high frequency and small signal $b(t)$ has disappeared. The envelop of the composite signal can be extracted for a known cutoff frequency of non linear behavior.

The previous demonstration can be extended to any kind of envelop, not only the sinusoidal ones. This can be shown just defining:

$$v(t) = \sum_q \alpha_q \text{Cos} \left(2\pi \frac{t}{T_q} \right)$$

with $T_q \gg T_0, \forall q$.

In order to facilitate the distinction between both kind of signal, it is sometimes interesting to use also both time and Laplace's operator in a same equation. The envelope is treated using the time and the small signal component using Laplace's operator. This appears naturally in the program where inside the frequency loop there are equations depending on time and

also depending on the pulsation. The time part of the equation changes the polarization and the impedance operator, while the small signal part evolves depending on the impedance sets by the envelope. The impedance changes because parameters like voltages or currents set domains that determine the expression of the impedance operator. This can be somewhere written: $\hat{e}_a = \zeta_{ab} [v(t)] \hat{i}^b$.

We imagine a fixed polarisation source $p_k(t)$ and a system passing in front of this source. If the system has N inputs, the source can illuminate various inputs of the system during time. If the covector e at the origine is given by (for three inputs in the system):

$$e = \begin{bmatrix} p_1(t) \\ 0 \\ 0 \end{bmatrix} \quad (7.41)$$

If, after sometime, the movement of the system exposes the second input to the source, the matrix γ defined by:

$$\gamma = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (7.42)$$

translates this process.

What is particularly interesting in this technique is that the matrix γ can have for components the probabilities that such a source illuminates a given input of the system. More often, these probabilities are bayesian probabilities. We write the probability that the source takes a value, knowing that the configuration

of the system is also a first one, etc.:

$$\gamma_{(k)1} = P(e_{(k)} = e_1 | x^k, \zeta, \dots)$$

The commands e_k drive the behavior of the system and the external commands depends on stochastic processes. A connectivity make links between a set of sources determined by a γ matrix that selects the concerned sources and the covector source applied to the system. We start from a known set of sources $\{e_a\}$. We identify in this set the sources concern by the system life in a given moment: $\gamma_b^a \{e_a\}$. Then a connectivity creates the adequate source covector for the studied manifold:

$$u_k = C_k^b \gamma_b^a \{e_a\}$$

The use of gamma matrices can be very fun for modeling of propagating waves. Let's take an example. We imagine a system with two vertices. The source vector is defined by:

$$U_\alpha = \begin{bmatrix} e_1 \\ 0 \end{bmatrix} \quad (7.43)$$

The propagation of the waves between these two vertices is defined by the gamma matrix:

$$\gamma_\alpha^\alpha = \begin{bmatrix} 0 & -G \\ -G & 0 \end{bmatrix} \quad (7.44)$$

G is a Green's function. If we compute $U_\alpha = \gamma_\alpha^\alpha U_\alpha$, this gives:

$$U_\alpha = \begin{bmatrix} 0 \\ -Ge_1 \end{bmatrix} \quad (7.45)$$

$U_\alpha = \gamma_\alpha^\alpha \gamma_\alpha^\alpha U_\alpha$ gives:

$$U_\alpha = \begin{bmatrix} G^2 e_1 \\ 0 \end{bmatrix} \quad (7.46)$$

$U_\alpha = \gamma_\alpha^\alpha \gamma_\alpha^\alpha \gamma_\alpha^\alpha U_\alpha$:

$$U_\alpha = \begin{bmatrix} 0 \\ -G^3 e_1 \end{bmatrix} \quad (7.47)$$

$U_\alpha = \gamma_\alpha^\alpha \gamma_\alpha^\alpha \gamma_\alpha^\alpha \gamma_\alpha^\alpha U_\alpha$:

$$U_\alpha = \begin{bmatrix} G^4 e_1 \\ 0 \end{bmatrix} \quad (7.48)$$

etc. This gamma matrix models the ping pong of a wave exchanged between the two vertices. If $G = 1$, it is an oscillator.

Let's take another example. We consider a system of three vertices linked by two channel of communication. The part of energy transmitted on vertex 2 is given by the coefficient $y_{21} = 1 + \sigma_{12}$, where σ_{12} is the reflexion coefficient at the border on vertex 2. Another part of the energy is reflected to vertex 1, etc. These coefficients are weighted by the delays of propagation between each vertex. Finally the gamma matrix is defined by:

$$\gamma_\alpha^\alpha = \begin{bmatrix} 0 & \sigma_{12} e^{-\tau_1 p} & 0 \\ y_{21} e^{-\tau_1 p} & 0 & \sigma_{23} e^{-\tau_2 p} \\ 0 & y_{32} e^{-\tau_2 p} & 0 \end{bmatrix} \quad (7.49)$$

τ_x are the delays of each channel.

Starting from the source vector:

$$U_\alpha = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (7.50)$$

we obtain:

$$\gamma_\alpha^\alpha U_\alpha = \begin{bmatrix} 0 \\ y_{21} e^{-\tau_1 p} \\ 0 \end{bmatrix} \quad (7.51)$$

$$\gamma_\alpha^\alpha \gamma_\alpha^\alpha U_\alpha = \begin{bmatrix} \sigma_{12} y_{21} e^{-2\tau_1 p} \\ 0 \\ y_{32} y_{21} e^{-(\tau_2 + \tau_1) p} \end{bmatrix} \quad (7.52)$$

etc.

The evolution of the source vector shows how the waves propagate in the system for a source in vertex 1.

7.5.6 Time and frequencies

It is clear that the harmonic domain allows to compute easier complicated phenomenon like the skin effect. Its big disadvantage concerns the non linear behaviors. If we consider a trapezoidal form of rise and fall time t_s and duration t_D , its Laplace's transform is:

$$s(p) = \left(\frac{1 - e^{-t_s p}}{t_s p^2} - \frac{1 - e^{-t_s p} e^{-t_D p}}{t_s p^2} \right) \quad (7.53)$$

We can sometimes approximate a function $u(t)$ as a sum of these kind of signal:

$$u(t) = \sum_\alpha A_\alpha s(p) e^{-\tau_\alpha p} = \sum_\alpha v_\alpha(t) \quad (7.54)$$

The amplitude of each signal v_α is given by the root mean square value:

$$A(v_\alpha) = \sqrt{\frac{1}{t_D} \int_{t_D} dt v_\alpha^2} \quad (7.55)$$

But following Parseval's formula, it is equivalent to write:

$$A(v_\alpha) = \sqrt{\frac{1}{2} \sum_n C_n^2 + C_0^2} \quad (7.56)$$

If the domains of the non linearity depends on the amplitude of the signal, we can compute the harmonic response across the non linear component, then computing the rms value using 7.56 we verify if the non linear component changes of domains or not. If the decomposition of the original signal is sufficiently thin, the error due to the delay in taking into account the domain changing can be acceptable.

Ideally, we try to bring us closer of a music sheet. On a music sheet, each moment is associated with a spectrum. But the frequency of the lower note is many more higher than the frequency of change in the spectrum. Wavelets are the theoretical technique to treat of this approach. Another very interesting representation is $A(v_\alpha, t)$ and the corresponding time window duration.

7.6 Trajectory, geodesics and curvatures

If we consider a very simple circuit made of two meshes. A first mesh with a generator of self impedance R_0 , source e and a load R_1 and a second mesh similar to the first one but with a load R_2 . Its impedance operator is:

$$\zeta = \begin{bmatrix} R_0 + R_1 & R_0 \\ R_0 & R_0 + R_2 \end{bmatrix} \quad (7.57)$$

The system can be represented by two vertices linked with a cord of value R_0 . The current of each vertices is given by:

$$J^k = \frac{1}{(R_0 + R_1)(R_0 + R_2) - R_0^2} [2R_0 - R_{\bar{k}}] e \quad (7.58)$$

$\bar{k} = 1$ if $k = 2$ and reciprocally. If $R_1 = 0$, it is clear that $J^1 > J^2$. In that case, the majority of the current follows the lower way in impedance. What about the power?

The power dissipated in each vertex is the current multiplied by e . In that case it is obvious that the power follows the current. This is logical because here, the power results in dissipations only.

We imagine now a system made of four vertices numbered from 1 to 4. We study the couplings between a source vertex: 1, and a receiver one: 4. To go from 1 to 4 we can go through the vertex 2 or through the vertex 3. For the first way, the EM force induced in 4 is e_4 which is equal by definition to $\zeta_{42}J^2$. We

can write also that $J^2 = y^{2x} e_x$. e_x can be e_4 itself, e_1 directly or e_3 linked with e_1 . We write:

$$e_4 = \zeta_{42} (y^{24} e_4 + y^{23} e_3 + y^{21} e_1) \quad (7.59)$$

If we make the assumption that there are no couplings between the vertices 2 and 3, we obtain:

$$(1 - \zeta_{42} y^{24}) e_4 = \zeta_{42} y^{21} \zeta_{11} J^1$$

The reasoning seems simple, but is that correct? If the vertices 2 and 3 have no self sources, they only modify the path of coupling. If we synthesize the first path with an operator ζ_{41} and the second path with an operator μ_{41} , the system can be represented by the operator:

$$\zeta = \begin{bmatrix} \zeta_{11} & \zeta_{14} + \mu_{14} \\ \zeta_{41} + \mu_{41} & \zeta_{44} \end{bmatrix} \quad (7.60)$$

With this form we can easily compute the expressions of the currents:

$$J^x = \frac{1}{\Delta} \begin{bmatrix} \zeta_{44} & -(\zeta_{14} + \mu_{14}) \\ -(\zeta_{41} + \mu_{41}) & \zeta_{11} \end{bmatrix} \begin{bmatrix} e_1 \\ 0 \end{bmatrix} \quad (7.61)$$

with $\Delta = \zeta_{11} \zeta_{44} - (\zeta_{14} + \mu_{14}) (\zeta_{41} + \mu_{41})$.

Finally

$$J^4 = -\frac{(\zeta_{41} + \mu_{41})}{\Delta} e_1 \quad (7.62)$$

Our observable is something like αJ^4 , α being some impedance that belongs to the fourth vertex. As the couplings are symmetric, it is not so easy to obtain the different relations implied in the path. For example, we may now replace e_1 by a function of the first current. But in fact, as we are not under the WCA, the previous system of equations tells us that:

$$J^1 = \frac{\zeta_{44}}{\Delta} e_1$$

so

$$\alpha J^4 = V_4 = -\alpha \frac{-(\zeta_{41} + \mu_{41})}{\Delta} \frac{\Delta}{\zeta_{44}} J^1$$

our result becomes:

$$g_{41} = \frac{V_4}{J^1} = \frac{\alpha (\zeta_{41} + \mu_{41})}{\zeta_{44}} \quad (7.63)$$

In this case we see that it's possible to make the distinction between the two paths of coupling:

$$g_{41}^1 = \frac{\alpha \zeta_{41}}{\zeta_{44}} \quad g_{41}^2 = \frac{\alpha \mu_{41}}{\zeta_{44}} \quad (7.64)$$

J^1 and J^2 being determined, the metrics $\sqrt{g_{41}^n J^1 J^2}$ give the normalized work of the interactions and the path preferred by the energy to go from 1 to 4.

The previous method is always true, whatever the expression of the system that has been chosen. It implies to compute the current, then to compute the ratio of the targeted current by the emitting current and to multiply the targeted current by the impedance operator that leads to the force observable.

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In this complicated space, the geodesics are also complicated and between the two paths $\sqrt{g_{41}^1 J^1 J^2}$ and $\sqrt{g_{41}^2 J^1 J^2}$ nothing intuitive allows to know which path is the shortest, i.e. communicates the higher level of power to the vertex 4. We cannot speak of curvature in this space if the two paths are not equals but anyway leads to the same vertex. This because the space remains flat. No Christoffel's symbols are involved in its expression. It means that the distance does not depend on the amplitude of the current, and indirectly, the distance does not depend on the amplitude of the generator. If the coupling do not leads to the same vertex, in that case we can speak of torsion.

The basic concept of these notions comes from the difference that can exist between the two paths of coupling between two vertices, but each path going in one direction only. For example if we have two loops coupled with two different mutual interactions: $M_{21} \neq M_{12}$. The operator is:

$$\zeta = \begin{bmatrix} z_1 & -M_{12}p \\ -M_{21}p & z_2 \end{bmatrix} \quad (7.65)$$

We proceed as previously. The currents are:

$$J^x = \frac{1}{\Delta} \begin{bmatrix} z_2 & M_{12}p \\ M_{21}p & z_1 \end{bmatrix} \begin{bmatrix} e \\ e \end{bmatrix} \quad (7.66)$$

with $\Delta = z_1 z_2 - M_{12} M_{21} p^2$.

If β is a part of z_2 that gives the observable, we can compute

the path between the two loops from 1 to 2:

$$g_{21} = \beta \frac{J^2}{J^1} = \beta \left(\frac{M_{21}p + z_1}{M_{12}p + z_2} \right) \quad (7.67)$$

for the other path we find:

$$g_{12} = \gamma \frac{J^1}{J^2} = \gamma \left(\frac{M_{12}p + z_2}{M_{21}p + z_1} \right) \quad (7.68)$$

g_{12} is the inverse of g_{21} if $\beta = \gamma$, what seems to be logical. Seen from the emitter, the reaction will depend on $\sqrt{g_{21}g_{12}} = \sqrt{\beta\gamma}$. As usual, the interaction depends on the cord, but not only, it depends also of the impedance of each mesh involved in the interaction. We see here clearly that the mutual inductance plays a role in the interaction. If we remain in the same example but with a first vertex of impedance $R_1 + L_1p$ and a second vertex of impedance $R_2 + L_2p$. There is also the mutual inductance coupling between the two vertices. This time we can define:

$$H = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \quad (7.69)$$

and

$$\mathcal{L} = \begin{bmatrix} L_1 & -M - N \\ -M - N & L_2 \end{bmatrix} \quad (7.70)$$

and $\zeta = H + \mathcal{L}p$. M and N are two paths of coupling between the two vertices. Both currents are given by (for a single source on the vertex 1):

$$J^x = \frac{1}{\Delta} \begin{bmatrix} R_2 + L_2p & (M + N)p \\ (M + N)p & R_1 + L_1p \end{bmatrix} \begin{bmatrix} e_1 \\ 0 \end{bmatrix} \quad (7.71)$$

with $\Delta = (R_1 + L_1p)(R_2 + L_2p) - (M + N)^2p^2$.

We obtain:

$$J^1 = \frac{1}{\Delta} (R_2 + L_2p) e_1 \quad J^2 = \frac{1}{\Delta} (M + N) p e_1 \quad (7.72)$$

Now we can define $V_2 = R_2 J^2$ and

$$g_{21} = \frac{V_2}{J^1} = R_2 \frac{J^2}{J^1} = R_2 \frac{(M + N)p}{R_2 + L_2p} \quad (7.73)$$

Now let's compute $g_{21} J^1$:

$$g_{21} J^1 = \frac{R_2}{\Delta} (M + N) p e_1$$

and $g_{21} J^1 J^2$:

$$g_{21} J^1 J^2 = \frac{R_2}{\Delta^2} (M + N)^2 p^2 (e_1)^2$$

In final we obtain:

$$s = \sqrt{g_{21} J^1 J^2} = \frac{M + N}{\Delta} p \sqrt{R_2} e_1 \quad (7.74)$$

Once more we can compare both paths: the path associated with the coupling coefficient M and the path associated with

the coupling coefficient N . We understand that the higher the mutual inductance, the higher the energy transmitted to the load R_2 . The same conclusion can be made for the action s . Depending on the criterion that is retained for the disturbed state on a system, the observable in action $s = \sqrt{V_2 J^2}$ or in voltage V_2 can be preferred. What may we found using the second geometrization?

7.6.1 Second geometrization writing

We determine the basis vectors:

$$\mathbf{b}_1 = \begin{bmatrix} R_1 \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ R_2 \end{bmatrix} \quad (7.75)$$

They generate a metric:

$$G = \begin{bmatrix} R_1^2 & 0 \\ 0 & R_2^2 \end{bmatrix} \quad (7.76)$$

The equation of the system is:

$$e_\mu = (b_{\mu x} + \mathcal{L}_{\mu x} p) J^x \quad (7.77)$$

$b_{\mu x}$ being the jacobian. If Q is the transposed matrix of b , we obtain:

$$Q_\sigma^\mu e_\mu = Q_\sigma^\mu b_{\mu x} J^x + Q_\sigma^\mu \mathcal{L}_{\mu x} p J^x \quad (7.78)$$

but $Q_\sigma^\mu b_{\mu x} = G_{\sigma x}$.

$$K_{\sigma x} = Q_\sigma^\mu \mathcal{L}_{\mu x} = \begin{bmatrix} R_1 L_1 & -R_1(M+N) \\ -R_2(M+N) & R_2 L_2 \end{bmatrix}$$

and

$$T_\sigma = Q_\sigma^\mu e_\mu = \begin{bmatrix} R_1 e_1 \\ 0 \end{bmatrix}$$

The equation $T_\sigma = (G_{\sigma x} + K_{\sigma x} p) J^x$ leads to the matricial equation:

$$\begin{bmatrix} R_1 e_1 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1^2 + R_1 L_1 p & -R_1(M+N)p \\ -R_2(M+N)p & R_2^2 + R_2 L_2 p \end{bmatrix} \begin{bmatrix} J^1 \\ J^2 \end{bmatrix}$$

And so, noting $\Delta' = R_1 R_2 \Delta$:

$$\begin{bmatrix} J^1 \\ J^2 \end{bmatrix} = \frac{1}{\Delta'} \begin{bmatrix} R_2^2 + R_2 L_2 p & R_1(M+N)p \\ R_2(M+N)p & R_1^2 + R_1 L_1 p \end{bmatrix} \begin{bmatrix} R_1 e_1 \\ 0 \end{bmatrix}$$

We see that we obtain exactly the same result as previously. This confirms that H can be considered as a pseudo-metric but \mathcal{L} more than a metric, as suggested by Kron, should more be considered as an inertia tensor. But that's the whole, H and \mathcal{L} which describes completely the space. It is clear when we look at our previous conclusion, the more the inertia, the more the

transmitted energy. The inertia for the interaction is given by Neumann's formula:

$$M = \frac{\mu}{4\pi} \int_{l_1} \int_{l_2} \frac{dl_1 \cdot dl_2}{r_{12}} \quad (7.79)$$

7.6.2 Topological analysis

If two paths are available depending on M and N , it means that to go from 1 to 2 we have two borders, and the system is $\mathcal{S}^{1/2}$. The ratio of the action of each path is simply M/N . What may happen if the space is curved?

Inertia curvature

If L_1, L_2, M, N are functions of J^1, J^2 , noting:

$$p(L_1 J^1) = L_1 p J^1 + \Omega_{11,1} p J^1 J^1 \quad (7.80)$$

with:

$$\Omega_{ij,x} = \frac{\partial \mathcal{L}_{ij}}{\partial J^x} \quad (7.81)$$

The other terms are:

$$p(L_2 J^2) = L_2 p J^2 + \Omega_{22,2} p J^2 J^2,$$

$$p(M J^1) = M p J^1 + \Omega_{21,1} p J^1 J^1 + \Omega_{21,2} p J^2 J^1$$

The same for N . To do the difference between M and N , we use Ω' for N . The operators H and \mathcal{L} are now completed by an

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operator Ω :

$$\Omega_{\mu x} = \begin{bmatrix} \Omega_{11,1} J^1 & (\Omega_{12,2} + \Omega'_{12,2}) J^2 + (\Omega_{12,1} + \Omega'_{12,1}) J^1 \\ (\Omega_{21,1} + \Omega'_{21,1}) J^1 + (\Omega_{21,2} + \Omega'_{21,2}) J^2 & \Omega_{22,2} J^2 \end{bmatrix} \quad (7.82)$$

and the equation of the problem becomes:

$$e_\mu = (H_{\mu x} + \mathcal{L}_{\mu x} p + \Omega_{\mu x} p) J^x \quad (7.83)$$

This formulation is quite different from the one we use before. Be careful that the operator Ω involves product with the currents. So the final expression is non linear and depends on the product $J^\sigma J^\beta$.

The first equation of the problem is:

$$\begin{aligned} e_1 = & H_{11} J^1 + L_{11} p J^1 + L_{12} p J^2 + \Omega_{11,1} J^1 p J^1 + \dots \\ & \dots + \Omega_{12,2} J^2 p J^1 + \Omega'_{12,2} J^2 p J^2 + \Omega_{12,1} J^1 p J^2 + \Omega'_{12,1} J^1 p J^2 \end{aligned} \quad (7.84)$$

and the second equation is:

$$\begin{aligned} 0 = & H_{22} J^2 + L_{22} p J^2 + L_{21} p J^1 + \Omega_{21,1} J^1 p J^1 + \dots \\ & \dots + \Omega'_{21,1} J^1 p J^1 + \Omega_{21,2} J^2 p J^1 + \Omega'_{21,2} J^2 p J^1 + \Omega_{22,2} J^2 p J^2 \end{aligned} \quad (7.85)$$

But as the inertia depends on the currents, it can be natural

to create domains apply to a linear inertia K with for example:

$$\begin{cases} (\Omega_{21,1}J^1 + \Omega_{21,2}J^2) pJ^1 = \mathcal{D}^{J^1, J^2}_n K_{21}^n pJ^1 \\ \Omega_{11,1}J^1 pJ^1 = \mathcal{D}^{J^1}_n K_{11}^n pJ^1 \end{cases} \quad (7.86)$$

The operator Ω becomes equal to:

$$\Omega_{\mu x} = \begin{bmatrix} \mathcal{D}^{J^1}_n K_{11}^n & \mathcal{D}^{J^1, J^2}_n K_{12}^n + \mathcal{D}'^{J^1, J^2}_n K_{12}^m \\ \mathcal{D}^{J^1, J^2}_n K_{21}^n + \mathcal{D}'^{J^1, J^2}_n K_{21}^m & \mathcal{D}^{J^2}_n K_{22}^n \end{bmatrix} \quad (7.87)$$

With this technique we remain in a classical expression of the tensorial equation writing :

$$e_\mu = (H_{\mu x} + [\mathcal{L}_{\mu x} + \Omega_{\mu x}] p) J^x$$

But it appears anyway the term Ω which does not exist usually. As no path reaches a vertex different from the two vertices included in the set of the beginning, we consider that there are no torsions. We can wonder what is the curvature of the space? There is mathematical definitions, but we can accept a definition that gives us information, without knowing the definitions that have been retains on mathematical criteria.

For us, the deformation of the space comes from the dependence of the inductance to the current. A difference with a flat

space that may speak to us, is the difference between the inductance of the flat space with the inductance of the curved space. This step is directly given by:

$$1 + \left| \frac{\Omega_{\mu x}}{\mathcal{L}_{\mu x}} \right| \tag{7.88}$$

By making the comparison between this step and the deviation $1 + tg\theta$ we obtain the expression of our curvature:

$$\theta = \text{arctg} \left(\left| \frac{\Omega_{\mu x}}{\mathcal{L}_{\mu x}} \right| \right) \tag{7.89}$$

In the expression of Ω 7.87, we can make the distinction between the two paths, and in fact we can define two operators Ω and Ω' . The first depends on \mathcal{D} and the second on \mathcal{D}' . Two curvatures can be computed also and if the two paths are of different lengths, we can compute the whole path coming from the trajectory 1 to 2 by one path, then the trajectory to go from 1 to 2 by the other path. In that approach we create:

$$\Omega_{\mu x} = \begin{bmatrix} J^1 & J^1, J^2 \\ \mathcal{D}_n K_{11}^n & \mathcal{D}'_n K_{12}^n \\ J^1, J^2 & J^2 \\ \mathcal{D}'_n K_{21}^n & \mathcal{D}_n K_{22}^n \end{bmatrix} \tag{7.90}$$

and

$$\Omega'_{\mu x} = \begin{bmatrix} J^1 & J^1, J^2 \\ \mathcal{D}_n K_{11}^n & \mathcal{D}'_n K'^n_{12} \\ J^1, J^2 & J^2 \\ \mathcal{D}'_n K'^n_{21} & \mathcal{D}_n K'^n_{22} \end{bmatrix} \tag{7.91}$$

The difference between the two paths is given by:

$$\left\| \left(R_2 \frac{J^2}{J^1} \right)_{\mathcal{D}} - \left(R_2 \frac{J^2}{J^1} \right)_{\mathcal{D}'} \right\| = |g_{21} - g'_{21}|$$

We note:

$$R_{2121} = |g_{21} - g'_{21}|$$

and in general:

$$R_{abcd} = |g_{ab} - g'_{cd}| \quad (7.92)$$

The tensor R_{abcd} is another expression of the curvature. In geometry, this tensor is in relation with Riemann's tensor.

