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# Acoustics of Multi-Porous Media: Investigation of Interscale Couplings

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## Abstract

This paper is devoted to the investigation, via homogenisation theory, of the dynamic behaviour of multiple scale porous media. Both the phenomenon and the structure of the multi-porous medium introduce distinct characteristic lengths from which scale ratios may be defined. The influence of the relationships between all scale ratios is highlighted through the analysis of interscale couplings. The derived macroscopic descriptions are presented in the case of high permeability contrasts and then for very high permeability contrasts.

## 1 Introduction

The continuous representation of a given heterogeneous medium makes sense only when there is a *separation of scales* or when a property of *local invariance* is verified. This actually imposes the two following requirements: i) The material must be sufficiently regular so that a Representative Elementary Volume can be defined; ii) The phenomenon must vary with respect to a characteristic length which is widely greater than the REV's size. If both conditions are fulfilled, then homogenisation methods allow the definition of the equivalent continuous medium from the characteristics of the REV and the interactions occurring between the REV constituents. Upscaling issues are usually - implicitly or explicitly - investigated on the basis of two scale media. The structure of these media is thus quite simple and is characterised by a single scale ratio.

The purpose of this work is to examine heterogeneous media of higher complexity that contain one or several intermediate or mesoscopic scales between the microscopic and the macroscopic scales. When the hierarchical heterogeneous medium is a porous medium, it is referred as a dual-porosity or a multi-porous medium (Figure 1). When the microscopic and the mesoscopic

Figure 1: *Two examples of dual-porosity media. Left: Porous medium that consists of porous grains. Right: Porous medium with high contrasts between constituent permeabilities*

sizes are distinct, the physics at both scales may strongly differ. A great variety of interscale couplings and of macroscopic behaviours can then be observed. This was investigated for instance for petroleum engineering applications by phenomenological approaches (Barenblatt et al. 1960) or via homogenisation (Royer and Auriault 1994).

In this paper, interscale couplings are investigated in the context of acoustics of multi-porous media saturated by air. The interest of this example is two-fold:

- i) The simplicity of the physics allows to clearly examine the possible interscale couplings and their consequences upon the associated macroscopic behaviour.
- ii) Artificial porous media are frequently used for reducing ambient noise, (Allard, 1993). According to the applications, the size of the constituents may range from  $10cm$  to  $10\mu m$ . This allows to suggest the use of artificial multi-porous media in order to increase the level of noise absorption.

This lecture is organised in five sections. In Section 2, the homogenisation method for multiple scale media is briefly presented. Section 3 is devoted to the investigation of acoustics in a single-porosity medium. The purpose of Section 4 is to examine the various possible dual-porosity situations. These situations are determined from the main features of acoustics in single-porosity media. Finally, section 5 focuses on the interpretation of the derived interscale couplings.

## 2 Homogenisation Applied to Multi-Scale Media

The homogenisation method as it is presented in (Sanchez-Palencia, 1980) is mainly devoted to two-scale upscaling. Anyhow, this approach that allows rigorous upscaling from the microscopic scale to the macroscopic scale can also be applied when several successive upscaling processes are required (Bensoussan et al., 1978), e.g. from the microscopic scale ( $l_\mu$ ) to the mesoscopic scale ( $l_m$ ) and then to the macroscopic scale ( $L$ ) (Auriault and Boutin, 1993, 1994). The three well distinct characteristic lengths  $l_\mu$ ,  $l_m$  and  $L$  are associated with the REV at the microscopic

scale,  $(REV)_\mu$ , the REV at the meso-scale,  $(REV)_m$ , and the phenomenon, respectively. From these three characteristic lengths, three space variables,  $x$ ,  $y$ ,  $z$  may be defined for describing the three scales. Let define the two following scale ratios:

$$\varepsilon = \frac{l_m}{L} \ll 1; \quad \varepsilon_m = \frac{l_\mu}{l_m} \ll 1.$$

With respect to these scale ratios the space variables are such that:

$$y = \varepsilon^{-1}x; \quad z = \varepsilon_m^{-1}y.$$

In dynamics,  $L$  is either the wavelength or related to the medium dimension, if this latter is lower than the wavelength (Boutin and Auriault, 1990). Thus, for a given medium, the actual "physical"  $\varepsilon$  varies in a wide range of values, whereas  $\varepsilon_m$  depends only upon the material and is therefore a constant.

As a result of the separation of scales, *each medium behaves as a continuum at the upper level*. Therefore, each quantity  $q_m$  defined at the meso-scale is looked for in the form of an asymptotic expansion in powers of  $\varepsilon$ , with respect to the macrovariable  $x$  and to the mesovvariable  $y$ , only:

$$q_m(x, y) = \sum \varepsilon^j q_m^j(x, y) \quad \text{where} \quad q_m^j/q_m^0 = O(1). \quad (1)$$

On the other hand, any quantity  $q_\mu$  defined at the microscale is developed in powers of both  $\varepsilon$  and  $\varepsilon_m$ , using the three space variables  $x$ ,  $y$  and  $z$ :

$$q_\mu(x, y, z) = \sum \varepsilon_m^j \varepsilon^i q_\mu^{j,i}(x, y, z) \quad \text{where} \quad q_\mu^{j,i}/q_\mu^0 = O(1). \quad (2)$$

With this homogenisation technique, the existence of the REV is replaced by the assumption of periodicity at the corresponding scale, which also entails the periodicity of the physical quantities. Thus, the functions  $q_m^j$  are periodic with respect to the variable  $y$  and the functions  $q_\mu$  are periodic with respect to  $y$  and  $z$ .

As a consequence of the separation of scales, the boundary conditions over the micro-meso boundary are expressed at the mesoscopic level, i.e. they relate quantities of the meso-domain to *homogenised* quantities of the micro-domain.

