# Gravitational Interaction between Photons and Unification of All the Fundamental Interactions 

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# Gravitational Interaction between Photons and Grand Unification 

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#### Abstract

Recently, it has been reported an experiment where a very weak laser beam passes through a dense cloud of ultracold rubidium atoms. Under these circumstances, it was observed that the photons bound together in pairs or triplets, suggesting an unexpected attractive interaction between them. Here, it is shown that mentioned interaction can be related to the gravitational interaction.


Key words: Interaction Gravitational, Casimir Force, Interaction between Photons.

## 1. Introduction

In a paper recently published in Science [1], researchers have reported that when they have put a very weak laser beam through a dense cloud of ultracold rubidium atoms (as a quantum nonlinear medium), the photons bound together in pairs or triplets, suggesting an unexpected attractive interaction between them.

Here, it is shown that mentioned interaction is related to the gravitational interaction.

## 2. Theory

I have show in the Mathematical Foundations of the Relativistic Theory of Quantum Gravity [2] that, by combination of Gravitation and the Uncertainty principle it is possible to derive the expression for the Casimir force. The starting point is the expression of correlation between gravitational mass $m_{g}$ and rest inertial mass, $m_{i 0}$, obtained in the mentioned paper, i.e.,

$$
\begin{equation*}
\chi=\frac{m_{g}}{m_{i 0}}=\left\{1-2\left[\sqrt{1+\left(\frac{p}{m_{i 0} c}\right)^{2}}-1\right]\right\} \tag{1}
\end{equation*}
$$

where $p$ is the variation in the particle's kinetic momentum; $c$ is the light speed.

Thus, an uncertainty $\Delta m_{i 0}$ in $m_{i 0}$ produces an uncertainty $\Delta p$ in $p$ and therefore an uncertainty $\Delta m_{g}$ in $m_{g}$, which according to Eq.(1) , is given by

$$
\begin{equation*}
\Delta m_{g}=\Delta m_{i 0}-2\left[\sqrt{1+\left(\frac{\Delta p}{\Delta m_{i 0} c}\right)^{2}}-1\right] \Delta m_{i 0} \tag{2}
\end{equation*}
$$

From the uncertainty principle for position and momentum, we know that the product of the uncertainties of the simultaneously measurable values of the corresponding position and momentum components are at least of the magnitude order of $\hbar$, i.e.,

$$
\Delta p \Delta r \sim \hbar
$$

Substitution of $\langle p \sim \hbar / \Delta r$ into (2) yields
$\Delta m_{g}=\Delta m_{i}-2\left[\sqrt{1+\left(\frac{\hbar / \Delta m_{i} C}{\Delta r}\right)^{2}}-1\right] \Delta m_{i}$
Therefore if

$$
\begin{equation*}
\Delta r \ll \frac{\hbar}{\Delta m_{i} c} \tag{4}
\end{equation*}
$$

Then the expression (3) reduces to:

$$
\begin{equation*}
\Delta m_{g} \cong-\frac{2 \hbar}{\Delta r c} \tag{5}
\end{equation*}
$$

Note that, $\Delta m_{g}$ does not depend on $m_{g}$.
Consequently, an uncertainty $\Delta F$ in the gravitational force $F=-G m_{g} m_{g}^{\prime} / r^{2}$, will be given by

$$
\begin{align*}
\Delta F & =-G \frac{\Delta m_{g} \Delta m_{g}^{\prime}}{(\Delta r)^{2}}= \\
& =-\left[\frac{2}{\pi(\Delta r)^{2}}\right] \frac{h c}{(\Delta r)^{2}}\left(\frac{G \hbar}{c^{3}}\right) \tag{6}
\end{align*}
$$

The amount $\left(G \hbar / c^{3}\right)^{1 / 2}=1.61 \times 10^{-35} \mathrm{~m} \quad$ is called the Planck length, $l_{\text {planck }}$, the length scale on which quantum fluctuations of the metric of the space time are expected to be of order unity).

