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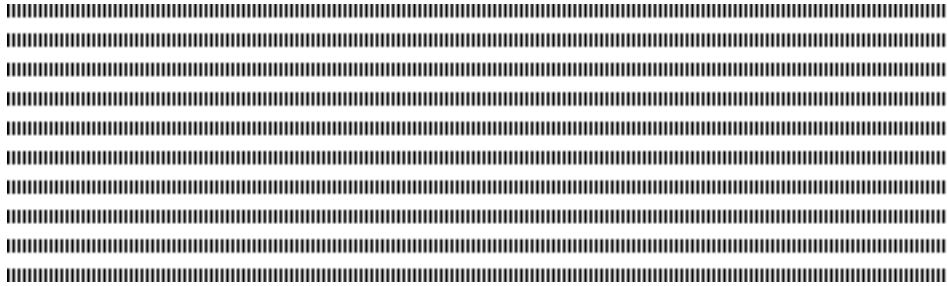
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Optimal Control of an SIR Model with delayed State variable

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RÉSUMÉ. Dans cet article, nous avons étudié une stratégie de contrôle optimal correspondante à un modèle épidémique de type SIR avec des retards dans l'état. Le modèle a été développé pour examiner plusieurs stratégies de traitement avec vaccination et déterminer leurs effets sur la propagation d'une infection. Le modèle prédit trois classes des personnes infectées et la dynamique de transmission des maladies avec certains scénarios de traitement. Selon le modèle, les sous-classes d'infection avec des retards dans l'état, peuvent être le meilleur moyen de réduire la propagation d'une épidémie. Les problèmes aux limites multi-points ont été obtenus en fonction de la version discrète de principe du maximum de Pontryagin, et résolus numériquement en utilisant un schéma discret progressif régressif qui converge suite à un test approprié lié à la méthode de balayage avant-arrière dans un contrôle optimal. Des simulations numériques sont également menées à confirmer et étendre les résultats analytiques.

ABSTRACT. In this paper, we investigate the optimal control strategy of an epidemic SIR model with delays in state. The model was developed to consider several treatment strategies with vaccination and determine their effects on the spread of the infection. The model predicts three classes of infected people and the transmission dynamics of diseases with certain treatment scenarios. According to the model, sub-classes of infective with delays in state, may be the best way to reduce the force of an epidemic. The multi-points boundary value problems are obtained based on discrete version of Pontryagin's maximum principle, and resolved numerically using a progressive-regressive discrete scheme that converges following an appropriate test related to the Forward-Backward Sweep Method (FBSM) in optimal control. Numerical simulations are also conducted to confirm and extend the analytic results.

MOTS-CLÉS : Modèles épidémiques avec retard, modèle discrète SIR, contrôle optimal, les problèmes aux limites multi-points

KEYWORDS : delay epidemic models, discrete SIR model, optimal control, multi-points boundary value problems



1. Introduction

Nowadays, mathematics play an important role in choosing the appropriate and effective means in the fighting against diseases. In 1925 Sir R.ROSS was the first to establish a mathematical model based on what we called compartmental model, the main goal of this model is to control the spread of malaria by controlling the vector population.

Recently, there have been determined efforts to encourage better communication between mathematicians and health professionals so they can better understand the situations in which simple models may be used. Mathematical models have provided a useful tool to gain insights into the transmission and control of diseases. These insights can potentially help guide us to evaluate the effectiveness and implications of various preventive and control strategies. Motivated by the importance of mathematical modeling of infectious diseases. We must consider more realistic and hence more complex models, which allow quantitative comparison with known data and let us make practical predictions with more confidence. In this paper we propose the standard SIR model, by introducing two new sub-classes I^2 and I^3 with delays in state in order to explain the stage of infectivity.

The aim of this work is not to consider one special disease but to set up an optimal control problem relative to discrete-time SIR models with delays in state, which makes it a genuine control application, providing a simple framework to analyze several treatment strategies with vaccination and determine their effects on the spread of the infection.

All the way through this paper, it is assumed that the analytical results are also used to guide our numerical studies, which illustrate how the combination of vaccination and several treatment used may change the total infection level.

Mathematical properties of the model system are studied both analytically and numerically. It is assumed that the system has three possible classes of infected people I_i^1 , I_i^2 , I_i^3 with delays in state. As the main purpose of this model is to investigate the effect of vaccination and several treatment, we study the interaction between vaccination and treatment of these different classes of infected persons, any kind of diseases that respects this rule, can be modelled and controlled with our approach. For instance, more recently, the threat of the spread of influenza pandemic [1, 4, 5, 8, 7, 12, 2].

The first control objective is to restrict the spread of the epidemic, by introducing a vaccination control variable in our discrete delayed SIR model, as a second task, we try to treat the infected classes to reduce the spread of the epidemic. Effective treatment will therefore be an adequate measure in the event of an epidemic.

We illustrate how the optimal control theory can be applied to minimize the infected individuals and increase the susceptible, so minimize the cost of applying treatment and vaccination controls.

The remainder of this paper is organized as follows : In section 2 we describe the mathematical model. Objective functional and the analysis of optimal control are given in Section 3, where section 4 includes the numerical simulations. Finally, we conclude in Section 5.

