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Stabilization of a magnetic island by localized heating in a tokamak with stiff temperature profile

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In tokamaks plasmas, turbulent transport is triggered above a threshold in the temperature gradient and leads to stiff profiles. This particularity, neglected so far in the problem of magnetic island stabilization by a localized heat source, is investigated analytically in the present paper. We show that the efficiency of the stabilization is deeply modified compared to previous estimates due to the strong dependence of the turbulence level on the additional heat source amplitude inside the island.

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I. INTRODUCTION

The production of fusion energy in tokamaks requires the confinement of a hot plasma medium in nested magnetic surfaces. Magnetic islands degrade the quality of this confinement by breaking locally the magnetic topology, but they can be damped using the injection of a localized current drive at their O-point¹⁸, or by a localized heating^{7,14,15}, as demonstrated experimentally²¹. In this later case, the stabilizing effect is provided by the reduction of the local plasma resistivity that depends on the temperature change produced by the local heating, and therefore on the properties of the heat transport that is mainly originating from turbulent processes. Theory^{1,6,19} and experiments^{10,17} show that turbulent transport is triggered above a critical temperature gradient, and leads to resilient (also referred to as stiff) profiles above this threshold, with a stiffness that is expected to be large in ITER¹³. Inside magnetic islands, where the temperature profile is flattened, a reduced diffusivity is expected⁸ and effectively measured^{9,11}. The consequences of this kind of transport rule has recently been investigated for nonlinear island saturation⁴. Here we show analytically that profile stiffness strongly impacts the stabilization efficiency by localized heating. It varies as $(P_{RF}/P_{eq})^{1/\sigma}$, with σ the stiffness parameter, P_{eq} the power injected inside the island position of a plasma at equilibrium and P_{RF} the additional heat source centered at the O-point of the island. In the most common case where the ratio (P_{RF}/P_{eq}) is small, the stabilization can be much larger than anticipated without profile stiffness.

II. STIFFNESS MODEL

We adopt a simple model for the heat diffusivity, that incorporates plasma stiffness in the vicinity of a reference

state where turbulent transport equilibrates the incoming heat flux:

$$\chi_{\perp} = \chi_{\perp}^0 \left| T'/T'_{eq} \right|^{\sigma-1} \quad (1)$$

where T is the temperature and the prime refers to the derivative relative to the radial co-ordinate, σ is the stiffness, the "eq" subscript refers to the equilibrium situation without magnetic island and without additional heating from RF waves, and χ_{\perp}^0 is the heat diffusivity in this reference case. This formulation is consistent with the definition of the stiffness parameter given in¹³. In this representation, anomalous transport starts growing above a critical gradient $|T'_{crit}/T'_{eq}| = 1 - 1/(\sigma - 1)$, with a soft transition between sub- and over-critical regimes. In this simple parametrization, the turbulent transport properties are assumed to be identical with and without island.

The equilibrium (no island) is assumed to be in a fully developed turbulent regime, above the threshold, with a distance to the threshold that depends on the stiffness parameter. In the realistic situation where the stiffness parameter is large, the temperature gradient cannot depart strongly from the threshold value. The variation of the temperature gradient when the power injected inside a given radial position is varied is illustrated in figure 1.

In the presence of an island, the above diffusivity model (equation 1) reproduces the fact that for a stiffness parameter σ larger than unity, the drive for turbulent modes (the temperature gradient here) may vanish and lead to a reduced diffusivity, as observed experimentally and expected theoretically. The effect of local heating on island stabilization has not been derived so far in this case (only $\sigma = 1$ has been considered in previous works), and this threshold property of turbulent transport leads to deep modifications of the island response to this control method, as we will show in the following.