Thus, we can write the expression of $\Delta F$ as follows

$$
\begin{align*}
\Delta F & =-\left(\frac{2}{\pi}\right) \frac{h c}{(\Delta r)^{4}} l_{\text {planck }}^{2}= \\
& =-\left(\frac{\pi}{480}\right) \frac{h c}{(\Delta r)^{4}}\left[\left(\frac{960}{\pi^{2}}\right) I_{\text {planck }}^{2}\right]= \\
& =-\left(\frac{\pi A_{0}}{480}\right) \frac{h c}{(\Delta r)^{4}} \tag{7}
\end{align*}
$$

or

$$
\begin{equation*}
F_{0}=-\left(\frac{\pi A_{0}}{480}\right) \frac{h c}{r^{4}} \tag{8}
\end{equation*}
$$

which is the expression of the Casimir force for $A=A_{0}=\left(960 / \pi^{2}\right) l_{\text {planck }}^{2}$.

Now, multiplying Eq. (8) (the expression of $F_{0}$ ) by $n^{2}$ we obtain
$F=n^{2} F_{0}=-\left(\frac{\pi n^{2} A_{0}}{480}\right) \frac{h c}{r^{4}}=-\left(\frac{\pi A}{480}\right) \frac{h c}{r^{4}}$
This is the general expression of the Casimir force.

We can then conclude that the Casimir effect is just a gravitational effect related to the uncertainty principle. In this context, the nature of the Casimir force is clearly gravitational as shown in the derivation of Eq. (9), which expresses, in turn, the intensity of the gravitational force in the case of very small scale ( $r$ very small) ${ }^{1}$.

Now consider the discovery reported recently in Science [1]. When the researchers have put a very weak laser beam through a dense cloud of ultracold rubidium atoms ${ }^{2}$, the photons bound together in pairs or triplets, suggesting an unexpected attractive interaction between them. Now, we will show that the nature of this interaction is gravitational.

According mentioned in the paper, the length of the cloud of ultracold rubidium atoms

[^0]were of approximately $130 \mu \mathrm{~m}$ (along the propagation direction), while the transverse extent of the probe beam waist had about $4.5 \mu \mathrm{~m}$. Therefore, the distances $r$ between the photons of the cloud were very small. As we have already seen, at very small scale, the gravitational interaction cannot be trated via usual Newton's equation of gravitation. In this case, Eq. (9) must be used. Thus, assuming $A \approx \lambda^{2}=(c / f)^{2} \cong 10^{-13} \mathrm{~m}^{2}$, and substituting this value into Eq. (9), we obtain:
\[

$$
\begin{equation*}
F \approx 10^{-40} / r^{4} \tag{10}
\end{equation*}
$$

\]

Using the above equation, and considering the dimensions of the mentioned cloud $(130 \mu m \times 4.5 \mu m)$, we can calculate the intensity of the gravitational force between two photons of the cloud, when the distance $r$ between them were, for example, of the order of $1 \mu \mathrm{~m}$, i.e.,

$$
\begin{equation*}
F \approx 10^{-16} \mathrm{~N} \tag{11}
\end{equation*}
$$

The intensity of this gravitational force is highly significative. Compare for example, with the Coulombian attractive force between an electron and a proton, separated by the same distance ( $r \approx 1 \mu m$ ), which is given by

$$
\begin{equation*}
F_{c}=\frac{e^{2}}{4 \pi \varepsilon_{0} r^{2}} \cong \frac{10^{-28}}{r^{2}} \approx 10^{-16} N \tag{12}
\end{equation*}
$$

The Coulombian repulsive force between two protons in an atomic nucleus, considering that, $r_{\text {proton }}=1.4 \times 10^{-15} \mathrm{~m}$, and that the distance between them is $r=4 \times 10^{-15} \mathrm{~m}$ [4], is given by

$$
\begin{equation*}
F_{c}=\frac{e^{2}}{4 \pi \varepsilon_{0} r^{2}} \cong 14 N \tag{13}
\end{equation*}
$$

This enormous repulsive force must be overcomed by the intense attractive nuclear force (strong nuclear force).