2. Mathematical Model

According to the disease transmission mechanism, a host can be Susceptible, Infective, or Recovered. Susceptible individuals are those who are healthy and do not carry the epidemic but can contract it from infective hosts, denoted by S_i . Infective hosts carry the contagion and are able to pass it on to another hosts, in this model we sub-divide the total

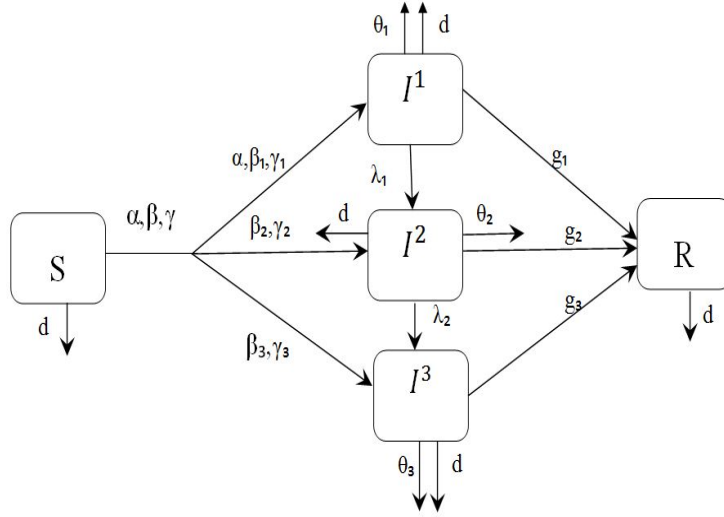


Figure 1. Schematic presentation of SIR model

human infected, into sub-populations of infective class one individuals I_i^1 , those infected class two I_i^2 , individuals with class three I_i^3 . Finally, recovered R_i , hosts are those who are no longer infective and have acquired immunity from future infection note that this immunity is not necessarily everlasting and that the transition states is probabilistic, with probabilities being determined by the observed characteristics of specific diseases.

In addition to the death, there are population movements among those three epidemiological compartments, from time unit i to time $i + 1$; $i = 1, \dots, N - 1$ thus, the infection transmission is assumed to occur between individuals present in a given population and is given by

$$[\beta I_i^1 S_i, \alpha I_i^2 S_i \text{ and } \gamma I_i^3 S_i]$$

where $\alpha > 0$, $\beta > 0$ and $\gamma > 0$ are constant parameters, α is the proportion of contacts between a susceptible and an infective class 1, β is the proportion of contacts between a susceptible and an infective class 2, γ is the proportion of contacts between a susceptible and an infective class 3. we assume that every fraction of β, γ are divided respectively into $\beta_1, \beta_2, \beta_3$ and $\gamma_1, \gamma_2, \gamma_3$ where β_i, γ_i is the gain in the infective class I^i . In order to have a realistic model, we need to take into account that the movement of the infected individuals I^1, I^2, I^3 from a class to another one is subjected to delays. Thus, the time delay is introduced in the system as follows : at time i only a percentage λ_1 of patiented individuals I^1 that have been transmitted p time unit ago, that is, at time, $i - p$, when the patiented condition is aggravated, they are removed from the infected class I^1 and added to the infected class I^2 . In addition at time i only a percentage λ_2 of patiented individuals I^2 that have been transmitted q time unit ago, that is, at time, $i - q$, when he passes to a more serious state of the disease, they are removed from the infected class I^2 and added to the infected class I^3 . Our model is based, in addition, on the following assumptions :

- 1) d is the natural mortality rate ;
- 2) The death rates due to the infection are θ_1, θ_2 and, θ_3 , for each class I^1, I^2 and I^3 respectively ;

3) Infective I^i recover their immunity and leave the infective compartments at rates g_1, g_2 and g_3 , and go to the removed compartment R ;

4) It is reasonable to assume that $\beta_1 > \beta_2 > \beta_3$ and $\gamma_1 > \gamma_2 > \gamma_3$;

$$S_{i+1} = S_i - \alpha I_i^1 S_i - \beta I_i^2 S_i - \gamma I_i^3 S_i - d S_i \quad [1]$$

$$I_{i+1}^1 = I_i^1 + \alpha I_i^1 S_i + \beta_1 I_i^2 S_i + \gamma_1 I_i^3 S_i - \lambda_1 I_{i-p}^1 - (\theta_1 + g_1 + d) I_i^1 \quad [2]$$

$$I_{i+1}^2 = I_i^2 + \lambda_1 I_{i-p}^1 + \beta_2 I_i^2 S_i + \gamma_2 I_i^3 S_i - \lambda_2 I_{i-q}^2 - (\theta_2 + g_2 + d) I_i^2 \quad [3]$$

$$I_{i+1}^3 = I_i^3 + \beta_3 I_i^2 + \gamma_3 I_i^3 S_i + \lambda_2 I_{i-q}^2 - (\theta_3 + g_3 + d) I_i^3 \quad [4]$$

$$R_{i+1} = R_i + g_1 I_i^1 + g_2 I_i^2 + g_3 I_i^3 - d R_i \quad [5]$$

$S_0 \quad I_0 \quad R_0 \quad \text{are given}$

3. Model with Treatment and Vaccination

As a strategy of control, we choose a vaccination program and several treatment, so into the model(1) we include a control v_i that represents the density of susceptible individuals being vaccinated per time i . We assume that all susceptible vaccinates are transferred directly to the removed class and into the model (2-3-4) we include a control (u_i^1, u_i^2, u_i^3) which is the control treatments of (I_i^1, I_i^2, I_i^3) respectively. The model is given by the following equations :

$$S_{i+1} = S_i - \alpha I_i^1 S_i - \beta I_i^2 S_i - \gamma I_i^3 S_i - \varepsilon_1 v_i S_i - d S_i$$

$$I_{i+1}^1 = I_i^1 + \alpha I_i^1 S_i + \beta_1 I_i^2 S_i + \gamma_1 I_i^3 S_i - \lambda_1 I_{i-p}^1 - (\theta_1 + g_1 + d) I_i^1 - \varepsilon_2 u_i^1 I_i^1$$

$$I_{i+1}^2 = I_i^2 + \lambda_1 I_{i-p}^1 + \beta_2 I_i^2 S_i + \gamma_2 I_i^3 S_i - \lambda_2 I_{i-q}^2 - (\theta_2 + g_2 + d) I_i^2 - \varepsilon_3 u_i^2 I_i^2$$

$$I_{i+1}^3 = I_i^3 + \beta_3 I_i^2 + \gamma_3 I_i^3 S_i + \lambda_2 I_{i-q}^2 - (\theta_3 + g_3 + d) I_i^3 - \varepsilon_4 u_i^3 I_i^3$$

$$R_{i+1} = R_i + g_1 I_i^1 + g_2 I_i^2 + g_3 I_i^3 + \varepsilon_1 v_i S_i + \varepsilon_2 u_i^1 I_i^1 + \varepsilon_3 u_i^2 I_i^2 + \varepsilon_4 u_i^3 I_i^3 - d R_i$$

Where the parameters $\{(\varepsilon_i \in \{0, 1\} / i = 1, 2, 3, 4)\}$ are used to consider several treatment strategies with vaccination to determine their effects on the spread of the infection. Our goal is obviously to try to minimize the population of the infected group and the cost of treatment, while increasing the population in the removed group. Our control function

is assumed taking values between $v_{max} < 1$, $u_{max}^1 < 1$, $u_{max}^2 < 1$, $u_{max}^3 < 1$, and $u_{min}^2 > 0$, $u_{min}^3 > 0$, $u_{min}^1 > 0$, $v_{min} > 0$.