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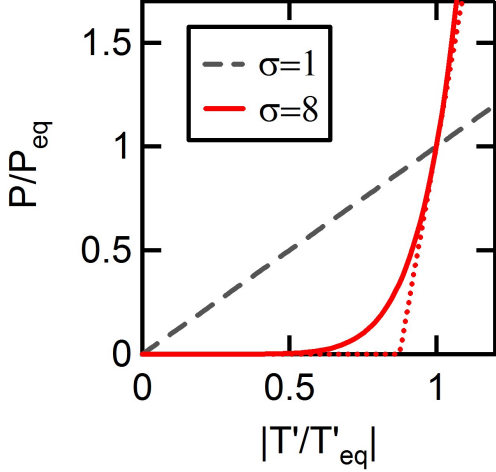


FIG. 1. Ratio of the power injected inside a given radial position relative to its equilibrium value, as a function of the temperature gradient relative to its equilibrium value, for a stiffness parameter $\sigma = 1$ and $\sigma = 8$, and in the absence of any island.

III. ADDITIONAL HEATING IN THE ABSENCE OF ISLAND

The energy balance equilibrates the heat flux with the volumic heat source H :

$$\nabla \cdot (N\chi_{\perp}\nabla T) = -H \quad (2)$$

with N the plasma density. Without island, after integrating on the plasma volume inside a radial position r we obtain

$$-T'(r) = \frac{\int_0^r dr' \mathcal{J}(r')H(r')}{\mathcal{J}(r)N(r)\chi_{\perp}(r)} = \frac{P(r)}{(2\pi)^2 \mathcal{J}(r)N(r)\chi_{\perp}(r)} \quad (3)$$

with $P(r)$ the power injected inside r and \mathcal{J} the Jacobian of the co-ordinate system (r, θ, φ) , with θ and φ the poloidal and toroidal angles, that in the large aspect ratio limit is $\mathcal{J} \approx rR$ with R the major radius of the torus. At equilibrium, $H = H_{eq}$ and

$$-T'_{eq}(r) = \frac{\int_0^r dr' \mathcal{J}(r')H_{eq}(r')}{\mathcal{J}(r)N(r)\chi_{\perp}^0(r)} = \frac{P_{eq}(r)}{(2\pi)^2 \mathcal{J}(r)N(r)\chi_{\perp}^0(r)} \quad (4)$$

$$\left(-\frac{dT}{d\Omega}\right) = \frac{(P_{eq}/(2\pi)^2)^{1-1/\sigma}}{N\chi_{\perp}^0 \mathcal{J}_s} \left(\frac{w}{4\sqrt{2}}\right)^{1+1/\sigma} \left\{ \frac{\int_{-1}^{\Omega} d\Omega' \mathcal{J} \oint d\alpha \frac{H(\Omega', \alpha)}{\sqrt{\Omega' + g}}}{\oint d\alpha (\Omega + g)^{\sigma/2}} \right\}^{1/\sigma} \quad (9)$$

with \mathcal{J}_s the Jacobian at the resonant surface. We assume

When the plasma is heated by RF waves, $H = H_{eq} + H_{RF}$, $P = P_{eq} + P_{RF}$ and

$$-T'(r) = \frac{P_{eq}(r)}{(2\pi)^2 \mathcal{J}(r)N(r)\chi_{\perp}^0(r)} \left(1 + \frac{P_{RF}}{P_{eq}}\right)^{1/\sigma} \quad (5)$$

The gradient increase due to the RF heating is reduced as expected when the stiffness is large.

IV. HEATING AT THE O-POINT OF AN ISLAND

Inside an island, the temperature gradient is reduced and may eventually go below the turbulent threshold, leading to a low level of diffusivity. In this context, the impact of a localized heat source strongly depends on the distance to the threshold, to which the equilibrium temperature gradient is close when the stiffness parameter is large. In the following we derive the evolution equation of the island width in the case of a localized heating source at the O-point, for an island that is large enough so that the temperature gradient without heating is zero inside the separatrix.

A. Temperature gradient increase due to localized heating

We consider the magnetic equilibrium of a tokamak with major radius R and minor radius a , a toroidal magnetic field B_z , a safety factor q and magnetic shear $s = (r/q)dq/dr$ in the large aspect ratio limit. The magnetic perturbation with poloidal and toroidal mode numbers m and n associated with the magnetic island localized at $r = r_s$ where $q = m/n$ is $\tilde{B} = \nabla \times \tilde{\psi} \mathbf{e}_z$. The magnetic flux surfaces are labelled by Ω with $\Omega = 8(x/w)^2 - \cos(m\alpha)$ with $x = r - r_s$, $\alpha = \theta - (n/m)\varphi$, and w the island total width. We have the relation

$$\frac{\tilde{\psi}}{B_z} = -\frac{w^2}{16} \frac{ns}{Rm} \cos(m\alpha) \quad (6)$$