Now consider Eq. (9), where we put $A=\pi r_{\text {proton }}^{2} \cong 6 \times 10^{-30} \mathrm{~m}^{2}$ and $r=4 \times 10^{-15} \mathrm{~m}$, then the result is

$$
\begin{equation*}
F=-\left(\frac{\pi A}{480}\right) \frac{h c}{r^{4}} \cong 30 N \tag{14}
\end{equation*}
$$

Comparing Eq. (14) with Eq. (13), we can conclude that the attractive gravitational force ( $30 N$ ) is sufficient to overcome the repulsive Coulombian force expressed by Eq. (13).

These results lead us to formulate the following question: What is the true nature of the "strong nuclear force"? Is it gravitational as shown above?

This possibility is reinforced by the derivation the Coupling Constants for the Fundamental Forces that we will make hereafter, starting from Eq. (9).

It is known that the weak force, $F_{W}$, which is related to the strong force, $F_{s}$, by means of the following expression:

$$
\begin{equation*}
\frac{F_{W}}{F_{S}}=\frac{\alpha_{W}}{\alpha_{S}} \tag{15}
\end{equation*}
$$

where $\alpha_{W}$ is the weak force coupling constant, and $\alpha_{s}$ is the strong force coupling constant ${ }^{3}$.

Assuming that $F_{S}=F$, where $F$ is given by Eq. (9), then Eq.(15) can be rewritten as follows

$$
\begin{equation*}
F_{W}=\left(\frac{\alpha_{W}}{\alpha_{S}}\right)\left(\frac{\pi A}{480}\right) \frac{h c}{r^{4}} \tag{16}
\end{equation*}
$$

At $r \cong 3 \times 10^{-18} \mathrm{~m}(0.1 \%$ of the diameter of $a$ proton), the weak interaction has a strength of a similar magnitude to electromagnetic force, $F_{E}=e^{2} / 4 \pi \varepsilon_{0} r^{2}$ [5]. Thus, making $F_{W}=F_{E}$, and substituting the above mentioned value of $r$, we obtain

[^1]$\frac{\alpha_{W}}{\alpha_{S}}=\frac{480 r^{2} e^{2}}{4 \pi^{2} \varepsilon_{0} A h c}=\frac{480 r^{2} e^{2}}{4 \pi^{3} \varepsilon_{0} r_{p}^{2} h c} \approx 3 \times 10^{-7}$
This is the same value mentioned in the literature for $\alpha_{W} / \alpha_{S}[\underline{6}]$.

Now, considering that $F_{W} / F_{E}=\alpha_{W} / \alpha_{E}$, where $\alpha_{E}$ is the electromagnetic force coupling constant, then we can write that

$$
\begin{equation*}
F_{W}=\left(\frac{\alpha_{W}}{\alpha_{E}}\right) \frac{e^{2}}{4 \pi \varepsilon_{0} r^{2}} \tag{18}
\end{equation*}
$$

At the maximum range of the weak interaction, $r_{\max }$, we have the minimum value of the weak force, $F_{W}^{\text {min }}$, which can be expressed by Eq. (16) or Eq. (18) as follows

$$
\begin{align*}
& F_{W}^{\min }=\left(\frac{\alpha_{W}}{\alpha_{S}}\right)\left(\frac{\pi A}{480}\right) \frac{h c}{r_{\max }^{4}}  \tag{19}\\
& F_{W}^{\min }=\left(\frac{\alpha_{W}}{\alpha_{E}}\right) \frac{e^{2}}{4 \pi \varepsilon_{0} r_{\max }^{2}} \tag{20}
\end{align*}
$$