3.1. Optimal Control problem.

Generally to minimize the number of sensitive and infected person, and maximize the number of an individual recovered during an epidemic. We are interested in controlling the population . Then, the problem is to minimize the objective functional given by

$$J(v, u^1, u^2, u^3) = \sum_{i=1}^{N-1} \left(\frac{a_i}{2}(v_i)^2 + \frac{b_i}{2}(u_i^1)^2 + \frac{c_i}{2}(u_i^2)^2 + \frac{d_i}{2}(u_i^3)^2 \right) + \sum_{i=1}^N (A_1 I_i^1 + A_2 I_i^2 + A_3 I_i^3 - A_4 R_i)$$

Where $A_1 > 0, A_2 > 0, A_3 > 0, A_4 > 0$, and $a > 0, b > 0, c > 0, d > 0$ are the weight constants of control, the infected and the removed group respectively, $u_i^1 = (u_0^1, \dots, u_{N-1}^1), u_i^2 = (u_0^2, \dots, u_{N-1}^2), u_i^3 = (u_0^3, \dots, u_{N-1}^3)$ and $v_i = (v_0, \dots, v_{N-1})$. Our goal is to minimize the infected, while minimizing the cost of applying controls. In other words, we are seeking an optimal control $u_i^{1*}, u_i^{2*}, u_i^{3*}$ and v_i^* such that

$$J(u_i^{1*}, v_i^*, u_i^{2*}, u_i^{3*}) = \min \{ J(u_i^1, v_i, u_i^2, u_i^3) / (u_i^1, u_i^2, u_i^3) \in U, v_i \in V \}$$

Where U and V are the control sets defined by

$$U = \{ u \text{ measurable} / u^{min} \leq u_i^m \leq u^{max}, i = 1, \dots, N-1, m \in (1, 2, 3) \}$$

$$V = \{ v \text{ measurable} / v^{min} \leq v_i \leq v^{max}, i = 1, \dots, N-1 \}$$

The sufficient condition for existence of an optimal control for the problem follows from theorem 1 in [10, 11] . At the same time by using Pontryagin's Maximum Principle[9] we derive necessary conditions for our optimal control. For this purpose we define the Hamiltonian as :

$$\begin{aligned} H(\Omega) = & (A_1 I_i^1 + A_2 I_i^2 + A_3 I_i^3 - A_4 R_i \\ & + \frac{a_i}{2}(v_i)^2 + \frac{b_i}{2}(u_i^1)^2 + \frac{c_i}{2}(u_i^2)^2 + \frac{d_i}{2}(u_i^3)^2) \\ & + \zeta_{1,i+1} [S_i - \alpha I_i^1 S_i - \beta I_i^2 S_i - \gamma I_i^3 S_i - \varepsilon_1 v_i S_i \\ & - d S_i] + \zeta_{2,i+1} [I_i^1 + \alpha I_i^1 S_i - (\theta_1 + g_1 + d) I_i^1 \\ & - \varepsilon_2 u_i^1 I_i^1 + \beta_1 I_i^2 S_i + \gamma_1 I_i^3 S_i - \lambda_1 I_{i-p}^1] \\ & + \zeta_{3,i+1} [I_i^2 + \lambda_1 I_{i-p}^1 + \beta_2 I_i^2 S_i + \gamma_2 I_i^3 S_i - \lambda_2 I_{i-q}^2] \\ & - (\theta_2 + g_2 + d) I_i^2 - \varepsilon_3 u_i^2 I_i^2] \\ & + \zeta_{4,i+1} [I_i^3 + \beta_3 I_i^2 S_i + \gamma_3 I_i^3 S_i + \lambda_2 I_{i-q}^2) \\ & - (\theta_3 + g_3 + d) I_i^3 - \varepsilon_4 u_i^3 I_i^3] \\ & + \zeta_{5,i+1} [R_i + g_3 I_i^3 + g_1 I_i^1 + g_2 I_i^2 - d R_i) \\ & + \varepsilon_2 u_i^1 I_i^1 + \varepsilon_3 u_i^2 I_i^2 + \varepsilon_4 u_i^3 I_i^3 + \varepsilon_1 v_i S_i \end{aligned}$$

Theorem 1 (Necessary Conditions). *Given an optimal control $u_i^{1*}, u_i^{2*}, u_i^{3*}, v_i^*$ and solutions $S^*, I_i^{1*}, I_i^{2*}, I_i^{3*}$*

and R^* , there exists $\zeta_{k,i}$, $i = 1 \dots N$, for $k = 1, 2, 3, 4, 5$, the adjoint variables satisfying the following equations :