Finally, whatever is the number of scales, the homogenisation process consists of two steps:

- i) To perform a physical analysis of the phenomenon and to rescale the equations by using powers of  $\varepsilon$  (or  $\varepsilon_m$ ) for expressing the order of magnitude of the dimensionless terms.
- ii) To incorporate the asymptotic expansions into the rescaled equations and to identify the terms of identical power in  $\varepsilon_m^j \varepsilon^i$ , and to solve the boundary-value problems obtained at the successive orders.

The purpose of the next sections is to apply this method for modelling acoustics of porous media saturated by air. The first step is explained in details and then the results obtained from the second step are presented.

# 3 Adiabatic Acoustics of a Single-Porosity Medium Saturated by Gas

## 3.1 Noise Absorbing Porous Media

Dry porous media, i.e. porous media saturated by air, present interesting acoustic properties for reducing the level of ambient noise. Inside buildings, such media are now very frequently used as wall lining. On the outside, the design of anti-noise walls, for instance, takes advantage of these properties to reduce sound intensity in the vicinity of railways or motorways. The materials used for this purpose are often coarse, made of granular aggregates and have a pore size ranging from few millimetres to few centimetres. Another example is that of porous road surfacing used in road engineering. These materials, which present a large intrinsic permeability ( $10^{-9}m^2$ ) are known for decreasing traffic noise from 20 to 10 dB (Bar and Delanne, 1993).

The phenomenon of sound absorption is due to the fact that when an acoustic wave arrives on a pervious surface, air is pushed within the pores. Therefore, only a partial reflection of the wave occurs, and the transmitted wave is damped. The description of these phenomena requires the physical analysis of gas flow within the pores. The reader is referred to (Allard, 1993) for a detailed presentation of these aspects.

## 3.2 Local Assumptions and Macroscopic Description

In this study, the four following main assumptions are made:

- i) As a result of the low pressure level, the porous skeleton is assumed to be perfectly rigid;
- ii) Air is assumed to be a viscous fluid and its flow in the pores is governed by Navier-Stokes equation;
- iii) Since acoustic motions are very small, convection or advection effects are negligible, and the non-linear terms are ignored.
- iv) For simplicity, heat exchanges are neglected and as a result the perturbations in the gas are assumed to be adiabatic. The treatment of thermal effects, (Boutin et al., 1998) is presented in Appendix A.

Under harmonic pulsations the local linearised governing equations are given below;  $v$  is the velocity,  $p$  is the pressure variation,  $\mu$  is the gas viscosity and  $\gamma$  its specific heat ratio,  $P$  and  $\rho$  are the pressure and the density at the equilibrium, respectively. The term  $e^{i\omega t}$  will be omitted.

**In the gas ( $\Omega$ )**

$$\text{Compressibility} \quad i\omega p/(\gamma P^e) + \text{div}(v) = 0 \quad (3)$$

$$\text{Navier-Stokes equation} \quad -\text{grad}(p) + \mu\Delta(v) = \rho^e i\omega v \quad (4)$$

**At the gas-solid interface ( $\Gamma$ )**

$$\text{No-slip condition} \quad v = 0 \quad (5)$$

Under wave propagation the pressure and the volume variation oscillate according to the wavelength (the macroscopic size) whereas the velocity varies at the pore scale. Then, using the macroscopic length as the reference length, only Navier-Stokes equation needs to be rescaled and when rescaled it is written as:

$$-\text{grad}(p) + \mu\varepsilon^2\Delta(v) = \rho^e i\omega v \quad (6)$$

This leads to the problem of dynamic flow through a porous medium [Biot, 1956], [Auriault, 1980]. The macroscopic behaviour is described by the following equations where  $P$  is the pressure - which is *homogeneous in the pores* - and  $V$  the average velocity ( $\phi$  is the porosity) :

$$\text{Compressibility} \quad \text{div}(V) + i\omega\phi P/\gamma p^e = 0 \quad (7)$$

$$\text{Dynamic Darcy's law} \quad V = -K^* \text{grad}(P) \quad (8)$$

### 3.3 Dynamic Permeability $K^*$

The acoustic characteristics of the system are related to the properties of the dynamic permeability  $K^*$ . These properties are described in details for example in (Auriault et al, 1985) for a medium saturated by a liquid.

At low frequencies, viscous effects are predominant and  $K^*$  tends towards  $k/\mu = O(\phi l^2)$ , where  $k$  is the intrinsic permeability. We therefore retrieve the classical Darcy's law.  $k$  is related to the flow resistivity,  $\sigma$ , which, in acoustics, is usually defined by:  $\sigma = \mu/k$ .

At high frequencies, inertial effects are dominant and then  $K^*$  tends towards a pure imaginary value:  $K^*(\infty) = O(\phi\mu/i\omega\rho^e\tau)$ . Hence, at high frequencies, the dynamic Darcy's law tends towards a classical dynamic equation in which air density is corrected by the tortuosity  $\tau$ , which highlights the influence of the "added" mass ( $\tau$  is generally  $O(1)$ ).

Low and high frequency domains are delimited by the characteristic pulsation  $\omega_c$  which is such that both viscous and inertial effects are of the same order of magnitude:  $\omega_c = O(\mu\phi/k\rho^e)$ .