Hamiltonian or Metric curvature

Under the second geometrization writing, the metric is associated with the operator H and the dissipation, $G = H^T H$. The resistances depend also of the currents. We can compute:

$$\frac{\partial G_{xy}}{\partial J^z} = \frac{\partial}{\partial J^z} \langle \mathbf{b}_x, \mathbf{b}_y \rangle = \Gamma_{xz,y} + \Gamma_{yz,x}$$

If the resistances evolve like $R_1(1 + \alpha J^1)$ (a similar expression for R_2), we have the basis vectors:

$$\mathbf{b}_1 = \begin{bmatrix} R_1(1 + 2\alpha J^1) \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ R_2(1 + 2\beta J^2) \end{bmatrix} \quad (7.93)$$

The Christoffel's coefficients that are not equal to zero are:

$$\Gamma_{11,1} = \alpha(R_1)^2(2\alpha + 4\alpha^2 J^1) \quad \Gamma_{22,2} = \beta(R_2)^2(2\beta + 4\beta^2 J^2) \quad (7.94)$$

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Starting from Kron's equation we have:

$$e_\sigma = H_{\sigma x} J^x + \mathcal{L}_{\sigma x} p J^x$$

We have seen that multiplying each member by Q makes appear the metric G , with $T = Qe$, $K = Q\mathcal{L}$. In our new problem, the H operator is the only one which changes:

$$H = \begin{bmatrix} R_1 (1 + \alpha J^1) & 0 \\ 0 & R_2 (1 + \beta J^2) \end{bmatrix}$$

The equation $e_\mu = H_{\mu x} J^x + \mathcal{L}_{\mu x} p J^x$ can be written, if $b_{\mu x}$ is the jacobian $\partial_{J^x} e$:

$$e_\mu = b_{\mu x} J^x - F_{\mu x} J^x + \mathcal{L}_{\mu x} p J^x \quad (7.95)$$

with:

$$F = \begin{bmatrix} -\alpha J^1 R_1 & 0 \\ 0 & -\beta J^2 R_2 \end{bmatrix}$$

If we multiply by $Q = b^T$ with $T_\sigma = Q_\sigma^\mu e_\mu - Q_\sigma^\mu \mathcal{L}_{\mu x} p J^x$, all members, it gives:

$$T_\sigma = Q_\sigma^\mu b_{\mu x} J^x - Q_\sigma^\mu F_{\mu x} J^x$$

but $Qb = G$ and

$$Q_\sigma^\mu F_{\mu x} J^x = \frac{1}{2} \Gamma_{\sigma y, x} J^y J^x$$

Finally:

$$T_\sigma = G_{\sigma x} J^x - \frac{1}{2} \Gamma_{\sigma y, x} J^y J^x \quad (7.96)$$

If various paths are available, we can test each of them depending on the flux J^γ to see their extrema. We compute:

$$\begin{aligned} \frac{\partial}{\partial J^\gamma} T_\sigma &= \frac{\partial}{\partial J^\gamma} G_{\sigma x} J^x + G_{\sigma x} \frac{\partial}{\partial J^\gamma} J^x - \dots \\ &\dots - \frac{\partial}{2\partial J^\gamma} \Gamma_{\sigma y, x} J^y J^x - \frac{1}{2} \Gamma_{\sigma y, x} \frac{\partial}{\partial J^\gamma} J^y J^x - \frac{1}{2} \Gamma_{\sigma y, x} \frac{\partial}{\partial J^\gamma} J^x J^y \end{aligned}$$

knowing that:

$$\begin{aligned} \frac{\partial}{\partial J^\gamma} G_{\sigma x} &= \Gamma_{\sigma x, \gamma}, \\ \frac{\partial}{\partial J^\gamma} J^x &= \delta_\gamma^x, \\ \frac{\partial}{\partial J^\gamma} J^x J^y &= \delta_\gamma^x J^y = J^x \text{ or } 0 \end{aligned}$$

with

$$g_{\sigma\gamma} = \frac{\partial}{\partial J^\gamma} T_\sigma$$

we obtain:

$$g_{\sigma\gamma} = \Gamma_{\sigma x, \gamma} J^x + G_{\sigma\gamma} - \frac{1}{2} \frac{\partial \Gamma_{\sigma y, x}}{\partial J^\gamma} J^y J^x - \Gamma_{\sigma x, \gamma} J^x$$

which gives finally:

$$g_{\sigma\gamma} = G_{\sigma\gamma} - \frac{1}{2} \frac{\partial \Gamma_{\sigma y, x}}{\partial J^\gamma} J^y J^x \quad (7.97)$$

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Now we may be interested in looking for the difference $|g_{\sigma\gamma} - g_{\zeta\chi}| = R_{\sigma\gamma\zeta\chi}$. We have:

$$R_{\sigma\gamma\zeta\chi} = G_{\sigma\gamma} - \frac{1}{2} \frac{\partial \Gamma_{\sigma y, x}}{\partial J^\gamma} J^y J^x - G_{\zeta\chi} + \frac{1}{2} \frac{\partial \Gamma_{\zeta y, x}}{\partial J^\chi} J^y J^x$$

Making the contraction with $\zeta = \chi$, which implies $R_{\sigma\gamma\zeta\chi} = R_{\sigma\gamma}$ and extracting the \mathcal{L} part of T we obtain:

$$R_{\sigma\gamma} - \frac{1}{2} \left(\frac{\partial \Gamma_{\gamma y, x}}{\partial J^\sigma} - \frac{\partial \Gamma_{\sigma y, x}}{\partial J^\gamma} \right) J^y J^x = \left(\frac{\partial \mathcal{L}_{\sigma x}}{\partial J^\gamma} - \partial \mathcal{L}_{\gamma x} \partial J^\sigma \right) J^x \quad (7.98)$$

The second terme can be seen as the tensor of the differential inertia $\mathcal{T}_{\sigma\gamma}$. The derivative of Γ leads to pure resistive impedance operator R . So we can create a tensor which definition is:

$$y_{\sigma\gamma} = \delta_{\sigma y} J^y \delta_{\gamma x} J^x \quad (7.99)$$

with these writings we obtain the remarkable equation:

$$\mathcal{T}_{\sigma\gamma} = \left(R_{\sigma\gamma} - \frac{1}{2} y_{\sigma\gamma} R \right) \quad (7.100)$$

In fact when we look to the difference between the various paths, we make an operation similar to a rotational. The Riemann's tensor can be seen for me as a kind of rotational.

7.6.3 Paths of higher orders

As we have seen, the problem of higher order paths leads to determine the expression of the whole path from one emitter

to one receiver. The figure 7.26 shows a situation where two paths of various types allow to reach the vertex 5, starting from a parasitic emission in a vertex 1.

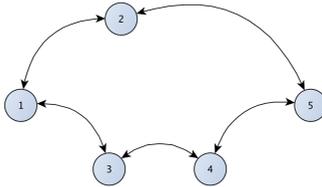


Figure 7.26: An EMC situation

The fact that the second path uses 2 vertices rather than one like the first path doesn't mean that this path is "longer" than the first. We cannot have "a priori" on the properties of each path and its contribution in the energy given to the final target, here the vertex 5. The whole expression of the first path can be obtained simply using our definition:

$$g_{51} = R_{55} \frac{J^5}{J_1} = R_{55} \frac{J^5 J^2}{J_2 J_1} \quad (7.101)$$

As usually, R_{55} is a function of the impedance operator of the vertex 5 that gives the voltage V_5 observed as criterion for the received energy of 5 coming from 1. Defining: $f_x^y = J^y (J^x)^{-1}$ we have:

$$g_{51} = R_{55} f_2^5 f_1^2 \quad (7.102)$$

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This is a general definition that can be used for any kind of path:

$$g_{\alpha\beta} = R_{\alpha\alpha}\Pi_{\sigma}f_{\sigma}^{\alpha} \dots f_{\beta}^{\sigma} \quad (7.103)$$

The problem is finally to solve the various transfer functions f_{χ}^{σ} , from χ to σ .

Each possibility gives a path to reach 5 through 2: $g_{51}^{(2)}$ or through 3 and 4: $g_{51}^{(3,4)}$, etc. The final structure is always a 2x2 matrix, but the extra-diagonal components can be complicated functions involving all the $g_{yx,\{\dots\}}$ paths.

Any path gives an action (you have noticed that I call action the root of a power, i.e. a potential). The energy is so given by:

$$\omega = \int_t dt \sqrt{g_{yx} J^y J^x} \quad (7.104)$$

In our previous case, this leads to:

$$\omega = \sqrt{\frac{R_{55}}{p^2} \{f_2^5 f_1^2 + f_4^5 f_3^4 f_1^3\} J^1 J^5} \quad (7.105)$$

All the possible paths appear clearly and give us the order n of the system of kind \mathcal{S}^n . The electromagnetic energy uses all these paths to reach the vertex 5. But some of them are more or less significant. If they are all undesired, it is very interesting to evaluate their contribution, to suppress, in priority the more influent paths. This can be sufficient to decrease the coupling between 1 and 5 and get out the risk of disturbing vertex 5. But it exists a difficulty in electromagnetism: one time a path is modified to decrease its influence, we have to compute again

all the system. After this modification perhaps another path will behave differently and partly destroy the effort made to separate the vertices 1 and 5. The behavior of the system is not linear. Why? Because when we change a coupling path without changing the impedance of the involved vertices, the energy has no other choice to dissipate than using more other paths and increasing the current in some vertices. Let's take an example to illustrate this problem.

We have seen that lines like waveguides in general or antennas are perceived by an emitter through some back impedances that translate the transmission of energy. In the case of the antennas, this is the radiation resistance linked with the whole energy radiated in the free space. In the case of lines, what is its expression? We have already discussed of this notion, but it is interesting to come back to it with a quite different approach.

If we look to the voltage anywhere in a line, at an abscisse z , this voltage can be written:

$$V(z) = V_i e^{-jkz} + V_r e^{-jk(2L-z)} \quad (7.106)$$

The current has a similar expression, with the characteristic impedance dividing the voltage:

$$J(z) = \frac{V_i}{z_c} e^{-jkz} - \frac{V_r}{z_c} e^{-jk(2L-z)} \quad (7.107)$$

The incident wave V_i defines the limit condition in $z = L$. We can write this coefficient in z :

$$\sigma(z) = \frac{V_r e^{-jk(2L-z)}}{V_i e^{-jkz}} = \frac{V_r}{V_i} e^{-2jk(L-z)} \quad (7.108)$$

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in $z = L$, the reflexion coefficient is defined by:

$$\sigma(L) = \frac{V_r}{V_i} = \frac{V_t}{V_i} - 1 = 2 \frac{Z_L}{Z_c + Z_L} - 1 = \frac{Z_L - Z_c}{Z_L + Z_c} \quad (7.109)$$

We obtain with this definition, the voltage and current at any abscisse z of the line:

$$\begin{cases} V(z) = V_i e^{-jkz} (1 + \sigma(L) e^{-2jk(L-z)}) \\ J(z) = \frac{V_i}{Z_c} e^{-jkz} (1 - \sigma(L) e^{-2jk(L-z)}) \end{cases} \quad (7.110)$$

and the backward impedance at any point z of the line:

$$Z_R(z) = \frac{V(z)}{J(z)} = Z_c \frac{[1 + \sigma(L) e^{-2jk(L-z)}]}{[1 - \sigma(L) e^{-2jk(L-z)}]} \quad (7.111)$$

If we want to have a symmetric model of the line with a backward impedance seen from the output of the line, due to its mismatching on the input, the reported EM force can be computed from $V(L)$, knowing V_i , the voltage for the matched output line. The model is a little more complicated than the one of Branin, but makes appearing the very interesting concept of backward impedance.

Now let's imagine two vertices that are connected by two different lines. From the first vertex, we can see these lines like two backward impedances ZR_1 and ZR_2 . The two lines have two characteristic impedances Z_1 and Z_2 . The first line reports on the first vertex the impedance:

$$ZR_1 = Z_1 \frac{[1 + \sigma_1 e^{-2jk(L)}]}{[1 - \sigma_1 e^{-2jk(L)}]}$$

and the second line reports on the same vertex the impedance:

$$ZR_2 = Z_2 \frac{[1 + \sigma_2 e^{-2jk(L)}]}{[1 - \sigma_2 e^{-2jk(L)}]}$$

and we have:

$$\sigma_1 = \frac{Z_L + XR_2 - Z_1}{Z_L + XR_2 + Z_1} \quad \sigma_2 = \frac{Z_L + XR_1 - Z_2}{Z_L + XR_1 + Z_2}$$

XR_1 and XR_2 are the impedance reported by the line and seen from their outputs. To simplify the analysis, we make the assumption that the generator is matched (i.e. $Z_0 = Z_1 + Z_2$). Then $XR_1 = Z_1$ and $XR_2 = Z_2$. We understand that any modification that we may realized on one of the lines, to decrease the coupling through this line will affect the impedance seen by the other line and as a consequence, will affect the response of this other line. That's the difficulty of the electromagnetic compatibility, systems are in general not easily separable.

When solutions will be tested to try to reduce the coupling between two vertices, we will often need the impedance operator of some discontinuity. The figure 7.27 shows this kind of structure.

With this circuit we have:

$$V_1 = (Z_{11} - Z_{12})i^1 + Z_{12}(i^1 - i^2) = Z_{11}i^1 - Z_{12}i^2$$

and

$$V_2 = Z_{12}(i^1 - i^2) - (Z_{22} - Z_{12})i^2 = Z_{12}i^1 - Z_{22}i^2$$

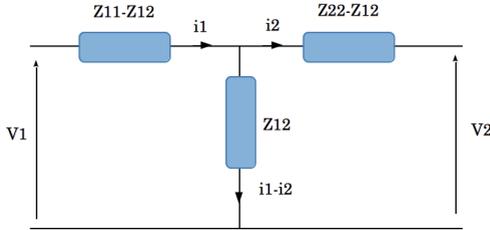


Figure 7.27: Discontinuity

7.6.4 Cavities

A cavity is a special environment that turns back at least partially the radiation created inside of it. A cavity is first of all, a short-circuited waveguide with discontinuities inside. We have not studied so accurately this ambiance previously.

A cavity behaves like a line, but where the modes used by the field to propagate are different. We imagine an electric field depending on the axis x in a rectangular waveguide of directions $x \times y = z$. Maxwell's equation tells us the relation between the electric and the magnetic field:

$$H_y = \frac{-1}{j\mu\omega} \frac{\partial E_x}{\partial z} = \frac{jk}{j\mu\omega} E_{0x} \text{Sin} \left(\pi \frac{y}{W} \right) e^{-jkz} \Rightarrow \frac{E_x}{H_y} = \eta_n = \mu \frac{\omega}{k_n}$$

The modes $\text{Sin} (n\pi y/W)$ of order n is the function ϕ_n . If a_n are the amplitude of each mode and a_m the amplitudes of modes

created by discontinuities in the cavity, the electric field is:

$$\mathbf{E} = \sum_n^N a_n (1 + \sigma(n)e^{-2jk_n L}) \phi_n + \sum_{N+1}^M a_m \phi_m e^{-2jk_m L} \quad (7.112)$$

the cavity being of length L and width W , height X .

If $Y_n = a_n/\eta_0$ and ψ represents the magnetic modes, the magnetic field is:

$$\mathbf{H} = \sum_n^N Y_n (1 - \sigma(n)e^{-2jk_n L}) \psi_n + \sum_{N+1}^M Y_m \psi_m e^{-2jk_m L} \quad (7.113)$$

The input impedance of the cavity is defined by:

$$Z_R = \frac{V_x}{J_y} = \int_x dx \mathbf{E}_x \left\{ \int_y dy \mathbf{H}_y \right\}^{-1} \quad (7.114)$$

If we consider that the added modes a_m and Y_m comes from residual diffractions on small discontinuities and are negligible, we have:

$$ZR(n) = \eta_0 \frac{(1 + \sigma(n)e^{-2jk_n L})}{(1 - \sigma(n)e^{-2jk_n L})} \quad (7.115)$$

The characteristic impedance can be obtained from the wave impedance (we have already seen that) using:

$$z_n = \eta_n \int_x d\mathbf{x} \cdot \frac{\mathbf{E}_x}{|E_x|} \left(\int_y d\mathbf{y} \cdot \frac{\mathbf{H}_y}{|H_y|} \right)^{-1} \quad (7.116)$$

η_n is the wave impedance of the mode n .

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It the first wall of the cavity is equipped with a slot, the equivalent schematic of this limit condition is the capacitor of the slot, associated with its inductance, in parallel with the resistances of the parts of the wall on each side of the slot. Once the energy is transmitted to the input of the cavity, we can study the propagation of the potentials (V, i) , as the propagation of the fields (E, H) , it's the same thing!

Now when the wave see an obstacle, it can be treated as a discontinuity. Over the discontinuity, there is a new waveguide (perhaps the same as before, perhaps not). The difference with lines is that the impedances must be computed for each mode.

The generator coming from the slot can be synthesized using Thévenin's theorem, with the source $h_s \mathbf{E}_0$, h_s height of the slot and \mathbf{E}_1 an external incident field in the x direction; and the impedance:

$$z_s = \frac{R_w}{R_w C_s p + 1}$$

R_w is the resistance of the wall and C_s the capacitor of the slot supposed to be small compare to the wavelength. Following Branin's modeling, we know that the generator presented at the input of the discontinuity is defined by:

$$e_2 = \left(h_s E_1 - z_s Y H_y + \eta_n \frac{X}{Y} Y H_y \right) e^{-j k_n z} \quad (7.117)$$

if $Y H_y = \bar{h}_y$, and $\eta_n X/Y = z_n$, the equation becomes:

$$e_2 = (h_s E_1 + [z_n - z_s] \bar{h}_y) e^{-j k_n z} \quad (7.118)$$

We call \bar{h}_y the current potential, perpendicular to the current, but having the same dimension.

This defines the impedance $z_{11} - z_{12}$ of the discontinuity. To determine the impedance of the border, a technique uses energies. There are two cases:

1. the half wavelength is larger than the dimensions of the discontinuity;
2. the half wavelength is equal or smaller than the dimensions of the discontinuity.

In the first case, we are in the same situation than for the slot: the discontinuity can be modeled using lumped elements, capacitors, inductances and resistances. In the second case, we must find the equivalent impedance of the discontinuity for each of its modes. An available technique remains always the same: making the equivalence between the energy stored in the volume of the discontinuity for one assumption of mode and the expressions Li^2 , CV^2 , and for the dissipations Ri^2 . This determines the impedance z_{21} of the discontinuity. If the first mode of the discontinuity is higher than the first mode of the waveguide, we should find for example a capacitor that acts as a filter for the mode of the cavity and transmits a low part of its energy to the volume behind it.

The whole technique can be applied to the wavefront defined by the E_y and E_x polarizations. Each direction of propagation should be studied also, to understand the cross coupling between the various modes. If we have two Branin's structures for each surface of propagation, it leads to 6 branins in order to model to whole cavity and its objects inside. It's not so heavy and it is the only way to model theoretically filled cavities.

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How to find the modes of a complicated discontinuity? If we report twice the width of the TE_{1y} mode, the frequencies generated by its Fourier's transform is:

$$\text{Sin} \left(n2\pi \frac{y}{2Y} \right) = \text{Sin} \left(n\pi \frac{y}{Y} \right)$$

The modes can be found making the Fourier's transform of the geometrical profile of the discontinuity. Now from what dimensions of discontinuities should they be regarded as obstacles modifying modes? If the energy they stored is negligible compared to the energy transported by the wave, we can consider that they act more like porosities than like discontinuities in the propagation of the waves. For example if the floor of the cavity is equipped with a printed circuit board with its electronic components. We wonder if the microprocessors can affect the field in the cavity. Locally, the height of the microprocessors up to the ground plane is one millimeter. Its thickness is 2 millimeters. It acts like a capacitor for 1 millimeter height h_m and like a resistance for 2 millimeters height h_r . The dimensions of the microprocessor are 2 cm x 2 cm, a square of surface s_m . The electrical power stored is:

$$\frac{p}{2} CV^2 = \frac{p}{2} \epsilon_0 \frac{\sqrt{s_m}}{h_m} \int_{y=0}^{\sqrt{s_m}} dy (h_m E_x(y))^2$$

and the electrical power dissipated is:

$$\frac{1}{2} Ri^2 = \frac{1}{2} \frac{1}{\sigma s_m} s_m^2 \sigma^2 \int_{h_m} d\mathbf{h}_m \cdot (\mathbf{E}_x(y))^2$$

It remains to appreciate if these powers are negligible or not compared to:

$$s_0 = \int_y \int_x dx dy \frac{1}{\eta_n} (\mathbf{E}_x(x, y) \times \mathbf{H}_y(x, y)) \quad (7.119)$$

The study must be conducted for each mode, but it is possible to consider that at one frequency, only one major mode can be taken into account. This mode is defined for example by

$$n = 2 \frac{Y}{\lambda} \quad (7.120)$$

Be convinced that due to many uncertainties, these approximations remain acceptable to evaluate the energy transmitted to one element in a filled cavity. The problem is anyway very difficult, and even the use of 3D solvers using meshing won't give more guarantees and trust in the results. In fact, both analytical and numerical approaches are necessary to trust the results.

The losses in the propagation can be evaluated computing the losses in the metallic parts, and taking into account the skin effect. We calculate for example the resistance r of losses using:

$$r i_0^2 = \int_y dy \left[\int_z dz \frac{1}{\sigma} \sqrt{\pi \frac{\omega}{2\pi} \mu \sigma} \left(\frac{X E_x(y)}{z_n} \right)^2 \right] \quad (7.121)$$

and $i_0 = V_{0x}/z_n$.

Others geometries can modify the modes, for example coupling E_x modes with E_y ones. It can be a simple waveguide rotating between two other waveguides connected to this transition. To model this kind of material, it is often interesting to

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use the chain matrix. Being able to write the relations between the fields at the input and output of the structure under the form:

$$\begin{bmatrix} \mathbf{E}_2 \\ \mathbf{H}^2 \end{bmatrix} = \begin{bmatrix} d_2^1 & f_{21} \\ j^{21} & d_1^2 \end{bmatrix} \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{H}^1 \end{bmatrix} \quad (7.122)$$

d, f, j are functions of the field.

we can after deduce the impedance matrix from this chain matrix by:

$$\begin{bmatrix} \mathbf{E}_2 \\ \mathbf{E}_1 \end{bmatrix} = j_{12} \begin{bmatrix} d_2^1 & d_2^1 d_1^2 - f_{21} j^{21} \\ 1 & d_1^2 \end{bmatrix} \begin{bmatrix} \mathbf{H}^2 \\ \mathbf{H}^1 \end{bmatrix} \quad (7.123)$$

with of course $j_{12} = (j^{21})^{-1}$. Thanks to Maxwell's equations, the magnetomotive and electromotive forces makes the link between the fields and the potentials:

$$e = \oint_c d\mathbf{c} \cdot \mathbf{E} \quad \bar{h} = \oint_c d\mathbf{c} \cdot \mathbf{H} \quad (7.124)$$

and so, all the techniques developed for the potentials (Branin's models, chain matrices, etc.) can be applied to the fields, through some geometrical integrals.

Evanescent and near fields

Near field or evanescent field interactions are modeled using lumped elements: capacitors or inductances and mutual inductances. There is no propagation of the field in these process that's why static representations are pertinents. For example,

it is the simplest model to translate the transmission in a waveguide under the cutoff frequency. An electronic system can be seen in global as a set of networks of evanescent fields communicating through cords and propagating fields. It is a continuous exchange between the energy stored in the evanescent fields and the energy dissipated in the resistances.

What is perhaps less evident is the problem of the near electric field. Forgetting the transverse part of the near field, we are interested in the longitudinal part of the electric field. It can be modeled using capacitors, but this implies to add a branch and this increases the network dimension. Another solution is to consider the loads $\pm q$ stored somewhere in another capacitor and to compute at the location we want the potential created by these loads:

$$\pm\psi = \pm \frac{q}{4\pi\epsilon_0 r} \quad (7.125)$$

r is the distance between the load and the location we consider. Once we have the two potentials ψ , we can compute the potential difference $\Delta\psi$ across the capacitor C there is in reception between the two nodes where we compute the potentials. Across this capacitor, the potential difference $\Delta\psi$ creates a source of current K given by:

$$K = pC\Delta\psi \quad (7.126)$$

This source must be added on the receiver, using nodes-pair current sources. Note that it exists only if the emitter has capacitors in its schematic, and the same for the receiver. All the other couplings are covered by the magnetic EM force.