$$\begin{aligned}
\Delta \zeta_{1,i} &= - \left(\zeta_{1,i+1} (1 - \alpha I_i^1 - \gamma I_i^3 - d) + \beta I_i^2 (\zeta_{2,i+1} - \zeta_{1,i+1}) \right. \\
&+ \zeta_{2,i+1} (\beta_1 I_i^2 + \gamma_1 I_i^3) + \zeta_{3,i+1} (\beta_2 I_i^2 + \gamma_2 I_i^3) \\
&+ \zeta_{4,i+1} (\beta_3 I_i^2 + \gamma_3 I_i^3) + \varepsilon_1 v_i (\zeta_{5,i+1} - \zeta_{1,i+1}) \left. \right) \\
\Delta \zeta_{2,i} &= - \left(A_1 + (\zeta_{2,i+1} - \zeta_{1,i+1}) \alpha S_i + \zeta_{2,i+1} (1 - (\theta_1 + d)) \right. \\
&+ (\zeta_{5,i+1} - \zeta_{2,i+1}) (\varepsilon_2 u_i^1 + g_1) \left. \right) \\
\Delta \zeta_{3,i} &= - \left(A_2 - \zeta_{1,i+1} \beta S_i + \zeta_{2,i+1} \beta_1 S_i \right. \\
&+ \zeta_{3,i+1} (1 + \beta_2 S_i - (\theta_2 + d)) \\
&+ \zeta_{4,i+1} \beta_3 S_i + (\zeta_{5,i+1} - \zeta_{3,i+1}) (\varepsilon_3 u_i^2 + g_2) \left. \right) \\
\Delta \zeta_{4,i} &= - \left(A_3 - \zeta_{1,i+1} \gamma S_i + \zeta_{2,i+1} \gamma_1 S_i + \zeta_{3,i+1} \gamma_2 S_i \right. \\
&+ \zeta_{4,i+1} (1 + \gamma_3 S_i - (g_3 + \theta_3 + d)) \\
&+ \zeta_{5,i+1} (\varepsilon_4 u_i^3 + g_3) \left. \right) \\
\Delta \zeta_{5,i} &= - \left(-A_4 + \zeta_{5,i+1} (1 - d) \right)
\end{aligned}$$

Démonstration. With $\zeta_{1,N} = 0$, $\zeta_{2,N} = A_1$, $\zeta_{3,N} = A_2$, $\zeta_{4,N} = A_3$, $\zeta_{5,N} = -A_4$,. To obtain the optimality conditions we take the variation with respect to control (u_i^m $m = 1, 2, 3$, and v_i) and set it equal to zero

$$\frac{\partial \mathcal{H}}{\partial u_i^1} = b_i u_i^1 - \zeta_{2,i+1} \varepsilon_2 I_i^1 + \zeta_{5,i+1} \varepsilon_2 I_i^1 = 0$$

$$\frac{\partial \mathcal{H}}{\partial u_i^2} = c_i u_i^2 + \zeta_{5,i+1} \varepsilon_3 I_i^2 - \zeta_{3,i+1} \varepsilon_3 I_i^2 = 0$$

$$\frac{\partial \mathcal{H}}{\partial u_i^3} = d_i u_i^3 + \zeta_{5,i+1} \varepsilon_4 I_i^3 - \zeta_{4,i+1} \varepsilon_4 I_i^3 = 0$$

$$\frac{\partial \mathcal{H}}{\partial v_i} = a_i v_i - \zeta_{1,i+1} \varepsilon_1 S_i + \zeta_{5,i+1} \varepsilon_1 S_i = 0$$

Then we obtain the optimal control pair

$$u_i^1 = \frac{(\zeta_{2,i+1} - \zeta_{5,i+1}) \varepsilon_2 I_i^1}{b_i}; \quad i = 0, \dots, N - 1$$

$$u_i^2 = \frac{(\zeta_{3,i+1} - \zeta_{5,i+1}) \varepsilon_3 I_i^2}{c_i}; \quad i = 0, \dots, N - 1$$

$$u_i^3 = \frac{(\zeta_{4,i+1} - \zeta_{5,i+1})\varepsilon_4 I_i^3}{d_i}; i = 0, \dots, N - 1$$

$$v_i = \frac{(\zeta_{1,i+1} - \zeta_{5,i+1})\varepsilon_1 S_i}{a_i}; i = 0, \dots, N - 1$$

By the bounds in U and V of the control, it is easy to obtain $u_i^{m*}; m = \{1, 2, 3\}$ and v_i^* in the following form \square

$$\begin{aligned} v_i^* &= \min\{ \max\{v^{min}, \frac{(\zeta_{1,i+1} - \zeta_{5,i+1})\varepsilon_1 S_i}{a_i}\}, v^{max} \}, \\ u_i^{1*} &= \min\{ \max\{u^{min}, \frac{(\zeta_{2,i+1} - \zeta_{5,i+1})\varepsilon_2 I_i^1}{b}\}, u^{max} \}, \\ u_i^{2*} &= \min\{ \max\{u^{min}, \frac{(\zeta_{3,i+1} - \zeta_{5,i+1})\varepsilon_3 I_i^2}{c}\}, u^{max} \}, \\ u_i^{3*} &= \min\{ \max\{u^{min}, \frac{(\zeta_{4,i+1} - \zeta_{5,i+1})\varepsilon_4 I_i^3}{d}\}, u^{max} \}, \\ i &= 0, \dots, N - 1 \end{aligned}$$

4. Numerical simulation.

In this section we first present an iterative method for the numerical solution of the optimality system. Next, we present numerical results obtained using Matlab.

4.1. Numerical simulation

We now present numerical simulations associated with the above mentioned optimal control problem. To solve this system we wrote a code in MATLABTM and simulated our results using different data. We solve the optimality systems using an iterative method. Where the state system with an initial guess is solved forward in time and then the adjoint system is solved backward in time because of the transversality conditions. Afterwards, we updated the optimal control values using the values of state and adjoint variables obtained in the previous steps. Finally, we execute the previous steps till a tolerance criterion is reached. we obtain this numerical simulations to illustrate our theoretical results using data cited in table 1.

4.2 Numerical results

4.1.1. Without controls

figure2 : the following figures show the dynamic of the population without any control strategy. We start with a population of 760 susceptible people, 50 infected ones in the first stage, 10 ones in the second and no one in the last stage. We analyze data over a period of 20 days, we remark that in the absence of control, the number of susceptible decrease exponentially to the benefit of infected people, thus the number of infected in the first stage increase exponentially after 5 days, while the number of infected in the second stage takes 12 days to show a remarkable growth, and the number of infected in the third stage grow slowly, which is reasonable according to our parameters and the dynamic of the disease. We remark also that the growth of people Recovering their immunity as well as I_2 and I_3 .

