

Spatio-temporal filtering of blood flow in 4D Phase-Contrast MRI

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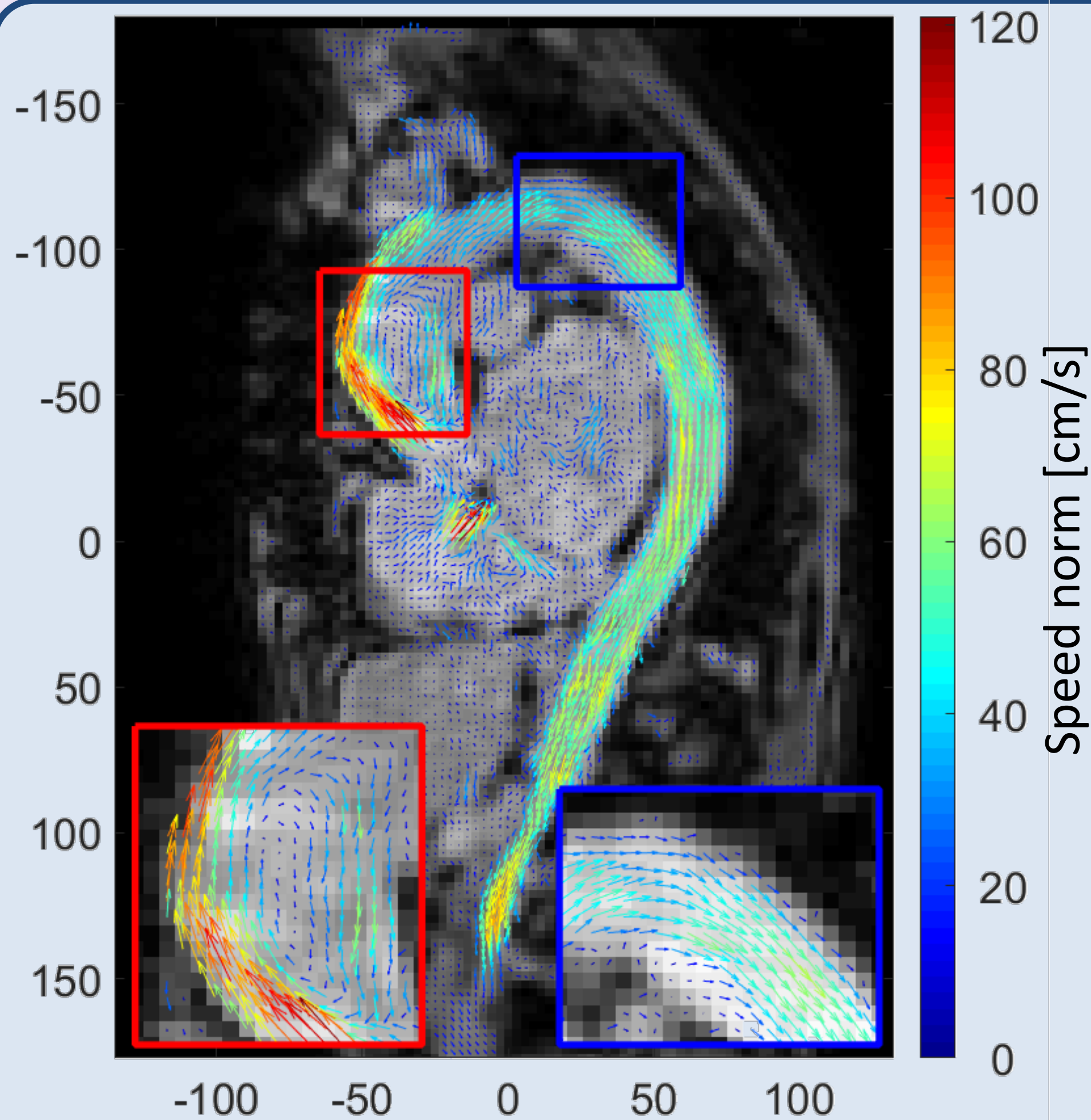


Figure: Cardiovascular image obtained by 4D flow Phase-Contrast MRI (anatomy and speed).