The specificity of this coupling is that it comes from the

integral of a current (the load $q = 1/pi$) and creates a current. It makes a link between a branch and a spanning tree. The term zJ where J is the nodes pair source is replaced by something like $(z/p)i$, i being computed elsewhere. The voltage developed across the receiving branch is an unknown that can be solved in a complete space resolution (see the chapter on Graphs for that).

7.6.5 Path number reduction

When branches are in parallel, we can replace them by an unique branch. For example, imagine a single generator (e_1, R_0) that power supplied two branches R_1 and $1/(Cp)$. We can define two meshes associated with the following operator H :

$$H = \begin{bmatrix} R_0 + R_1 & R_0 \\ R_0 & R_0 + \frac{1}{Cp} \end{bmatrix} \quad (7.127)$$

With $\Delta = (R_0 + R_1)(R_0 + 1/(Cp)) - R_0^2$, the solutions for J^1 and J^2 the currents of the meshes are:

$$\begin{cases} J^1 = \frac{1}{\Delta} \left(R_0 + \frac{1}{Cp} \right) e_1 - \frac{R_0}{\Delta} e_1 \\ J^2 = -\frac{R_0}{\Delta} e_1 + \frac{R_0 + R_1}{\Delta} e_1 \end{cases}$$

So

$$J = J^1 + J^2 = e_1 \left(\frac{R_1 Cp + 1}{R_1 R_0 Cp + R_0 + R_1} \right) \quad (7.128)$$

But we may have considered an unique mesh with R_0 in series with the impedance made of the resistance R_1 in parallel with the capacitor C :

$$z_{RC} = \frac{R_1}{R_1 Cp + 1}$$

The current of the mesh is directly given by:

$$J = \frac{e_1}{R_0 + z_{RC}} = e_1 \left(\frac{R_1 Cp + 1}{R_1 R_0 Cp + R_0 + R_1} \right) \quad (7.129)$$

we obtain, of course, exactly the same result as equation 7.128. A method to reduce the number of meshes is to benefit of impedances that are in parallel, to replace them by an unique one having for function, the function that results from this management in parallel.

It is clear that we won't use this technique is we want to distinguish the importance of various path in parallel. But as we have compacted series of vertices on a same path to reduce the analysis to a 2x2 matrix, we can reduce known and functional path in parallel to reduce also the dimensions of the matrix that must be studied. The first action may be called "series reduction" while the second action may be called "parallel reduction".

7.6.6 Stationary wave ratio and the mix of quantum and dissipated fields

Starting from the potential along a line:

$$V(z) = V_i e^{-jkz} \left(1 + \sigma e^{-2jk(L-z)} \right) \quad (7.130)$$

we develop the expression, replacing σ by its equation to find:

$$V(z) = \frac{V_i}{R_0 + \eta} e^{-jkL} (2R_0 \text{Cos} [-k(L - z)] + 2j\eta \text{Sin} [-k(L - z)])$$

with

$$\sigma = \frac{R_0 - \eta}{R_0 + \eta}$$

to obtain finally:

$$V(z) = \frac{2}{R_0 + \eta} (R_0 + j\eta tg [-k(L - z)]) V_i \text{Cos} [-k(L - z)] e^{-jkL} \quad (7.131)$$

if $k = \omega/c$, that to say $kL = \pi$ at the resonance frequency $L = \lambda/2$; the potential becomes:

$$V(z) = \frac{2}{R_0 + \eta} V_i \text{Cos} (kz) (R_0 + j\eta tg kz) \quad (7.132)$$

For the current we obtain:

$$J(z) = \frac{2}{R_0 + \eta} \frac{V_i}{\eta} \text{Cos} (kz) (jR_0 tg kz + \eta) \quad (7.133)$$

If we make the product $1/2V(z)J^*(z)$ using the relations 7.130 for $V(z)$ and $J(z)$, it gives:

$$\frac{1}{2} V(z) J^*(z) = \frac{V_i^2}{2\eta} e^{-2jkz} \left(1 - \sigma \sigma^* e^{-4jk(L-z)} \right) \quad (7.134)$$

which at the resonance frequency $\lambda/2$ and the abscissa $z = L$ gives:

$$\frac{1}{2}V(L)J^*(L) = \frac{V_i^2}{2\eta} (1 - |\sigma|^2) = \mathcal{W}_i (1 - |\sigma|^2) = \mathcal{W}_t \quad (7.135)$$

\mathcal{W}_i being the incident power. If we use our previous results on $V(z)$ and $J(z)$ we can write the power as:

$$\mathcal{W}_z = 2 \frac{V_i^2}{\eta(R_0 + \eta)^2} \text{Cos}^2 kz (R_0 + j\eta \text{tg} kz) (\eta - jR_0 \text{tg} kz) \quad (7.136)$$

This leads to

$$\mathcal{W}_z = \frac{V_i^2}{\eta(R_0 + \eta)^2} (1 + \text{Cos} 2kz) [j(\eta^2 - R_0^2) \text{tg} kz + \eta R_0 (1 + \text{tg}^2 kz)]$$

if $z = L$ and $L = q\lambda/2$, then $kL = q\pi$ and \mathcal{W}_z is equal to \mathcal{W}_t given by:

$$\mathcal{W}_t = \frac{R_0}{(R_0 + \eta)^2} V_i^2 \quad (7.137)$$

If we replace σ by its expression in 7.135 we find the same result. And this result is also the same for \mathcal{W}_0 or $\mathcal{W}_{L/2}$. But the power transmitted from the input of the waveguide to its output has two components: a real one and an imaginary one. The real one is:

$$\mathcal{W}_z^{\text{R}} = \frac{1 + \text{Cos} 2kz}{(R_0 + \eta)^2} V_i^2 (R_0 [1 + \text{tg}^2 kz]) \quad (7.138)$$

and the imaginary one is:

$$\mathcal{W}_z^{\mathbb{J}} = j \frac{1 + \text{Cos}2kz}{(R_0 + \eta)^2} V_i^2 \left(\eta - \frac{R_0^2}{\eta} \right) tgkz \quad (7.139)$$

A remark is that if the waveguide is matched, there are no imaginary energies. The imaginary term translates the part of energy which is stored in the waveguide in standing waves. It doesn't generate Joule's losses. We may define a ratio between these two kinds of energies, without looking for the official definition of the "standing wave ratio":

$$\left| \frac{\mathcal{W}_z^{\mathbb{J}}}{\mathcal{W}_z^{\mathbb{R}}} \right| (z) = \left| \frac{\eta^2 - R_0^2}{\eta R_0} \right| \frac{tgkz}{[1 + tg^2kz]} \quad (7.140)$$

The ratio is zero on the input and output because at these limit conditions, the energy must be dissipated in the loads but also for the modes q where $kz = q\pi/2$. On the points where the stationary wave is at its maxima, the electric and magnetic fields behave like a TEM wave transmitting the energy through a real flux. For the half modes where $kz = q\pi/4$ the stored energy is a part of the real one with a ratio given by:

$$\left| \frac{\mathcal{W}_z^{\mathbb{J}}}{\mathcal{W}_z^{\mathbb{R}}} \right| (z = q\lambda/8) = \left| \frac{\eta^2 - R_0^2}{2\eta R_0} \right|$$

This result says us that the field can exist under two kinds of nature: a quantum one with modes defining how it is distributed in the space, and a continuous and real one associated with the losses. Somewhere the direction of the space can be identified

through the continuous flux of energy that doesn't depend on modes. On these directions, it is possible that the field installes itself following modes. That's what is suggested by the second geometrization. Under this formalism, the metric is first of all linked with the resistances, i.e. the losses while the inductances appear as inertia.

Let's take an example. We consider the following hamiltonian:

$$H = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 + \frac{1}{C_2 p} \end{bmatrix} \quad (7.141)$$

It generates the basis:

$$\mathbf{b}_1 = \begin{bmatrix} R_1 \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ R_2 + \frac{1}{C_2 p} \end{bmatrix} \quad (7.142)$$

and the metric

$$G = \begin{bmatrix} (R_1)^2 & 0 \\ 0 & \left(R_2 + \frac{1}{C_2 p}\right)^2 \end{bmatrix} \quad (7.143)$$

and the sources

$$T = \begin{bmatrix} R_1 e_1 \\ \left(R_2 + \frac{1}{C_2 p}\right) e_2 \end{bmatrix} \quad (7.144)$$

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The space here has two independent directions that include the losses, plus the memory associated with the capacitor C_2 . How to interpret this kind of axis? In fact if we write the relations in the fundamental time domain, we have:

$$H = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 + \frac{1}{C_2} \int_t dt(\cdot) \end{bmatrix} \quad (7.145)$$

When we look to the values of the current at each time step, we can write:

$$\begin{cases} e_1 = R_1 J^1 \\ e_2 = R_2 J^2 + \frac{1}{C_2} \int_t dt J^2(t) \end{cases} \quad (7.146)$$

but

$$\frac{1}{C_2} \int_t dt J^2(t) = \frac{1}{C_2} \int_0^{t-dt} dt J^2 + \frac{dt}{C_2} J^2$$

Now we can define:

$$E = \begin{bmatrix} e_1 \\ e_2 - \frac{1}{C_2} \int_0^{t-dt} dt J^2 \end{bmatrix} \quad (7.147)$$

and

$$H = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \quad (7.148)$$

because $dt/C_2 \rightarrow 0$. In that case

$$G = \begin{bmatrix} (R_1)^2 & 0 \\ 0 & (R_2)^2 \end{bmatrix} \quad (7.149)$$

and

$$T = H^T E = \begin{bmatrix} R_1 e_1 \\ R_2 \left(e_2 - \frac{1}{C_2} \int_0^{t-dt} dt J^2 \right) \end{bmatrix} \quad (7.150)$$

The space remains driven by the losses and the capacitor translates the impact of a potential energy that changes with time: the more the current increases, the more the source is decreased. If we imagine a constant source (a step voltage) $e_1 = e_1/p$ that gives power to a capacitor C through a resistance R_1 , we have only one direction in that case: the value of the resistance R_1 . At the beginning the current is at its maximum e_1/R_1 . So across the resistance where we can observe the projection of J on R_1 the instantaneous value is e_1 . The current is multiplied by dt/C and retired from e_1 . Next value will be less, and so on... When the time becomes far enough, the amplitude of b_2 tends to zero. $1/C$ times the integral of the current over the time increases with time and gives the voltage across the capacitor.

What is happening if the resistances are equal to zero? At $t = 0$ the current value is e_1/R_1 . If $R_1 = 0$ the current is infinite. There are no physical solutions without losses. At least the resistances exist on the limit conditions, like for antennas, to give the relations between the radiated energy and the sources

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to create these radiations. After what, the energy transformed in radiation can be identified by its imaginary property and can be stored. But to reach this order and organization I must pay for an equivalent disorder. The entropy is taken in charge during the operations of loading the energy or when we want to extract it through resistances and Joule's effects.

The fluxes associated with kinetic energy and exchanges of energies go through losses, even if they are very thin. These losses can be seen as defining the basis vectors to go from points to points, and these directions can be seen as the directions of the space where we describe these exchanges. Inertia are translated by magnetic energies and memories in states are translated by capacitors where the potential energy can be stored. The inertia can be seen as a capacity to store movement, like in a spring and translates accelerations. There's no hazard if the elasticity is similar to the inductance.

In a space without inductances ($\mathcal{L} = 0$), the next equation gives the **geodesic** of the space (by definition the solutions without acceleration).

$$T_{\mu}J^{\mu} - G_{\mu\nu}J^{\nu}J^{\mu} = 0 \quad (7.151)$$

We have previously discussed that the quantum field was firstly identified by stationary waves and absence of losses. In fact losses are radiations at frequencies where the matter behaves like oscillators. When they are excited because of compressions, interactions with free particles, etc., they radiate. The more they are, the more the losses are. The macroscopic world results in a change of scale applied on the atomic or molecular

scale. Starting from the wave function, the basic expression is:

$$\psi = A_0 e^{j\omega(t - \frac{x}{c})} \quad (7.152)$$

Using the first quantum rule, i.e. $E = h\nu$, it becomes (with η the impulse):

$$\psi = A_0 \exp\left(\frac{j}{\hbar} [Et - \eta x]\right) \quad (7.153)$$

but

$$E = \int_t dt P = \frac{1}{p} P \Rightarrow Et = \frac{1}{p^2} P$$

P being the power, and

$$P = H_{ab} k^a k^b - p \mathcal{L}_{ab} k^a k^b$$

so:

$$\psi = A_0 \exp\left(\frac{j}{p^2 \hbar} [H_{ab} k^a k^b - p \mathcal{L}_{ab} k^a k^b]\right) \quad (7.154)$$

H and \mathcal{L} are operators at the atomic scale. They take in charge the models of the oscillators and the coupling between them. ηx corresponds to $p^{-1} \mathcal{L}_{ab} k^a k^b$. These operators act on current densities or moving particles. To do a measurement, we use the operator:

$$\mathcal{H} = p^2 \frac{\hbar}{j} \frac{\partial}{\partial k^a} \psi \Rightarrow \mathcal{H} \cdot \psi = [H_{ab} k^b - p \mathcal{L}_{ab} k^b] \psi \quad (7.155)$$

With this operator, the measurement of a potential A can be realized making:

$$A = \int_{\nu} d\nu \mathcal{H} \cdot \psi \psi^* \quad (7.156)$$

Generally, 7.156 leads to macroscopic behaviors far from their quantum bases. Sometimes, with particular organizations of the matter (adequate periodicities, particular couplings, etc.) some quantum behaviors can be directly exploited. We can think in superconductivity for example. In these cases, the measurement A makes appear these properties directly in its function. But in any case, it is very difficult to write the complete operator ζ that includes all the phenomenons involved at any energies.

Second geometrization on Branin

It is interesting to apply the technique of the second geometrization to Branin's structure.

We start from a typical Branin's impedance operator:

$$\zeta = \begin{bmatrix} z_0 + \eta & (z_L - \eta) e^{-\gamma x} \\ (z_0 - \eta) e^{-\gamma x} & z_L + \eta \end{bmatrix} \quad (7.157)$$

in this definition $\gamma = p/v + \alpha$, v being the energy speed and α the losses in propagation. This impedance operator leads to the next basis vectors:

$$\mathbf{b}_1 = \begin{bmatrix} z_0 + \eta \\ (z_0 - \eta) e^{-\gamma x} \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} (z_L - \eta) e^{-\gamma x} \\ z_L + \eta \end{bmatrix} \quad (7.158)$$

This gives the metric:

$$G = \begin{bmatrix} (z_0 + \eta)^2 [1 + \sigma_{ls}^2 e^{-2\gamma x}] & (z_0 z_L - \eta^2) e^{-\gamma x} \\ (z_0 z_L - \eta^2) e^{-\gamma x} & (z_L + \eta)^2 [1 + \sigma_{lc}^2 e^{-2\gamma x}] \end{bmatrix} \quad (7.159)$$

with

$$\sigma_{ls} = \frac{z_0 - \eta}{z_0 + \eta} \quad \sigma_{lc} = \frac{z_L - \eta}{z_L + \eta} \quad (7.160)$$

while computing the sources T we find:

$$T = \begin{bmatrix} (z_0 + \eta) e_1 [1 + \sigma_{ls} e^{-2\gamma x}] \\ (z_L + \eta) e_1 e^{-\gamma x} [1 + \sigma_{lc}] \end{bmatrix} \quad (7.161)$$

What is interesting in this formulation $T_\mu = G_{\mu\nu} J^\nu$ is that the metric here involves clearly both limit conditions of the waveguide and the Branin's structure becomes symmetric.

The metric can be separate in two parts: the dissipative and stationary part G^o and a propagating kernel or Green's function K :

$$G^o = \begin{bmatrix} (z_0 + \eta)^2 & 0 \\ 0 & (z_L + \eta)^2 \end{bmatrix} \quad (7.162)$$

$$K = \begin{bmatrix} (z_0 + \eta)^2 \sigma_{ls}^2 e^{-2\gamma x} & (z_0 z_L - \eta^2) e^{-\gamma x} \\ (z_0 z_L - \eta^2) e^{-\gamma x} & (z_L + \eta)^2 \sigma_{lc}^2 e^{-2\gamma x} \end{bmatrix} \quad (7.163)$$

Due to the delay operators involved in K , it reuses the past values of the current in the time domain. So the system can be

written

$$T_\mu - K_{\mu\nu} = G_{\mu\nu}^o J^\nu \quad (7.164)$$

Once more the system is driven by its dissipations on both meshes at each extremity of the waveguide, remembering that η can even be a radiation resistance. If the line is matched, $K = 0$. The space has two directions and is flat. These two directions have the same basis vector component different from zero: 2η . We may represent what is happening on this space. The values of J^1 and J^2 are the curvilinear coordinates of the vector \mathbf{J} . We can visualize a third axis which can be the delay. If we use for e_1 the function $(p + a)^{-1}$, we obtain something that seems like the diagram given figure 7.28⁴. The receiver see a changed incident wave due to the losses of the medium described in α .

The action s given by $\sqrt{G_{yx}^o J^y J^x}$ is equal to

$$\sqrt{G_{11}^o (J^1)^2 + G_{22}^o (J^2)^2}$$

Due to the distance between the two vertices, the action involves both currents during a limited time. If the distance may be longer than the time duration of the waveform, at any moment, the action will depend only of one current, attached with only one direction of the space.

In this system there is only one way to go from one vertex to the other. The parasitic coupling (we imagine it is) depends primarily on K and γ . As the second vertex has non self source,

⁴We note $R(J^1)$ the amplitude R of the basis vector following the curvilinear direction J^1

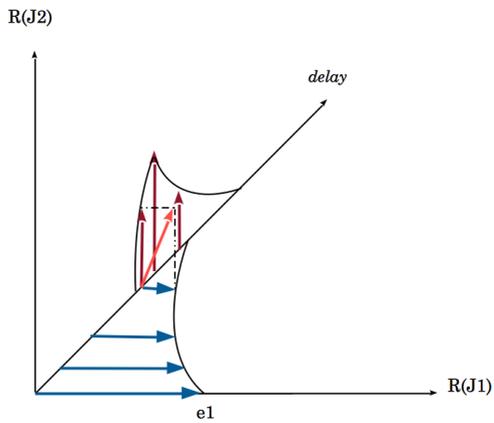


Figure 7.28: R-R diagram versus time

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if we suppress the current J^2 it is equivalent to suppress one direction of the space. We have two solutions:

- to suppress the second vertex, i.e. to make it far - it is the prevention solution;
- to suppress the coupling increasing α or making k real, it is a protection solution.

Often the prevention solution is hard to implement, because we don't chose the presence or not of materials. The second option is often used, for example using "honeycomb" with waveguides of cutoff frequencies very high in comparison with the higher frequency generated by the environment.

It is interesting to remember the exact expression of the losses in propagation. Imagine two potentials U and V no matter what they are. They respect the hamiltonian:

$$\begin{cases} \frac{\partial U}{\partial z} = -(R + Lp) V \\ \frac{\partial V}{\partial z} = -(g + Cp) U \end{cases} \quad (7.165)$$

These two equations leads to the Helmholtz's equation:

$$\frac{\partial^2 U}{\partial z^2} - (R + Lp)(g + Cp)U = 0 \quad (7.166)$$

Noting

$$\gamma^2 = (\alpha + jk)^2 = (R + Lp)(g + Cp)$$

we obtain the system of equations:

$$\begin{cases} \alpha^2 - k^2 = Rg - LC\omega^2 \\ 2\alpha k = (Lg + RC)\omega \end{cases} \quad (7.167)$$

Replacing k by its expression that depends on α we find

$$\begin{cases} \alpha^2 = \frac{1}{2} \left\{ \sqrt{(Rg - LC\omega^2)^2 + \omega^2(Lg + RC)^2} + Rg - LC\omega^2 \right\} \\ k^2 = \frac{1}{2} \left\{ \sqrt{(Rg - LC\omega^2)^2 + \omega^2(Lg + RC)^2} - Rg + LC\omega^2 \right\} \end{cases} \quad (7.168)$$

Without losses, i.e. if $R = 0$ and $g = 0$, $\alpha = 0$ and $k = \omega/v$ with $v = \sqrt{LC}^{-1}$.

Similar equations can be found for the free field, with the equivalences

- $R \rightarrow \rho$;
- $g \rightarrow \sigma$;
- $L \rightarrow \mu$;
- $C \rightarrow \epsilon$.

To go from potentials to fields is just a question of geometrical functions.

Chapter 8

Group $(\mathcal{M}, +)$

The matrix group provided with the addition operation is an Abelian group. But to construct systems we need to make direct addition between matrices. It remains possible to use the group $(\mathcal{M}, +)$ using the technique

$$if \zeta_{\alpha\beta} \in \mathcal{M}_1, \chi_{\sigma\nu} \in \mathcal{M}_2, \mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 \Rightarrow$$

$$\zeta' = \zeta \oplus [0]_{d(\mathcal{M}_1)}, \chi' = [0]_{d(\mathcal{M}_2)} \oplus \chi \quad (8.1)$$

$$\omega_{\mu\nu} \in \mathcal{M} \& \omega = \zeta' + \chi'$$

With these operations of "pre-formatted" and with some other points to define, we can consider that it is possible to apply operations on our manifolds to construct systems knowing they belong to a group with the addition.

That is to say that having a set of manifolds, we can make a system with this set by additions, but after having prepared these manifolds to this operation.

The construction of the system goes through three steps:

1. preparing its elements;
2. making their addition;
3. adding additional couplings.

It is similar to the construction of a mock-up. You prepare the pieces. After you fit them together and you add glue. That's exactly what we want to do. But the operation is not as easy as it might seem.

We must verify that the pieces are well done and in accordance each others. We must verify also that they are compatibles with the environment where they will work. These verifications are enclosed in the domains giving the intervals of operating of each element.

8.1 Manifold gluing requirements

First requirements for manifolds is to be well defined, i.e. having as many equations as many variables. The matrix corresponding to the operators should be well dimensioned. For that, it must be clear that the model is not the reality in a meaning that it is better to suppress some very small values than to keep them and take the risk to pollute its study. The correspondance between the real object and its theoretical manifold must be

homomorphic and not isomorphic. In a general approach, the model should give same tendencies as the real object, but it doesn't pretend to behave like the real object. The problem is to know if a brain is able to understand its own operation?

An impedance operator is in fact a couple of functions $\left(\zeta, \overset{a}{\mathcal{D}}\right)$. The first one ζ , gives a link between the dual space and the natural one; it can be a metric, an hamiltonian, ζ , etc. The second one $\overset{a}{\mathcal{D}}$ is the domains associated with the operators. The domains specify the intervals of belonging of the parameters a : temperature, pressure, etc. A manifold that represents the model of a real object should be bounded. Even in the case where domains are not used, they should exist at least to detail the intervals of validity of the model. The second requirement is to determine the domains through gluing operations.

Often people think that a 3D code is exact and reproduces exactly the reality. This does not make sense. The metric in electromagnetism is not what we see. We have studied geodesics and seen how they are complicated and far from any easy image able to be perceived. The difficulty is not to extract the geometrical forms of a system, but to embed the correct values in conductivity, permeability, etc., and with their dependences in frequency, saturations, ... A code that solves Maxwell's equations using meshing is not better than an analytical determination by graph for this difficulty. The question that arises is: how to know if the model is correct to represent the real object? The only way is to reframe the model by comparison with the experiment. But an experiment is not a demonstration. So we

have to determine in what conditions it should be conducted and for what objectives?

