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Abstract: Designing strategies for temperature control requires an efficient model which accurately describes building dynamics. The occurrence of different events in a room like opening/closing the windows, the doors and the shades affects the dynamic evolution of the indoor temperature. This makes state-of-the-art building models yield an incomplete description of the temperature dynamics. Hence, we propose in this work the use of hybrid (switching) models to characterize the thermal behavior of buildings based on the idea that, for every new configuration, hence, for each discrete state, a different continuous model is estimated. We present the motivation behind using hybrid models via a physical Resistor Capacitor (RC) model of the building and show how the values of the resistance changes depending on the considered configuration. We employ a PieceWise Autoregressive eXogeneous (PWARX) model for a scenario we consider. A preliminary result is presented at the end of the document.

Keywords: Building thermal modeling, system identification, switching models, PWARX.

1. INTRODUCTION

During the last decade, a lot of research has been done in the area of building thermal modeling with the goal of meeting the requirements for improving their energy efficiency and reduce their environmental impact. Developing an accurate dynamic building thermal model is beneficial in many ways: it gives a suitable description of their energy performance, it helps identify the sources of excessive energy consumption and therefore, establish the most effective actions to be taken for improving their efficiency; and last but not least, it allows to predict future performance, but also control indoor climate and energy consumption. Existing modeling approaches are divided into "white box", "black box" and "grey box". "White box" models are based on physics and are mostly used in the preliminary design phase where a detailed description of the building is available. However, for existing buildings, a detailed description is not always available, and errors due to the aging of materials might lead to an inappropriate model. In this case, "black box" models, being constructed only from input-output data, seem more suitable for describing the building heat dynamics, but no physical explanation lies behind the model. "Black box" models are known for their ease of implementation and low computational cost. "Grey box" models take the advantages of both techniques in a way to have a physics based easy to implement model, without the need of a detailed description of physical properties nor a big amount of data.

In this work, we propose the use of PWARX, a system identification "black box" technique for modeling the thermal dynamics of buildings. This technique is based on hybrid models, namely models that combine both continuous and discrete dynamics. The motivation behind using this type of modeling is that, in a building, different events like the opening/closing of windows, etc. occur, thus leading to different dynamic behaviors. Therefore, we propose a model consisting of several continuous submodels, each corresponding to a different event, thus generating a hybrid model describing the continuous and discrete (event-based) dynamics of the building. Previous knowledge about the number of submodels and information about the configuration (opened windows or shutters for example) is not necessary. The technique is based on data classification, i.e. assigning data to a specific subclass, according to a similarity criterion. Each new data is assigned to an existing class (submodel) or a new one based only on the criterion value. The estimation of the submodels parameters is done simultaneously with the data classification as they are updated with every new data attributed, thus leading to an online identification process. This modeling technique can be further applied for control purposes as it predicts the indoor thermal behavior changing from one dynamic to another. At this preliminary stage of our work, we test the ability of switching models to detect different configurations in a building.

The rest of the document is structured as follows: in section 2, we present a short review of state-of-the-art techniques for building thermal modeling. In section 3 we present the motivation behind using hybrid switching models for modeling the thermal behavior of buildings and the suggested model. Preliminary results are presented

in section 4 while the last section is dedicated to the conclusion and presentation of future work.

2. STATE OF THE ART

In this section, we present a non-exhaustive state-of-the-art techniques for modeling buildings thermal behavior (sections 2.1 to 2.3) and a short overview of system identification approaches for hybrid systems (section 2.4).

2.1 "White box" models

"White box" models are based on physical equations describing the energy transfer between the building and its environment. These equations are implemented in software tools aimed at different applications. Building simulation tools can be generally divided into three main categories. Approaches based on Computational Fluid Dynamics (CFD)((Fluent, 2012), etc.) are considered the most complete but also the most complicated to implement. CFD is particularly adapted to the description of fluid flow fields in buildings. The zonal approach(SPARK (2003)) is a simplification of the CFD while the multi-zonal or nodal approach EnergyPlus (2015), etc.) is the simplest one.