In summary, two domains may be distinguished:

#### **The viscous domain**

$$\omega < \omega_c; \quad K^* = O(k/\mu)$$

#### **The inertial domain**

$$\omega > \omega_c; \quad K^* = O(\phi\mu/i\omega\rho^e\tau)$$

The dynamic permeability may also be related to the effective density, which, in acoustics, is commonly defined by:

$$\rho_{eff}^* = \mu\phi/K^*$$

Analytical expressions can be derived for very simple duct geometries (Biot, 1956). For any pore geometry, an expression for  $\rho_{eff}^*$  is proposed in (Allard et al., 1993), which gives the correct asymptotic behaviour at low and high frequencies and that includes viscous layer effects:

$$\rho_{eff}^* \cong \tau\rho^e [1 + (1 + i\omega^*/F^2)^{1/2}/i\omega^*] \quad \text{with} \quad \omega^* = \omega/\omega_c,$$

where  $F$  is a shape ratio ranging from  $\sqrt{2}$  (for cylindrical pores) to 4, with respect to the pore geometry (Ping Sheng and Min-Yao Zhou, 1988).

### 3.4 Wave Propagation

The wave propagation equation is derived from equations (7) and (8), by eliminating the velocity  $v$ :

$$K^* \Delta(P) - i\omega\phi P/\gamma P^e = 0 \quad (9)$$

This macroscopic behaviour can be compared to Biot's theory. However, since the solid is rigid, the single acoustic wave is of  $P_2$  type (Biot, 1956). The complex valued velocity  $C(\omega)$  may directly be deduced from equation (9). In the isotropic case we get:

$$C^2 = -\gamma P^e K^* \omega / i\phi\mu = C_a^2 i K^* \omega^*,$$

where  $C_a$  is the sound velocity and is defined by:

$$C_a = (\gamma P^e / \rho^e)^{1/2}$$

Also from the wave propagation equation, it can be shown that the wave characteristic length is such that:

$$(\lambda/2\pi)^2 = K^* \gamma P^e / i\omega\phi.$$

At low frequencies ( $\omega < \omega_c$ ), we get  $C \cong C_a \sqrt{i\omega^*}$ , which shows that the waves are diffusive with a diffusion skin depth  $\lambda^v/2\pi = O(\gamma P^e k / \omega\phi\mu)^{1/2}$ .

At high frequencies ( $\omega > \omega_c$ ), the acoustic celerity is reduced by the tortuosity and can be approximated by  $C \cong C_a [1/\sqrt{\tau} + O(1/\sqrt{i\omega^*})]$ . Therefore, the waves are propagative and attenuated with a wavelength  $\lambda^i/2\pi = O(C_a/\omega)$ .

## 4 Interscale Couplings in a Dual-Porosity Medium

### 4.1 Analysis of a Dual-Porosity Medium

Let now consider a dual-porosity medium. At the meso-scale, the REV,  $(REV)_m$  consists of a pore network  $(\Omega_m)$  of dynamic permeability  $K_m^*$ , and of a microporous domain  $(\Omega_s)$ , which is characterised by  $(REV)_\mu$  and by the dynamic permeability  $K_\mu^*$ , which is due to the micropores  $(\Omega_\mu)$ . Figure 2 shows the acoustic characteristics of both media when considered separately.

The **low frequency** behaviours of both media are **very different**:

Since  $l_\mu \ll l_m$ , we have:

$$k_\mu/k_m = O((l_\mu/l_m)^2) = \varepsilon_m^2 \ll 1.$$

Moreover,

$$\omega_{cm}/\omega_{c\mu} = k_\mu/k_m = \varepsilon_m^2 \ll 1.$$

Figure 2: Acoustic characteristic of two single-porosity media of intrinsic permeability  $k_m$  (bold line),  $k_\mu$  (dashed line),  $k_m \gg k_\mu$ . Log-log scale representation of approximate values. Left: Dynamic permeability versus pulsation.  $V$  and  $I$  stand for the viscous and the Inertial domains, respectively. Right: Wavelength versus pulsation.  $D$  and  $P$  stand for the Diffusive and the propagative domains, respectively.

In contrast, the **high frequency** behaviours of both media are **very similar**:

$$(K_\mu^*/\phi_\mu)/(K_m^*/\phi_m) = O(\tau_m/\tau_\mu) = O(1).$$

Therefore, a priori three regimes of interscale couplings should be observed (Olny, 1999).

<b>Low frequency</b>	<b>Intermediate frequency</b>	<b>High frequency</b>
$\omega < \omega_{cm}$	$\omega_{cm} < \omega < \omega_{c\mu}$	$\omega > \omega_{c\mu}$
$K_m^* = O(k_m/\mu)$	$K_m^* = O(k_m/\mu)$	$K_m^* = O(\phi_m/i\omega\rho^e\tau_m)$
$K_\mu^* = O(k_\mu/\mu)$	$K_\mu^* = O(\phi_\mu/i\omega\rho^e\tau_\mu)$	$K_\mu^* = O(\phi_\mu/i\omega\rho^e\tau_\mu)$

The nature of the interscale couplings is furthermore influenced by the order of magnitude of the permeability contrast with respect to  $\varepsilon$ . The two following cases will be successively considered:

i) The generic situation, i.e. identical scale ratios between the three scales:  $\varepsilon_m = \varepsilon$ . This situation corresponds to a high permeability contrast:  $k_\mu/k_m = O((l_\mu/l_m)^2) = \varepsilon^2$ .

ii) The case of a very high permeability contrast,  $k_\mu/k_m = O((l_\mu/l_m)^2) = \varepsilon^4$ , which means that  $\varepsilon_m = \varepsilon^2$ .

The separation of scales and the relationship between both scale ratios provides a guideline for identifying the physics at both scales for whatever the situation under consideration is.

The perturbations at the meso-scale can be significant but the orders of magnitude of the different terms remain the same as in a single porosity medium. Therefore, the order of magnitude of the characteristic macroscopic length  $\lambda$  is such that  $(\lambda/2\pi) = O((\gamma P^e K^*/\omega\phi)^{1/2})$ , with:  $K^*/\phi = O(K_m^*/\phi_m)$ . This also means that the rescaled equation (3) remains valid for any other



situation that will be considered.