Notations	Value	Description(Units)
S_0	760	Initial susceptible population
I_0^1	50	Initial infected population
I_0^2	15	Initial infected population
I_0^3	0	Initial infected population
R_0	0	Initial immune population
α	0.0014	Transmission rate between S and $I^1(day^{-1})$
β	$\beta = \beta_1 + \beta_2 + \beta_3 = 0.0016$	Transmission rate between S and $I^2(day^{-1})$
γ	$\gamma = \gamma_1 + \gamma_2 + \gamma_3 = 0.004$	Transmission rate between S and $I^3(day^{-1})$
d	0.003	Natural death rate
g_1	$g_1 = 0.05$	Recovery rate of $I^1(day^{-1})$
g_2	$g_2 = 0.04$	Recovery rate of $I^2(day^{-1})$
g_3	$g_3 = 0.02$	Recovery rate of $I^3(day^{-1})$
λ_1	$\lambda_1 = 0.1$,	Probability of Transmission the infected in I^1 to I^2 at time p
λ_2	$\lambda_2 = 0.03$	Probability of Transmission the infected in I^2 to I^3 at time q
(p, q)	(1, 4)	Time of Transmission the infected in class to another
$\theta_i, i = 1, 2, 3$	0.005	The disease induced death rate (day^{-1})

Tableau 1. *Initial conditions and parameters values*

As a control strategy, we adopt a vaccination of susceptible people and a treatment infected ones. In order to show the impact and thus the importance of each control, we suggest 4 scenarios where we put our controls one by one. The following paragraphs present the numerical results.

4.1.2. With controls

Figure3 : Now we discuss the first scenario where we put our first control which is the vaccination. We remark that, in comparison with results without control, the number of susceptible individuals decreases more rapidly, and the growth of infected people within control is less than the other. The great benefit is on the number of recovered people which is reasonable and remarkable due to the effect of vaccination program.

Figure4 : At present, we put in treatment the infected people in the first stage of infection. Figures show that the evolution of susceptible is the same as without control (see Fig.2). The important difference is in the evolution of I_1 infected in the 1st stage, thus we remark that the number of infected individuals I_1 with control (solid curve) and without control (dashed curve). It shows that in the presence of a control, the number of infected individuals I_1 decreases greatly. The maximum number of infected individuals in the case with control is about 350 and is about 150 in the case without control ; moreover we see a small decrease in infected I_2 , and I_3 as a consequence of the decrease of the infection , the last Figure shows that the number of people removed begins to grow as well as at I_1 ; then the efficiency of our strategy in reducing the spread of infection.

Figure5 : Here we put in treatment the I_2 people, from this and from figures 2, there are no changes in the evolution of S and I_1 in comparison with the cases without control, which is reasonable. The benefit is in the number of people in the class I_2 , there is a notable decrease of them and as a result, the number of infected people in

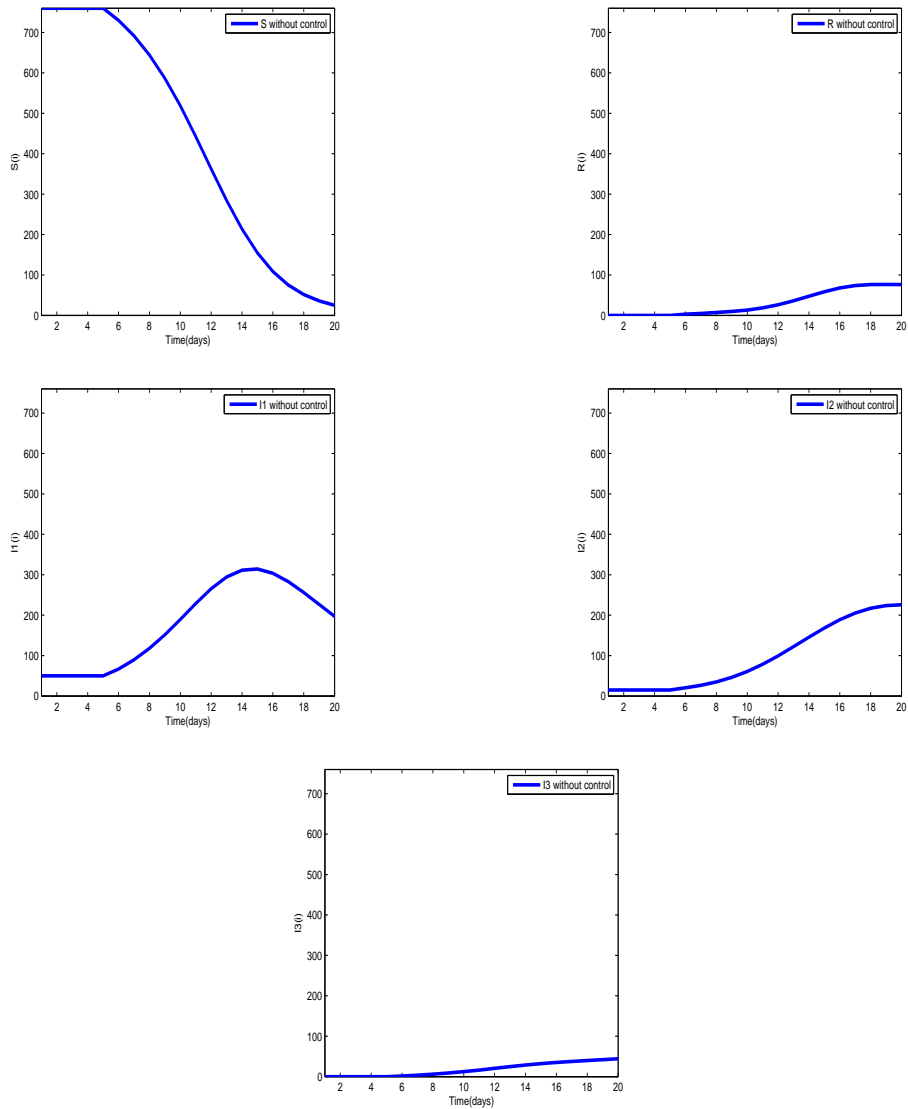


Figure 2. The function S, I^1, I^2, I^3, R without control. the class I_3 decrease. The number of recovered increase as well as the last case.