By comparing these equations, we obtain

$$
\begin{equation*}
\left(\frac{\alpha_{W}}{\alpha_{S}}\right)\left(\frac{\pi A}{480}\right) \frac{h c}{r_{\max }^{2}}=\left(\frac{\alpha_{W}}{\alpha_{E}}\right) \frac{e^{2}}{4 \pi \varepsilon_{0}} \tag{21}
\end{equation*}
$$

or

$$
\begin{align*}
\frac{\alpha_{S}}{\alpha_{E}} & =\frac{4 \pi^{2} A \varepsilon_{0} h c}{480 e^{2} r_{\max }^{2}}=\frac{4 \pi^{3} r_{p}^{2} \varepsilon_{0} h c}{480 e^{2} r_{\max }^{2}}= \\
& =\left(\frac{4 \pi \varepsilon_{0} \hbar c}{e^{2}}\right)\left(\frac{2 \pi^{3} r_{p}^{2}}{480 r_{\max }^{2}}\right) \tag{22}
\end{align*}
$$

Experimental data, describing the strong force between nucleons is consistent with a strong force coupling constant of about 1 [6]. Thus, making $\alpha_{S}=1$ (strong force coupling constant) in Eq. (22), we obtain

$$
\begin{equation*}
\alpha_{E}=\left(\frac{e^{2}}{4 \pi \varepsilon_{0} \hbar c}\right)\left(\frac{480 r_{\max }^{2}}{2 \pi^{3} r_{p}^{2}}\right) \tag{23}
\end{equation*}
$$

The maximum range of the weak interaction, $r_{\max }$, is of the order of $10^{-16} \mathrm{~m}$ [ $\underline{7}$ ]. Equation above shows that, for $r_{\text {max }} \cong 5 \times 10^{-16} \mathrm{~m}$ the term

$$
\begin{equation*}
\left(\frac{480 r_{\max }^{2}}{2 \pi^{3} r_{p}^{2}}\right) \cong 1 \tag{24}
\end{equation*}
$$

Consequently, Eq. (23) reduces to

$$
\begin{equation*}
\alpha_{E}=\frac{e^{2}}{4 \pi \varepsilon_{0} \hbar c} \cong \frac{1}{137} \tag{25}
\end{equation*}
$$

this is the expression of the electromagnetic force coupling constant.

Multiplying $\alpha_{W} / \alpha_{S}$ (given by Eq. (17)) by $\alpha_{S} / \alpha_{E}$ (given by Eq. (22)), we get

$$
\frac{\alpha_{W}}{\alpha_{E}}=\left(\frac{480 r^{2} e^{2}}{4 \pi^{3} \varepsilon_{0} r_{p}^{2} h c}\right)\left(\frac{4 \pi \varepsilon_{0} \hbar c}{e^{2}}\right)\left(\frac{2 \pi^{3} r_{p}^{2}}{480 r_{\max }^{2}}\right)
$$

whence we obtain

$$
\begin{align*}
\alpha_{W} & =\left(\frac{480 r^{2} e^{2}}{4 \pi^{3} \varepsilon_{0} r_{p}^{2} h c}\right)\left(\frac{4 \pi \varepsilon_{0} \hbar c}{e^{2}}\right)\left(\frac{2 \pi^{3} r_{p}^{2}}{480 r_{\max }^{2}}\right) \alpha_{E}= \\
& =\left(\frac{480 r^{2} e^{2}}{4 \pi^{3} \varepsilon_{0} r_{p}^{2} h c}\right)\left(\frac{2 \pi^{3} r_{p}^{2}}{480 r_{\max }^{2}}\right) \cong \\
& \cong\left(\frac{r^{2} e^{2}}{2 \varepsilon_{0} r_{\max }^{2} h c}\right) \cong 3 \times 10^{-7} \tag{26}
\end{align*}
$$