In the following we use the notations

$$g(\alpha) \equiv \cos(m\alpha) \quad (7)$$

$$\psi_1 \equiv \frac{w^2}{16} \frac{B_z ns}{Rm} \quad (8)$$

After integrating equation (2) in the interval $(2\sqrt{2}x_{\pm}/w) = \pm\Omega$, we obtain:

that there is no other heat source in the island than RF

power, with a constant value of H_{RF} in the region $\Omega \in [-1, \Omega_c]$ or $\mu = (\Omega + 1)/2 \in [0, \mu_c]$, with $\mu_c < 1$. This

mimics a perfect O-point heating over a width δ_H , with $\mu_c = (\delta_H/w)^2$ (see figure 2), and leads to

$$-\frac{dT}{d\Omega} = \frac{P_{eq}^{1-1/\sigma} P_{RF}^{1/\sigma}(\Omega)}{(2\pi)^2 N \chi_{\perp}^0 \mathcal{J}_s} \frac{w}{4\sqrt{2}} \left\{ \frac{\pi}{I_{\sigma}(\Omega)} \right\}^{1/\sigma} = (-T'_s) \left(\frac{P_{RF}^{tot}}{P_{eq}} \right)^{1/\sigma} \frac{w}{4\sqrt{2}} \left\{ \frac{\pi}{I_{\sigma}(\Omega)} \frac{E(\mu) + (\mu - 1)K(\mu)}{E(\mu_c) + (\mu_c - 1)K(\mu_c)} \right\}^{1/\sigma} \quad (10)$$

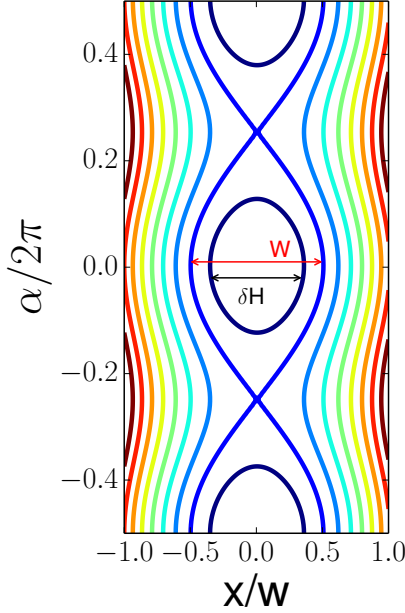


FIG. 2. Schematic view of the island geometry and heating area of width δ_H .

where $T'_s = T'_{eq}(r_s)$ and

$$I_{\sigma}(\Omega) = \oint d\alpha (\Omega + g(\alpha))^{\sigma/2} \quad (11)$$

$$= 2^{2+\sigma/2} \mu^{1/2+\sigma/2} J_{\sigma}(\mu) \quad (12)$$

$$J_{\sigma}(\mu) = \int_0^{\pi/2} d\theta \frac{\cos^{\sigma+1} \theta}{\sqrt{1 - \mu \sin^2 \theta}} \quad (13)$$

and K and E are the complete elliptic integrals of the first and second kind respectively:

$$K(\mu) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - \mu \sin^2 \theta}} \quad (14)$$

$$E(\mu) = \int_0^{\pi/2} d\theta \sqrt{1 - \mu \sin^2 \theta} \quad (15)$$

B. Rutherford equation

The Rutherford equation is obtained via the integration of the Maxwell-Ampère law

$$-\nabla_{\perp}^2 \tilde{\psi} = \mu_0 \tilde{J}_{\parallel} \quad (16)$$

Here we focus on the contribution of the perturbed ohmic current, due to the perturbed parallel electric field \tilde{E}_{\parallel} and perturbed plasma resistivity $\tilde{\eta}$, using the Spitzer resistivity dependence on temperature $\eta \propto T^{-3/2}$:

$$\tilde{J}_{\Omega} = \tilde{E}_{\parallel}/\eta - J_{\Omega} \tilde{\eta}/\eta \quad (17)$$

$$= \eta^{-1} \partial_t \psi_1 \langle g \rangle + J_{\Omega} \frac{3}{2} \langle \tilde{T}/T \rangle \quad (18)$$

with the flux surface average operator:

$$\langle A \rangle = \left(\oint d\alpha A / \sqrt{\Omega + g} \right) / \left(\oint d\alpha / \sqrt{\Omega + g} \right) \quad (19)$$