8.1.1 Well dimensioned matrix

We take a simple example. A π filter structure has its border with a very small impedance value and the other impedance have high values. The impedances are from left to right: A, B, C . If we take the option of this connectivity:

$$C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (8.2)$$

it leads to the operator:

$$\zeta = \begin{bmatrix} A+B & A \\ A & A+C \end{bmatrix} \quad (8.3)$$

but $B \ll A$, so the effective operator is:

$$\zeta \approx \begin{bmatrix} A & A \\ A & A+C \end{bmatrix} \quad (8.4)$$

which for a canonical source $[e, e]$ gives the fluxes:

$$\begin{bmatrix} J^1 \\ J^2 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} Ce \\ 0 \end{bmatrix} \quad (8.5)$$

($\Delta = AC$). Now if we take the connectivity:

$$C = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \quad (8.6)$$

with the source $[e, 0]$, we obtain

$$\zeta = \begin{bmatrix} A+B & -B \\ -B & B+C \end{bmatrix} \quad (8.7)$$

and the fluxes

$$\begin{bmatrix} J^1 \\ J^2 \end{bmatrix} = \begin{bmatrix} \frac{2B+C}{(A+B)(B+C)-B^2} e \\ \frac{2B+A}{(A+B)(B+C)-B^2} e \end{bmatrix} \approx \begin{bmatrix} \frac{e}{A} \\ \frac{e}{C} \end{bmatrix} \quad (8.8)$$

If we compute the current in the second branch $i = CJ$. In the first case we have:

$$i = J^1 = \frac{e}{A}$$

and in the first case:

$$i = J^1 - J^2 = \frac{e}{A} - \frac{e}{C}$$

Regardless of the approximations done, the results are completely different. You may say that if we calculate the exact expressions, we will find the same result. And you're right. But

this is in contradiction with the willingness to reduce complexity by suppressing low values. And this willingness is legitimate because if you do some numerical applications, the approximations will be done without your control. For the first branch, the exact result is:

$$i = \frac{C + B}{AC + AB + BC}$$

but

$$\frac{C + B}{AC + AB + BC} \approx \frac{1}{A}$$

The second choice was clearly the best one. We may give a rule:

Between various connectivities, the best one is the connectivity that doesn't lead to mix operators of low and high values.

After what all the connectivities are availables.

8.1.2 Domain for vertices and systems

Considering now the operators like couples $\left(\zeta, \overset{a}{\mathcal{D}}\right)$, how the connectivities act on them?

While the addition acts on the operator functions, the intersection acts on the domains. It is quite easy to understand with a simple example. Imagine two resistances $\left(R_{11}, \overset{t}{\mathcal{D}}_1\right)$ and $\left(R_{22}, \overset{t}{\mathcal{Q}}_1\right)$ defined in temperatures t on the domains $\overset{t}{\mathcal{D}}_1$ and $\overset{t}{\mathcal{Q}}_1$, if we connect these resistances in the same vertex, the temperature that the vertex can support comes from the intersection of

the two domains:

$$\overset{t}{\mathcal{D}}_1 \cap \overset{t}{\mathcal{Q}}_1 \quad (8.9)$$

but for a set of domains, the situation is more complicated. Each intersection can lead to a new composition of laws on each new domain defined through the intersections. And each resistance must keep its particularity in the new cutting of the space. If the couples are $\left(R_{11}, \overset{t}{\mathcal{D}}_x\right)$ and $\left(R_{22}, \overset{t}{\mathcal{Q}}_y\right)$, $R_{oo} = C_o^x R_{xx}$ will be defined on the domain $\overset{t}{\mathcal{O}}_z$ which definition is:

$$\overset{t}{\mathcal{O}}_z/z = f(x + y), \& \overset{t}{\mathcal{O}}_z = \overset{t}{\mathcal{D}}_x \otimes \overset{t}{\mathcal{Q}}_y, \otimes \leftrightarrow \cap \quad (8.10)$$

This relation means two things: first the index of \mathcal{O} is a function of the indices of \mathcal{D} and \mathcal{Q} . For example, $z = 1$ comes from $x = 1, y = 1$. $z = 2$ corresponds to $x = 1, y = 2$, etc.

Secondly, the tensorial products of the domains leads to all the domains that concern the resistance R_{oo} . Each product is in fact an intersection between the intervals of both \mathcal{D} and \mathcal{Q} . The law associated with the resistance for each of these intersections comes from the addition of each law of the original resistances. For example:

$$\overset{t}{\mathcal{O}}_1 = \overset{t}{\mathcal{D}}_1 \overset{t}{\mathcal{Q}}_1 = \overset{t}{\mathcal{D}}_1 \cap \overset{t}{\mathcal{Q}}_1$$

and

$$R_{oo} \left(\overset{t}{\mathcal{O}}_1 \right) = R_{11} \left(\overset{t}{\mathcal{D}}_1 \right) + R_{22} \left(\overset{t}{\mathcal{Q}}_1 \right)$$

The product of domains coming from a set of parameters is also more complicated. If we have:

$$R_{11} = \mathcal{D}_1^t \mathcal{D}_1^m R_{11}^{11} + \mathcal{D}_1^t \mathcal{D}_2^m R_{11}^{12} + \dots \quad (8.11)$$

and

$$R_{22} = \mathcal{Q}_1^t \mathcal{Q}_1^m R_{22}^{11} + \mathcal{Q}_1^t \mathcal{Q}_2^m R_{22}^{12} + \dots \quad (8.12)$$

A new set of domains is generated for each parameter q (here $q = t$ or $q = m$):

$$\mathcal{O}^q = \mathcal{D}^q \otimes \mathcal{Q}^q \quad (8.13)$$

and the number of possible functions for the resulting operator comes from the tensorial product of the resulting intervals for each parameter:

$$\zeta_{ij} = \left(\mathcal{O}_x^t \otimes \mathcal{O}_y^m \right) \zeta_{ij}^{xy} \quad (8.14)$$

When we see how it becomes difficult at a vertex scale, we imagine how it is difficult at the system scale! Often we forget these intervals of validities and uncertainties under the assumptions that we place ourselves in the conditions where they can be forgotten. A better management when possible may be to chose common domains before the assembling to simplify the study, because in this case the expressions are invariants. The addition coming from the connectivity acts identically on the domains. If this is not the case, the reflexion impose by the previous equations obliges to be careful with the possibility there is

in the assembling to connect components that are not in accordance with the needs for the system. We clearly see here that the system is more than the simple addition of its components. It can be true only if these components are defined on the same domains, which is of a small probability.

When we try to forget the domains, it is equivalent in fact to work with a single one on common parameters. That's near always the case in temperature. We can have one component with a restricted domain $\overset{t}{Q}_1$ of working in temperature, the others supporting larger intervals. In that case, $\overset{t}{O}$ can be restricted to the two domains of this component. We have:

$$\overset{t}{O}_1 = \overset{t}{D}_1 \overset{t}{Q}_1 \quad \overset{t}{O}_2 = \overset{t}{D}_1 \overset{t}{Q}_2 \quad (8.15)$$

Some particular components can be isolated following this principle in order to avoid increasing the complexity of the system in its theoretical analysis.

When we want to compute the system, all the definitions of the elementary operators can be written before to make the assembling and there is no need for a theoretical expression of this assembling. Each component will evolve during the time depending on its own definition in the transformation coming from the connectivity.

8.1.3 Experiment versus models convergences

A model can be compared with the object it is supposed to represent through some observables. We can try to calculate

the quadratic approximation to evaluate the pertinence of the model.

Let G be the metric of the system model. If we observe a measured current $\nu(p)$ versus the frequency p (Laplace's operator), we try to minimize the function L defined by:

$$L = \sum_p (\nu(p) - J^1(p))^2 \quad (8.16)$$

J^1 here is supposed to be the current that corresponds to ν . So we calculate

$$L = \sum_p (\nu(p))^2 + (J^1(p))^2 - 2\nu J^1$$

but $J^1 = y^{1x}e_x$, so:

$$L = \sum_p (\nu(p))^2 + (y^{1x}(p)e_x(p))^2 - 2\nu y^{1x}(p)e_1(p)$$

If the space is not inertial, $y = G^{-1}$ is purely dissipative and does not depend on p . It remains

$$L = \sum_p (\nu(p))^2 + (y^{1x}e_x(p))^2 - 2\nu y^{1x}e_1(p)$$

We imagine for example that $J^1 = y^{11}e_1 + y^{12}e_2$, the previous equation gives (we remember the variables that depend on p : ν and e_x):

$$L = \sum_p \nu^2 + (y^{11}e_1 + y^{12}e_2)^2 - 2\nu (y^{11}e_1 + y^{12}e_2)$$

To minimize L we look for

$$\frac{\partial L}{\partial y^{1x}} = 0 \quad (8.17)$$

We obtain here a first equation

$$\frac{\partial L}{\partial y^{11}} = 0 \Rightarrow 2y^{11} \sum_p (e_1)^2 + 2y^{12} \sum_p e_1 e_2 = 2 \sum_p \nu e_1$$

and

$$\frac{\partial L}{\partial y^{12}} = 0 \Rightarrow 2y^{12} \sum_p (e_2)^2 + 2y^{11} \sum_p e_1 e_2 = 2 \sum_p \nu e_2$$

Writing

$$\left\{ \begin{array}{l} \bar{e}_1^2 = \sum_p (e_1)^2 \\ \bar{e}_{12} = \sum_p e_1 e_2 \\ \bar{\nu}_1 = \sum_p \nu e_1 \\ \bar{e}_2^2 = \sum_p (e_2)^2 \\ \bar{\nu}_2 = \sum_p \nu e_2 \end{array} \right.$$

the previous set of equations can be written:

$$\begin{bmatrix} \bar{e}_1^2 & \bar{e}_{12} \\ \bar{e}_{12} & \bar{e}_2^2 \end{bmatrix} \begin{bmatrix} y^{11} \\ y^{12} \end{bmatrix} = \begin{bmatrix} \bar{\nu}_1 \\ \bar{\nu}_2 \end{bmatrix} \quad (8.18)$$

We can compare the y^{1x} values obtained from this system of equations to the values determined through G^{-1} and see if the correspondance is correct, i.e. if the chosen metric seems to answer to the problem. It is clear that in this exercise, the sources e_i are known. More complicated expressions of operators y can be solved of course, even depending on p . The method can also be used to find approximations of the metric coefficients when these coefficients cannot be obtained from the information on the system (i.e. conductivity, etc.).

8.2 What's \mathcal{M}^{-1} for ”+”?

We wanted to attach our manifold definition to the classical group of matrices with the addition as internal operation. This implies the existence of a neutral element, which is the zero matrix. We can define the opposite matrix to the matrix associate with any impedance operator ζ of any manifold \mathcal{M} : $-\zeta$ which must be the operator of \mathcal{M}^{-1} . But is this has sense?

The domains remain unchanged between ζ and $-\zeta$. If the temperature of a resistance R belongs to $[t_1, t_2]$, the negative resistance $-R$ should respect the same intervals. But this negative resistance has no physical meanings. It gives us two theoretical advantages

1. having a group, we can construct the system regardless of the order in which we take its elements;
2. if adding an element makes disappearing another, this can be translated using some negative operator added to a

positive one, the result giving 0.

For example if we have an electronic and making the system, we plunge it into liquid helium. This will suppress the resistances of the circuit. To translate this effect we just have to add a matrix, where all the resistances are equal to those of the circuit, but with a negative sign. It can be seen like the symmetric of the metric of the circuit relatively to the neutral element [0].

8.3 Multiscale concept

The multiscale problems are simply real problems! All in life involves multiscale process. And multiscale is also the natural process of the dual space. When we search for a form giving a number starting from the component of a vector, we apply an integration on one variable of the configuration space. Let's consider a simple resistance. We can write its formula like:

$$R_{AB} = \int_A^B d\mathbf{x} \cdot \frac{1}{\sigma S} \mathbf{n} \quad (8.19)$$

At a lower scale, an elementary resistance is given by $dx/(\sigma S)$. Finally, the operator that leads to the metric comes from the n-form that integrates the phenomenons at a lower scale. Multiscale process is equivalent to go from one space to a higher one using n-form scalar product and one definition of metric:

$$d\mathbf{x} \rightarrow R = f \left(\int d\mathbf{x} \right) \rightarrow e_a = R_{aa} i^a \quad (8.20)$$

The scale relation is intrinsic to the n-form processus:

$$d\mathbf{x}_1 d\mathbf{x}_2 \dots d\mathbf{x}_n \rightarrow \int \int \dots \int d\mathbf{x}_1 d\mathbf{x}_2 \dots d\mathbf{x}_n \quad (8.21)$$

This mechanism of integration can be transposed into the direct summation technique. If we imagine a resonator of operator ζ (it can be a single mesh with a LC model of an elementary line). Now we create a set of N of these resonators. It is described by $\oplus_{i=0}^N \zeta_i$. These resonators can be coupled, connecting them each other (it makes a part of telegraph's line or similar lumped structure). Then we can power supplied this set by injecting a nodes-pair source J on the first mesh (a resistance on the first mesh) that goes out on the last mesh. The unknown voltage V developed across the all structure gives the operator relation of the macrostructure. The N meshes are linked with N currents K^n and due to the fact that J must be common to the whole, the connectivity makes relations between the branches of all meshes and the K^n and also the shared J .

We obtain an equation similar to:

$$\begin{bmatrix} 0 \\ V \end{bmatrix} = \left[\left(\oplus_{i=0}^N \zeta_i \right)_{qm} + \mu_{qm} \quad \chi_{qm} \right] \begin{bmatrix} K^m \\ J \end{bmatrix} \quad (8.22)$$

μ_{qm} is the tensor of interactions appearing once the system beginning to be constructed. Computing the K^m we can after compute V . Then the operator Θ is obtained making:

$$\Theta = \frac{V}{J} \quad (8.23)$$

The multiscale mechanism is somewhere completely defined in this operator and in the n-form principle.

We have noticed that the tensor μ adds the interactions between the primitive systems pointed out by the summation on ζ_i . We know that a way to make a system is to use a connectivity to manage the branches as desired. But cords can also give a way to construct a system. A particular aspect of cords is that they allow to make links between circuits coming from various scales. Through cords we can study the interaction between an antenna and a single photon coming from the deep space, a far satellite. As cords wear mathematical operators, they can easily report physical properties from one system in a first scale into a second system at a second scale.

The interactions make part of the system. The μ skeleton has its own existence, once the system described. Somewhere the system is constructed on the skeleton given by μ . Due to this behavior of completion, $\zeta + \mu = \mu + \zeta$. But a cord is immaterial and can appear as it can disappear. This capability allows to make evolving systems. And periodic changing can be modeled using a complementary tensor for the interactions μ' , the system being translated by successive and alterned of additions of μ and μ' . As a matter of fact the tensors of interactions μ constitute a group.

Chapter 9

Lightning

Lightning is probably the most important natural phenomenon for electromagnetic compatibility. It is an extremely complex mechanism of the nature to equilibrate the charges at the earth level. Our purpose here is not to study this complexity, but just to speak of some of the aspects attached to this phenomenon.

9.1 Lightning impact

As we said without looking in detail to the lightning process, main effect of lightning is to apply a high level current on structures. This current can act directly on these structures or through diffusion or radiated magnetic field, influence other part of a system. Using a lightning rode, the lightning current is

directed to the ground. The ground return and building ground impedances are very important element of the protections installed to avoid disturbances coming from lightning.

The nature of the ground and of its geological structure give informations to determine the ground conductivity. Depending on humidity, kind of earth, etc., the resistivity can vary from $10\Omega/m$ to $10k\Omega/m$. The ground on a given location is often very heterogeneous. The resistivity of the up layers change depending on the weather, if the water is frozen for example. The depth must be superior to 2 meters to be independent of these effects. The conductivity of the ground is principally due to the electrolytic effect. So, it depends firstly of humidity. We note the ground resistance R_G . Figure 9.1 shows the curve $R = f(T)$ and figure 9.2 the curve $R = f(h)$ ¹.

As we don't want to model all the mechanisms involved in the lightning spark gap, we only consider the source of current coming from the lightning. The current source is a nodes-pair source. In a space having N branches, once the meshes are defined, if we have M meshes, the potential number of nodes-pairs P is $N - M$. For modeling one lightning impact, only one nodes-pair is usefull. This nodes-pair translates the circulation of the lightning current on a part of the system structure. So, it is implicitly defined by the impact location. All the other sources can be neglected. This is equivalent to set the value of connection of all the branches in relation with the other nodes-pairs to zero. Let's consider M meshes K^1, K^2, \dots and N nodes-

¹These measurements was done on light earth in England in 1980

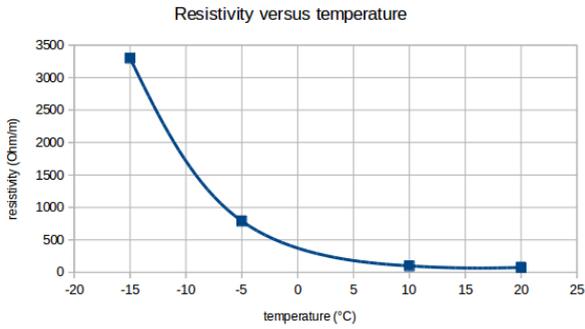


Figure 9.1: Resistivity versus temperature

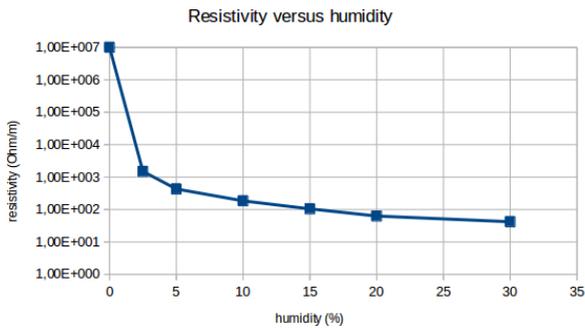


Figure 9.2: Resistivity versus humidity

pair J^1, J^2, \dots . If e is the source covector the manifold is:

$$e_\mu = \zeta_{\mu\nu}|_{\nu \in M} K^\nu + \zeta_{\mu\beta}|_{\beta \in P} J^\beta \quad (9.1)$$

All the currents J^β are known. If all the J^β are equal to zero except $J^{(\beta)}$ the manifold can be reduced to:

$$e_\mu = \zeta_{\mu\nu}|_{\nu \in M} K^\nu + \zeta_{\mu(\beta)} J^{(\beta)} \quad (9.2)$$

The existence of an unique nodes-pair source implies the topology of the spanning tree. A part of this spanning tree is considered, the one including the known current source. This source can be placed in the left member of the system of equation of the manifold, and complete the source covector.

If a lightning rode is present on a building, its circuit includes its own impedance, the impedance of the wire conducting the current and the impedance of the earth connector. This source circuit is connected to the rest of the building through various mechanisms. It can be conducted connections or radiated connections. In the first case, a mesh connects one branch of the source circuit to the building circuit. In the second case, a cord of principally magnetic field connects the source circuit to the building circuit.

9.2 Radiated interaction

The conducted interaction is simple to treat. It is more interesting here to discuss the radiated case.

The spark gap can be seen as an infinite wire coming from the sky. An approximation of the potential vector emitted by this current is given by:

$$\mathbf{A}_x = \frac{\mu_0 J^{(\beta)}}{4\pi} \int_0^x \frac{dx}{(x^2 + y^2)^{1/2}} \quad (9.3)$$

which gives:

$$\mathbf{A}_x = \frac{\mu_0 J^{(\beta)}}{4\pi} \text{Log} \left[x + (x^2 + y^2)^{1/2} \right]_0^x \quad (9.4)$$

and:

$$\mathbf{A}_x = \frac{\mu_0 J^{(\beta)}}{4\pi} \text{Log} \left[\frac{\chi + (\chi^2 + y^2)^{1/2}}{y} \right] \quad (9.5)$$

Now if we look at the figure 9.3, we see the lightning rode on the left of a building.

The current flowing in this circuit radiates a magnetic field through a vector potential to the vertical part of the steel bars of reinforced concrete. The electromagnetic force e_l induced on the left vertical part is given by:

$$e_l = -p\mathbf{A}_x(a) = -p \frac{\mu_0 J^{(\beta)}}{4\pi} \text{Log} \left[\frac{\chi + (\chi^2 + y_a^2)^{1/2}}{y_a} \right] \quad (9.6)$$

and on the right vertical part:

$$e_r = -p\mathbf{A}_x(b) = -p \frac{\mu_0 J^{(\beta)}}{4\pi} \text{Log} \left[\frac{\chi + (\chi^2 + y_b^2)^{1/2}}{y_b} \right] \quad (9.7)$$

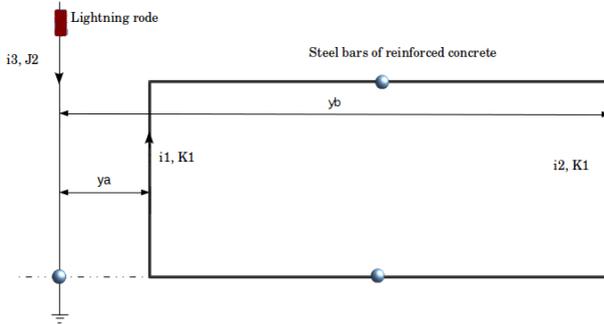


Figure 9.3: Lightning rod and the building

These two electromagnetic forces creates two cords. The first is between the branch 1 and the branch 3 (note that $i_3 = J^{(\beta)}$):

$$z_{13} = \frac{e_l}{i_3} = -p \frac{\mu_0}{4\pi} \text{Log} \left[\frac{\chi + (\chi^2 + y_a^2)^{1/2}}{y_a} \right] \quad (9.8)$$

The second is between branch 1 and 2:

$$z_{12} = \frac{e_r}{i_3} = -p \frac{\mu_0}{4\pi} \text{Log} \left[\frac{\chi + (\chi^2 + y_b^2)^{1/2}}{y_b} \right] \quad (9.9)$$

In the branch space, the impedance matrix is:

$$z = \begin{bmatrix} R_v + L_v p & z_{12} & z_{13} \\ 0 & R_v + L_v p & 0 \\ 0 & 0 & R_G + R_M + Z_{lr} \end{bmatrix} \quad (9.10)$$

R_M is the mast of the lightning rode impedance and Z_{lr} the impedance of the lightning rode itself. L_v is the self inductance of the bars. The connectivity C with the two currents (K^1, J^2) is:

$$C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (9.11)$$

Making $\zeta = C^T z C$ we obtain:

$$\zeta = \begin{bmatrix} 2(R_v + L_v p) + z_{13} & z_{13} \\ 0 & R_G + R_M + Z_{lr} \end{bmatrix} \quad (9.12)$$

while the source is given by $e = C^T u^T$. u is the source covector in the branch space with all the elements equal to zero except the unknown voltage across the lightning source V . The system to solve is:

$$e_\mu = (\zeta_{\mu\nu} + \mathcal{L}_{\mu\nu} p) Q^\nu \quad (9.13)$$

$Q^\nu = (K^1, J^2)$. $\mathcal{L}_{\mu\nu}$ is the inductance of the loop made by the steel bars.

The magnetic field inside the building is given by:

$$\mathbf{B}_z = \left(-\frac{\partial \mathbf{A}_x}{\partial y} - \frac{\mathbf{u}_z}{xy} \mathcal{L}_{\mu\nu} K^1 \right) e^{-\epsilon/\delta} \quad (9.14)$$

δ is the skin depth and ϵ the depth of the steel bars. The current K^1 is of the form:

$$K^1 = \frac{z_{13} J^3}{R + Lp} \quad (9.15)$$

At low frequencies, this current is near to zero. The diffracted magnetic field is so also near to zero and the field inside the building is equal to the external magnetic field. After what, the current K^1 increasing, the internal field decreases and the skin effect makes the internal magnetic field equal to zero (Bode's diagram shows very well this mechanism).

The resistance R_G of the steel bars can be computed knowing the steel bars depth and height, etc. and their permeability for the skin effect. The previous considerations for the magnetic field are those of a metallic plate without any holes. When the frequency increases, the holes radiate a magnetic field that makes the internal field of the building different to zero.

If we consider two wires of currents separate by a distance D , the magnetic field at a distance y from these wires is given by:

$$d\mathbf{B}_z = -\mu_0 \frac{dK^1}{\pi} \left[\frac{y}{\left(\frac{D}{2}\right)^2 + y^2 + x^2} \right] \mathbf{u}_z \quad (9.16)$$

To obtain the complete field radiated by the holes, it remains

to integrate the previous relation:

$$\mathbf{B}_z = \sum_n \int_x d\mathbf{B}_z \quad (9.17)$$

n is the number of steel bars. The assumption made in all these computations is that the wavelength is superior to the dimensions of the building. If not, the current distribution on the structure, as the magnetic field distribution must be considered. And the electric field may be regarded also. But for lightning, the spectrum is low enough to remain under our assumption in general.

9.3 Circuits inside the building

Circuits of impedance ζ^c being installed inside the building, two phenomenons create EM force on these circuits:

1. diffusion through common mode impedance;
2. EM force induced by the field radiated inside the building.