The microporous domain, that is embedded within the mesopores, behaves as a single-porosity medium which is submitted, at its boundary, to harmonic *meso-homogeneous* conditions. Thus, the characteristic size of the oscillations in this domain is given by  $(\lambda_\mu/2\pi) = O((\gamma P^e K_\mu^*/\omega \phi_\mu)^{1/2})$ .

## 4.2 High Permeability Contrast ( $\varepsilon_m = \varepsilon$ )

An important issue is the determination of the order of magnitude of the fluid velocity in the micropores,  $v_\mu$ , relatively to the order of magnitude of the velocity in the mesopores,  $v_m$ . The fact that both scale ratios are of the same order ( $\varepsilon_m = \varepsilon$ ), implies that the thickness of the diffusion skin in the microporous domain,  $\lambda_m^v/2\pi$ , is larger than or of the same order as the characteristic dimension of the microporous regions  $d_m = O(l_m)$ . In effect, situations for which  $\lambda_m^v/2\pi \ll l_m$  are not homogenisable since they can occur only for a frequency range of  $(\lambda/2\pi) = O(l_m)$ , which corresponds to a diffraction regime. Now, as a result of the pressure continuity over the meso-micro boundary, the pressure is of the same order in both media:  $p_\mu = O(p_m)$ . As a consequence of the compressibility equation, we get:

$$\text{div}(v_m) = O(\text{div}(v_\mu)) \quad , \text{ hence } \quad v_m/(\lambda/2\pi) = O(v_\mu/(\lambda_m/2\pi)),$$

from which we deduce

$$v_\mu/v_m = O((K_m^*/K_\mu^*)^{1/2}).$$

This leads to distinguish two distinct cases, for which the rescaled form of Navier-Stokes equation in the micropores (using  $v_m$  and  $\lambda$  as reference values) will be different:

### Low and Intermediate frequencies

$$v_\mu/v_m = O(\varepsilon)$$

$$-\text{grad}(p_\mu) + \mu\varepsilon^2\Delta(v_\mu) = \rho^e i\omega v_\mu$$

### High frequencies

$$v_\mu/v_m = O(1)$$

$$-\text{grad}(p_\mu) + \mu\varepsilon^4\Delta(v_\mu) = \rho^e i\omega v_\mu$$

## 4.3 Very High Permeability Contrasts ( $\varepsilon_m = \varepsilon^2$ )

When  $\varepsilon_m = \varepsilon^2$ , according to the value of  $d_m$ , the depth of the diffusion skin in the microporous domains  $\lambda_m^v/2\pi$ , can be - in the intermediate frequency range - of one order lower than  $d_m$  for avoiding the diffraction regime.

Then, there is no pressure variation in the microporous with the exception of the very thin diffusion skin at its boundary. Therefore, the velocity ratio is of one order lower than  $O(\sqrt{K_m^*/K_\mu^*})$  and we get the following rescaled Navier-Stokes equation at intermediate frequencies:

$$v_\mu/v_m = O(\varepsilon^3) \quad - \text{grad}(p_\mu) + \mu\varepsilon^2\Delta(v_\mu) = \rho^e i\omega v_\mu$$

## 4.4 Comments on the Thermal Effects

The above analysis is based upon the comparison of the dynamic permeabilities  $K_m^*$  and  $K_\mu^*$ , which are fully independent of thermal effects (see Appendix A). Therefore, the reasoning remains valid even in presence of non-adiabatic perturbations in the gas.

## 5 Acoustics of Double and Multi-Porous Media

### 5.1 The Macroscopic Descriptions

Applying the homogenization procedure described in section 2 leads to the following macroscopic descriptions with respect to the frequency range and to the the permeability contrast (Boutin et al., 1998), (Olny, 1999). Here again the perturbations are assumed to be adiabatic. The description that account for thermal effects is given in Appendix B.

**High permeability contrast:**  $\varepsilon_m = \varepsilon$

Frequency	Compressibility	Dynamic Darcy's law
Low	$div(V) + i\omega(\phi_m + (1 - \phi_m)\phi_\mu)P/\gamma P^e = 0$	$V = -K_m^* \cdot grad(P)$
Intermediate	$div(V) + i\omega[\phi_m + (1 - \phi_m)\phi_\mu(1 - \Pi^*)] \cdot P/\gamma P^e = 0$	$V = -K_m^* \cdot grad(P)$
High	$div(V) + i\omega(\phi_m + (1 - \phi_m)\phi_\mu)P/\gamma P^e = 0$	$V = -L_m^* \cdot grad(P)$

**Very high permeability contrast:**  $\varepsilon_m = \varepsilon^2$

Frequency	Compressibility	Dynamic Darcy's law
Intermediate	$div(V) + i\omega\phi_m P/\gamma P^e = 0$	$V = -K_m^* \cdot grad(P)$

The interscale couplings in theses four cases can be summarised in the following manner:

At *high frequencies* ( $\omega > \omega_{c\mu}$ ), the physics is the same at both levels and then there is a full interscale flux coupling. The macroscopic dynamic Darcy's law ( $L^*$ ) accounts for the presence of the micropores.

At *intermediate and low frequencies* ( $\omega < \omega_{c\mu}$ ), the coupling is lower so that the dynamic macroscopic Darcy's law is that given by the mesopores only ( $K^*$ ). The coupling appears on the macroscopic effective compressibility, which takes into account the micropores gas flux coming from the pressure diffusion in micropores. This effect, characterised by the new complex valued and frequency dependent diffusion function  $\Pi^*$ , is examined in details hereafter.

### 5.2 Pressure Diffusion at the Meso-scale

The evidence of pressure diffusion at the mesoscale is obtained in three steps. The flows within the micropores and the pores are successively analysed and finally the coupling flux between both scales is derived.