Now, we will obtain the gravitational force coupling constant, $\alpha_{G}$, starting of the fact that the strong force, $F_{G}$, is related to the electromagnetic force, $F_{E}$, by means of the following expression:

$$
\begin{equation*}
\frac{F_{G}}{F_{E}}=\frac{\alpha_{G}}{\alpha_{E}} \tag{27}
\end{equation*}
$$

Then, we can write that
$\alpha_{G}=\alpha_{E}\left(\frac{F_{G}}{F_{E}}\right)=\alpha_{E}\left(\frac{G m_{p}^{2}}{\frac{e^{2}}{4 \pi \varepsilon_{0}}}\right) \cong 5.9 \times 10^{-39}$
The relative strength of interactions varies with distance [ $\underline{8}$ ]. Here, starting from the fact that the strong nuclear force and the
weak nuclear force are gravitational forces expressed by Eq. (9), we have showed that, at the range of about $10^{-15} \mathrm{~m}$ $\left(r_{\text {max }} \cong 5 \times 10^{-16} \mathrm{~m}\right)$, the $\operatorname{strong}$ force $\left(\alpha_{s}=1\right)$ is approximately 137 times as strong as electromagnetic force ( $\alpha_{E}=1 / 137$ ), about $a$ million times as strong as the weak force $\left(\alpha_{W} \cong 3 \times 10^{-7}\right)$, and about $10^{38}$ times as strong as gravitation $\left(\alpha_{G} \cong 5.9 \times 10^{-39}\right)$. All these values are in strong accordance with the values widely mentioned in the literature [ $\underline{9}$, 10], given below

$$
\begin{aligned}
& \alpha_{S}=1 \\
& \alpha_{E}=1 / 137 \\
& \alpha_{W} \approx 3 \times 10^{-7} \\
& \alpha_{G} \cong 5.9 \times 10^{-39}
\end{aligned}
$$

Finally, we complete the unification of the Fundamental Forces of the Universe, by deriving from Eq. (9) the equations of the Coulombian Force and of the Newtonian Force.

Consider two electric charges $q_{1}$ and $q_{2}$ separated by a distance $r$. If we define the area $A$ in Eq. (9) by means of the following expression

$$
\begin{align*}
A & =\sqrt{A_{1} A_{2}}=\sqrt{k_{e}\left(\frac{q_{1}}{e}\right)^{2} r^{2} \times k_{e}\left(\frac{q_{2}}{e}\right)^{2} r^{2}}= \\
& =k_{e}\left(\frac{q_{1} q_{2}}{e^{2}}\right) r^{2} \tag{29}
\end{align*}
$$

where $k_{e}$ is a constant to be determined, and $e=1.6 \times 10^{-19} \mathrm{C}$, then Eq. (9) can rewritten as follows

$$
\begin{equation*}
F=\frac{\pi h c k_{e} q_{1} q_{2}}{480 e^{2} r^{2}}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{4 \pi^{2} h c k_{e} \varepsilon_{0}}{480 e^{2}}\right) \frac{q_{1} q_{2}}{r^{2}} \tag{30}
\end{equation*}
$$

Note that, the term in brackets is equal to 1 for $k_{e}=120 e^{2} / \pi^{2} h c \varepsilon_{0} \cong 0.1769$. In this case, Eq. (30) reduces to

$$
\begin{equation*}
F=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} q_{2}}{r^{2}} \tag{31}
\end{equation*}
$$

which is the expression of the Coulombian Force.

In a similar way, we can derive the expression of the Newtonian Force for two particles with masses $m_{1}$ and $m_{2}$ respectively, separated by a distance $r$. First we define the area $A$ in Eq. (9) by means of the following expression

$$
\begin{align*}
A & =\sqrt{A_{1} A_{2}}=\sqrt{k_{g}\left(\frac{m_{1}}{m_{0}}\right)^{2} r^{2} \times k_{g}\left(\frac{m_{2}}{m_{0}}\right)^{2} r^{2}}= \\
& =k_{g}\left(\frac{m_{1} m_{2}}{m_{0}^{2}}\right) r^{2} \tag{32}
\end{align*}
$$