The first manifold is added to the circuit definition: $\zeta \oplus \zeta^c$, and the cords of the interactions between these two circuits are added. The process is very similar to the process used for shielded cables. The diffusion process makes a cord with the form: $R + Lp$. Both R and L including the skin effect and its particular impedance. This approach can be followed whatever the number of layers that are involved since the external to the deepest internal layer.

9.4 Impact location

For a length x of spark gap, the elementary work developed by the lightning can be written under the second geometrization process and with the metric G :

$$dw = q\alpha\sqrt{\bar{G}_{\mu\nu}J^\mu J^\nu} \quad (9.18)$$

$\bar{G}_{\mu\nu}$ is the metric normalized for the length (for example for a resistance, $R = (\sigma S) = 1$ and α some coefficient). If we accept that the probability for a point on the structure to be touched by a spark gap follows the equation:

$$P = e^{-q\alpha \int_x dx \sqrt{\bar{G}_{\mu\nu}J^\mu J^\nu}} \quad (9.19)$$

With this expression, we understand that between two points, the lower one can be touched by the spark gap, because if the length is higher, for lower currents, the probability becomes similar, especially since the law evolves depending on the square of the current. Figure 9.4 illustrates this mechanism.

This is a very important consideration. It means that even with a lightning rode, the protection against the lightning spark gap is a probabilistic one. It is never sure that some points on the structure won't never be impacted by the lightning. This law may says also that in the case of spark gap starting from the ground, the probability to have higher currents is superior because the charge density q will be in general lower. The relation between the seed distance d and the current intensity I

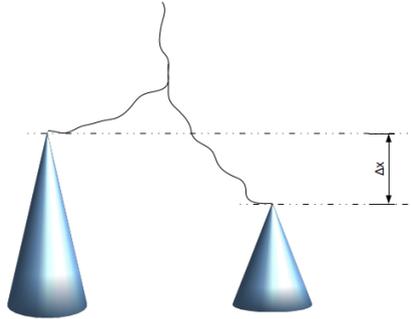


Figure 9.4: Lightning impact

can be estimated taking simply²:

$$d = 1.5 \cdot 10^{-6} I \quad (9.20)$$

For negative current we use $d = 10^{-6} I$.

9.5 Risk for humans and animals

A difficult aspect of EMC is the risk for humans and animals. If a person is inside the building or outside near the lightning rode, she can be affected by the currents in the ground. The interaction between the circuits is the process of diffusion (common mode impedance coupling). For a resistance per meter R_G

²From *Haute tension*, Aguet, Ianoz, Romand polytechnic press, 1987

of the ground, the potential developed across the alive is simply $R_G J^2$. But how this potential can act on the circuit of the alive? Persons or animals are not metallic conductors. A suspension of cells can be modeled using Fricke's circuit. It is a resistance r_e in parallel with a resistance r_i in series with a capacitor C . The resistance r_e represents the conductivity of the medium at the exterior of the cells. The capacitor C represents the membranes of the cells and the resistance r_i the conductivity of the cytoplasm of the cells. This model can be applied for blood for example. The Fricke's circuit has for real part ($r_s = r_i + r_e$):

$$z_R = \frac{r_e}{1 + r_s^2 C^2 \omega^2} (1 + r_i r_s C^2 \omega^2) \quad (9.21)$$

and the active power W dissipated in the tissues by the lightning current of Fricke's impedance ζ_t is:

$$W = \frac{1}{2} z_R |K^2|^2 \quad (9.22)$$

K^2 is the current in the tissues:

$$K_2 \approx \frac{R_G J^2}{\zeta_t} \quad (9.23)$$

The cord with the nodes-pair source is directly given by R_G . A risk is determined is $R_G J^2$ goes over 30 volts or if W goes over around 1 watt.

Chapter 10

Disturbed Cyber-Physical Systems

Let's give us some definitions. A cyber-physical system (CPS) is a set of systems. Each of these systems embeds at least a microprocessor and can make automatically some actions. They exchange information between them through various communication networks. They are all made of:

1. an energy distribution network;
2. an actuators network;

3. a bus network to share internal datas;
4. a network of sensors;
5. different nodes of digital signal processing and memories.

A CPS can interact with humans, with animals, with plants, with space, with the Earth. It may use "big datas".

The story begins with automates. There was mechanical systems, then with some electric or electronic parts, able to reproduce gestures.

After the emerging of numeric and microprocessors, appear robots. A robot can do gestures but has the capacity to decide by itself some parameters in relation with these gestures, see, with some mission. The microprocessors capacities growing enormously, cybernetic borns. Cybernetics was invented by Robert Wiener, then was the purpose of deep Russian's works.¹ In France, Robert Vallée was one of the researcher that write deep thoughts on cybernetics.²

I want here to submit some thoughts on the thematic "EMC and CPS", a problem of today and I think a real problem to come with our future systems.

¹Korchounov: "Fondements mathématiques de la cybernétiques. MIR edition, Moscou 1975.

²Robert VALLÉE, "Cognition et Systèmes: essai d'épistémopraxéologie". Interdisciplinaire editor, Lyon-Limonest, 1995. Founder of the circle of cybernetic studies whose president was Louis de Broglie - 1950.

10.1 Complexity

A CPS system interacts with the nature. The nature is a complex system. So, CPS are complex systems. What does it mean? A complex system is a system that:

- has many interactions with outside as with inside;
- behaves for part following stochastic process;
- shows systemic properties.

A systemic property is the fact that the system presents properties that comes from it own existence. If you suppress one of its component, this property disappears. I often take for example, Dali's painting "*Le marché aux esclaves*" (the slave market). The painting being finished, bust of Voltaire appears. But it was impossible before the painting was finished to guess that this bust may appear. It's remarkable and it is a systemic behavior. CPS involving numbers of systems through communication networks can make proof of systemic behaviors. It can be seen sometimes also like meta-heuristic approaches. The word "complexity" is often used where "complicated" may be more pertinent. One deep difference is that stochastic behaviors are intrinsic for complex systems and complicated systems don't have systemic properties. For modeling complex systems and CPS we need to know how to model wire networks and to incorporate each sub-system of a CPS in a global set of reference frames.

10.2 Wire networks

Thirty years after, modeling wire networks remains one of the biggest problem for EMC. It is really difficult to model hundred of wires shared in hundred of harnesses without regular management and well defined ways. I am convinced that there is no sense to be rigorous with this problem. And as we have seen, uncertainties in EMC are important. The real objective is to be able to predict the transport of parasitic waves versus functional signals.

Each line is made of two conductors. Sometimes, a conductor is shared between some signals, sometimes each signal has its own pair of wires. We can start modeling low frequency networks to understand the mechanisms of coupling between signals and how the signals are identified.

10.2.1 Signal families

We have list four networks in a CPS. Whatever the CPS, these networks always exist more or less. To each kind of network, is attached a kind of signal. For EMC, an interaction matrix helps to manage the various disturbances that each kind of signal can create in its neighbors.

- Power network should not be noisy because it distributes this noise to all the equipments of the system. For this reason we try to isolate it from noises. Intrinsically, it is not a susceptible network;

- actuators network creates noise: noises from the command law of motors, noises in the machine activation, etc. It is not by itself susceptible;
- buses are robust networks. They emit noises due to their short rise time signals or high frequency modulations and are medium susceptible to external noises. As they are in charge of the sharing of information, it is of major target to keep them safe. Another important information is that Buses are also distributed in all the system;
- microprocessors today are big sources of noises. It must be known that a modern microprocessors can call pulse of currents of 1 ns of time duration and 100 ampere peak of amplitude. It would be very difficult to create such a generator with classical techniques! It is important also not to disturb microprocessors for the same reasons than the buses;
- Sensors are very susceptibles and use low level signals. It is also a priority to keep their integrity to avoid false measurements to be used by the system. They are not sources of noises.

From power networks to sensor networks, some mixings should be avoid to guarantee the good working of the whole system, independently of the noises coming from its environment.

10.2.2 Low frequencies modeling for wire network

Under the low frequency assumption, a link using wires is a single mesh. Both loads of input and output, added to the impedances of the wires and the inductance or capacitance of the line make this mesh. This structure is a single Telegraph's cell. The output load encloses the line capacitor. For example if R_0 is the generator impedance, L the line inductance, C the capacitance and R_L the load we obtain the mesh:

$$R_0 + Lp + \frac{R_L}{R_L Cp + 1} \quad (10.1)$$

There are as many meshes as there are links. When they share the same harness or the same background conductor, couplings are added between these meshes. There are four kinds of couplings:

1. mutual couplings;
2. capacitive couplings;
3. transfer impedance couplings;
4. common mode impedance couplings.

The energy can be transported using common mode or differential mode. But the differential mode can always be written depending on two common modes.

In a first step we must draw the various ways that are followed by the signals, and to identify what are their neighbors

in these ways. Some signals use two wires, some others three wires or two wires and a shield. The engineer must establish the list of these signals and identify where they are located in the system structure. When various signals share the same path, he must compute the couplings between these signals. After what he just have to add cords to report these couplings. It is important to understand that in this method we start from the separate links. We look after to their management without taking care of some modifications of their properties coming from their grouping. So the method is not a rigorous one (if any exists!). Remember that if the electrical properties of the line may be significantly modified, the functional use may alert on this modification. Let's take an example. We consider three links: a first link with a simple wire using the common ground for return, a second link which is a twisted pair and a third link which is a shielded cable with one wire.

Single wire The inductance can be calculated knowing its diameter ϕ and length x and the height h of the wire over the ground plane. We have:

$$Z_c = 60 \text{Log} \left(\frac{4h}{\phi} \right), L = \frac{Z_c}{v} x$$

The capacitance is obtained making:

$$C = \frac{x}{vZ_c}$$

v is the speed in the line. For a dielectric of permittivity ϵ_r , it is equal to $c\sqrt{\epsilon_r}^{-1}$. If R_1 and Q_1 are the source impedance and

load, the impedance operator for the corresponding mesh of this link is

$$\zeta_{sw} = R_1 + Lp + \frac{Q_1}{Q_1 C p + 1}$$

Twisted pair The approach is similar. For a twisted pair, the characteristic impedance between the two wires is:

$$Z_c = 120 \text{Log} \left(\frac{D}{d} \right)$$

D is the distance between the wire and d their diameters. This characteristic impedance will be weakly modified by the installation of the twisted pair over the ground plane. This because in general the distance D is smaller than the height. This kind of line is often used for low speed bus and their match is verified, being a confirmation of our assumption. The corresponding differential mode inductance is Ld . The common mode impedance characteristic, inductance and capacitance of the twisted pair line can be computed using the same formulas as for the single wire, taking $\phi = 2d$ rather than ϕ for diameter. It generates a common mode inductance Le and capacitance Ce (Le includes the skin effect inductance). The twisted pair is modeled by 3 meshes. 2 meshes represent the common mode of each wire over the ground plane. They are coupled by the differential mode inductance of the twisted pair. When we make the models, we should take care of the relative values of the capacitors compared to the loads. The capacitors of the line are given by:

$$\frac{1}{C|p|} = \frac{Z_c \lambda}{2\pi x} \quad (10.2)$$

which by assumption is high (the low frequency assumption implies $\lambda \gg x$). Each time the capacitor impedance is very high compare to a load in parallel, we may forget it in the model. The figure 10.1 shows the management of a twisted pair for its modeling. R_g is the ground resistance in common impedance coupling (it includes its skin effect). L_f the inductance of each wire and M the mutual inductance between the common mode and the differential mode.

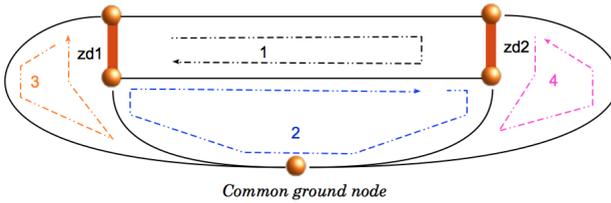


Figure 10.1: Twisted pair model

If Zd_1 and Zd_2 are the differential mode loads, R_f the resistance of the wires (including the skin effect), supposed to be small compare to $1/(Cp)$, the twisted pair is modeled by the impedance operator:

$$\zeta_{tp} = \begin{bmatrix} Z d_1 + Z d_2 + L d p + 2 R f & -M p - L f p - R f & Z d_1 & Z d_2 \\ -M p - L f p - R f & R f + R g + \frac{2}{C e p} + L e p & \frac{1}{C e p} & \frac{1}{C e p} \\ Z d_1 & \frac{1}{C e p} & Z d_1 + \frac{2}{C e p} & 0 \\ Z d_2 & \frac{1}{C e p} & 0 & Z d_2 + \frac{2}{C e p} \end{bmatrix} \quad (10.3)$$

Note that we have supposed here that the common mode impedances are equal, and purely capacitives. In reality, they are determined by the electronics connected to each extremity of the twisted pair line and often are not equal. One will present a low impedance to the ground while the other often presents a higher impedance to the ground. But each case is a particular case.

Another choice can be made for the twisted pair (or any pair of wires in general). Rather than taking one common mode and one differential mode for the meshes, we keep the extremities but we take the two common modes for the two first meshes (we need 4 meshes anyway: $M=B-N+R=8-5+1=4$). The advantage of this option is to make clearly in evidence the importance of the common impedance coupling, often forgotten. The impedance operator in this case becomes:

$$\zeta_{tp} = \begin{bmatrix} Rf + \frac{2}{C_{ep}} + Lep + Rg & -Rg - Mp & \frac{1}{C_{ep}} & \frac{1}{C_{ep}} \\ -Rg - Mp & Rf + \frac{2}{C_{ep}} + Lep + Rg & \frac{1}{C_{ep}} & \frac{1}{C_{ep}} \\ \frac{1}{C_{ep}} & \frac{1}{C_{ep}} & Zd_1 + \frac{2}{C_{ep}} & 0 \\ \frac{1}{C_{ep}} & \frac{1}{C_{ep}} & \frac{1}{C_{ep}} & Zd_2 + \frac{2}{C_{ep}} \end{bmatrix} \quad (10.4)$$

Each time we will need a model for a twisted pair, we will recall this operator.

Shielded cable The characteristic impedance of the link using a shielded cable is

$$Z_c = 60 \text{Log} \left(\frac{d_2}{d_1} \right)$$

d_2 is the shield diameter and d_1 the wire diameter. The common mode characteristic impedance between the shield and the

ground plane is the characteristic impedance of a wire of diameter d_2 located at the height h' over a ground plane. The equation is the same as for the single wire with these datas. So a shielded link with one wire inside is a system of two meshes. The coupling between the external domain and the internal one is well defined by a function named transfer function of the shield. Often noticed ZT . The model for a shielded one wire link is finally:

$$\zeta_{S1} = \begin{bmatrix} Rg + Lep + 2Rcc & -ZT \\ -ZT & Rf + Lip + Zd_1 + \frac{Zd_2}{Zd_2Ci_{p+1}} \end{bmatrix} \quad (10.5)$$

Li is the inductance between the wire and the shield, Ci the capacitor and Rf the wire resistance. As previously Zd_x are the loads. Rg is the ground resistance, Le the inductance between the shield and the ground and Rcc the impedance of the short-circuits of the shield with the ground. If the shield has more than one wire inside, the simplest way is similar to the twisted pair: to consider each wire as a particular line over the shield and a particular common mode for the internal domain. Then considering the meshes made with the differential loads and the coupling between the common modes of each wire inside the shield.

Constructing the harness

Once each model for each link is determined, first step is to realize the direct addition of all the links enclosed in the harness:

$$\oplus \zeta_{xy} = (\oplus_{sw} \zeta_{sw}) \oplus (\oplus_{tp} \zeta_{tp}) \oplus (\oplus_{Sn} \zeta_{Sn}) \quad (10.6)$$

It remains to add the coupling between these links. Always in the spirit to simplify the model in order to keep its essential parts, we consider the coupling between the common modes. Mutual couplings are computed using Neumann's formula and capacitive couplings are computed using Poisson's formula. Remember that the electrostatic forces induced by the time variation of the voltage on the culprit line can be reported using nodes pair sources. If γ is the coupling capacitor,

$$J^2 = \gamma_{21} p V_1$$

This nodes pair source is added on the branch of the spanning tree, at each extremity of the victim line.

We may wonder if the variation of height along the line path could impact significantly the results? In fact, by considering constant height on all the harness path we can be sure of computing the maximum coupling taking the maximum height. Our computation will give the majorant result and covers various cases. The point on which we should be careful is the assumption of ratio between x and λ .

Coupling with external fields

If u is the curvilinear abscissa of the harness, h the direction of its height and B an external field, the coupling between the

harness common mode and the external magnetic field is given by:

$$\check{e} = \int_u d\mathbf{u} \cdot (\mathbf{h} \times \mathbf{B}) \quad (10.7)$$

The EM force e is reported on the meshes of the common modes of the harness. The coupling with the electric field, often negligible, is obtain adding a node pair source on the extremity, which value if given by:

$$\bar{h} = J^e = p \frac{\sqrt{\epsilon_r}}{cZ_c} \int_u du \mathbf{h} \cdot \mathbf{E} \quad (10.8)$$

These sources (\check{e}, \bar{h}) are added to all the common mode lines of the harness coupled with the external field. Splices connecting various harnesses are represented by a border: the branch shared between the harnesses.

10.2.3 High frequencies modeling for wire network

The modeling of harnesses in high frequencies follows the same method as for low frequencies. The only difference is in the models of the lines, which reproduces the high frequencies behavior of the lines. We have seen to do that Branin's model. This model remains the best in the time domain simulation. For theory and Laplace's formulation the model using backward impedance can be more interesting. It is a model very stable, and that reproduces with the minimum of effort the resonances on mismatched lines.

We have already seen the impedance presented at the input of the line of length r for a load Z_T :

$$Z_E = \eta \left(\frac{Z_T + \eta th \gamma r}{\eta + Z_T th \gamma r} \right) \quad (10.9)$$

η is the characteristic impedance of the line.

For a source impedance Z_0 , similar input impedance can be seen from the output of the line:

$$Z_S = \eta \left(\frac{Z_0 + \eta th \gamma r}{\eta + Z_0 th \gamma r} \right) \quad (10.10)$$

We know the expression of the transmitted voltage at the output, coming from a functional source e_i on the input:

$$V(r) = (1 + \sigma) e^{-\gamma r} \left(\frac{\eta}{\eta + Z_0} \right) e_i \quad (10.11)$$

If the model of the line incorporates the impedance Z_E and Z_S , a generators e_r must be added on the output to reproduce the same current of same transmitted voltage. It leads to:

$$V(r) = \frac{Z_T}{Z_T + Z_S} e_r \Rightarrow e_r = \left(1 + \frac{Z_S}{Z_T} \right) V(r) \quad (10.12)$$

Finally a link becomes two meshes with two generators defined by:

$$\begin{cases} \zeta = \begin{bmatrix} Z_0 + Z_E & 0 \\ 0 & Z_T + Z_S \end{bmatrix} \\ e = \left[1 \quad \left[\frac{2\eta(Z_T + Z_S)}{(\eta + Z_0)(\eta + Z_T)} \right] e^{-\gamma r} \right] e_i \end{cases} \quad (10.13)$$

Each link before to be enclosed in a harness is a set of two meshes with previous definitions (ζ, e) .

The same strategy is used than for low frequencies. Differential mode is obtained from coupled common modes. The coupling is more complicated to calculate because we must take into account the delays on each coupling on each location of the target line.

Two wires line We have now to obtain the expression of coupled lines in high frequencies. This was done by Vabre. Vabre defines two coefficients:

$$\begin{cases} \alpha = \frac{\gamma}{\gamma+C} \\ K = \frac{M}{\alpha L} \end{cases} \quad (10.14)$$

C and L are the common inductances and capacitances of the coupled lines, M the mutual inductance and γ the coupling capacitor. Having a current i^1 on the source line, we can compute the transmitted voltage on the receiving line in near end cross-talk (V_n^c) and far end cross-talk (V_f^c) with:

$$\begin{cases} V_n^c(0) = (1 + \sigma^G) \alpha \frac{p(K-1)r}{2v} \eta p e^{-\frac{r}{v} p} i^1 \\ V_f^c(r) = (1 + \sigma^D) \alpha \frac{K+1}{4} [1 - e^{-2\frac{r}{v} p}] \eta i^1 \end{cases} \quad (10.15)$$

$$\sigma^G = \frac{Z_0 - \eta}{Z_0 + \eta} \quad \sigma^D = \frac{Z_T - \eta}{Z_T + \eta}$$

v is the wave speed in the victim line. As previously, knowing the transmitted voltage we can compute the EM force that must

be reported as extrinsic sources on the victim line:

$$\begin{cases} e_n = V_n^c(0) \left(1 + \frac{Z_E}{Z_0}\right) \\ e_f = V_f^c(r) \left(1 + \frac{Z_S}{Z_T}\right) \end{cases} \quad (10.16)$$

which leads to two coupling impedance operators:

$$\begin{cases} e_n = \left[\left(1 + \frac{Z_E}{Z_0}\right) (1 + \sigma^G) \alpha \frac{p(K-1)r}{2v} \eta p e^{-\frac{r}{v}p} \right] i^1 \\ e_f = \left[\left(1 + \frac{Z_S}{Z_T}\right) (1 + \sigma^D) \alpha \frac{K+1}{4} [1 - e^{-2\frac{r}{v}p}] \eta \right] i^1 \end{cases} \quad (10.17)$$

Shielded cables

For shielded cables we can neglect in general the transfer admittance. So just exist the magnetic and resistive couplings. The resistive coupling only concerns the low frequencies. For the high frequencies the situation is the same if we consider the previous ones with $\alpha = 0$. In that assumption, $\alpha(K-1)/2$ is replaced by $M/(2L)$ and $\alpha(K+1)/4$ by $M/(4L)$. M is the transfer impedance high frequency part often represented by an inductance of small value (typically 1 to 10 pH).

10.2.4 Coupling with external fields

Principles remain always the same but once more, we have to take into account the delays. The reported EM forces are determined in three phases:

1. integrating the coupling to obtain the incident wave on the extremities of the line;
2. to compute the transmitted voltage including Z_E and Z_S ;
3. to compute the EM force reported to obtain the same transmitted voltage across the loads.

Let's consider for example an incident magnetic field B_i on a line. Locally, the EM force can be given by:

$$de(x) = -pd\mathbf{x} \cdot \mathbf{h} \times \mathbf{B}_i \quad (10.18)$$

h is the height of the line and x its direction. This coupling reported on the left, at the input of the line:

$$e_G = - \int_x pd\mathbf{x} \cdot \mathbf{h} \times \mathbf{B}_i e^{-\frac{x}{v}p} \quad (10.19)$$

while on the output, it gives:

$$e_D = \int_x pd\mathbf{x} \cdot \mathbf{h} \times \mathbf{B}_i e^{-\frac{L-x}{v}p} \quad (10.20)$$

L being the length of the line. Having e_G and e_D we calculate

$$\begin{cases} V_n^c(0) = (1 + \sigma^G) e_G \\ V_f^c(r) = (1 + \sigma^D) e_D \end{cases} \quad (10.21)$$

and finally e_n and e_f as previously. This can be made for all the links of the same harness where the coupling with the external

field is nearly the same for all the common mode lines. Note that if there are differential modes, the coupling with the external field must be computed for this mode also, in high frequencies but also in low frequencies. To do that, the first configuration considered in the twisted pair model is perhaps simpler to use.

10.2.5 Low frequencies versus high frequencies limit

In low frequencies, the inductance includes the magnetic energy of the free space, L_0 and the magnetic energy stored in the metallic part L_m . This part disappears while the frequency increases: this is the imaginary part of the skin effect. For a metallic part of length x and depth d , it is given by (ν is the frequency):

$$L_m = \mu \frac{x}{d \sqrt{\pi \nu \mu \sigma}} \quad (10.22)$$

At the contrary, the resistance of a conductor increases with the frequencies:

$$R(\nu) = R_0 + \frac{1}{\sigma} \frac{x}{2\pi r} \sqrt{\pi \nu \mu \sigma} \quad (10.23)$$

r is here the radius of the conductor.

The limit of the low frequency is when we reach the first resonance of the system, under the condition that it is reachable by the model which has been constructed. But even if the model cannot reverberate, we can study the system until this value. We know that for this value, the result is not correct, but it gives a simple and clear limit for the low frequency domain. The

high frequency domain begins also from this limit. The fact to have a common domain allows to verify the robustness of the analysis. On the border, the results should be similar using the two models in high and low frequencies.