Context

4D flow PC-MRI:

- Acquisition time: 8-20 min
- Resolution: [2,8x2,8x2,2 mm³ ; 30ms]

Technological constraint:

Measurements are subjected to :

$$SNR \propto \log \left(K \times \frac{T_{\text{acquisition}}}{N_{\text{voxels}}^3} \right)$$

with K a system and patient linked constant.

- **Low reliability of biomarkers values**
- Wall Shear Stress [Boussel et al., 2009]

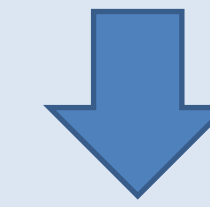
MRI-Quantif project objective:

Enhance biomarkers estimation reliability in 4D flow PC-MRI.

Contribution

Denoise the speed vector field in the region of interest on the same data grid.

Strategy: Inverse problems theory



A priori knowledge on the vector field

- The vector field represents the speed vectors of the blood flow in the vessels.

- Simplest fluid mechanics hypothesis:
 - Incompressibility [Dehghani Tafti et al., 2011; Busch et al., 2013; Song et al. 1993; Bostan et al., 2015]

Problem statement

Hypotheses:

- Incompressible fluid
- Time regularity

$$\hat{V} = \underset{s.c. V \in \mathcal{M}_0}{\operatorname{argmin}} \{ \|V_m - V\|^2 + \lambda \|D_t V\|^2 \}$$

with V_m the data vector, λ the time penalization weight, D_t a finite-difference matrix in time such that $D_t V = V_{t+1} - V_t$.

\mathcal{M}_0 is the divergence-free vector field:

$$\mathcal{M}_0 = \{ v \in \mathbb{R}^{3 \times N_{\text{voxels}}} \mid D_s v = \vec{0} \}$$

with $D_s = (D_x \ D_y \ D_z)$ a discrete version of the divergence operator.

Solution:

Application of two separable operators:

$$\hat{V} = (I + \lambda D_t^T D_t)^{-1} P V_m$$

with I the identity matrix and P the orthogonal projector on \mathcal{M}_0 for all acquisition times [Song et al., 1993]. This projector can be written for a given instant as: $P_0 = I - D_s^t (D_s D_s^T)^{-1} D_s$.

Results

Synthetic data: Womersley flow [Womersley, 1955]

Hypotheses:

- Incompressible, homogeneous, Newtonian and non-stationary
- Circulates in a cylinder (1cm radius)
- Input: Cardiac flow [Puelz et al., 2017]