### 5.2.1 In the Micropores

The first significant orders for the average flow ( $V_\mu^1$ ) through the micropores ( $\Omega_\mu$ ) lead to a classical *steady-state* Darcy's law:

$$V_\mu^1 = -(k_\mu/\mu)grad(P_\mu^0)$$

Now, incorporating the compressibility equation over the microporous period ( $\Omega_s$ ) yields the mass balance equation in this domain:

$$div_y(V_\mu^1) + i\omega\phi P_\mu^0/\gamma P^e = 0$$

given by:

$$div_y(-(k_\mu/\mu) \cdot grad(P_\mu^0)) + i\omega\phi_\mu P_\mu^0/\gamma P^e = 0$$

### 5.2.2 In the pores

The resolution procedure for the flow is classical and yields:

$$V_m^0 = -(K_m^*) \cdot grad(P_m^0) \quad \text{with} \quad P_m^0 = P_m^0(x)$$

Now, the mass balance obtained over the pore volume  $\Omega_m$  gives:

$$div_x(V_m^0) + \langle div_y(V_\mu^1) \rangle_\Omega + i\omega\phi_m P_m^0/\gamma P^e = 0$$

In order to determine the additional term due to  $V_\mu^1$ , we must go back to the micropore scale. The actual pressure distribution in the whole micropore domain ( $\Omega_s$ ) is defined by the following boundary-value problem:

$$div_y(-(k_\mu/\mu) \cdot grad(P_\mu^0)) + i\omega\phi_\mu P_\mu^0/\gamma P^e = 0$$

with

$$P_\mu^0 = P_m^0(x) \quad \text{over the pore-micropore interface.}$$

Using the pressure difference  $P' = P_\mu^0 - P_m^0$ , this problem can be rewritten in the following equivalent form:

$$div_y(-(k_\mu/\mu) \cdot grad(P')) + i\omega\phi_\mu(P' + P_m^0)/\gamma P^e = 0$$

with

$$P' = 0 \quad \text{over the pore-micropore interface.}$$

It turns out that the pressure difference is governed by a diffusion equation with a forcing term. Note that this boundary value problem looks like the heat transfer problem encountered for the temperature field (see Appendix A). We deduce that the pressure fields in the micropores and in the pores are related by:

$$P' = -\pi P_m^0 \quad \text{so that: } P_\mu^0 = (1 - \pi)P_m^0$$

where the function  $\pi(y)$  is complex, depends on the local variable  $y$  and on the pulsation  $\Omega$ . From the average mass balance equation on the whole micropores ( $\Omega_s$ ) and the boundary condition at the pore-micropore interface we deduces that:

$$\langle \text{div}(V_\mu^1) \rangle_{\Omega_s} = i\omega\phi_\mu(1 - \phi_m)(1 - \Pi^*)P_m^0/\gamma P^e$$

where

$$\Pi^* = \langle \pi \rangle_{\Omega_s}$$

### 5.3 The Diffusion Function $\Pi^*$

The difference between the macroscopic mass balance for the dual-porosity model and the single porosity model consists in replacing the porosity by new complex valued term. This latter corresponds to the influence of air that saturates all the micropores of the grains. It highlights a coupling effect between pore and micropore air fluxes, which is due to the diffusion of the pore pressure in the micropores. This phenomenon is expressed by the complex valued function  $\Pi^*$ , which depends on the dimensionless pulsation  $\omega/\omega_d$ . The characteristic pulsation is such that, when  $\omega = \omega_d$ , the thickness of the diffusion skin  $\lambda_m^v/2\pi$  is equal to the grain size,  $d_m$ . Let define the length  $\Lambda_d$  as the ratio of the grain volume to the surface. A good estimation of  $\omega_d$  is given by :

$$\omega_d = k_m P^e / \mu \phi_\mu (\Lambda_d)^2$$

Three possibilities may exist according to the frequency and the dimension  $d_m$  of the microporous domain. The classification from low to intermediate frequency is deduced from the comparison of the diffusion skin  $\lambda_m^v/2\pi$  with  $d_m$ :

$\lambda_m^v/2\pi \gg d_m$ : The transient diffusion effects are negligible. Therefore, the micropore pressure is homogeneously diffused through the micropores with the same level as in the pores. The total gas volume contributes to the compressibility and the coupling appears in the macroscopic mass balance which includes both meso and microporosity ( $\phi_m + (1 - \phi_m)\phi_\mu$ ). Note that since the dynamic Darcy's law affects the pore volume only the description does not reduce to a single porosity.

$\lambda_m^v/2\pi = O(d_m)$ : The pressure is diffused through the micropores, out of phase with the pore pressure. The coupling appears in the macroscopic compressibility which becomes complex ( $\phi_m + (1 - \phi_m)\phi_\mu(1 - \Pi^*)/\gamma P^e$ ).

$\lambda_m^v/2\pi \ll O(d_m)$ : Diffusion affects air only in the vicinity of the pore walls so that the pressure does not vary in the micropores. Then the effective compressibility reduces to the compressibility given by the pores. In this case the macroscopic description is totally ignoring the presence of the microporous domain.

To conclude, at the macroscopic scale, a new dissipation effect appears in the frequency range

where the influence of diffusion is maximum i.e. for pulsation close to  $\omega_d$ .

Obviously, for a given dual-porosity medium, a particular analysis based on the value of  $k_m$ ,  $k_\mu$ ,  $l_m$ ,  $d_m$  must be performed to identify which of these four cases can actually be observed in the various frequency ranges.

In the case of multiple porosity this analysis is also applicable for each level. A great number of situations can be encountered according to the characteristics of each porous domain.