Thermal networks are another type of modeling techniques that offer a physics based simple approach with low computational cost. The building is approximated by a Resistance Capacitance (RC) network that represents an analogue to an electric circuit. The electric resistances and capacitances are equivalent to thermal ones, the electric potential is the temperature gradient and the electric current is the heat flux. This method is widely coupled with data driven techniques as will be seen in section 2.3.

2.2 "Black box" models

A "black box" model is based on a mathematical equation establishing the relationship between the inputs and outputs of a system, without any prior knowledge of its internal functioning. Data driven models are constructed using system identification or machine learning techniques. System identification techniques applied to building thermal modeling mainly include linear parametric models. (Parti and Parti, 1980) were the first to propose a new method for the prediction of energy consumption in buildings using linear regression and many applications followed. Machine learning techniques include, among others, Artificial Neural Networks (ANN), and are mainly applied to predicting heating or cooling demand (Kalogirou and Bojic, 2000).

2.3 "Grey box" models

"Grey box" models are a mix of "white" and "black box" models. They combine physics and statistics and need a rough description of the building geometry and limited number of data. Researchers tend to apply "grey box" models on a building model described as a Resistance Capacitance (RC) network. First, the model structure is identified and afterwards, the parameter values are identified from measurements or simulations. Numerous applications employ this kind of modeling (Bacher and Madsen, 2011), (Wang and Xu, 2006), etc.

2.4 Hybrid dynamical systems

A hybrid system is a system that combines simultaneously continuous dynamics and discrete events. This type of systems can be represented by a model consisting of continuous submodels. Several models exist to describe hybrid systems (Vidal et al., 2002), (Doucet et al., 2001). One of the most commonly used models are PieceWise Affine (PWA) models (Sontag, 1981). PWA models are a collection of affine models that share the same continuous state variables and are related by switches that are determined by a polyhedral partition of the input+state group (Paoletti et al., 2007). They are dedicated to the modeling of non-linear systems with more than one functioning mode, by linearizing them around operating points. Many approaches exist in the literature for the identification of PWA systems and they are classified according to the technique used for finding the partitions, (Ohlsson and Ljung, 2011). The approach we are using (Boukharouba, 2011) is detailed in section 3.2: it simultaneously classifies the data into a corresponding submodel and finds its parameter vector, the number of submodels being initially unknown.

3. OUR CONTRIBUTION

This paper proposes to employ switching models for building thermal modeling in the context of system identification from experimental data. To the authors' knowledge, the closest studies to our own were the studies in which change point models or piecewise linear regression models were employed to predict the energy consumption in buildings (Paulus et al., 2015). Section 3.1 offers the motivation from the physical point of view, while section 3.2 summarizes our algorithm of choice.

3.1 Why use switching models for buildings?

When modeling complex systems, employing a single mathematical model might lead to a complex non-linear structure which is hard to exploit in reality. Hybrid models are used for modeling complex systems by combining continuous and discrete dynamics. They are represented by several continuous differential equations.

In a building system, events like opening/closing doors, windows and shades affect the dynamic evolution of the zone temperature. Therefore, a single model is not capable of describing these different configurations. In this context, Fazenda et al. (2016) presented a context-based thermodynamic modeling of building. The building was modeled by a thermal RC network, and transitions between different contexts and models are described by context-connecting switches. Inspired by this way of representing the building in different operating conditions, we will show why the transition from one configuration to another leads to a transition between different dynamical models.