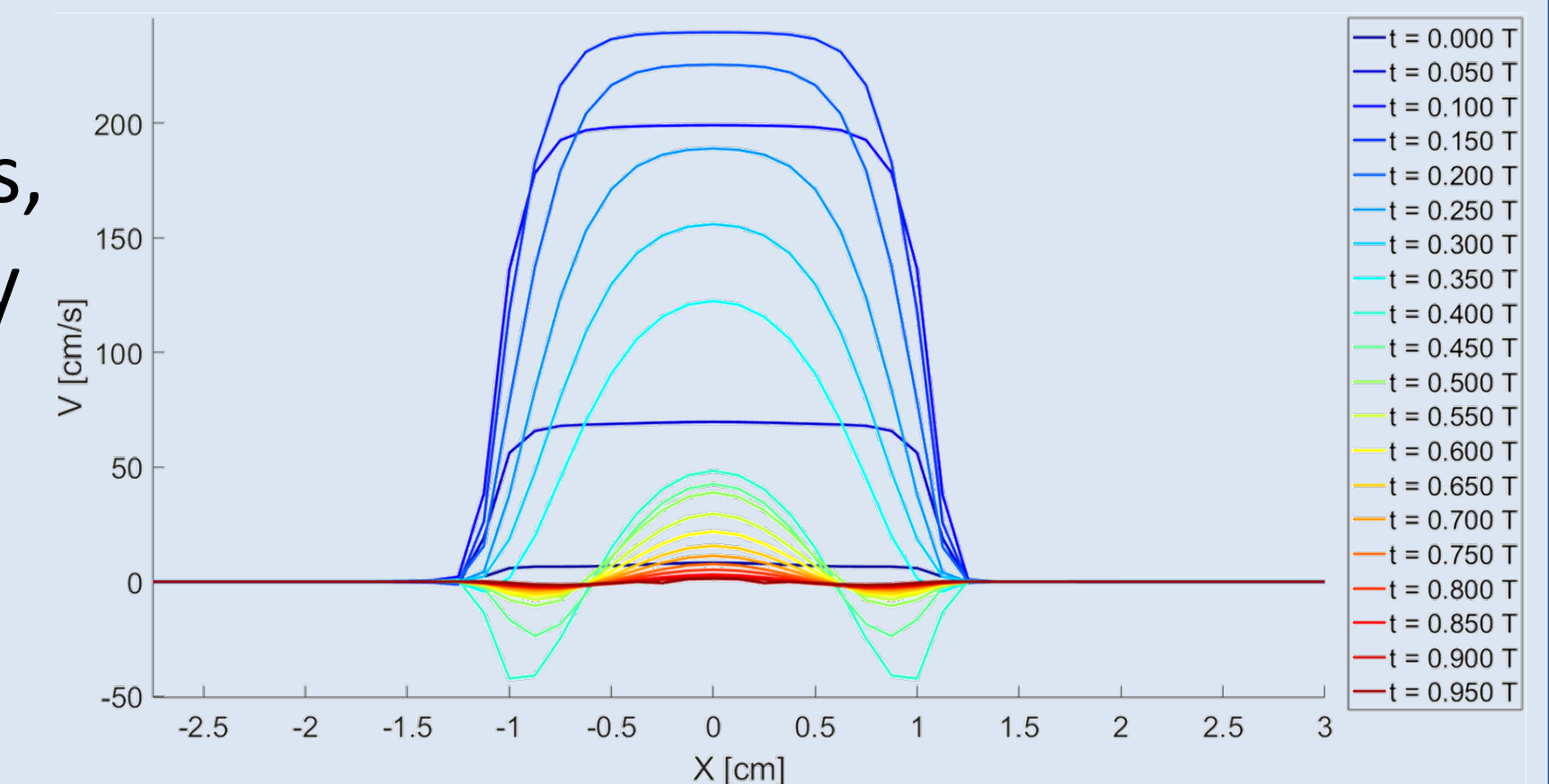


Figure: Analytical flow of Womersley over time

Studied problem: 2D+time simulation [2.5x2.5mm²; 46ms]