Introducing the tearing stability index Δ'^5 and the normalized island width $W = w/a$, we obtain from the standard asymptotic matching procedure²⁰

$$I_1 \tau_R \partial_t W = a \Delta' + a \Delta'_{\Omega} \quad (20)$$

with $\tau_R = \mu_0 a^2 / \eta$ the resistive time and

$$I_1 = \sqrt{2} \int_{-1}^{\infty} d\Omega \oint \frac{d\alpha}{2\pi} \frac{g \langle g \rangle}{\sqrt{\Omega + g}} \approx 0.82 \quad (21)$$

$$a \Delta'_{\Omega} = \frac{24}{W \sqrt{2}} \frac{q \mu_0 R J_{\Omega}}{s B_z} \int_{-1}^{\infty} d\Omega \oint \frac{d\alpha}{2\pi} \frac{g \langle \tilde{T}/T \rangle}{\sqrt{\Omega + g}} \quad (22)$$

We consider a large island (compared with the characteristic transport scale length³), so that the temperature profile without RF heating is flat inside the island. The perturbed temperature is determined from $\langle \tilde{T}/T \rangle = (T(\Omega) - T(\Omega = 1))/T_s$, leading to

$$\left\langle \frac{\tilde{T}}{T} \right\rangle (\mu) = \frac{-T'_s}{T_s} \frac{w}{4} \left(\frac{\pi P_{RF}^{tot}}{4 P_{eq}} \right)^{1/\sigma} F_{\sigma}(\mu, \mu_c) \quad (23)$$

with P_{RF}^{tot} the total RF power, and

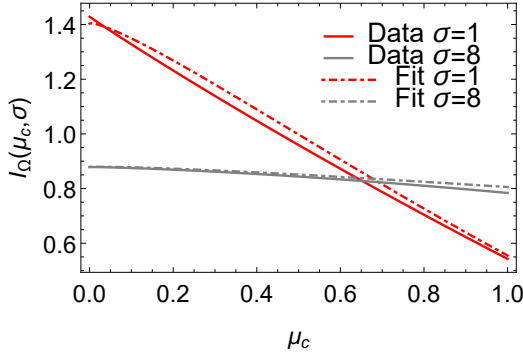


FIG. 3. $\mathcal{I}_\Omega = f(\mu_c)$ and the fit given by equation 32 for $\sigma = 1$ and $\sigma = 8$.

$$F_\sigma(\mu < \mu_c, \mu_c) = \int_\mu^{\mu_c} \frac{d\mu'}{\mu'^{1/2}} \left\{ \frac{1}{\mu'^{1/2} J_\sigma(\mu')} \frac{E(\mu') + (\mu' - 1)K(\mu')}{E(\mu_c) + (\mu_c - 1)K(\mu_c)} \right\}^{1/\sigma} + \int_{\mu_c}^1 \frac{d\mu'}{\mu'^{1/2}} \left\{ \frac{1}{\mu'^{1/2} J_\sigma(\mu')} \right\}^{1/\sigma} \quad (24)$$

$$F_\sigma(\mu > \mu_c, \mu_c) = \int_\mu^1 \frac{d\mu'}{\mu'^{1/2}} \left\{ \frac{1}{\mu'^{1/2} J_\sigma(\mu')} \right\}^{1/\sigma} \quad (25)$$

Note that the symmetry in x of the temperature perturbation that is assumed here implies that the impact of the O-point heating on the bootstrap current perturbation³ is zero within this model.