Let us consider the thermal network in Figure **??**. It should be noted that this is not a representation for a specific building and it is only used for interpretation. For this lumped model, all building envelopes are considered in one resistance (R_E) and one capacitance (C_E). T_E is the envelope temperature considered equal to the mean of all envelope surfaces temperatures. R_{cext} and R_{cint}

are the convective resistances of external and internal surfaces, respectively. Windows and doors resistances are represented by R_w and R_d. Shades are represented by R_{sh} with a switch indicating their state: if they are closed, their resistance is added to the windows resistance. Switches are also used to indicate the state of windows and doors. R_{atw} and R_{atd} are the air transmission resistances for windows and doors respectively when they are open with $R_{atd} << R_d$ and $R_{atw} << R_w$. The heating system is modeled by the heater surface temperature T_h, its resistance and capacitance (R_h and C_h) and a switch which indicates if heating is on or off. Occupants are modeled in the same way. T_{bs} is the temperature of the body surface, R_{occ} is the resistance of the heat transfer between the body and the surroundings and C_{occ}, its capacitance. A switch indicates the presence of occupants or not. Solar effects are considered as input currents with ϕ_s, the solar radiation acting on the envelope, and ϕ_{sw}, the solar radiation going inside the building through the windows. C_{int} is the internal capacitance (indoor air, furniture, etc.). T_{ext} and T_{int} are the exterior and indoor temperatures, respectively.

The differential equations governing the evolution of the temperatures when heating is on, occupants are present and windows and doors are closed are:

$$\begin{aligned}
C_E\dot{T}_E &= \frac{T_{ext}-T_E}{R_{cext}} + \frac{T_{int}-T_E}{R_{cint}} + \phi_s \\
C_h\dot{T}_h &= \frac{T_{int}-T_h}{R_h} + \phi_h \\
C_{occ}\dot{T}_{bs} &= \frac{T_{int}-T_{bs}}{R_{occ}} + \phi_{occ} \\
C_{int}\dot{T}_{int} &= \frac{T_{ext}-T_{int}}{R_{eq_1}} + \frac{T_E-T_{int}}{R_{cint}} + \\
&\quad \frac{T_h-T_{int}}{R_h} + \frac{T_{bs}-T_{int}}{R_{occ}} + \phi_{sw}
\end{aligned} \tag{1}$$

These can be rewritten as state equations by defining the state vector $x = [T_E \quad T_h \quad T_{bs} \quad T_{int}]^\top$:

$$\underbrace{\begin{bmatrix}\dot{T}_E\\ \dot{T}_h\\ \dot{T}_{bs}\\ \dot{T}_{int}\end{bmatrix}}_{\dot{x}(t)} = \underbrace{\begin{bmatrix} -\frac{1}{C_E}(\frac{1}{R_{cext}}+\frac{1}{R_{cint}}) & 0 & 0 & \frac{1}{C_E R_{cint}} \\ 0 & -\frac{1}{C_h R_h} & 0 & \frac{1}{C_h R_h} \\ 0 & 0 & -\frac{1}{C_{occ}R_{occ}} & \frac{1}{C_{occ}R_{occ}} \\ \frac{1}{C_{int}R_{cint}} & \frac{1}{C_{int}R_h} & \frac{1}{C_{int}R_{occ}} & -\frac{1}{C_{int}}(\frac{1}{R_{eq_1}}+\frac{1}{R_{cint}}+\frac{1}{R_h}+\frac{1}{R_{occ}}) \end{bmatrix}}_{A} \underbrace{\begin{bmatrix}T_E\\ T_h\\ T_{bs}\\ T_{int}\end{bmatrix}}_{x(t)}$$
$$+ \underbrace{\begin{bmatrix} \frac{1}{R_{cext}C_E} & \frac{1}{C_E} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_h} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_{occ}} & 0 \\ \frac{1}{R_{eq_1}C_{int}} & 0 & 0 & 0 & \frac{1}{C_{int}} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix}T_{ext}\\ \phi_s\\ \phi_h\\ \phi_{occ}\\ \phi_{sw}\end{bmatrix}}_{u(t)} \tag{2}$$