Figure6 : Finally, we put in treatment the infected people in the 3th stage, we can see from figures that there are no changes in S, I_1, I_2 and R , the effect of our control is view in the number of class I_3 , there is a small decrease.

Figure7 : Finally, we combine all previous control strategies, we can see from the following figures the great changes in all classes. The numbers S, I_1, I_2 and I_3 decrease notably, while the number of removed R increases.

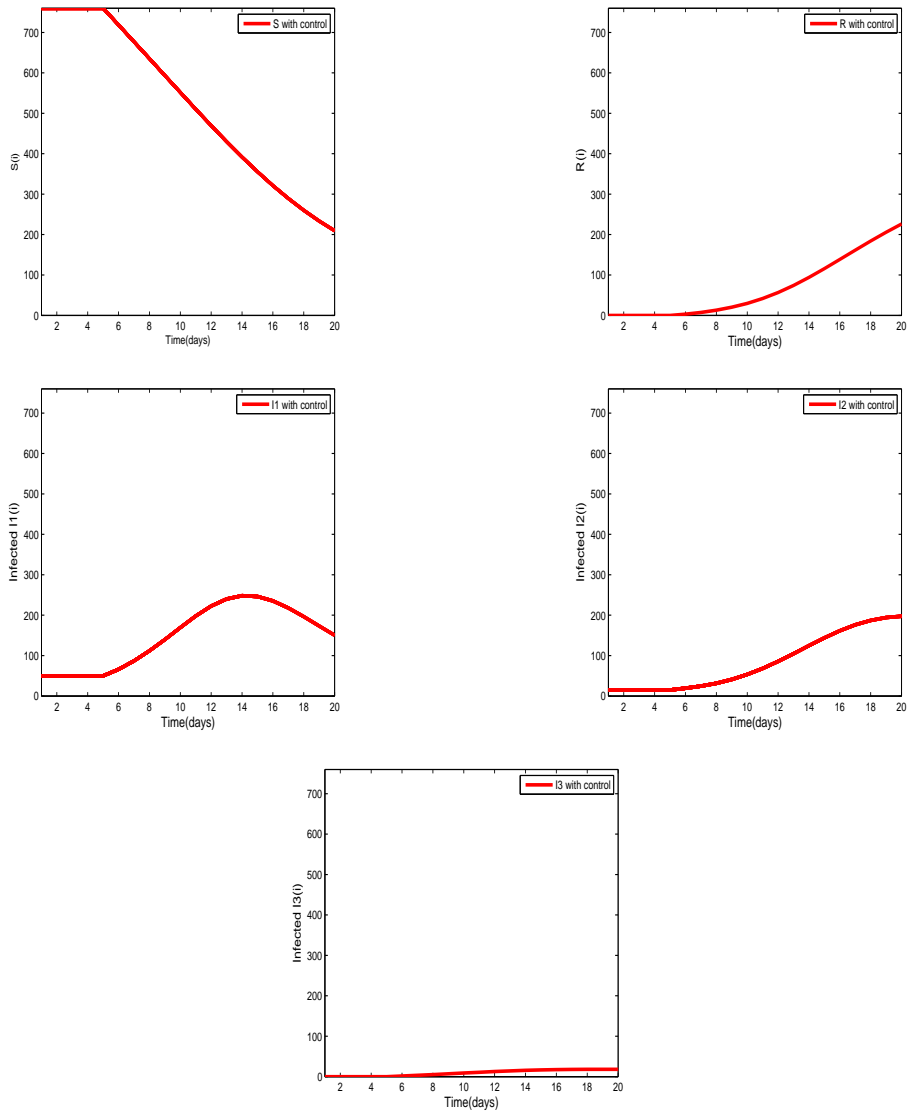


Figure 3. The function S, I^1, I^2, I^3, R with control v

5. Conclusion

In this work we discussed an efficient numerical method based on optimal control to study the several treatment strategies with vaccination in our discrete delayed SIR model, on the spread of the infection. The numerical simulation of both the systems i.e with control and without control, shows that the several strategies helps to reduce the number of infected and susceptible individuals and increase the number of the removed individuals greatly. The results obtained shows also that the effectiveness of vaccination campaign.