The stabilizing contribution due to a localized heating can finally be expressed as

$$a\Delta'_\Omega = -\mathcal{F}_\Omega \left(\mu_c, \frac{P_{RF}^{tot}}{P_{eq}}, \sigma \right) \frac{a}{\mathcal{J}} \frac{q}{s} \frac{\mu_0 R J_\Omega}{B_z} \frac{P_{eq}}{N \chi_\perp^0 T_s} \quad (26)$$

$$\mathcal{F}_\Omega = \frac{3}{4\pi^2} \mathcal{I}_\Omega(\mu_c, \sigma) \left(\frac{4}{\pi} \right)^{1-1/\sigma} \left(\frac{P_{RF}^{tot}}{P_{eq}} \right)^{1/\sigma} \quad (27)$$

with $\mathcal{I}_\Omega(\mu_c, \sigma) = \mathcal{I}_1(\mu_c, \sigma) + \mathcal{I}_2(\mu_c, \sigma) + \mathcal{I}_3(\mu_c, \sigma)$ and

$$\mathcal{I}_1(\mu_c, \sigma) = \int_0^{\mu_c} d\mu [2E(\mu) - K(\mu)] \int_\mu^{\mu_c} \frac{d\mu'}{\mu'^{1/2}} \left\{ \frac{1}{\mu'^{1/2} J_\sigma(\mu')} \frac{E(\mu') + (\mu' - 1)K(\mu')}{E(\mu_c) + (\mu_c - 1)K(\mu_c)} \right\}^{1/\sigma} \quad (28)$$

$$\mathcal{I}_2(\mu_c, \sigma) = \left(\int_0^{\mu_c} d\mu [2E(\mu) - K(\mu)] \right) \left(\int_{\mu_c}^1 \frac{d\mu'}{\mu'^{1/2}} \left\{ \frac{1}{\mu'^{1/2} J_\sigma(\mu')} \right\}^{1/\sigma} \right) \quad (29)$$

$$\mathcal{I}_3(\mu_c, \sigma) = \int_{\mu_c}^1 d\mu [2E(\mu) - K(\mu)] \int_\mu^1 \frac{d\mu'}{\mu'^{1/2}} \left\{ \frac{1}{\mu'^{1/2} J_\sigma(\mu')} \right\}^{1/\sigma} \quad (30)$$

Note that using equation 4, and introducing the temperature gradient length $L_T \equiv (-T_s/T'_s)$, we can also express the stabilization effect as

$$a\Delta'_\Omega = -\mathcal{F}_\Omega \left(\mu_c, \frac{P_{RF}^{tot}}{P_{eq}}, \sigma \right) (2\pi)^2 \frac{a}{L_T} \frac{q}{s} \frac{\mu_0 R J_\Omega}{B_z} \quad (31)$$

showing that the knowledge of the equilibrium profiles

and of the function \mathcal{F}_Ω fully characterizes the expected island decay rate.

The function \mathcal{I}_Ω is computed numerically and fitted with a simple formulae in the range $\mu_c \in]0, 1[$ (39 points) and $\sigma \in [1, 9]$ (9 points) using the Mathematica software¹² (figure 3), so that the function \mathcal{F}_Ω can be approximated by:

$$\mathcal{F}_\Omega \left(\mu_c, \frac{P_{RF}^{tot}}{P_{eq}}, \sigma \right) \approx \frac{3}{4\pi^2} \left[0.804 + \frac{0.600}{\sigma} - 1.091 \frac{\mu_c}{\sigma} + 0.242 \left(\frac{\mu_c}{\sigma} \right)^2 - 0.228 \frac{\mu_c}{\sigma} \ln \frac{\mu_c}{\sigma} \right] \left(\frac{P_{RF}^{tot}}{P_{eq}} \right)^{1/\sigma} \quad (32)$$

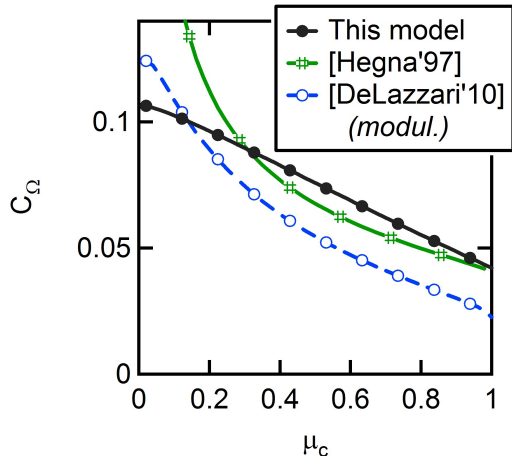


FIG. 4. Case $\sigma = 1$: comparison of the $C_\Omega(\mu_c)$ term with references⁷ (we take $C_0 = 1$) and^{15,16}.