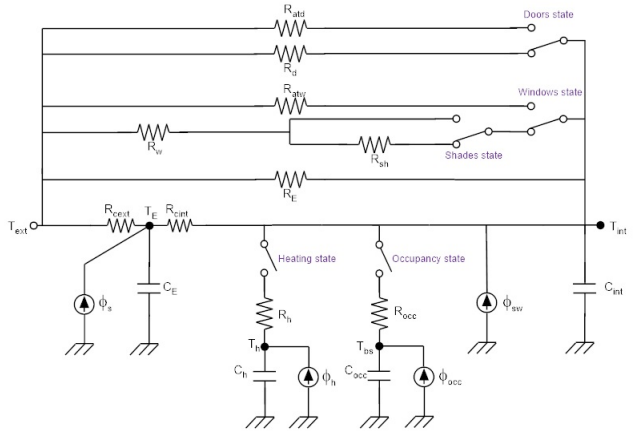

Fig. 1. Building thermal network

$$y(t) = \underbrace{[0\ 0\ 0\ 1]}_{C} \underbrace{\begin{bmatrix}T_E\\ T_h\\ T_{bs}\\ T_{int}\end{bmatrix}}_{x(t)} \tag{3}$$

is the output equation. For the case where the windows, doors and shades are closed, R_{eq1} is calculated as follows:

$$\frac{1}{R_{eq_1}} = \frac{1}{R_d} + \frac{1}{R_w + R_{sh}} + \frac{1}{R_E}. \tag{4}$$

Let us now consider the case for which the doors and windows are opened. The new equivalent resistance becomes:

$$\frac{1}{R_{eq_2}} = \frac{1}{R_{atd}} + \frac{1}{R_{atw}} + \frac{1}{R_E}. \tag{5}$$

One can see that $R_{eq_2} \neq R_{eq_1}$, which implies that the parameters in the system matrix A change, hence, the time constants (obtained from this matrix) change and therefore, the dynamic model changes. In fact, opening windows or doors will increase the air change between zones in the building or between the building and its environment. Moreover, the heat transfer by air change is characterized by a rapid dynamic response, it directly affects the indoor temperature and therefore, leads to a change in its underlying dynamics. Similarly, if the shades are opened when the windows are closed, a new value of the equivalent resistance is obtained:

$$\frac{1}{R_{eq_3}} = \frac{1}{R_d} + \frac{1}{R_w} + \frac{1}{R_E}. \tag{6}$$

Closing the shades will add an extra resistance to the heat transfer through windows ϕ_{sw}, it will also reduce the effect of solar radiation, thus changing temperature dynamics.

Given the above observations, we propose to use a hybrid switching model to describe these different configurations in a building. This model will join the discrete aspect of the system related to the transition between different configurations, and the continuous aspect of the temperature evolution that lies behind each configuration.

3.2 PWARX

Considering the fact that different dynamic behaviors occur in a building system, the approach here aims to model these different configurations given the input-output data of the system, using a Piece Wise AutoRegressive eXogeneous (PWARX) model. This model is based on input output observations in discrete-time, since this is the typical data-acquisition mode, we, thus, assume outputs to be observed at the sampling instants. The PWARX algorithm we are using for modeling the thermal behavior was introduced by Boukharouba (2011). It simultaneously treats classification of data and parameter estimation. The classification of data uses a novel method based on the euclidean distance between data in the regression vector, on one hand, and the error between the output of each submodel and the true output of the system, on the other hand. The parameter estimation is done using least squares. The model of interest is defined by

$$y(t) = f(\varphi(t)) + e(t), \tag{7}$$

where $e(t)$ refers to zero-mean noise samples and $\varphi(t) \in \mathbb{R}^n$ is the regression vector defined as

$$\varphi(t) = \left[y(t-1)...y(t-n_a)\ u(t)^\top ...u(t-n_b)^\top\right]^\top, \tag{8}$$

where $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}$ are the input vector (for a MISO model) and the measured output of the system at time $t \in \mathbb{Z}$ respectively. The parameters n_a and n_b are the model orders (maximum relevant past terms). In (7), f is a PieceWise Affine map of the form