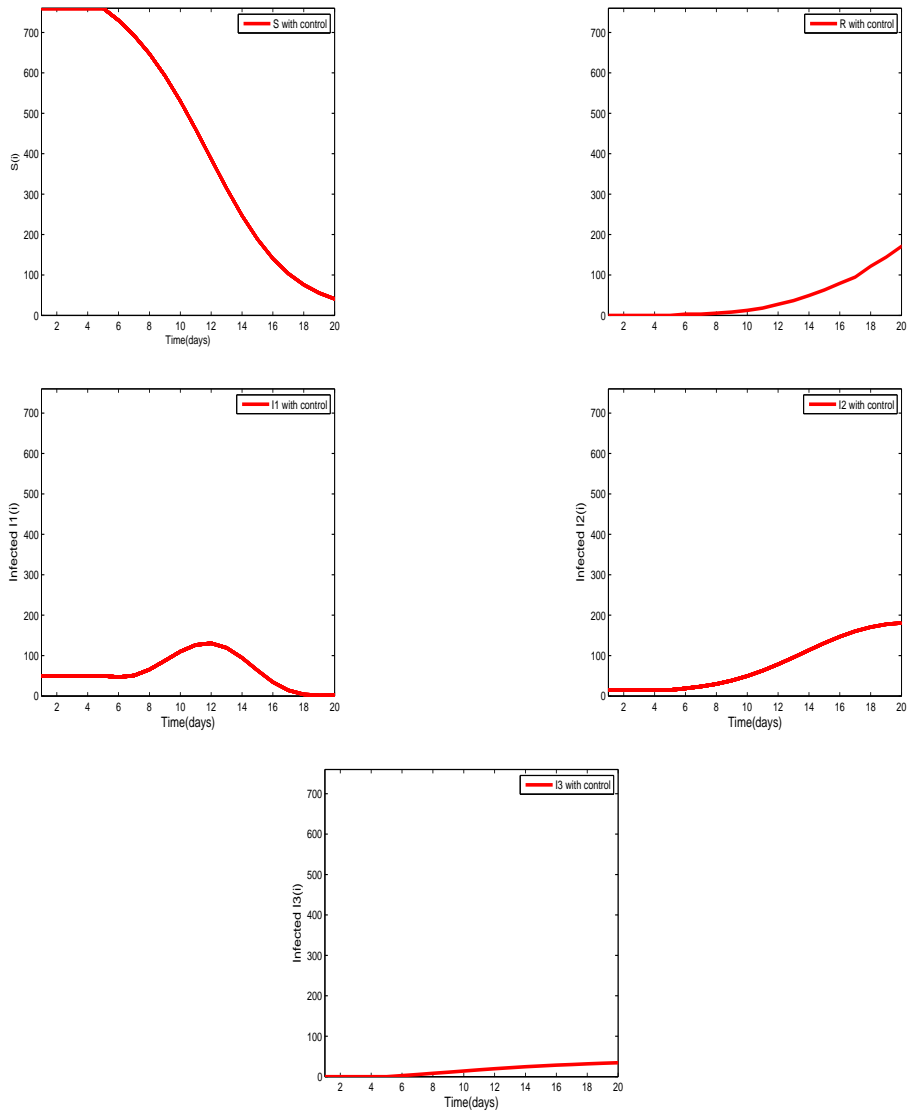


Figure 4. The function S, I^1, I^2, I^3, R with control u^1 .

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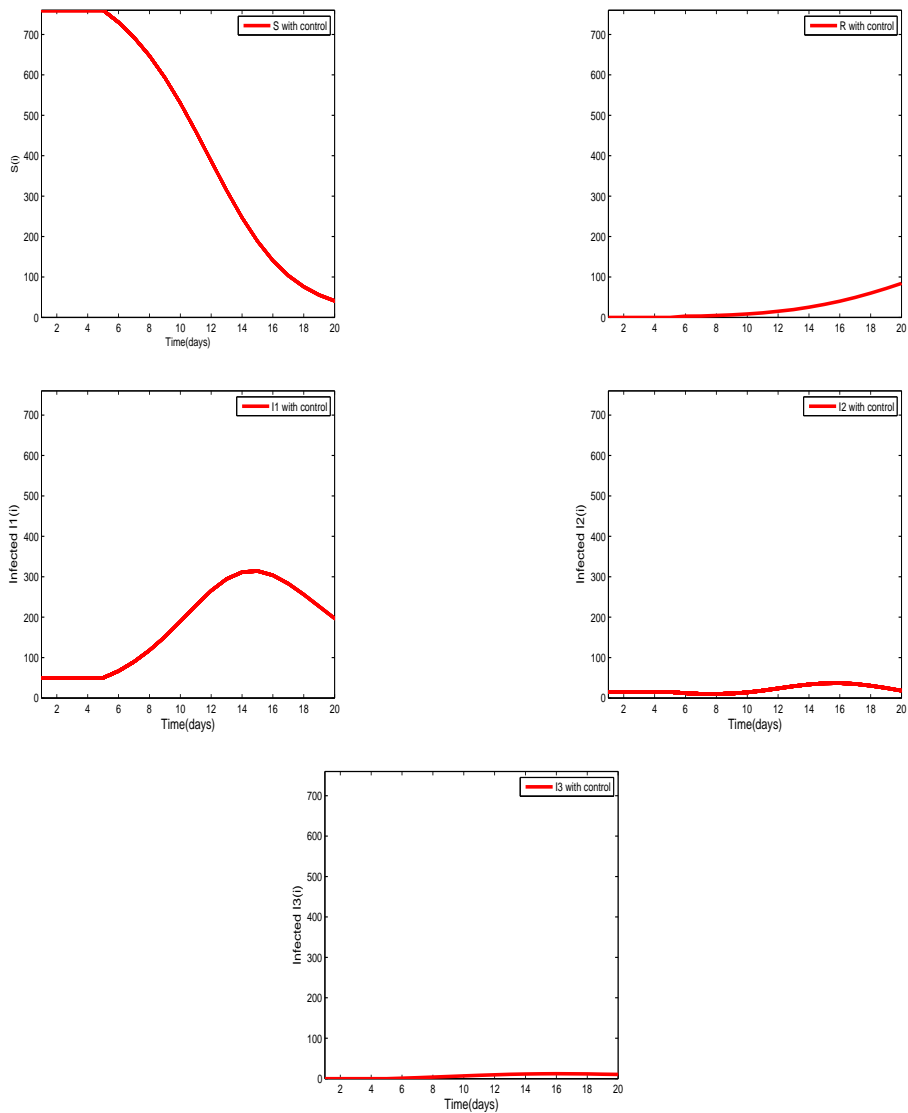


Figure 5. The function S, I^1, I^2, I^3, R with control u^2 .