The first resonance is given by $L = \lambda/4$ for lines or $L = \lambda/2$ for waveguides, \sqrt{LC}^{-1} for oscillators, etc. It is in general quite easy to evaluate its value. In more complicated cases, we must remember that the first resonance is also the first eigenvalue of the system. They can be obtained using:

$$\left| p\mathcal{L}^{-1} \left[H - \sqrt{G} \right] - \omega^2 I \right| = 0 \quad (10.24)$$

\mathcal{L} is the inertia tensor, H the hamiltonian, G the metric reduced to the dissipation, and I the identity matrix. Solving the characteristic equation we obtain the eigenvalues and the first of them should be the first resonance frequency of the system studied.

10.3 Another approach for lines

Lines networks can be modeled using many various techniques. We have evocated the use of backward impedance. If we imagine a network of many lines (they can represent a network of information, a network of energy distribution, etc.), a first step consists in representing each line as a separate element of information transport. This line (of length x) has two or three conductors. The signal is transported through this line with a given characteristic impedance η_c . If the source excites one ex-

tremity, from this extremity, the other extremity of the line is seen through the backward impedance Z_b defined by:

$$Z_b = \eta_c \left(\frac{Z_T + \eta_c \operatorname{th} \gamma x}{\eta_c + Z_T \operatorname{th} \gamma x} \right) \quad (10.25)$$

On the other extremity, the generator must be transmitted. The voltage across the load Z_T is:

$$V_T = (1 + \sigma) u_i \quad (10.26)$$

If the second extremity is represented by the backward impedance seen from the output:

$$Z'_b = \eta_c \left(\frac{Z_0 + \eta_c \operatorname{th} \gamma x}{\eta_c + Z_0 \operatorname{th} \gamma x} \right) \quad (10.27)$$

To obtain V_T the reported voltage e_1 must be:

$$e_1 \frac{Z_T}{Z_T + Z'_b} = V_T = (1 + \sigma) u_i \quad (10.28)$$

but

$$u_i = \eta_c \frac{e_0}{Z_0 + \eta_c} = \eta_c \frac{1}{Z_0 + \eta_c} (Z_0 + Z_b) K^0 \quad (10.29)$$

Finally,

$$\frac{e_1}{K^0} = \eta_c (1 + \sigma) \frac{(Z_T + Z'_b)(Z_0 + Z_b)}{Z_T (Z_0 + \eta_c)} \quad (10.30)$$

In the matched case, $e_1 = 2\eta_c K^0$. Finally, the metric associated with one line under this approach is:

$$\zeta = \begin{bmatrix} Z_0 + Z_b & 0 \\ \eta_c (1 + \sigma) \frac{(Z_T + Z'_b)(Z_0 + Z_b)}{Z_T(Z_0 + \eta_c)} & Z_T + Z'_b \end{bmatrix} \quad (10.31)$$

Each line is modeled by this way. It remains after to add the couplings between the lines. Two kinds of coupling must be computed: a first coming from the near magnetic field coupling and a second coming from the near electrical field coupling.

10.3.1 Magnetic coupling

The principle is always the same. But at high frequencies, Neumann's relation has to be completed by the delays that the field takes to go from the source to the target. For an element of current on the line 1 of curvilinear coordinates x , the radiated potential vector is:

$$\mathbf{A}_x = \frac{\mu i_1 e^{-\gamma x}}{4\pi R_{12}} d\mathbf{x} e^{-jkR_{12}} \quad (10.32)$$

At the level of the line 2, this potential vector creates the EM force:

$$\delta e_2 = -phd\mathbf{x}' \cdot \nabla \times \mathbf{A}_x \quad (10.33)$$

This EM force propagates on the second line and arrives at its extremity with the value:

$$\delta e_2 e^{-\gamma' x'} \quad (10.34)$$

The distance R_{12} is optimized in order to minimize the duration:

$$\omega \left(\frac{x}{v} + \frac{x'}{v'} \right) + kR_{12} \quad (10.35)$$

In empty space, R_{12} is a straight line. The whole mutual inductance is obtained making the integration

$$Mp = \frac{e_2}{i_1} = \frac{1}{i_1} \int_x \int_{x'} \delta e_2 e^{-\gamma' x'} \quad (10.36)$$

i_1 comes from both wires of the first line. The rotational of the potential vector is computed between the two wires of the second line.

10.3.2 Electric coupling

The scalar potential ψ at the level of the second line is created by the charges accumulated on the first line. Always taking into account both wires:

$$\psi_{2x'} = \frac{dx}{Cp} \left(\frac{i_1}{4\pi\epsilon_0 R_{12}} \right) e^{-[jkR_{12} + \gamma x]} \quad (10.37)$$

The elementary voltage developed on the second line is obtained through

$$\delta V_2 = \Delta \psi_{2x'} e^{-\gamma x'}$$

The current source δg locally induced is then given by:

$$\delta g = C' dx' p \delta V_2 \quad (10.38)$$

and the whole coupling t_C is defined by:

$$t_C = \frac{g}{i_1} = \frac{1}{i_1} \int_x \int_{x'} \delta g e^{-\gamma' x'} \quad (10.39)$$

Through this approach we define a nodes-pair current source driven by the mesh current of the culprit line.

10.3.3 General approach

The need for EMC is to determine risks. Accurate representation of the lines is in general useless, because the difference between the real distribution of the wires and their idealized representation is wide. In fact a simplified representation has made its proof. It consists in modeling first each line using 10.31 impedance operator. These definitions depend on the lines only and not of the couplings between the lines. It is obviously an approximation, the couplings changing the characteristic impedance of the lines. If ζ_i is the impedance operator of the line i , the whole network of the uncoupled lines is given by $h = \oplus_i \zeta_i$.

Once the lines are modeled, we have to add the couplings. Depending on the neighborhood of the lines, we compute the mutual inductance couplings and the t_C couplings. These functions are added as extra-diagonal elements of the global impedance operator h of our problem. In a first run, to determine the EMC risk, it is in general sufficient to consider the lines running in a common harness.

The functional signals of each line is considered as source for the covector e , and nodes-pair source g are also added on

the spanning trees of the lines extremities. In final we solve the equation

$$e_\mu = h_{\mu\nu} \begin{bmatrix} K \\ g \end{bmatrix}^\nu \quad (10.40)$$

10.3.4 Waveguides and cavities

The backward impedance is also a good model for waveguides and cavities. If we know the equivalent circuit of the boundaries, we can calculate the backward impedance Z_r at the level of the emitter. Then, from the imaginary part of Z_r we can compute the overvoltage coefficient Q , knowing the losses R :

$$Q = \frac{\Im(Z_r)}{R} \quad (10.41)$$

If z_0 is the self impedance of the emitter e , the voltage developed in the cavity of height Z is $Z_r/(Z_r + z_0)e$, and the maximum amplitude of the field in the cavity can be approximated by:

$$E_0 = Q \left(\frac{Z_r}{Z_r + z_0} \right) \frac{e}{Z} \quad (10.42)$$

for modes developed in the polarisation Z following standing waves in the plan x, y . The receiver, a slot, integrate the field on its height h at its location: $u = h(x, y).E_0 f(x, y)$, $f(x, y)$ being the modal distribution of the field.

Knowing the maximum value of the field in the cavity can be sufficient to conclude on the EMC risk. If the susceptibility threshold of the electronics existing in the volume of the cavity

is known in field, it can be compared with this value, without looking in detail on the receiver location. If the level of field in that case is higher, then we can take a look on the impact of the location.

If the emitter is not centered in the cavity, the backward impedance must be computed for each direction of waveguide propagation. Typically for a rectangular cavity, four backward impedances are computed on the emitter.

The advantage of this formalism is to translate simply the resonator equivalent to the cavity. If the limit condition is a short circuit, the backward impedance gives an inductance $L\omega$ at low frequencies. With the resistance of the generator, it leads to an overvoltage coefficient equal to $L\omega/R$. If four axes are represented, the principle is the same, but four admittances are in parallel and must be added to contribute to the final impedance presented to the emitter.

10.4 Equipments

Once we know how to model the harnesses, we have to connect them to the equipments. An equipment is a behavioral kernel surrounded by interfaces that makes the links with the external world. Some parts of the kernel can also be affected by external fields through equivalent antennas. It is impossible to model a complete equipment. And this has perhaps no sense. What is important is to be able to model the signals that go in or go out the equipment and how undesired signals can change its working. Usually, it is interesting to separate the analysis of

the equipment in two parts: the conducted interactions and the radiated interactions.

10.4.1 Conducted interactions

Signals transported by the previously studied harnesses comes into the equipment. They first transit through an interface stage that we can call "Passive Description Network"³. It is a network of passive components like inductances, capacitors, that translates the transmission of the signal through the layers of connectors and wires to the printed circuit board etc. After what the signals goes in an electronic component whose input impedance is known. The model can stop here and the whole component can then be modeled with all the chain that makes the processing of the signal in an unique behavioral function. If a parasitic signal comes from one of the wire used by the equipment, this behavioral part must give the answer that the equipment may give in response to this parasitic sollicitation. It's a little bit more complicated if the signal is over the working frequency band of the equipment. In that case, non linear components transform the out-band signal in in-band one. This is a non linear operator applied to the fast signal and which transposed it in the working frequency of the equipment. All these details are driven by the electronic schematic of the equipment and result in impedances and cords for ζ that gives the whole transfer function of the equipment, even in disturbed state. Let's take

³I reuse here the naming submit by the ICEM model for the emissions of the microprocessors - see UTE standards for details

a look to the structure of ζ . A series of input impedance and networks take in charge the interfaces. For example we have impedances like $Lp + R/(RCp + 1)$. L is the lead inductance and C the input capacitor with R the input impedance of some operational amplifier. This input stage generates a voltage Ri that is processed by all an electronic behind the interface. This electronic executes a function f , applied to Ri and gives on another output of impedance R_O this EM force in series with an output inductance L_O . The equivalent schematic of the transfer function between access 1 and 2 is shown figure 10.2. The cord $f.R$ takes in charge the behavioral model.

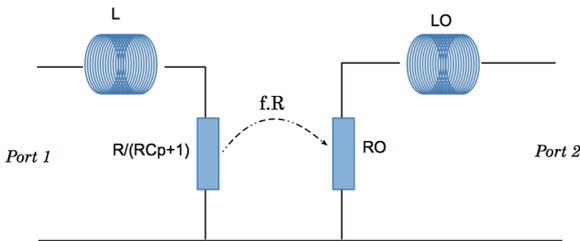


Figure 10.2: Equipment model

The impedance operator of such a device is:

$$\zeta = \begin{bmatrix} Lp + \frac{R}{RC^{p+1}} & 0 \\ f.R() & R_O + L_{Op} \end{bmatrix} \quad (10.43)$$

10.4.2 Radiated interactions

The radiated interactions proceed following the same principle. An antenna that reproduces the equipment reception of the external field transmits some noise to the input output of the device. These antennas integrating slots, internal couplings, etc., have their own gain and radiation diagram. They are finally represented by one or more meshes including their properties and having cords with the access of the device. By these cords, it translates the impact of the external field on the internal working of the equipment. For example on the previous model, we could easily add a mesh like a dipole, which current acts through a function $k(E)$ on the output of the equipment. The impedance operator of such a device is:

$$\zeta = \begin{bmatrix} R_a + L_a p + \frac{1}{C_a p} & 0 & 0 \\ 0 & Lp + \frac{R}{RC^{p+1}} & 0 \\ k(E) & f.R() & R_O + L_{Op} \end{bmatrix} \quad (10.44)$$

The equivalent antenna of the slot of the equipment is modeled here by an oscillator (R_a, L_a, C_a) .

10.4.3 Equipments as filled cavities

An equipment is often a metallic box with printed circuit boards inside. These printed circuit boards have plenty of electronic components soldered on their coppers. All these components appear like disturbances on the flat surface of the printed circuit board. How act these disturbances on the cavity modes? How the printed circuit board (PCB) changes the modes? Answering to these questions gives response to a complete category of problems where electromagnetic fields are enclosed in volumes that include other parts.

Major principles

We have seen that the fields in a cavity behave like in a short-circuited waveguide. They are a little different from the free space but not so much. The major difference comes from the fact that they exist under modes defined by the geometry of the cavity. A first consequence of this is that the field propagates depending on the dispersion coefficient k and not with the speed of light.

Each component on the PCB receives the field and diffracts it in various ways. We will look at this process and wonder how it influences the transmission of energy from outside the equipment to some particular component inside the equipment.

Major principles are:

- diffraction depends on the ratio between the component dimension and the wavelength;

- the PCB modifies the modes in particular it creates a TEM mode with a low cutoff frequency.

Various methodologies use ABCD matrix or scattering S matrix, then ABCD to Z conversion or S to Z conversion.

Component influence on the interior field

All the components fixed on the PCB behaves like small antennas. If their heights are very small, they store or dissipate a little part of the field energy. We have to study this first behavior. If they are high enough, they can diffract the field. This is a second behavior we have to study.

Porosity. If we consider a PCB inside a cavity, its length being in the direction x , the width of the cavity in the direction y and the height of the cavity in the direction z is Z , we take a look to a particular resistance soldered on the PCB, and making a little loop with the ground plane of the PCB with its track. The resistance value is very much higher than the losses in the track. A modal wave of magnetic field B_0 go through the cavity. If the length x of the resistance is smaller than the wavelength we can consider the field constant all over the resistance. The EM force induced in the loop is:

$$e = -xhpB_0 \quad (10.45)$$

and the power dissipated in the resistance is:

$$P = Ri^2 = R \frac{x^2 h^2 p^2 B_0^2}{R^2} \quad (10.46)$$

The power associated with the wave before the resistance is:

$$P_0 = \frac{c^2 B_0^2}{\eta} Z \int_y dy \left[\text{Sin} \left(n\pi \frac{y}{Y} \right) \right]^2 = c^2 \frac{B_0^2 Z}{2\eta} \quad (10.47)$$

η being the wave impedance.

Finally:

$$B_0^2 = 2 \frac{\eta}{c^2 Z} P_0 \quad (10.48)$$

and

$$dP = 2 \frac{h^2 p^2 \eta s^2 \sigma^2}{c^2 Z s \sigma} dx \quad (10.49)$$

replacing R by $\sigma^{-1}x/s$. This gives:

$$\frac{dP}{P} = 2 \frac{p^2 h^2 \eta s \sigma}{c^2 Z} dx \quad (10.50)$$

which leads to:

$$P = P_0 e^{-\alpha x} \quad (10.51)$$

and:

$$\alpha = -2\omega^2 h^2 \frac{\eta \sigma}{c^2 Z} s \quad (10.52)$$

with N components we may have:

$$\alpha = -2N\omega^2 \langle h \rangle^2 \frac{\eta \sigma}{c^2 Z} \langle s \rangle \quad (10.53)$$

This leads to small values of losses. Figure 10.3 shows the typical curve obtained for one component. Except at very high frequencies, near 1 GHz, the losses are weak. But considering the number of components on a real PCB, we obtain anyway

losses of around 1 for 1000 components at 100 MHz. It means that 1/3 of the level is lost for a cavity of 10 cm at 1 GHz. Clearly, the what I call the porosity of the PCB inside the cavity decreases the quality factor of the cavity in hyper frequencies. But we mustn't forget that this behavior is in average. Particular cases can exist giving very different results.

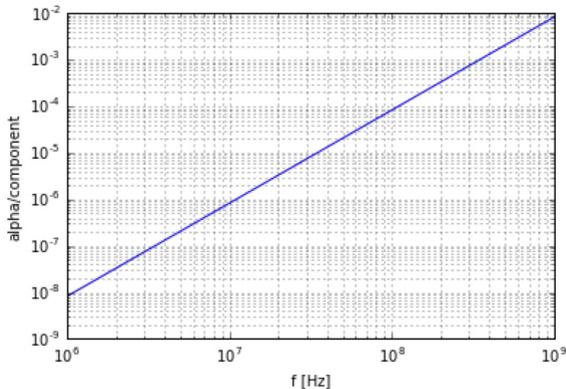


Figure 10.3: α curve for one component

To confirm or not this tendency, it can be quite easy. Using one input of a microprocessor that has been characterized before for its response in high frequencies, we can use it as a field sensor. This can be done also using the same process with the power supply lines. Measuring the level induced on these lines, we can evaluate the field level inside the cavity, and then the quality

factor. This measurement can be made with the equipment in power off. In general we can consider that the functional state doesn't change the components behavior in front of high frequency field illumination. This is not always true: first when these components work at the same frequencies or when their impedances in hyper frequency change radically when they are powered. In general that's not the case.

Diffraction and high height components. General principle says that an antenna receiving energy through an incident field is then traveled by currents. These currents radiate diffracted fields in the free space. These diffracted field come in opposition with the incident field. Another rule is that Nature tries also to increase its entropy. If she cannot benefit of a high level of freedom inside the matter, she will try to increase the radiation to the environment. The skin effect is a translation of this behavior. A high component inside the equipment, up to the PCB receives the field of the modal wave inside the equipment cavity. It behaves like a small antenna.

A small antenna like a component of small length compared to the wavelength, is a capacitor. Basic reasoning consider a cone. For an original radius r at the height y on the base of the cone, the capacitor with the ground under the cone is:

$$dC = \epsilon_0 \frac{2\pi r(y)dy}{y\theta} \quad (10.54)$$

θ is the angle of the cone, i.e. $\arctan(Y/R)$ if Y is the height of the cone and R its maximum radius. Increasing the height, all

the capacitors are in parallel and:

$$C = \int_y dC = \int_y \epsilon_0 \frac{2\pi r(y) dy}{y\theta} \quad (10.55)$$

if $r(y) = \alpha y + r_0$ and the minimum height is y_0 , we obtain:

$$C = \frac{\epsilon_0}{\arctan\left(\frac{Y}{R}\right)} \left[\alpha(Y - y_0) + r_0 \ln\left(\frac{Y}{y_0}\right) \right] \quad (10.56)$$

$$\alpha = dr/dy = \tan(\theta)^{-1}.$$

Knowing the capacitor equivalent to the small antenna, the circuit realized when the field illuminates this antenna is a simple RC circuit. The resistance comes from the losses in the capacitor conductors and from the part of radiated energy. For an incident field \mathbf{A}_0 (we use the potential vector which is powerful for this kind of problem). The current induced in the antenna is:

$$\mathbf{i} = \frac{-pY \mathbf{A}_0}{R + \frac{1}{Cp}} = -pY \left(\frac{Cp}{RCp + 1} \right) \mathbf{A}_0 \quad (10.57)$$

This current radiates a diffracted field \mathbf{A}_d :

$$\mathbf{A}_d = -\mu_0 G p^2 Y^2 \left(\frac{C}{RCp + 1} \right) \mathbf{A}_0 \quad (10.58)$$

G is a Green's function associated with the field attenuation and propagation with the distance. This diffracted field is added to the incident field \mathbf{A}_0 with various phases depending on the direction. If we take a look to the electric transverse field $-p\mathbf{A}_0$, the

diffracted field depends on a function that evolves with f and depends on a function that evolves with $(1+f)^{-1}$. Bode's diagram associated with the diffracted field shows a field that increases up to $\omega_0 = (2\pi RC)^{-1}$ then becomes constant in frequencies. As we are below the first resonance of the currents associated with the field on the components, R is principally due to the resistive part of the component and the total field is decreased by the field diffracted by the component. But this impact plays a role principally at the component layer level. Up to the component, the diffracted field changes of direction and amplitude with $\cos(\beta)$, β being the angle between the PCB and the vertical axe associated with the component height. So, without considering the near field interactions and until the frequency is low enough compared to the $\lambda/4$ criterion associated with the component heights, it seems that the disturbance created by the high components acts at the layer level enclosed between the PCB and the height of these components. As $C \approx \text{some } pF$ and $R \approx \text{some } m\Omega$, ω_0 is around 100 THz! So, the diffracted field increases with the frequency ($\lambda/4$ criterion reaches around 15 GHz).

What does it mean?

It means that under the major energy stored in the modes of the empty cavity, some higher modes are created at the component level coming from the field diffracted by these components and associated with the typical distance between them. Until this distance is far from $\lambda/2$, the stored energy in these modes will be neglected. Up to this frequency, this stored energy must be withdrawn from the stored energy of the major mode in the cavity. This appears like an added losses for this majority mode.

But a paradox is that the distance between the component is many more small than the PCB dimensions (in general). A resonance will appear between the components at very high modes for the cavity. For these frequencies, the losses inside the cavity are very high (we have seen the order of values coming just from the resistances on the PCB). The problem changes of scale and what becomes interesting is the direct interactions between the sources of transmitted field inside the equipment and the structures at the component level that may be excited by this direct field.

10.4.4 Low frequency TEM mode coming from the PCB presence

An important influence of the PCB is that it creates a conductor inside the cavity. The cavity cannot be longer considered as an empty one with a low cutoff frequency associated with the first trans-electrical (TE) mode. It behaves more like a waveguide with a conductor inside, like a coaxial line for example. It means that low frequency TEM mode exists. The limit conditions are determined by the connexions between the PCB and the metallic box of the equipment. It creates also lower first TE modes, associated with the whole distance that the field can follow between the PCB and the box. For example for a coaxial line of radius r , the first TE mode is given by the relation:

$$\frac{\lambda}{2} = 2\pi r \quad (10.59)$$

Except for this particularity, the approach is exactly the

same as for empty cavities, the problem being to determine the dispersion k and the characteristic impedance η . After what, many models are available to compute the interactions and the field inside the equipment.

10.4.5 Hyper frequency behavior

Due to the losses in the walls and in the PCB, the field ambiance in the equipment can be seen as a pseudo-free field ambiance when the frequency reaches the component scale. In that case we have said that direct interactions must be considered to compute the energy transmission from outside the equipment to the components of the equipment. Direct flows of electromagnetic energy between elements can be quite easily translated using scattering matrix and S parameters. A way to represent these paths of energy exchanges is to use graphs of fluxes. Looking at figure 10.4, we can see S parameters relations and normalized loads like G_s or G_L . Following the rules of use for these graphs, each node is a point of convergence for the waves a_x , b_x , and the branches are the loads or the S parameters. The waves have for dimensions squares of powers.

To find transfer functions like for example the ratio between the incident wave b_s and the wave on node 2 b_1 , we apply Mason's rules. To start, we search the ways between b_s and b_1 : C_x . Here we recognize a first way $C_1 = S_{11}$ and a second way $C_2 = S_{21}G_L S_{12}$.

Then we look for loops L^1 of first order. A loop of first order is a succession of branches starting from one node and coming back to the same node. Here we have for first order

loops: $G_c S_{11}$, $G_L S_{22}$ and $S_{21} G_L S_{12} G_s$.

Then we look for second order loops L^2 , it means products of first order loops but that have not common branches. Here we have only one second order loop: $S_{11} G_s S_{22} G_L$.

Then same research is made for loops of order n : L^n , with the same definition: product of n loops of first order that have no common branches. Here we don't have loops of third order or higher orders.

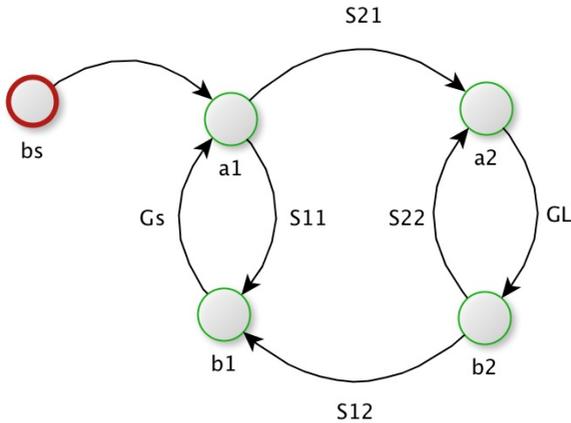


Figure 10.4: Fluxes graph

Mason's rule says that the transfer function $T = b_1/b_s$ is

given by:

$$\frac{b_1}{b_s} = \frac{c_1(1 - \sum L_1^1 + \sum L_1^2 - \sum L_1^3 + \dots) + c_2(1 - \sum L_2^1 + \sum L_2^2 - \sum L_2^3 + \dots) + \dots}{1 - \sum L^1 + \sum L^2 - \sum L^3 + \dots} \tag{10.60}$$

L_q^n is a loop of order n that has no relations with the way q . For example here $\sum L_1^1 = G_L S_{22}$. In our case, applying Mason's rule we obtain:

$$T = \frac{S_{11}(1 - S_{22}G_L) + S_{12}G_L S_{21}}{1 - (S_{11}G_s + S_{22}G_L + S_{21}G_s S_{12}G_L) + S_{11}G_s S_{22}G_L} \tag{10.61}$$

But this result should be easily obtained using our formalism. Replacing branches by cords, and making the relations:

- $a_1 \rightarrow node_1$
- $b_1 \rightarrow node_1$
- $a_2 \rightarrow node_3$
- $b_2 \rightarrow node_4$

We normalize all the meshes impedance to 1Ω defining the metric:

$$\zeta = \begin{bmatrix} 1 & G_s & 0 & 0 \\ S_{11} & 1 & 0 & S_{12} \\ S_{21} & 0 & 1 & S_{22} \\ 0 & 0 & G_L & 1 \end{bmatrix} \tag{10.62}$$

The source covector is:

$$e = [b_s \quad 0 \quad 0 \quad 0] \quad (10.63)$$

The transfer function T_2 is defined by:

$$T_2 = \frac{b_1}{b_s} = \frac{y^{21} e_1}{b_s} = y^{21} \quad (10.64)$$

Using any tool able to make formal computation we find immediately $y^{21} = T_2 = T$. When the graph is complicated or too much abstract Mason's rule becomes difficult to apply. In that case, our technique can be powerful.