and $\mathcal{F}_\Omega \left(\mu_c, \frac{P_{RF}^{tot}}{P_{eq}}, \sigma \rightarrow \infty \right)$ tends to be a constant, independent of both P_{RF}^{tot} and μ_c .

C. Particular case $\sigma = 1$: comparison with existing results

For $\sigma = 1$ (no profile resilience), the stabilizing contribution of RF heating writes

$$a\Delta'_\Omega = -C_\Omega(\mu_c) \frac{a}{\bar{J}} \frac{q}{s} \frac{\mu_0 R J_\Omega}{B_z} \frac{P_{RF}^{tot}}{N \chi_\perp^0 T_s} \quad (33)$$

$$C_\Omega \approx \frac{3}{4\pi^2} [1.40 - 1.09\mu_c + 0.24\mu_c^2 - 0.23\mu_c \ln \mu_c] \quad (34)$$

The stabilization effect is linear in P_{RF}^{tot} in this case. The expression for $C_\Omega(\mu_c)$ can be compared with previous analytical⁷ and numerical^{15,16} works:

$$C_\Omega^{HC}(\mu_c) = \frac{2C_0}{5\pi^2} \frac{\mu_c^{1/2}}{E(\mu_c) + (\mu_c - 1)K(\mu_c)} \quad (35)$$

$$C_\Omega^{DL}(\mu_c) = \frac{6}{\pi^2} N_H(\mu_c) M_H(\mu_c) / f^{on}(\mu_c) \quad (36)$$

where the superscript *HC* refers to reference⁷ and *DL* to references^{15,16}. The result of⁷ (equation (33)) exhibits a divergence for an extremely localized source ($\mu_c \rightarrow 0$), but is otherwise quantitatively similar to our result. For the comparison with references^{15,16}, we need to take into account the fact that the situation comparable (but not strictly identical) to the one we consider is a modulation

with an on-time $f^{on}(\mu_c) = \pi^{-1} \arccos(1 - 2\mu_c)$. The effective stabilization is then $a\Delta'_\Omega(\sigma) \times f^{on}$, thus explaining the division by f^{on} in the above expression. This procedure cannot reproduce exactly the modulation scheme, because in this latter case the deposition width in terms of flux surface (our variable μ_c) tends to be larger as the heat source departs from the O-point. We expect therefore the coefficient C_Ω^{DL} to be lower. Also, the RF heat source is Gaussian in reference¹⁵ instead of Heaviside in our model. We need therefore to express the Gaussian deposition width as a function of $\mu_c \equiv (\delta_H/w)^2$, and we choose to identify the width at half the maximum of the Gaussian with δ_H , as was done in² where the difference between Heaviside and Gaussian shapes on island stabilization efficiency by current drive was found to be moderate. The comparison of the factor C_Ω for $\sigma = 1$ (equations 34, 35 and 36) is displayed in figure 4, showing a reasonable agreement.