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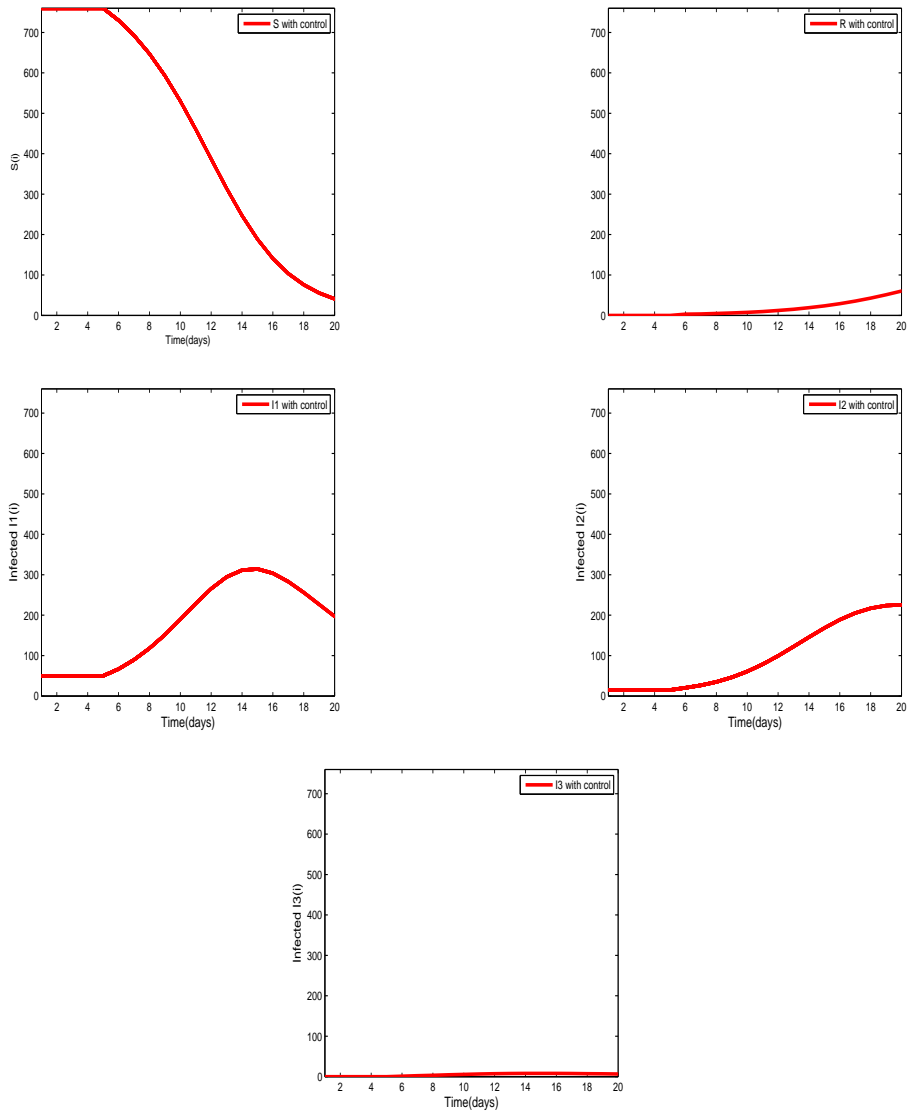


Figure 6. The function S, I^1, I^2, I^3, R with control u^3 .
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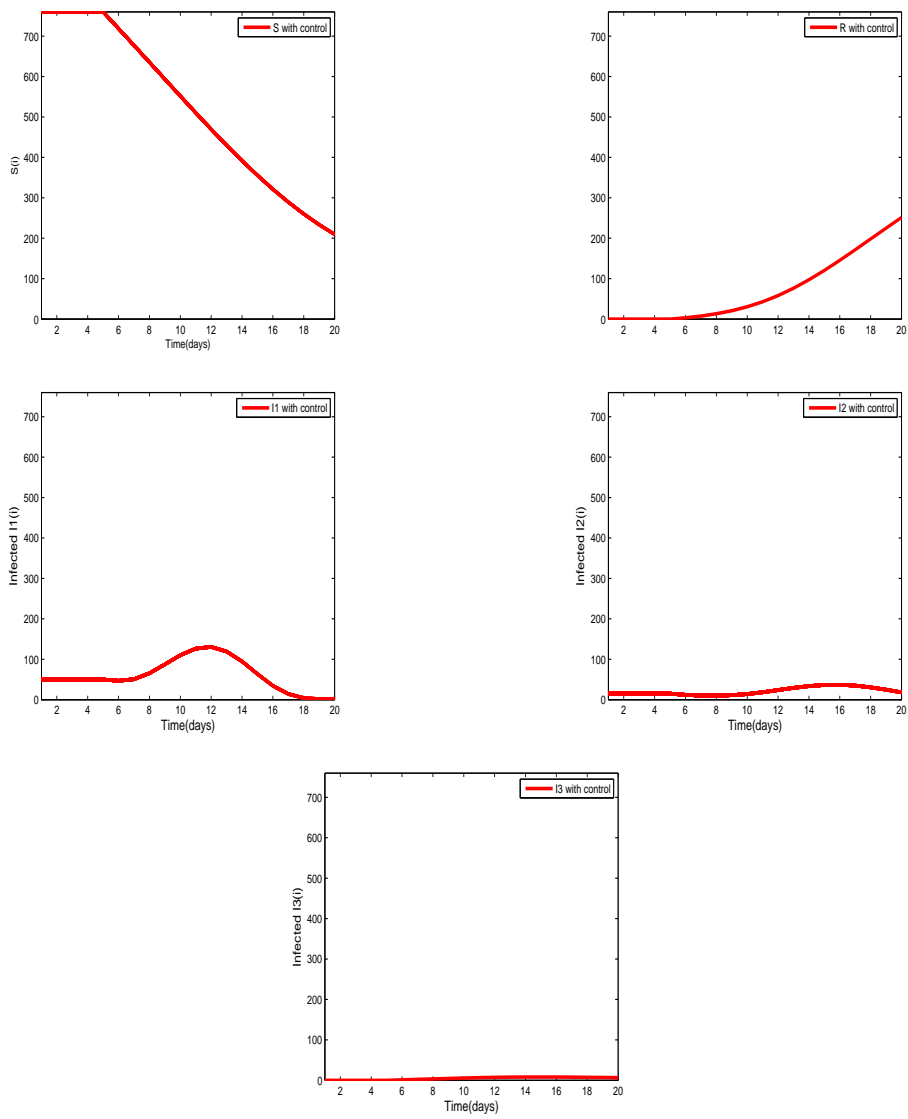


Figure 7. The function S, I^1, I^2, I^3, R with controls v, u^1, u^2, u^3 .