For a normalized impedance z_{0j} on port j , the waves are defined by:

$$\begin{cases} a_j = \frac{e_j + z_{0j} i^j}{\sqrt{2(z_{0j} + z_{0j}^*)}} \\ b_j = \frac{e_j - z_{0j}^* i^j}{\sqrt{2(z_{0j} + z_{0j}^*)}} \end{cases} \quad (10.65)$$

Knowing the correspondence between S parameters matrix and impedance matrix Z :

$$Z = \frac{1 + S}{1 - S} \quad (10.66)$$

Once the transfer function is obtained using the scattering matrix, we can change it in the impedance expression using the previous relations.

Our problem in EMC is to identify the possibility that a direct interaction occurs between some openers in the equipment

box and a target component on the PCB. In hyper frequency, the risk can be evaluated using field paths and fluxes graph. Once the transfer function is written, we can change it in Kron's form to include it in a larger problem involving various objects at the system level.

10.5 How to study the whole system?

A system can be imagined starting from the customer needs. Once the missions well understood, it is possible to construct a set of equipments that grouped, will make the missions. Once these equipments listed, it remains to link them using the previous networks.

For a known kind of system, Future evolutions consist in changing some elements inside the system without starting from zero. Often, the conception is limited to the revision of some parts of the system. We consider here the case where we start from zero.

If O is a set of objectives written by a customer, O is a set of function of the space-time and of materials making movements, communications, transformations. Knowing O and its elements O_k we must define for each O_k the couples *Commands - Actions* they imply. If we are able to define these couples, we can imagine that more or less, we can define equipments that reproduces these requirements. So we make a correspondence between each O_k and a set of manifolds able to answer to the requirements (they are abstract models of these parts of missions). To illustrate this we imagine a system able to manage a part of the

lights in a group of buildings, using the energy of the Sun. To reach its mission the system must:

- O_1 : receive the energy of the Sun, has actuators that switch on the light and the light itself;
- O_2 : understand the needs in lights;
- O_3 : control the lights activation;

In a first step we create the subsystems able to cover the missions O_k .

The first one is a resistance that transforms the sun light in electricity to charge a capacitor. Its impedance operator is a simple RC circuit, one port is the sun light and the output is the voltage across the capacitor. It works for given intervals of temperature, light, etc. To the first mesh we add a second one to communicate the status of charge to the central unit that will manage the system. It is a simple mesh with a measurement of the current in the capacitor, and a message sends to the central unit through an antenna which is an oscillator. A third mesh is added which is the light control and the light itself is a fourth mesh. The manifold is \mathcal{M}_{1q} for q devices and is the correspondence for O_1 .

A second system detects the light near various buildings to decide is it is necessary to switch on some lights. It is a first mesh that measure the presence of light in the environment and a second mesh that sends the need for lights to the system decision organ (central processing unit: CPU). There are as many of these systems as it is needed light detections: $O_2 \rightarrow \mathcal{M}_{2n}$. It is an autonomous system for energy.

O_3 is the CPU. It receives the messages coming from O_1 and O_2 , commands coming from a panel that interacts with the users and send commands to the lights and the panel. It is made of a CPU, receives messages and send others. So it is made of two antennas, a screen and a keyboard which is a set of buttons sending words to the CPU. It needs power, as O_1 . The corresponding manifold is \mathcal{M}_3 .

10.5.1 Constructing the system

First action means to complete the source vectors, including external resources for power supplies, etc. This completes the definitions of our manifolds. Then we realize the addition of the various manifolds involved in the whole system \mathcal{M} :

$$\mathcal{M} = \sum_q \mathcal{M}_{1q} + \sum_n \mathcal{M}_{2n} + \mathcal{M}_3 \quad (10.67)$$

The addition includes the operation of increasing the dimension of each manifold and realizing the direct summation through a simple addition (remember that thanks to this technique, the order doesn't change the result, contrary to the classical direct summation). These additions imply also the operations made on the intervals with intersections (see *Domains for vertices and systems*).

Next action consists in adding the networks of communication (in this example, because there is not networks here for controls, energy, this last network being embedded through the sources. The modeling strategy may have been different). Cord functions is added in the extra-diagonal terms of the matrix ζ

of \mathcal{M} to translate communications through antennas. At this level, \mathcal{M} is near to model the real system in a quite large sense. It lacks the interaction with the users.

That's the next action. It means sending to the panel some command to display some words (W^x). These words are the choice of the CPU player. Depending on his needs, the user can make choices c_y in response to these solicitations. For each couple (c_y, W^x) we define a bayesian probability associates with the gain ω_x^y the user win choosing the command c_y in response to the solicitation W^x . The function h_{yx} applied to W^x is interpreted by the CPU through the synthesis Q_a^y send to it in order to provoke next communication to activate some actuators. These last cords are the more complex ones and end the definition of the theoretical system. To detail each of its mechanism the engineer is obliged to go deeper in all the definitions, but the structure defined symbolically helps him to manage his work as system engineer. More, the formalism allows him to develop the details remaining inside it. Having defined the networks, he can after describe various signals involved and study their transport using harnesses. The computation of the coupling functions will help him to decide wether or not shields are necessities or what may be the best architecture for his system.

Here we have imagine the method of construction for a fixed system. The CPU can exchange with a cloud in order to make better prediction of the need of light, taking into account stormy weather, etc. This is already a CPS system for sure and its development can reach today capacities that wasn't at least imaginable yesterday.

10.5.2 System communication network

The elements of our imagined system are connected through antennas. There is no wire network. The communication between antennas, under the far field assumption is quite easy to model. The impedance operator is:

$$\zeta_{21} = h \frac{\sqrt{30R_0G}}{d} e^{-d/cp}$$

d is the distance between the antennas, the first of gain G and the second of effective height h . It is always possible to separate the tensor that takes in charge the coupling between the antennas from the other operators. Let \mathcal{G} being this tensor (similar to a Green's function). If we forget the power supply of the CPU and the actuators, the weak side of this system is its communication network. There are various kinds of possible attacks:

1. natural ones, like lightning;
2. artificial ones like in-band noises using same frequency of work (the available frequencies for civil applications are not so many).

External sources can be represented by external vertices. The transmission to the N antennas of communication is computed by another tensor B where all the cords are located on the same column going from the source to each antenna.

With low frequencies natural noises like lightning, two kinds of aggressions can occur. The first one is a direct lightning strike on one of the antennas. In that case that's not B which

is concerned but a specific circuit where an equivalent circuit of lightning is directly connected to an antenna. This is a classical job for any engineer in EMC. The objective here is to avoid the destruction of the electronic of the antenna.

If a lightning strike arises far from the antennas (but not so far!), all the antennas will be exposed to the lightning magnetic field. The disturbance has a short time duration, around 300 to 400 μs . But it can affect the electronics of the antennas: destroying them if the EM force is too high and provoke spark gap or over voltage on components; saturating the antenna and blocking for this duration the emission or reception of signals.

A first observation can be made. If the network is constructed point to point, i.e. that there are only one link between each antenna and the CPU, in that case the loss of one sensor is easily found, one communication being broken with the CPU (we make the assumption that the CPU is not disturbed). In this architecture, ζ has a particular structure, all its terms are located on the first column and each first element of all the rows. For example for 3 elements we have:

$$\zeta_{ab} = \begin{bmatrix} CPU & \mathcal{G}_{12} & \mathcal{G}_{13} \\ \mathcal{G}_{21} & Sensor_1 & 0 \\ \mathcal{G}_{31} & 0 & Sensor_2 \end{bmatrix} \quad (10.68)$$

This architecture has a weak point: if the CPU is disturbed, all the system is affected. The figure 10.5 shows this kind of architecture where it appears clearly that all the cords go or start from the CPU.

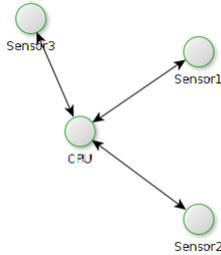


Figure 10.5: Star architecture

Note that regardless of the numbers associated with the vertices, the \mathcal{G} structure indicates also clearly that all the cords converges to the CPU. Another example of such a tensor may be (a is any \mathcal{G}_{ij}):

$$\mathcal{G}_{ab} = \begin{bmatrix} 0 & 0 & a & 0 & 0 \\ 0 & 0 & a & s & s \\ a & a & 0 & a & a \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & a & 0 & 0 \end{bmatrix} \quad (10.69)$$

We identify a "cross" of "a" on the row and column associated with the impedance of the CPU C .

We can generalize this finding. If in a structure of impedance operator, the cords follow the figure of a cross around a particular impedance, it means that an interaction exists between this

element and all the other. This can be the testimony of a weakness in the system.

Another choice for the architecture may be to use antennas as relays to have a better coverage of the domain, even if the CPU is not directly in front of some sensors. This flexibility is translated by a network where cycles appear, like in the figure 10.6.

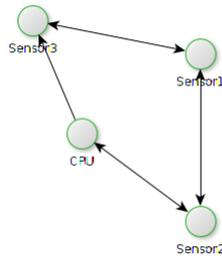


Figure 10.6: Circular architecture

This management has for impedance operator:

$$\mathcal{G}_{ab} = \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ a & 0 & a & 0 & 0 \\ 0 & a & 0 & a & 0 \\ 0 & 0 & a & 0 & a \\ 0 & 0 & 0 & a & 0 \end{bmatrix} \quad (10.70)$$

The matrix of \mathcal{G} is diagonal. If one sensor is lost, the whole

network is affected. The same weakness versus the CPU as in the "star" architecture is observed.

Another possibility is to distribute the CPU through many little ones added in the electronics of the sensors. The architecture can be meshed as shown figure 10.7.

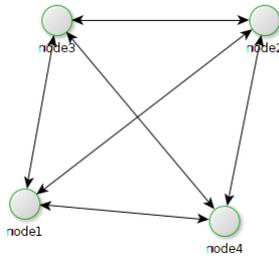


Figure 10.7: Meshed architecture

The impedance operator for this network architecture is (b for the interactions):

$$\mathcal{G}_{ab} = \begin{bmatrix} 0 & b & b & b \\ b & 0 & b & b \\ b & b & 0 & b \\ b & b & b & 0 \end{bmatrix} \quad (10.71)$$

the matrix is filled. This kind of system is very robust against electromagnetic aggressions. As the signal processing is distribute all over the network, some nodes can be lost with-

out blocking the network working. But its conception is harder than for the others organization and its cost can be higher.

The robustness of each architecture can be tested by replacing some components of \mathcal{G} by 0. On the star architecture 10.69, if one sensor loses its antenna, the rest of the network can be functional. You can confirm that taking $\mathcal{G}_{32} = \mathcal{G}_{23} = 0$ for example let the CPU communicating with the other sensors. While if the emitter of the CPU is touched, all \mathcal{G} becomes equal to zero. For the circular architecture 10.70, the loss of one vertex, for example the second one, destroys half of the network. The impact of one loss of an antenna can be evaluated in this architecture using WCA. We have under this assumption for the vertex 3:

$$e_3 = \mathcal{G}_{32} J^2 = \frac{\mathcal{G}_{32}}{\zeta_{22}} \mathcal{G}_{21} J^1$$

If \mathcal{G}_{32} is forced to zero to translate the loss of the antenna 3, the communication between the vertices 1 and 3 is lost. But also with all vertices which have in its communication function the operator \mathcal{G}_{32} .

In the meshing architecture, the lost of one antenna has for impact only the functions associated with this antenna. Making one element, for example $\mathcal{G}_{21} = 0$, the rest of the network works as if it has been reduced to this residual part.

10.6 CPS in spaces

We want to add in our considerations on systems, the dynamic of these systems. To do that we create a global cartesian referential

attached to the coordinates $x^\alpha = x^1, x^2, x^3$. A mobile follows a curvilinear trajectory. Its speed on this trajectory u^c can be developed on the basis of the speeds in the cartesian space:

$$u^c = u^c (v^1, v^2, v^3) = \Lambda_\alpha^c v^\alpha \quad (10.72)$$

When this speed changes, we have to project these changes in the common space:

$$\frac{du^c}{d\tau} = \frac{d}{d\tau} \Lambda_\alpha^c v^\alpha = \Lambda_\alpha^c \frac{dv^\alpha}{d\tau} + \frac{\partial \Lambda_\alpha^c}{\partial x^\sigma} \frac{dx^\sigma}{d\tau} v^\alpha \quad (10.73)$$

The figure 10.8 illustrates in two dimensions the mechanism. We see at each point $p_x = (x^1, x^2)$ that the trajectory follows a vector u^c whose transformation from the vector v^α changes with time τ .

Finally we obtain:

$$\gamma^c = \frac{du^c}{d\tau} + \Gamma_{\alpha,\sigma}^c v^\sigma v^\alpha \quad (10.74)$$

The Christoffel's coefficients here come from the time derivative of the transformation matrix of the basis vectors and not from their direct derivatives. There are of another kind. And we will see that we will retrieve the first kind we have seen.

Knowing γ^c we can define a force f_b using Newton's law and an inertia m by:

$$f_b = m_{bc} \gamma^c = m_{bc} \frac{du^c}{d\tau} + m_{bc} \Gamma_{\alpha,\sigma}^c v^\sigma v^\alpha \quad (10.75)$$

but $m_{bc} \Gamma_{\alpha,\sigma}^c = \Gamma_{b\alpha,\sigma}$ and

$$f_b = m_{bc} \frac{d}{d\tau} u^c + \Gamma_{b\alpha,\sigma} v^\alpha v^\sigma \quad (10.76)$$

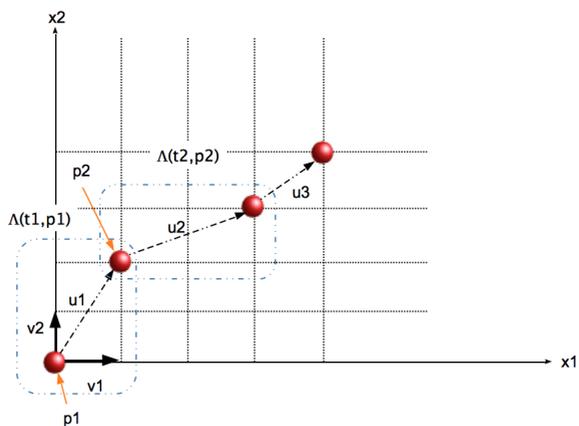


Figure 10.8: Curvilinear dynamic

If the matrix Λ doesn't depend on x^α , we can integrate both member to obtain:

$$\int_t dt f_b = m_{bc} u^c \Rightarrow s^2 = \int_t dt f_b u^b = m_{bc} u^c u^b \quad (10.77)$$

giving

$$s = \sqrt{m_{bc} u^c u^b}$$

if not:

$$s = \sqrt{m_{bc} u^b u^c + \int_t dt \Gamma_{b\alpha, \sigma} v^\alpha v^\sigma u^b}$$

The geodesic of the space are given by the equation 10.75 with no acceleration, i.e. no force:

$$m_{bc} \frac{du^c}{d\tau} + m_{bc} \Gamma_{\alpha, \sigma}^c v^\sigma v^\alpha = 0 \Leftrightarrow \frac{du^c}{d\tau} + \Gamma_{\alpha, \sigma}^c v^\sigma v^\alpha = 0 \quad (10.78)$$

10.7 CPS group behavior

A group of systems moving in a common space can be described by the union of the vector J^k giving the current in all the electronics and the vector u^α giving all the speeds of the mobiles. Temperature, pressure, etc., are taken in charge by using domains. The sources are composed of the EM forces e_q and the mechanical forces f_β . The mechanical forces can come from motors that give the mobilities. The decisions to move can be taken by humans or processors.

The coupling between the moving systems comes from two kinds of techniques:

1. coupling by communications in a CPS where a group of mobiles is guided by a central unit or a distributed unit;
2. coupling by game theory cords where actions from a mobile influence the decision of a pilot that decides of the direction to take with his mobile. This is extended to the whole group.

The system can be modeled with a general covector ϵ_μ and a general vector η^ν . The inertia \mathcal{L} exists for both electronics and mechanics. For mechanics, inductances are replaced by the inertia mass m_{bc} . Losses R exist in both electronics and mechanics as storage of potential energy. As we have seen previously the metric is associated first with the losses. These losses can take various forms: frictions, dissipations, radiations, etc. But in mechanics, frictions are proportional to the speed $-R_{b\sigma}v^\sigma$. We obtain exactly the same structure than for electronics. And as the frictions are proportional to the speed, it is logical through a second geometrization to find them defining the metric.

We consider a projectile. It see the gravitational force of the Earth f_g and the force coming from the wind, proportional to its speed. The general aspect of the system of equations that describes the system is:

$$f_g + m_{gu} \frac{d}{dt} V^u = K_{gu} V^u \quad (10.79)$$

K is the tensor of frictions. Looking for $\mathcal{J} = \partial f_g / \partial V^w$, this equation can also be written

$$T_g = G_{gu} V^u \quad (10.80)$$

with

$$T_w = \delta_w^w \{ \mathcal{J}_{gw} \}^{-1} \left(f_g + m_{gu} \frac{d}{dt} V^u \right)$$

and

$$G = \{ \mathcal{J} \}^{-1} \mathcal{J}$$

The vectors V^u includes the three directions of the projectile, but also the speed of the wind.

10.8 Fake datas from EM attack

We start accepting the existence of a space with N directions defined in a metric \sqrt{G} . At any moment, the state of any mobile that belongs to a set A can be projected on these axis as an image of its activity both electrical and mechanical. We can draw on a space (R_a, K) the activity of communication which uses the radiation resistance R_a and a factor of movement K . This can be made for a set A of N_A mobiles. At each moment, a vector of activity u^A attached with one mobile can be defined by

$$u^A = \begin{bmatrix} i(t) \\ v(t) \end{bmatrix} \quad (10.81)$$

The curvilinear coordinates i associated with the radiated communication and v associated with the speed of the mobile. This speed can be projected on a cartesian and common space ω^α and the current i can be projected on mesh currents associated

with magnetic moments in the three directions of the cartesian space. So:

$$\begin{cases} v^k = \Lambda_x^k \omega^x \\ i^q = \Lambda_x^q J^x \end{cases} \quad (10.82)$$

If the cartesian space is associated with the generalized vector c :

$$c^x = \begin{bmatrix} J^x \\ \omega^x \end{bmatrix} \quad (10.83)$$

having six dimensions, three for the moments⁴ and three for the speeds. The previous relations can be written:

$$u^A = \Lambda_x^A c^x \quad (10.84)$$

We suppose that the information i influences the speed v . i.e. that it exists a function χ such as a coupling between i and v is defined by $-\chi i$. Knowing that, the impedance operator may be inferred from the invariant:

$$s^2 = g_{\beta\sigma} u^\beta u^\sigma \equiv R_a i^2 - \chi i v + K v^2 \quad (10.85)$$

which suggests the pseudo-metric (for one mobile and pseudo because g is not extracted form a riemanian space):

$$g = \begin{bmatrix} R_a & 0 \\ -\chi & K \end{bmatrix} \quad (10.86)$$

⁴I have submitted the use of the moment space first time in "Compatibilité électromagnétique des systèmes complexes". Lavoisier editor, 2007.

For a source covector f_b the generalized equation adding the inertia is:

$$f_b - \mathcal{L}_{bA} \frac{d}{dt} u^A = g_{bA} u^A \quad (10.87)$$

By replacement we obtain:

$$f_b - \mathcal{L}_{bA} \frac{d}{dt} (\Lambda_n^A c^n) = g_{bA} \Lambda_n^A c^n \quad (10.88)$$

but

$$\frac{d}{dt} (\Lambda_n^A c^n) = \Lambda_n^A \frac{d}{dt} c^n + c^n \Gamma_{nm}^A c^m$$

and so:

$$f_b - \mathcal{L}_{bA} \Lambda_n^A \frac{d}{dt} c^n - \mathcal{L}_{bA} \Gamma_{nm}^A c^m c^n = g_{bA} \Lambda_n^A c^n$$

now we multiply all the members by Λ_q^b :

$$\Lambda_q^b f_b - \Lambda_q^b \mathcal{L}_{bA} \Lambda_n^A \frac{d}{dt} c^n - \Lambda_q^b \mathcal{L}_{bA} \Gamma_{nm}^A c^m c^n = \Lambda_q^b g_{bA} \Lambda_n^A c^n$$

which gives:

$$f'_q - \mathcal{L}'_{qn} \frac{dc^n}{dt} - Q_{qA} \Gamma_{nm}^A c^m c^n = g'_{qn} c^n \quad (10.89)$$

As we are in that case interested by the radio communication, each current i has a correspondence with three currents J^x, J^y, J^z of c^n . What is the advantage of this representation? Having the speeds versus time, we know the locations of the

moments associated with the mesh currents at any time and of course their intensities. A consequence is that the distribution of power D_p in emission in one direction (for example $g'_{xx} J^x J^{*x}$) associated with the distribution of speeds D_v of the same moments in each direction gives the whole activity of the network. The distribution of speeds shows how the "gaz" of mobiles moves and if their movements are stochastic or organized. The distribution of radio activity shows with which rhythm the informations are exchanged and if some particular mobiles take in majority the role of emitters or not. But there is another important information included in this equation: the Christoffel's symbols. They say if the location of the mobiles respectively to the cartesian space axes are constant or not. In other words if the mobiles turn more or less in a distribution D_t .

The definitions of each distribution are:

$$\begin{cases} D_p = (P, \langle g'_{xx} J^x J^{*x} \rangle) \\ D_v = (v, \langle c^n \rangle) \\ D_t = (\dot{\theta}, \langle \Gamma_{nm}^A \rangle) \end{cases} \quad (10.90)$$

The necessary averages to describe the distribution are computed on a given time duration:

$$\langle g'_{xx} J^x J^{*x} \rangle = \frac{1}{T} \sum_{t=0}^T g'_{xx} J^x(p, t) J^{*x}(p, t)$$

knowing these distributions it becomes possible to elaborate a strategy to attack massively the network. If the distributions

are Dirac's functions, it means that by radiating at the moment when the vertices are weakly emitting, the attacker can hope to spread fake information, it's a first possibility. This can be done because the attacker can look at the variation of the distribution versus time. But to reach effectively the receivers, the attacker takes care of the speeds and angles of the vertices. To transmit as best as possible energy to the receiver, the attacker should align its moment with the moments of the receivers. Another strategy may be to generate emissions synchronously with the higher levels of emission of the group. This allows the attacker to blur the communications. The various strategies can be constructed by making distributions linked with those of the group. The first strategy we discuss uses \bar{D}_p , etc. But as a central point of the success of the strategy is based on the alignment of the antennas, the object Γ_{nm}^A plays a major role in these determinations.

The CPS can use the same distributions to prevent attacks. The strategies for the CPS may be to avoid any predictable suite of configurations to exist. A first rule can be to change the sources of emission and angles following a law that changes regularly but without a regular pattern. The less the distribution presents recognizable patterns, the less the attacker can find a strategy to blur communications or pretending being a member of the group, communicating fake information.