$$f(x) = \begin{cases} \theta_1^\top \bar{\varphi}, & \text{if } \varphi \in \Re_1 \\ \quad\vdots & \\ \theta_s^\top \bar{\varphi}, & \text{if } \varphi \in \Re_s \end{cases} \tag{9}$$

where $\bar{\varphi} = [\varphi^\top\ 1]^\top$ is the extended regression vector, s is the number of submodels and $\{\theta_i\}_{i=1}^s$ are the parameter vectors containing polynomial coefficients defining each of the submodels of (7). One can notice that each submodel is no other than an ARX model, a linear parametric model that is frequently used in building modeling (section 2.2). However, instead of using one single ARX model to represent the heat dynamics of the building, we propose a set of ARX models, each describing a different configuration. In other words, each new configuration physically interpreted in section 3 will be modeled by an ARX model. The sets $\{\Re_i\}_{i=1}^s$ in (9) form a complete partition of the regression space $\Re \subseteq \mathbb{R}^n$, with $n = n_a + mn_b$. Each region $\Re_i$ is a convex polyhedron (see figure 2) with $\Re_i = \{\bar{\varphi} \in \mathbb{R}^{n+1} : H_i\bar{\varphi} \leq \mathbf{0}\}$, with H_i, a matrix of appropriate dimensions and $\mathbf{0}$, the null vector. For example, in figure 2, the regression data is distributed between three regions $\Re_{i=1,2,3}$ and a parameter vector is associated to each region. Data belonging to region $\Re_1$ follows the model defined by the parameter vector θ_1, and so on.

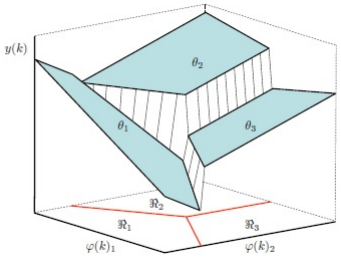

Fig. 2. Repartition of a PWARX system with 3 submodels (Boukharouba, 2011)

The algorithm treats simultaneously the problem of data clustering and parameter estimation. Given N data vectors $X(i) = [\varphi(i)^\top, y(i)]^\top;\ i = 1, ..., N$ from our system, the approach consists of three steps: initialization, data reaffectation and convergence test.
We set initially the number of submodels to $\bar{s} = N$, i.e. each data point is a submodel. We obtain N clusters $C = \{C_1, ..., C_N\}$, where $C_i = \{X(i)\}$. To these N clusters we assign the initial parameter vectors $\Theta^{(0)} = [\theta_1^{(0)}, ..., \theta_N^{(0)}]$ where $\theta_i^{(0)}$ is calculated by considering the data $X(i)$ of the submodel C_i and its c nearest neighbors, denoted as cNN, and by using the least squares technique on these c+1 data.

The initialization being done, the aim now is to reduce the number of clusters by reclassifying each data. Data will migrate towards the most representative clusters according to a specific decision rule. The less representative clusters become empty and thus are eliminated. Let $\Gamma_c(X(i))$ be the set of the cNN of $X(i)$ and let $X(j) \in \Gamma_c(X(i)), j = 1, ..., c$, be one of its neighbors that belongs to $C_p, p \in \{1, ..., \bar{s}\}$. We introduce the measure

$$\phi_j^i = exp(-\alpha_p (d_j^i)^2 - \beta_p (y(i) - \theta_p^\top \bar{\varphi}(i))^2), \tag{10}$$

where $d_j^i = [(X(i) - X(j))^\top (X(i) - X(j))]^{\frac{1}{2}}$ is the Euclidean distance between $X(i)$ and the neighbor $X(j)$ and θ_p is the parameter vector associated to cluster C_p. β_p and α_p are positive parameters associated to each cluster C_p and are computed by

$$\alpha_p = 1/d_p^2, \qquad \beta_p = 1/e_p^2, \tag{11}$$

where d_p is the average distance between data belonging to cluster C_p and e_p is the average error between the measured and the submodel output. We define now the probability for which data point $X(i)$ belongs to C_p

$$P(X(i) \in C_p) = \frac{\sum\limits_{j/X(j) \in \{\Gamma_c(X(i)) \cap C_p\}} \phi_j^i}{\sum\limits_{j=1}^{c} \phi_j^i},\ p \in \{1, ..., \bar{s}\}. \tag{12}$$