## 6 Conclusions

In this work, various macroscopic descriptions of sound propagation through a rigid porous medium saturated by air are derived using homogenisation theory. The results presented are valid as long as the wavelength is large in comparison with the pore size.

The first part of the paper deals with single porosity materials for which thermal exchanges are neglected. The thermal effects are included in the descriptions given in Appendix A. The derived descriptions are the same as those already obtained via phenomenological approaches (Allard, 1993), (Attenborough, 1983). Air flow is governed by a dynamic Darcy's law where both viscous and inertial effects act, and the effective compressibility is influenced by thermal exchanges.

In the second part, we focus on dual porosity media, i.e. media in which the grains of the skeleton are microporous. Let us notice that the results could also be applied to granular media that consist of grains of very different sizes.

The macroscopic descriptions are obtained by neglecting heat transfer effects (thermal transfers are included in the description given in Appendix B). It is shown that the physics in the micropores and in the pores are very different. Due to the low micropore permeability and to air compressibility, the pressure is inhomogeneously diffused in the micropores. This phenomenon implies increased acoustic attenuation for pulsation of the order of magnitude of  $\omega_d$ . These results prove that the introduction of a microporosity could be used designing new porous materials that would provide enhanced absorption properties in a given frequency range.

The presence of multiple scales can significantly modify the macroscopic behaviour. Moreover the nature of the equivalent continuous medium cannot be obtained with a two-scale upscaling procedure. The several descriptions are related to the diversity of the interscale couplings. Such results are mainly due to the fact that the phenomenon presents an intrinsic length (the viscous layer), and therefore the physics - which is governed by the ratio of this length to the pore size - is different at the different scales.

More generally for any physical problem which involves an intrinsic length at the local scale (depth of diffusion layer, wavelength,...), the presence of multiple scales should *modify the nature* of the macroscopic description. Such possibility is rather frequent when studying harmonic dynamics or transient regimes.

## 7 References

- Allard J.F (1993): *Propagation of Sound in Porous Media. Modelling Sound Absorbing Materials*, Chapman and Hall, London.
- Attenborough K. (1993): Acoustical Characteristics of Rigid Fibrous Absorbents and Granular Media, *J. Acoust. Soc. Am.*, **73** (3), 785-799.
- Auriault J.L. (1980): Dynamic Behaviour of Porous Media saturated by a Newtonian Fluid, *Int. J. Eng. Sci.*, **18**, 775-785.
- Auriault J.-L., Boutin C. (1993-94): Deformable Porous media with Double Porosity. I: Quasi-Statics; II: Memory Effects; III: Acoustics, *TIPM*, **7**, 63-82; **10** 153-169; **14**, 143-162.
- Auriault J.-L., Royer P. (1993): Ecoulement d'un gaz dans un milieu double porosité, *Compte rendu l'Académie des Sciences*, Paris, **317**, (II), 431-436.
- Bar P., Delanne Y. (1993): *Réduire le bruit Pneumatique-Chaussées*, LCPC. Presses des Ponts et Chaussées.
- Barenblatt G.I., Zheltov I.P., Kochnina I.N. (1960): Basic Concepts in the Theory of Seepage of Homogeneous Liquids in Fissured Rocks, *PMM*, **24** (5), 852-864.
- Bensoussan A., Lions J.-L., Papanicolaou G. (1978): *Asymptotic Analysis of periodic Structures*, North Holland Publishing Company, Amsterdam.
- Biot M.a. (1956): The theory of Propagation of Elastic Waves in a Fluid Saturated Porous Solid, I Low Frequency range, II Higher Frequency Range, *Acoust. Soc. Am.*, **28**, (2), 168-191.
- Boutin C., Auriault J.-L. (1990): Dynamic behaviour of Porous Media Saturated by a Viscoelastic Fluid. Application to Bituminous Concrete, *Int. J. Eng. Sci.*, **28** (11), 1157-1181.
- Boutin C., Royer P., Auriault J.-L. (1998): Acoustic Absorption of Porous Surfacing with Dual Porosity. *Int. J. Solid Structures*, **35** (34-35), 4709-4737.
- Olny X. (1999): *Absorption acoustique des milieux poreux à simple et à double porosité. Modélisation et validation expérimentale*, Thèse ENTPE/INSA, Lyon, France.
- Royer P., Auriault J.-L. (1994): Transient Quasi-Static Flow through a Porous rigid Medium with Double Porosity, *TIPM*, **17**, (33), 33-57.

Sanchez-Palencia E. (1980): *Non-Homogeneous Media and Vibration Theory*, Lecture Notes in Physics, 127, Springer-Verlag, Berlin.

## A Acoustics of Single-Porosity Media with Thermal Effects

### A.1 Introduction

The adiabatic analysis is valid for large pores. For a more general description, and in particular for smaller pores, thermal effects must be accounted for. In this aim, we may incorporate the equation of heat conduction together with the air state equation:

$$\operatorname{div}(\kappa.\operatorname{grad}(T)) = i\omega(\rho^e c_p T - P)$$

with

$$P = P^e(\rho/\rho^e + T/T^e) \tag{A1}$$

where  $T^e$  is the equilibrium temperature,  $\kappa$  and  $c_p$  the thermal air conductivity and the specific heat per mass. Since the air thermal impedance is much smaller than the solid thermal impedance it will be assumed hereafter that the skeleton remains under isothermal conditions and that thermal effects are significant only in the pores. As a consequence, the thermal condition on solid-gaz boundary may be expressed as :  $T = 0$ . The relative variations of pressure, temperature and density are of the same order of magnitude. Therefore the air state equation (A1) is already scaled and  $O(\rho^e c_p T) = O(P)$ . Since thermal exchange are assumed to occur at the pore scale, conduction and transient terms in Fourier equation are of the same order of magnitude, which expresses the fact that the thickness of the thermal layer is of the order of the pore size, i.e.,  $O(\omega\rho^e c_p T) = O(\kappa T/l^2)$ . Then using the macroscopic length as reference length, Fourier's equation is rescaled in the form :

$$\varepsilon^2 \operatorname{div}(\kappa.\operatorname{grad}(T)) = i\omega(\rho^e c_p T - P) \tag{A2}$$