V. APPLICATIONS WITH PLASMA PARAMETERS OF LARGE TOKAMAKS

The importance of profile stiffness on the island stabilization capability by RF heating is illustrated by computing the quantity $a\Delta'_\Omega(\sigma) \times f^{on}$ as a function of μ_c and P_{RF}^{tot}/P_{eq} (figure 5), as well as the ratio $\Delta'_\Omega(\sigma)/\Delta'_\Omega(\sigma = 1)$ (figure 6). We take as a typical value $\sigma = 8$ expected in ITER¹³. For moderate values of P_{RF}^{tot}/P_{eq} (typically below unity), profile resilience is favourable since the heat diffusivity remains low inside the island as long as the temperature gradient stays below its equilibrium value, thus enhancing the effect of the RF power. For a typical medium size tokamak experiment (i.e. Asdex-Upgrade), taking $B = 2.5 T$, $R = 1.7 m$, $a = 0.5 m$, $r = 0.2 m$, $q = 3/2$, $s = 1$, $J_\Omega = 5 \times 10^5 A/m^2$, $N = 6 \times 10^{19} m^{-3}$, $T = 2 keV$, $\chi_\perp^0 = 2 m^2/s$, $P_{RF}^{tot} = 2 MW$, $P_{eq} = 10 MW$, and a 50% modulation scheme, we have $\mu_c = 0.5$, and an average stabilizing contribution from ohmic heating of $a\Delta'_\Omega(\sigma) \times f^{on} \approx -85$ for $\sigma = 8$ instead of (-24) for $\sigma = 1$. In this example where P_{RF}^{tot}/P_{eq} is small (as is generally the case), the importance of the heating contribution to island stabilization with RF is therefore much larger than computed so far without profile stiffness, and the contrast between small and large profile stiffness is expected to increase as this ratio will be lower.

VI. SUMMARY

The nature of turbulent transport in tokamak plasmas, that manifest itself by stiff temperature profiles, strongly impacts the response of a magnetic island to a heating

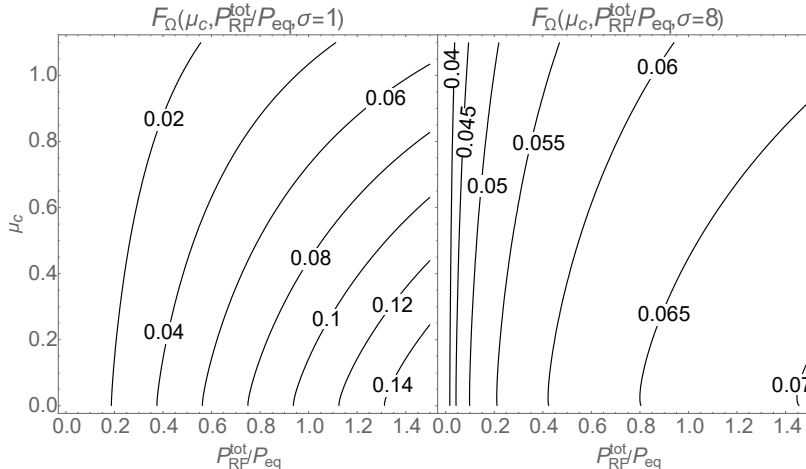


FIG. 5. The function $\mathcal{F}_\Omega\left(\mu_c, \frac{P_{RF}^{tot}}{P_{eq}}, \sigma\right)$ for $\sigma = 1$ (left) and $\sigma = 8$ (right).

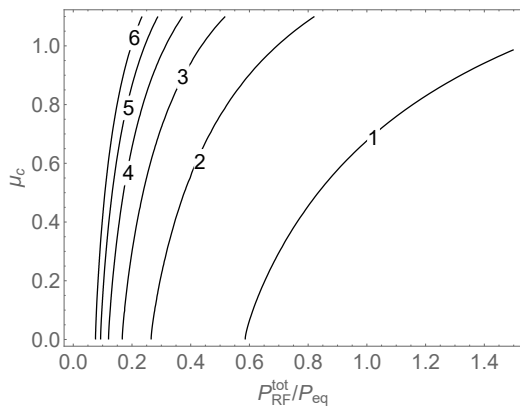


FIG. 6. The ratio $\mathcal{F}_\Omega\left(\mu_c, \frac{P_{RF}^{tot}}{P_{eq}}, \sigma = 8\right) / \mathcal{F}_\Omega\left(\mu_c, \frac{P_{RF}^{tot}}{P_{eq}}, \sigma = 1\right)$.

source localized at its O-point. We have shown, using a simple model for the heat diffusivity in the vicinity of an equilibrium point, that the stabilization efficiency of this control method can be larger than predicted without profile stiffness, in the usual situation where the power used for the control is less than the power already circulating in the island region. An expression for the contribution of localized heating to the island dynamics has been derived for arbitrary stiffness strength, and we find reasonable agreement with previously published results in the limit without stiffness.

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