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Homogenisation of Pollutant Transport in Fractured Porous Media

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ABSTRACT

1 Introduction

The objective of this work is to derive the mathematical models that describe pollutant migration through fractured porous media. For this purpose, we use an homogenisation method, i.e. we rigorously deduce the macroscopic models from the physical description which is valid within a Representative Elementary Volume (REV). The fundamental assumption behind homogenisation is the separation of scales, which we express as: $l \ll L$. In the present work, l denotes the characteristic size of the REV, i.e. at the fracture's scale, whereas L represents the characteristic macroscopic size. We apply the approach introduced in [1], which is on the basis of definition and estimation of dimensionless numbers that arise from the description at the REV's scale. The validity domains of the derived macroscopic descriptions are provided by means of orders of magnitude of local dimensionless numbers

2 Dimensionless local description

The REV consists of a porous matrix domain, Ω_m , and of a fracture domain, Ω_f , whose common boundary is denoted by Γ . The medium is saturated by water and a solute is diluted in water. In the fracture's domain, fluid flow is described by Stokes equations and solute transport by the diffusion-convection equation. In the porous ma-

trix, the filtration of the liquid is described by Darcy's law and solute transport is described by a diffusion-advection law that accounts for diffusion in the solid and diffusion and convection in the pores. Note that the behaviour at the porous matrix scale of the process of diffusion in both the solid and the micropores may be different from that considered here [2]. The methodology consists in writing the local description in a dimensionless form and then in estimating the dimensionless numbers with respect to the length-scale ratio ε . This leads to the following formulation of the equations:

- In the fractures (Ω_f)

$$\varepsilon^2 \mu \Delta \vec{v}_f - \vec{\nabla} p_f = \vec{0}$$

$$\vec{\nabla} \cdot \vec{v}_f = 0$$

$$N \frac{\partial c_f}{\partial t} - \vec{\nabla} \cdot (\tilde{D} \vec{\nabla} c_f - P e_f c_f \vec{v}_f) = 0$$

- In the porous matrix (Ω_m)

$$\vec{v}_m = -\tilde{K}_m \vec{\nabla} p_m$$

$$\vec{\nabla} \cdot \vec{v}_m = 0$$

$$N \frac{\partial c_m}{\partial t} - \vec{\nabla} \cdot (\tilde{D}_m \vec{\nabla} c_m - \alpha^2 P e_f c_m \vec{v}_m) = 0$$

- On the boundary (Γ)

$$\vec{v}_f = \alpha^2 \vec{v}_m$$

$$P_f = P_m$$

$$(\tilde{D}_f \vec{\nabla} c_f) \cdot \vec{n} = (\tilde{D}_f \vec{\nabla} c_m) \cdot \vec{n}$$

$$c_f = c_m$$

This dimensionless local description depends upon the following parameters:

$$Pe = \frac{LV_{fc}}{D_{fc}}; \quad N = \frac{L^2}{D_{fc}T_f}; \quad \alpha = \frac{l_p}{l}$$

Pe_f represents the Péclet number in the fractures, l_p denotes the characteristic pore-size, V_{fc} and D_{fc} are characteristic values of the fluid velocity and of the molecular diffusion in the fractures, and T_f represents the characteristic time of the transport process in fractures. It can be shown that the cases of interest, i.e the cases that lead to distinct macroscopic behaviours are the following:

- Case 1:

$Pe_f = O(\varepsilon)$; $N = O(1)$; $\alpha = O(\varepsilon)$
(predominant diffusion in the fractures)

- Case 2:

$Pe_f = O(1)$; $N = O(1)$; $\alpha = O(\varepsilon)$
(equivalent diffusion and convection in the fractures)

- Case 3: $Pe_f = O(\varepsilon^{-1})$; $N = O(\varepsilon^{-1})$

(predominant convection in the fractures)
Case 3.a: $\alpha = O(\varepsilon^{1/2})$; Case 3.b: $\alpha = O(\varepsilon)$

3 Derived upscaled models

The derived macroscopic models are the following:

- Fluid flow

$$\vec{V}_f = -\tilde{K}_f \vec{\nabla} P$$

$$\vec{\nabla} \cdot \vec{V}_f = 0$$

- Solute transport

Case 1: purely diffusive behaviour

$$\frac{\partial C}{\partial t} - \vec{\nabla} \cdot (\tilde{D}_{eff} \vec{\nabla} C) = 0$$

Case 2: convection-diffusion

$$\frac{\partial C}{\partial t} - \vec{\nabla} \cdot (\tilde{D}_{eff} \vec{\nabla} C - C \vec{V}_f)$$

Case 3.a: dispersion with influence of the porous matrix

$$\frac{\partial C}{\partial t} - \varepsilon \vec{\nabla} \cdot \left[\tilde{D}_{disp} \vec{\nabla} C - \frac{1}{\varepsilon} C (\vec{V}_f + \varepsilon \vec{V}_m) \right] = 0$$

Case 3.b: dispersion

$$\frac{\partial C}{\partial t} - \varepsilon \vec{\nabla} \cdot \left[\tilde{D}_{disp} \vec{\nabla} C - \frac{1}{\varepsilon} C \vec{V}_f \right] = 0$$

In the above equations, \tilde{K}_f represents the effective permeability tensor, \tilde{D}_{eff} and \tilde{D}_{disp} are the effective diffusion tensor and the apparent dispersion tensor, respectively. These three effective tensors are the solutions to boundary-value problems defined over the REV. It can be proved [3] that: i) \tilde{K}_f is a symmetrical tensor and depends upon the geometry of the REV and fluid viscosity; ii) \tilde{D}_{eff} is also symmetrical and depends upon the cell-geometry and the molecular diffusion; iii) \tilde{D}_{disp} is not symmetrical in the general case and depends upon the pressure gradient.

4 Conclusion

The above results show that the macroscopic description strongly depends upon the local transport regime (Pe_f) and, in a less extent, upon the order of magnitude of l_p/l . We have derived four distinct continuous models for solute transport in fractured porous media. Since the flow is steady-state, there is no dual-porosity effect in the macroscopic fluid flow equations (single-porosity model for fluid flow). On the contrary, since the solute transport process is time-dependent, dual-porosity effects are present in all macroscopic transport equations, through the expressions obtained for \tilde{D}_{eff} (models 1 and 2) and \tilde{D}_{disp} (models 3a and 3b). An important feature of model 3a is that it shows an influence of the advection in the porous matrix at the macroscopic scale.

References

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