### A.2 Macroscopic compressibility

The set of local dimensionless equations is the same as in section 3, but for the compressibility equation, which is replaced by the state equation (A1) and Fourier equation (A2). Therefore, concerning the flow, the dynamic Darcy law (8) remains valid. However the temperature  $T$  appears as an additional unknown, which, like the other unknowns, is looked for in the form of an asymptotic expansion in power of  $\varepsilon$ . The following heat transfer problem in the periodic cell problem arises:

$$\operatorname{div}_y(\kappa \operatorname{grad}_y(T^0)) - i\omega\rho^e c_p T^0 = -i\omega P^0 \tag{A3}$$

with

$$T^0 = 0 \tag{A4}$$

at the gas-solid boundary. The solution can be written in the following form (Auriault, 1983) and (Boutin and Auriault, 1993]):

$$T^0 = T^e(1 - 1/\gamma)gP^0/P^e$$

and therefore

$$\rho^0 = \rho^e[1 - (1 - 1/\gamma)g]P^0/P^e$$

where  $g$  is a complex valued function, which depends on the local variable  $y$  and on the pulsation. The macroscopic mass balance is altered. We get now, using the notation  $P$  instead of  $P^0$ :

$$\text{div}(V) + i\omega\Phi[1 - (1 - 1/\gamma)G^*]P/\gamma P^e = 0 \quad (\text{A5})$$

where

$$G^* = \langle \gamma \rangle_\Omega$$

Equations (8), and (A3) give the macroscopic behaviour which depend on the tensor  $K^*$  - the same as in the adiabatic case - and on the function  $G^*$ .

### A.3 Features of function $G^*$

The complex valued function  $G^*$  describes the frequency dependence of the thermal exchanges and the term  $\Phi[1 - (1 - 1/\gamma)G^*]/\gamma P^e$  gives the macroscopic effective complex valued compressibility. It depends on the dimensionless pulsation  $\omega/\omega_t$ . The pulsation  $\omega_t$  is such that, when  $\omega = \omega_t$ , the thickness of the thermal skin is equal the pore size. At low frequencies, transient thermal effects are negligible. Thus, considering the isothermal condition for the solid, the temperature variation tends towards zero. Then,  $G(0) = 0$ , and the effective compressibility tends towards the isothermal compressibility ( $\Phi/P^e$ ). At high frequencies, conduction effects are negligible except in close proximity to the solid. The perturbations occur in an adiabatic way in air. Then  $G(\infty) = 1$ , and the compressibility tends towards the adiabatic value ( $\Phi/\gamma P^e$ ). At medium frequencies, i.e. for pulsation of the order of  $\omega_t$ , there is a phase shift between temperature and pressure, and hence between density and pressure. Consequently, the effective compressibility is complex. Introducing the length  $\Lambda_t$  defined as the ratio of the of the volume of the pores to their surface (Champoux and Allard, 1991), a good estimation of  $\omega_t$  is given by :

$$\omega_t = \kappa/\rho^e c_p (\Lambda_t)^2$$

For spherical or cylindrical pores,  $G^*$  can be expressed analytically (Attenborough, 1983), (Auriault, 1983). For other geometries, the following expression gives the correct asymptotic behaviour at low and large frequencies (including the thermal layer effects) (Allard et al, 1993):

$$G^* \cong [1 + (F'2 + i\omega_t^*)^{1/2}/i\omega_t^*]^{-1}$$

with  $\omega_t^* = \omega/\omega_t$ . Where  $F'$  is a shape ratio of the pore structure ( $F' = 2$  for cylindrical pores and  $F' = 5/3$  for spherical pores).



## A.4 Wave propagation

The wave equation is obtained from (8) and (A4) by eliminating  $V$  :

$$K * \Delta(P) - i\omega\Phi[1 - (1 - 1/\gamma)G^*]P/\gamma P^e = 0$$

The velocity of harmonic waves is given, for isotropic cases, by :

$$C^2 = -\gamma P^e K^* \omega / i\Phi[1 - (1 - 1/\gamma)G^*]$$

Note that the macroscopic description includes two dissipative effects, one due to the viscosity, the other one due to thermal exchanges. However, the thermal dissipation is lower than the viscous dissipation and has a more limited range of frequencies. From the preceding analysis, these effects are maximum for pulsation close to  $\omega_t$ . It is important to note that whereas viscous and thermal layer thickness are of the same order in air, the characteristic frequencies associated with viscous and thermal dissipation can be very different. As a matter of fact, the permeability essentially depends on the small ducts in the media, while thermal effects involve all the pores. In consequence we have the inequalities:  $k < (\Lambda_t)^2$  and  $\omega_t < \omega_c$ . This approach is in agreement with the phenomenological approaches of (Attenborough, 1983) and (Allard et al., 1993).

## B Acoustics of Dual-Porosity Media with Thermal Effects

The descriptions given in section 5 may be improved by considering thermal exchanges. As in the case of single porosity, thermal effects do not act on the description of the flow, but only modify mass balances. Therefore, the results obtained in the treatment of Navier-Stokes equations in section 5 are still valid here.