This probability is equal to 1, if all the cNN of $X(i)$ belong to the cluster C_p and it is equal to 0 if none of them belongs to C_p. The decision is made by assigning the data $X(i)$ to the cluster C_{ret}, $ret \in \{1, ..., \bar{s}\}$, that achieves the maximum of P. The procedure ends when the stopping criterion $\| \Theta^{(r+1)} - \Theta^{(r)} \| \leq \eta$ is achieved, r being the iteration index and η is a threshold set by the user.

4. TEST CASE

We consider a room located in Douai, France. Data generated by the numerical model of the room created with Dymola (Wetter et al., 2015) is used for this application example. To study the ability of PWA models to describe the thermal behavior of the room, we tested a scenario in which the air change rate switches between 0.2 vol/h and 20 vol/h every 6 days and is assumed unknown. This scenario is equivalent to the transition between a configuration with closed windows and another one where they are open, i.e, changing the position of the windows switch (see 3.1). The room temperature is the output variable, the exterior influence (temperature, wind velocity, relative humidity, solar radiation) and the heating power are input parameters. The heat flow changes randomly every 15 min between 0 and 3000 W and is assumed known. The simulation is run for 36 days, the sample interval is 15 min.

The model's accuracy is computed by means of the Fit that should be close to 100% for a good match.

$$Fit = 100\left(1 - \frac{norm(y-\hat{y})}{norm(y-\bar{y})}\right), \tag{13}$$

where y is the measured output (temperature), $\bar{y}$ its mean and $\hat{y}$ the simulated temperature.

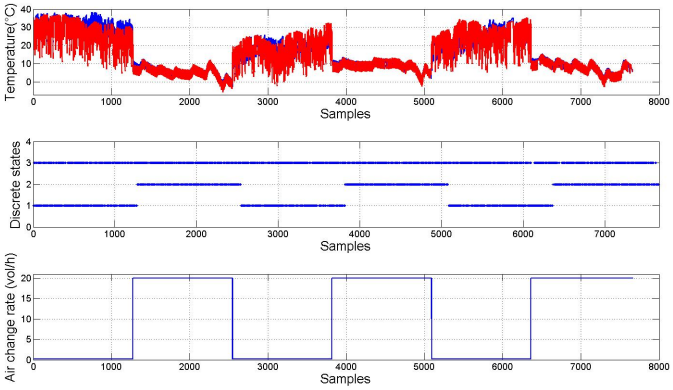

Fig. 3. (a) True output of the system (blue), estimated output of PWA models (red) for the entire data set; (b) discrete states of the system = number of submodels = number of clusters; (c) air change rate

Fig. 3 shows that PWA models are able to identify two functioning modes corresponding to the two values of the air change rate and the estimated temperature follows the output with a fit of 64%. Several orders were tested and the previous result was obtained for $n_a = n_b = 1$ and $c = 1500$. This same figure shows that the clusters identified correspond to the regions where the air change rate switches values. A third class is related to the heating power when it is at its maximal value. This preliminary result shows that using the PWARX we are able to identify different functionning modes that correspond to various configurations in a building system.

5. CONCLUSION AND FUTURE WORK

In this work, we presented a new technique for the thermal modeling of buildings based on the fact that different configurations might occur inside of the building. It was shown that for each new configuration a new dynamic model can be used to describe the behavior. The suggested technique allows switching between different linear regression models based on some criteria calculated from input data. The estimated model can be further used for temperature control.
For future work, we will continue investigating the capability of PWARX for describing the thermal behavior inside a building with physical interpretation of the switching using thermal networks identified for the case study.

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