The clear advantage of working in the cartesian space is to synthesize the behavior of the whole group and to allow the study of the distributions. We have two options. A first option consists in the consideration of a common curvilinear space \check{S} on which we make slip a local cartesian space \bar{S} . From loca-

tion to location, the transformation of \check{S} to \bar{S} explains how the curvilinear space evolves, as it is impossible to embrace it globally. Another option is to make \check{S} sliding in front of a standard cartesian space \bar{S} . That's what we do here. The figure 10.9 illustrates the two mechanisms.

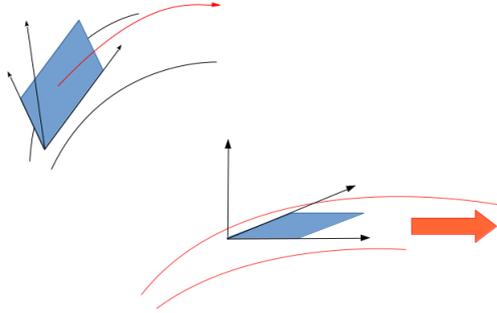


Figure 10.9: Two kinds of slippings

Note that the geodesics followed by the system are defined without sources, forcing the acceleration to zero which leads to:

$$g'_{qn}c^n + Q_{qA}\Gamma_{nm}^A c^m c^n = 0 \quad (10.91)$$

For a flat space, this is reduced to $g'_{qn}c^n = 0$. One more time we find a purely resistive network which doesn't depend on the current values. Let us illustrate our reflections by a two-fold example.

The natural space is the vector:

$$u^k = \begin{bmatrix} i^1 \\ i^2 \\ v^1 \\ v^2 \end{bmatrix} \quad (10.92)$$

We associate with this vector the hamiltonian

$$H = \begin{bmatrix} \sigma_{11} & 0 & 0 & 0 \\ 0 & \sigma_{22} & 0 & 0 \\ -\chi_{31} & 0 & K_{33} & 0 \\ 0 & -\chi_{42} & 0 & K_{44} \end{bmatrix} \quad (10.93)$$

and the sources:

$$f_n = [e_1 \quad 0 \quad f_3 \quad f_4] \quad (10.94)$$

Each impedance is linked with a vertex (we have already made the transformation from the branch space to the mesh one). The inertia is:

$$\mathcal{L} = \begin{bmatrix} L_{11} & -M_{12} & 0 & 0 \\ -M_{21} & L_{22} & 0 & 0 \\ 0 & 0 & m_{33} & 0 \\ 0 & 0 & 0 & m_{44} \end{bmatrix} \quad (10.95)$$

We project each current on a basis of three currents in the x, y, z directions of one standard cartesian space. We connect i^1 and J^x, J^y, J^z , and i^2 with Q^x, Q^y, Q^z . We make similar correspondences between $v^1 \rightarrow w^x, w^y, w^z$ and $v^2 \rightarrow n^x, n^y, n^z$.

These relations define the matrix Λ that transforms \check{S} in \bar{S} for a defined value of time. It means that rather than one value of i^1 we have three values of J^x, J^y, J^z , with $i^1 = \Lambda_k^1 J^k$, etc. And the values of these three currents change with time at the same rythm than i^1 , etc. Exactly the same reasoning is conducted for speeds.

If the two mobiles keep their speed and moment during all the time: for example (J^x, Q^x, w^x, n^y) . If we look to the distribution D_v , it presents two Dirac's functions for the values w^x and n^y . The same for the distribution D_P depending on J^x and Q^x . With these informations it becomes easy to reach the targets and to try to disturb them. It will be the same for any regular profiles of the distributions, revealing periodic positions of the mobiles.

10.9 Entropy

In any system there is a part of cinetic power \mathcal{L} , a part of the power is the radiated power \mathcal{G} and internal power which is the stored power \mathcal{H} . The system benefit of external sources $eJ = w$. We can write:

$$w - \mathcal{L}_{\mu\nu} \frac{d}{dt} J^\mu J^\nu = \mathcal{H}_{\mu\nu} J^\mu J^\nu + \sqrt{\mathcal{G}_{\mu\nu}} J^\mu J^\nu \quad (10.96)$$

The heat Q corresponds to the radiated part of the power. The heat is a process of exchange of energy between two bodys or more. The heat transfer is easily identified through the radiation resistances or any resistances.

A system can receive heat, i.e. radiation from another system. This radiation can be more or less efficient depending on the radiation efficiency of the system. Antennas that are the more efficient systems for radiating, works with established modes. Their number of states are quite easy to numbered, it corresponds to the eigenvalues of the system. This number can be high but in any case is less than the number of degree of freedom of the free radiated field. Equation 10.96 can be written:

$$w - Lp = H + Q = H + \sqrt{G} \quad (10.97)$$

which is similar to the thermodynamic equation. The quantity Q is associated with the radiated field and with the thermal field. The invariance of the power whatever the representation, leads to Parseval's relation. The average radiated power $\langle P \rangle$ is defined by

$$\langle P \rangle = \frac{R_a}{\tau} \int_0^\tau dt i_0^2(t) \quad (10.98)$$

(R_a is the radiation resistance). If we develop the signal using Fourier's serie:

$$i_0(t) = \hat{i}_{00} + \hat{i}_{01} \text{Cos}(\omega_0 t) + \hat{i}_{10} \text{Sin}(\omega_0 t) + \dots \quad (10.99)$$

($\omega_0 = 2\pi/\tau$). By replacing 10.99 into 10.98 we obtain:

$$\langle P \rangle = R_a \left\{ \hat{i}_{00}^2 + \frac{1}{2} \hat{i}_{01}^2 + \frac{1}{2} \hat{i}_{10}^2 + \dots \right\} \quad (10.100)$$

or in general:

$$\langle P \rangle = R_a \left[\hat{i}_{00}^2 + \frac{1}{2} \sum_{n=1}^N \hat{i}_n^2 \right] \quad (10.101)$$

which gives:

$$Q = \frac{1}{2} \sum_{n=1}^N \hat{i}_n^2 \quad (10.102)$$

for the unique radiated part (the thermal part is included in the continuous component that generates infrared radiation associated with Joule's effect). We can choose a signal $i_0(t)$ without any continuous component. For a rectangular signal, each component can be defined depending on the thermal theory of noise with:

$$kT\delta f = \frac{1}{2} R_a i_n^2 \quad (10.103)$$

For the N components this gives $NkT\delta f$. To reach the expression of an energy we integrate versus time our relation to obtain:

$$\int_t dt Q = \int_t NkT\delta f \delta t = NkT \quad (10.104)$$

Creating a quantity S called entropy, defined by:

$$\int_t dt Q = ST \quad (10.105)$$

this leads to: $S = Nk$. The entropy value says how many states the system has. But as N can be enormous, it was chosen to use the logarithm of this number rather than N directly. So:

$$S = k \ln(\Omega) \quad (10.106)$$

with $N = \ln(\Omega)$.

Once entropy was created, it gives one of the major law of physics: when various bodies interact, the total entropy of the system tends to be maximum.

When there is transformation in entropy, this transformation cannot be reversed. When an antenna radiates in the direction of a second antenna, the part of radiation lost in the universe cannot be recuperated. So even if the second antenna radiates the energy it have received from the first antenna, it cannot give it all the energy that the first antenna has radiated. It's again worst with thermal radiation.

If a resonator is excited outside its resonance frequency, the current in the resonator is given by:

$$J = \frac{w}{Lp + \frac{1}{Cp} + R} \quad (10.107)$$

The temperature increasing θ is given by the electrical power dissipated in the resistance:

$$RJ^2 = \sigma_{th}\theta \quad (10.108)$$

The energy stored in magnetic field $1/2LJJ^*$ is not equal to the energy stored in the electric field $1/2CVV^*$.

Now if the source is set at the resonant frequency, the current becomes equal to e/R and is maximum. As a consequence, the dissipation is maximum also. In that case the magnetic energy stored is equal to the electric energy stored. The number of state is drastically reduced: it's the first eigenvalue of the circuit. It means that S decreases and by the fact T must increase: that's the case. The number of degree of freedom of free field is

enormous, while that when it is organized in modes, it is very reduced. And increasing the temperature means to increase the consumption of current in the resistance. This consumption is taken in charge by the increasing of the modal field. Exchanges arise between the source and the modal field, then between the modal field and the thermal field. As a cavity can be seen like a set of resonators, this reasoning can be applied to cavities. The entropy of the universe is increased by this thermal field radiation and respects the thermodynamic law which says that the global entropy must be at the maximum. As said Feynman, the gain in organization inside is compensated by the disorder created at the exterior.

When we think of our cyber physical system, when it exchanges information it decreases the entropy by organizing the electromagnetic space with a controlled field. As the culprit wants to disturb this exchange of information, he tries to increase the entropy. The unique solution for the victim consists in isolating its system, i.e. to make a cavity where he can create various organization of the field through modes giving only disordered field outside. Major problem comes from the limited volume that a cavity can enclose. But this reasoning shows how entropy can be interpreted in electromagnetic compatibility. Shields allow to protect organized fields from outside disturbances which objective is to create disorder.

Finally we write:

$$\frac{dS}{dt} = \frac{1}{T} \left\{ w - \mathcal{L}_{\mu\nu} \frac{d}{dt} J^\mu J^\nu \right\} \quad (10.109)$$

Now we have also the relation⁵:

$$\frac{1}{T} = \frac{\partial S}{\partial E}$$

and so:

$$\frac{dS}{dt} - \left\{ w - \mathcal{L}_{\mu\nu} \frac{d}{dt} J^\mu J^\nu \right\} \frac{\partial S}{\partial E} = 0 \quad (10.110)$$

If we make the assumption that:

$$S = S_0 e^{\alpha t + \gamma E} \quad (10.111)$$

which means that the entropy increases with time and with the energy. Replacing in 10.110, this leads to:

$$\alpha = \gamma \left\{ w - \mathcal{L}_{\mu\nu} \frac{d}{dt} J^\mu J^\nu \right\} \quad (10.112)$$

As long as a source gives energy to the system, the entropy must increase. This increasing comes from the part of dissipation of the system that radiates in the universe, power supplied by the current given by the source. Now if the source disappears, the entropy decreases if only stored energy remains as in this case, the number of state is drastically reduced. The relation can be generalized by:

$$\alpha = \gamma \left\{ w - \left(\mathcal{L}_{\mu\nu} \frac{d}{dt} + \mathcal{H}_{\mu\nu} \right) J^\mu J^\nu \right\} \quad (10.113)$$

⁵See Berkeley's course on statistical physics, page 145 and next

and the metric \mathcal{G} appears clearly to be the operator that enclosed the radiations and dissipations, i.e. the entropy. The magnetic and kinetic energies - in other word the inertia - are the reactions that limit the increasing of entropy.

So, we may have think that movement increases the entropy. But that's not so simple. The movement increases the distance between the systems and so decreases their interactions. But movements increase the number of states, and so the entropy. In fact, total entropy tries to be maximum, but systems increase their organizations, paying for that by increasing their radiations into the universe. Somewhere in another far system, these radiations gives energy under sources form, which creates organization...

The cyber physical systems, the world and the electromagnetic compatibility are entropically linked.

10.10 Systemic

The xTAN⁶ formalism I present in this book can be a good base for system modeling. Robert Vallée was one of the great scientist that has tried to submit models for the systems. His major book *Cognition and System - essai on epistemo-praxeology* gives innovative tracks for this objective. In memory of his work, I present here some thoughts that have made the material of an article for Res-Systemica⁷.

⁶extended tensorial analysis of networks

⁷The journal of the AFSCET French association of systemic: www.afscet.asso.fr

10.10.1 Dynamical systems

Dynamical systems not only imply variables of position, load, etc., but also their time derivatives. A dynamical system is characterized by a second order differential equation, or an integro-differential equation. The time delays exist through the time derivative of the variables that calls for the past values of the variables. This appears clearly in a bloc-diagram drawn by Vallée in [16]. This bloc-diagram is given figure 10.10.

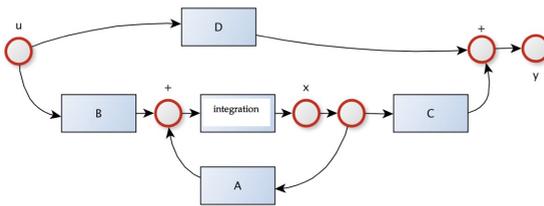


Figure 10.10: Bloc diagram

The equation associated with this bloc-diagram is:

$$\begin{cases} \frac{d}{dt}x = A.x + B.u \\ y = C.x + D.u \end{cases} \quad (10.114)$$

where A, B, C, D are matrices and x, y, u are vectors. This kind of system is classically used in automatic. We see on this figure that, while we use the time derivative in the equation, an

integration appears on the bloc-diagram. This comes the various possibilities that exist to define the space configuration of the problem. The general approach starts from an equation, for example:

$$m \frac{d^2 x}{dt^2} + k \frac{dx}{dt} - mgx = 0 \quad (10.115)$$

(the exact meaning of each variable here has no importance). We can define a new variable:

$$v = \frac{dx}{dt} \quad (10.116)$$

to obtain a system of two coupled equations:

$$\begin{cases} mgx = kv + m \frac{d}{dt} v \\ \frac{d}{dt} x = v \end{cases} \quad (10.117)$$

Looking to the bloc-diagram 10.10, we see that the past of the system evolution is implied through the loop using the operator A . This operator sends the current value of x to a summation implying the next value of x . This operation doesn't appear in the system of equation 10.117 that has the advantage of using only first order time derivative. This form being obtained from the relation 10.116 we may call it "hamiltonian". But we can define two transformations:

$$\begin{cases} \frac{d}{dt} x = v \\ mgx = \int_x dx mg = f \end{cases} \quad (10.118)$$

The problem equations becomes:

$$m \frac{d}{dt} v + kv = f \quad (10.119)$$

The difference may seem harmless but is in fact very important. The force f coming from an external influence g appears clearly here. It results from a form applied to the space and may be called "lagrangian".

Looking back to the system 10.114, we understand that locating the time derivative as member on the left, it gives an easy way to solve the system using finite difference in time domain. It is a classical organization for automaticians. The environment variable u is not separated from the other variables in this writing. Let's write this system locating the environment variables on the left:

$$\begin{cases} B.u = \frac{d}{dt}x - A.x \\ D.u = -C.x + y \end{cases} \quad (10.120)$$

This new system can be represented by a graph shown figure 10.11. This graph is very different from the bloc-diagram. Nodes becomes circles taking in charge various objects, including the sources $B.u$, $D.u$ and operators $p - A$, 1 , while a cord takes in charge the operator $-C$. The bloc of the bloc-diagram becomes cords in this new representation. The first representation leads to the matrices:

$$\begin{bmatrix} \frac{d}{dt}x \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \quad (10.121)$$

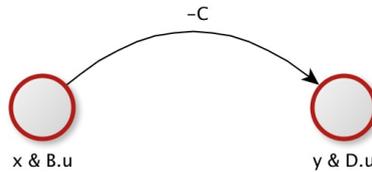


Figure 10.11: System 10.114 graph

The second representation leads to the matrices:

$$\begin{bmatrix} B.u \\ D.u \end{bmatrix} = \begin{bmatrix} p - A & 0 \\ -C & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (10.122)$$

We can call the first representation "ABCD" while the second one can be called a "lagrangian" representation.

Having defined the bases of a dynamical system, we can wonder now how we can control them, observe them and identify them as cybernetic systems (these cybernetic systems are at the origine of the cyber-physical systems).

10.10.2 Controllable and observable cybernetic systems

Neither Vallée nor Wiener uses bio-chemical equations in their descriptions of systems. These equations seems like:

$$A + \epsilon \rightarrow 2A \quad (10.123)$$

where A is a given substance and ϵ some energy. We have seen that xTAN allows to model biological systems like bacteria. Anyway, Vallée definition of systems seems to me a very good one. I recall this definition here:

A system will be called cybernetic if it is possible to distinguish in this system an observation chain, followed by a decision chain. It must be clear that the observation chain allows the system to perceive itself and its environment and some acting organs modify the system and the environment. What a beautiful definition in fact! It contains all the material to define a cybernetic system, even if it appears short. The observation chain includes all the mechanisms of the perception, of the attention, and the understanding of the signals received through the sensors. The decision chain encloses all the thoughts used to choice one action rather than another, then to command actions in response to the sollicitations of the environment. For the observation, Vallée used the concept of "epistemological territory". We note the signals coming from the environment u_α . The state variables of the system are noticed x^σ . In the lagrangian formalism, we clearly separate the exogenous sources u_α from the endogenous states x^σ . In automatic, both variables are considered together. We will see that by separating them we can reach

an deeper understanding of the processus. Vallée suggests the creation of an operator for the observation that transforms the value observed in a understood value. The system 10.122 can be written:

$$v_\alpha = \zeta_{\alpha\sigma} x^\sigma \quad (10.124)$$

v_α gives the exogenous excitations coming from the environment, x^σ are all the state variables of the system, including the inputs, outputs and the hidden ones. The operator ζ links the excitations of the system u_α to its responses and actions x^σ . We have already seen in the classical writing 10.121 that $v_1 = B.u$, $v_2 = D.u$. B and D are operators that give sens for the system to the variables coming from the environment. These operators can act on the passed values of u and v_1 can be a development on u like $B_k u^k$. But how may we take into account the hidden influences of the environment. The ABCD writing is not adequate for this, as it doesn't make the difference between all the state variables. With the xTAN approach we can define domains. This means to develop the operator ζ on functions \mathcal{D} with:

$$\sum_i \mathcal{D}_i = 1 \quad (10.125)$$

To each hidden parameter of the environment q is associated a partition function $\overset{q}{\mathcal{D}}_i$. We can so write:

$$\zeta_{\alpha\sigma} = \overset{q_1}{\mathcal{D}}_1 \overset{q_2}{\mathcal{D}}_1 \dots \zeta_{\alpha\sigma}^{11\dots} + \overset{q_1}{\mathcal{D}}_1 \overset{q_2}{\mathcal{D}}_2 \dots \zeta_{\alpha\sigma}^{12\dots} + \dots \quad (10.126)$$

We know that the whole system of equations 10.124 and the associated interval definitions can be seen as a manifold. Having

defined the observability, it remains to define the controllability and the decision process. We have here one of the key of the epistemo-praxeological loop (EPL). In automatic is often defined the optimal approach as the best approach of systems to reach their objectives. To control the systems there are some automatic feedback like the operator A in 10.10. Similarly, system 10.122 can be written:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - p \begin{bmatrix} \mathcal{L}_{11} & 0 \\ 0 & \mathcal{L}_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -A & 0 \\ -C & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (10.127)$$

where \mathcal{L}_{ij} is a tensor that takes in charge an automatic opposition to the impulse applied to v_α . Here appears all the advantage to separate the environment variables v, q from the state variables x^σ . Because if in general we can influence the state variables x^σ , we cannot influence the environment variables v, q . Even if the system can influence its environment, it is not by the same mechanisms.

10.10.3 Coming to a decision

For Vallée, both environment and states are enclosed in a single vector ϵ . He named this vector a "pragmatic chain". In our lagrangian approach they are separated. The observation operator acts on u_α to generate the usable observations v_α : $v_\alpha = \theta_\alpha^k u_k$. The command comes from the observed value through another operator δ : $\delta [\theta (\epsilon)]$. This new operator changes the observation in an appropriate command. It is clear that

various observations can lead to the same command. There is a kind of integration between the observation and the action. For example, if your hand is on a hot plate, you will retire your hand without asking if the temperature is 80° or 120° ! The xTAN formalism will give us other possibilities to describe the decision process.

The decision comes from a game. Knowing v_1, v_2, \dots, v_a the action of the environment named "nature" in the game, the possible responses of the system are y_1, y_2, \dots, y_a . Actions come from translated perceptions $v_x = \theta(u_x)$. The reactions are visible states of the system $y_x \in x^\sigma$. For each action-reaction (v_x, y_x) we can define the system earning, i.e. a positive or negative consequence for the system in comparison with the objective it wants to reach. We note G_y^v the payoff matrix of the game which contains the earnings for all action-reaction couples. The choices of the system in response to the environment solicitations are bayesian probabilities. We can associate to each response of the system a probability of occurrence given by:

$$P(y)_v [y_x | v_x, \dots] \quad (10.128)$$

Then we can establish the hope of earning E_y for the system gamer for each of its choices y :

$$E_y = G_y^v P(y)_v [y_x | v_x, \dots] \quad (10.129)$$

Between all these hopes of earning, the system will come to the decision k through an operator χ given by:

$$k^w = \chi^{wy} E_y \quad (10.130)$$

this reaction k being associated with a command v through a pragmatic operator ζ for which:

$$v_\beta = \zeta_{\beta w} k^w \quad (10.131)$$

When we compare the two expression under the "ABCD" formalism $\delta [\theta (\epsilon)]$ and under the lagrangian formalism

$$\zeta_{\beta w} \chi^{wy} G_y^v P(y)_v$$

it suggests to consider the observation operator θ applied to the function $\epsilon = f(v_\alpha, y^\sigma)$ as the bayesian probability

$$P(y)_v [y_x | v_x, \dots]$$

The operator δ is associated with the product $\zeta_{\beta w} \chi^{wy} G_y^v$. This identification seems to be legitimate. It makes in front of the couples ϵ the decisions k^w through the operation $\chi^{wy} G_y^v P(y)_v [\epsilon]$ which Vallée named "command \mathcal{P} "⁸. The fact that the environment and the action are linked in a common bayesian probability confirms Vallée's proposal. But they keep to different nature in the xTAN formalism, contrary to the ABCD formalism. We will see that it leads to a larger description of the process that exists between perception and decision.

10.10.4 EPL

The system state can finally be identified through a vector \mathbf{V} in a co-space $(u_\alpha, \zeta_{\alpha\nu} x^\nu, \zeta_{\alpha w} k^w)$. The inverse of $\zeta_{\alpha\nu}$ is called by

⁸"Cognition and System", page 87

Vallée "effecton operator" and noticed \mathcal{A}_s . But this operator that acts on the past and current states must create after the chronicle x^ν of the system evolution. This operator corresponds to the difference between commands and actions. It sends back the state covector \mathbf{V} to the state x . But the component of \mathbf{V} linked with x is $\zeta_{\alpha\nu}$. That's why $\mathcal{A}_s = (\zeta_{\alpha\nu})^{-1}$. By writing now: $u_e = u_\alpha$, $u_a = \zeta_{\alpha\nu}x^\nu$ and $u_d = \zeta_{\alpha w}k^w$, we arrive to the representation given figure 10.12 and submitted by Vallée in [16].

Vallée has well understood the importance of the link between environment and states. We have seen the earning of the system. But the nature plays also, and its earning is the influence of the system on the environment. Vallée has added an operator \mathcal{A}_e in a second step to take into account this feedback. We can define an environment decision h^w and follow the same reasonings as for the system. It makes the projection of ϵ on u_e as shown figure 10.12. Vallée's invention is synthesized in the covector \mathbf{V} . This covector is a complete image of the system state, including its interactions with the environment and its decisions in response to these sollicitations. Perhaps a mathematical image but a reliable image of what is a system. And xTAN formalism gives all the material to go from the system to the whole universe without asking how to proceed. It gives the cord based on game theory, essential piece of Vallée's EPL. All the complex concepts of a real system, perception, action, decision are included in \mathbf{V} under the EPL representation. It remains always difficult to define what is a system, what systemic covers. Rather than giving long sentences often meaningless, Vallée mathematical concepts give here a self consistant defini-

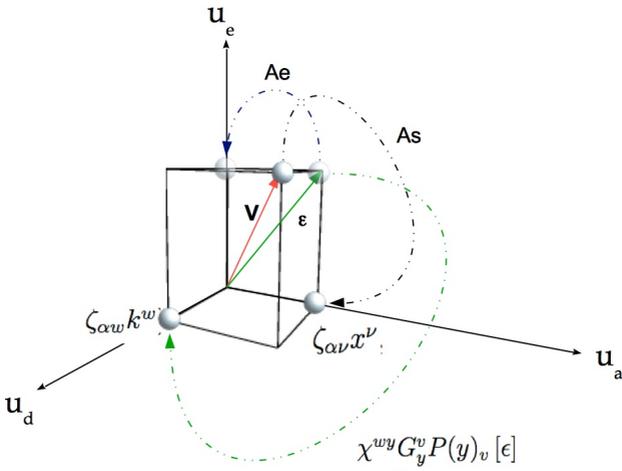


Figure 10.12: EPL draw

tion and xTAN formalism provides the mathematical objects to clearly write the equations. We have confirm that a system is a manifold, but a new kind of manifolds enclosing probabilities. A new field to continue the research on systems and mathematics.

This is my conclusion.

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