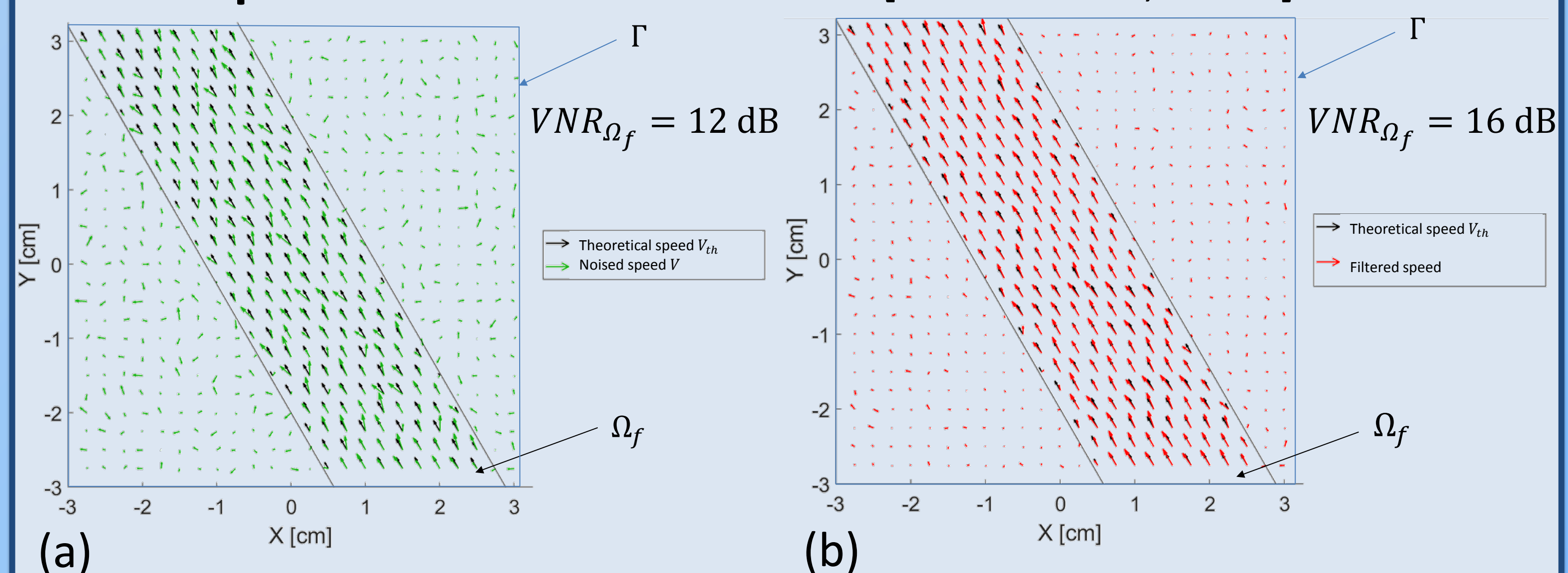


Figure: Noised vector field (a) and filtered vector field (b) at a given instant with the velocity-to-noise ratio defined as: $VNR = 20 \log \left(\frac{\|V_{th}\|}{\|V - V_{th}\|} \right)$.

VNR_{Ω_f} input	VNR_{Ω_f} after spatial-only filtering	VNR_{Ω_f} after spatio-temporal filtering
9 dB	12 dB	13,9 dB
12 dB	14,7 dB	16,2 dB
15 dB	17,7 dB	18,3 dB
30 dB	27,4 dB	27,4 dB

Table: Velocity-to-noise ratio after spatial (incompressibility projection) and spatio-temporal filtering for different VNR_{Ω_f} inputs (λ is chosen to optimize the VNR_{Ω_f}).

References

- [Boussel et al., 2009] L. Boussel, V. Rayz, A. Martin, G. Acevedo-Bolton, M. T. Lawton, R. Higashida, W. S. Smith, W. L. Young & D. Saloner, « Phase-Contrast magnetic resonance imaging measurements in intracranial aneurysms in vivo of flow patterns, velocity fields, and wall shear stress: comparison with computational fluid dynamics. », *Magn. Reson. Med.*, vol. 61, n°2, pp. 409-417.
- [Bostan et al., 2015] E. Bostan, M. Unser & J. P. Ward, « Divergence-free wavelets frames. », *IEEE Signal Processing Letters*, vol. 22, n°8, pp. 1142-1146, 2015.
- [Busch et al., 2013] J. Busch, D. Giese, L. Wiessman & S. Kozerke, « Reconstruction of divergence-free velocity fields from CINE 3D phase-contrast flow measurements. », *Magn. Reson. Med.*, vol. 69, n°1, pp. 200-210, 2013.
- [Dehghani Tafti et al., 2011] P. Dehghani Tafti & M. Unser. « On regularized reconstruction of vector fields », *IEEE Trans. Image Process.*, vol. 20, n°11, pp. 3163-3178, 2011.
- [Puelz et al., 2017] C. Puelz, S. Canic, B. Rivère & C. G. Rusin, « Comparison of reduced models for blood flow using Runge-Kutta discontinuous Galerkin methods. », *Appl. Num. Math.*, vol. 115, pp. 114-141, 2017.
- [Song et al., 1993] S. M. Song, S. Napel, G. H. Glover & N. J. Pelc, « Noise reduction in three-dimensional phase-contrast MR velocity measurements. », *J. Magn. Reson. Imaging*, vol. 3, n°4, pp. 587-596, 1993.
- [Womersley, 1955] J. R. Womersley, « Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known. », *J. Physiol.*, vol. 127, n°3, pp. 553-563, 1955.

Conclusion

- Solving of a problem including incompressibility and time regularity hypotheses;
- Direct and simple solution using two separable operators;
- Significant improvement of the VNR_{Ω_f} within the vessels.