### B.1 Macroscopic compressibility

In the pores and in the micropores, the governing equations for heat transfer are the same as in Appendix A, but variables are now indexed by  $p$  or  $m$ . Because of the contrast in conductivity, it is assumed here again, that the solid remains in isothermal conditions. In comparison with the single porosity case, the description of thermal exchanges in the pores is unchanged, i.e., the thermal skin is of the order of the pore size. As a consequence, the scaled equations (A1), (A2) remain the same. Now, due to the separation of scales, the micropore size is one order lower than the thermal skin and the continuity of heat flux at the grain boundary implies that:

$$\kappa T_\mu / l_\mu = \kappa T_m / l_m$$

which shows that:  $T_\mu = \varepsilon T_m$ .

This analysis leads to quasi-static exchanges at the microscopic scale. However, in order to treat the largest frequency domain as possible, we will keep the transient terms at the micropore scale.

These estimations yield the following scaled equations in the micropores, where  $L$  and  $T_p$  are used as reference quantities:

$$P_\mu = P^e(\rho_\mu/\rho^e + \varepsilon^{-1}T_\mu/T^e) \quad (B1)$$

$$\varepsilon^3 \text{div}(\kappa.\text{grad}(T_\mu)) = i\omega(\rho^e c_p T_\mu - \varepsilon P_\mu) \quad (B2)$$

with the boundary conditions

$$T_\mu = 0 \text{ at the gas-solid boundary}$$

$$T_\mu = T_m \text{ at the pores -micropores boundary.}$$

Firstly, the influence of thermal effects is derived by establishing the mass balance at both scales, and then by determining the inter-scale coupling term. At the first significant orders, the two following transient heat transfer problems must be solved:

In the micropores:

$$\text{div}_z(\kappa.\text{grad}_z(T_\mu^1)) - i\omega\rho^e c_p T_\mu^1 = -i\omega P_\mu^0$$

with

$$T_\mu^1 = 0 \text{ at the gas-solid boundary.}$$

In the pores:

$$\text{div}_y(\kappa.\text{grad}_y(T_m^0)) - i\omega\rho^e c_p T_m^0 = -i\omega P_m^0$$

with

$$T_m = 0 \text{ at the pores-micropores boundary.}$$

Both problems are similar to that defined for the single porosity problem and the solutions are:

$$T_m^0 = T^e(1 - 1/\gamma)g_m P_m^0/P^e \text{ and therefore } \rho_m^0 = \rho^e[1 - (1 - 1/\gamma)g_m]P_m^0/P^e$$

$$T_\mu^0 = T^e(1 - 1/\gamma)g_\mu P_\mu^0/P^e \text{ and } \rho_\mu^0 = \rho^e[1 - (1 - 1/\gamma)g_\mu]P_\mu^0/P^e$$

where  $g_m$  and  $g_\mu$  are complex valued functions, which depends on the pulsation and on the local variable  $y$  and  $z$  respectively. This leads to the average density variations:

$$\langle \rho_m^0 \rangle = \rho^e[1 - (1 - 1/\gamma)G_m^*]P_m^0/P^e \text{ and } \langle \rho_\mu^0 \rangle = \rho^e[1 - (1 - 1/\gamma)G_\mu^*]P_\mu^0/P^e$$

$$\text{where } G_m^* = \langle g_m \rangle_\Omega \text{ and } G_\mu^* = \langle g_\mu \rangle_{\Omega_\mu}.$$

These complex valued functions  $G_m^*$  and  $G_\mu^*$  play exactly the same role as the function  $G^*$  in the single porosity case and same comments (transposed to the micropores for  $G_\mu^*$ ) are valid. The descriptions of the flow in the pores and in the micropores remain unchanged by the thermal effects. But, the mass balances for the micropores and pores respectively become:

$$\text{div}_y(V_\mu^1) + i\omega\Phi_\mu[1 - (1 - 1/\gamma)G_\mu^*]P_\mu^0/\gamma P^e = 0 \quad (B3)$$

$$\text{div}_x(V_m^0) + \langle \text{div}_y(v_m^1) \rangle_\Omega + i\omega\Phi_m[1 - (1 - 1/\gamma)G_m^*]P_m^0/\gamma P^e = 0$$

Let us now determine the interscale coupling term  $\langle \text{div}_y(v_m^1) \rangle_\Omega$ . The mass balance (B3), Darcy's law in the micropores, and the boundary condition  $P_\mu^0 = P_m^0$ , constitute a boundary value problem in  $\Omega_s$ . This problem looks like the pressure diffusion problem encountered in section 5. However, due to the thermal non-equilibrium in the micropores, the diffusion coefficient

is now complex and frequency dependent. Nevertheless the solution can still be expressed on the form:

$$P_\mu^0 = (1 - \psi)P_m^0 \quad (B4)$$

where  $\Psi$  is a complex function depending on  $y$  and on the pulsation. As for  $\langle div_y(v_m^1) \rangle_\Omega$ , it is derived using flux continuity at the pores-micropore interface and integrating the mass balance (B3) over the whole microporous domain while considering expression (B4):

$$\langle div_y(v_m^1) \rangle_\Omega = i\omega[1 - (1 - 1/\gamma)G_\mu^*][1 - \Psi]P_m^0/\gamma P^e$$

where  $\Psi = \langle \psi \rangle_\Omega^s$ .

Finally, the macroscopic mass balance is (using  $P$  instead of  $P_m^0$ ) :

$$div(V) + i\omega\Phi_m[1 - (1 - 1/\gamma)G_m^*] + (1 - \Phi_m)\Phi_\mu[1 - (1 - 1/\gamma)G_\mu^*][1 - \Psi]P/\gamma P^e = 0$$

Adding thermal effects modify the inter-scale coupling term. Both thermal and pressure diffusion effects are now involved at the micropore scale, which is particularly highlighted through the function  $\Psi$  in which both phenomena are mixed. The behaviours of functions  $\Psi$  and  $\Pi^*$  are different because of the complex valued diffusion coefficient.