where $k_{g}$ is a constant to be determined, and $m_{0}$, is a minimum value of mass that will be calculated hereafter. Then substitution of Eq. (32) into Eq. (9) yields

$$
\begin{equation*}
F=\frac{\pi h c k_{g} m_{1} m_{2}}{480 m_{0}^{2} r^{2}}=G\left(\frac{2 \pi^{2} \hbar c k_{g}}{480 G m_{0}^{2}}\right) \frac{m_{1} m_{2}}{r^{2}} \tag{33}
\end{equation*}
$$

The term in brackets is equal to 1 for

$$
\begin{equation*}
\frac{k_{g}}{m_{0}^{2}}=\left(\frac{60}{\pi^{2}}\right)\left(\frac{4 G}{\hbar c}\right) \tag{34}
\end{equation*}
$$

Equation (34) can be rewritten as follows

$$
\begin{equation*}
\frac{k_{g}}{m_{0}^{2}}=\frac{\left(\frac{60}{\pi^{2}}\right)}{\left(\sqrt{\frac{\hbar c}{4 G}}\right)^{2}} \tag{35}
\end{equation*}
$$

where $\sqrt{\hbar c / 4 G}=1.08 \times 10^{-8} \mathrm{~kg}$.
Equation (35) shows that, the term $\left(60 / \pi^{2}\right)$ is a pure number such as $k_{g}$, and the term $\sqrt{\hbar c / 4 G}$ is expressed in kg such as $m_{0}$, then we can conclude that

$$
\begin{equation*}
k_{g}=\frac{60}{\pi^{2}} \tag{36}
\end{equation*}
$$

and

$$
\begin{equation*}
m_{0}=\sqrt{\frac{\hbar c}{4 G}} \tag{37}
\end{equation*}
$$

This expression it was first derivated by Hawking (1971) [11], and it is known as Hawking mass limit. Starting from the principle that the gravitational collapse is a process essentially classic, Hawking have concluded that black-holes could not exist with radius less than the Planck length $\left(\sqrt{G \hbar / c^{3}}\right)$ (limit for which quantum fluctuations in the metric of the spacetime are considered of the order of 1). In this way, the minimum radius of Schwarzschild, $r_{S}=2 G m_{0} / c^{2}$, would have this value and, to this radius would correspond to a minimum value of mass $m_{0}$, given by

$$
\begin{equation*}
m_{0}=\frac{r_{s} c^{2}}{2 G}=\frac{c^{2} \sqrt{G \hbar / c^{3}}}{2 G}=\sqrt{\frac{\hbar c}{4 G}} \tag{38}
\end{equation*}
$$

This would be, obviously, the smaller mass value for any macroscopic gravitational systems (black-holes, etc).

Now, just substitute Eq. (36) and Eq. (37) into Eq.(33), in order to obtain the expression of the Newtonian Force.

$$
\begin{equation*}
F=G \frac{m_{1} m_{2}}{r^{2}} \tag{39}
\end{equation*}
$$

The derivation of the Equations (31) and (39) via Eq. (9), shows clearly the unification of the Fundamental Forces of the Universe, i.e. shows that the nature of all the fundamental interactions is Gravitational.

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[^0]:    ${ }^{1}$ The Casimir force is only significative when the value of $r$ is very small (microcosm scale).
    ${ }^{2}$ The velocities of the photons through the cloud of ultracold rubidium atoms are strongly reduced. This is the reason for the laser to pass through the mentioned cloud. Lene Hau et al., [3] showed that light speed through a cloud of ultracold rubidium atoms reduces to values much smaller than $100 \mathrm{~m} \cdot \mathrm{~s}^{-1}$.

[^1]:    ${ }^{3}$ Similarly, the weak force is related to the electromagnetic force, $F_{E}$, by means of the expression: $F_{W} / F_{E}=\alpha_{W} / \alpha_{E} ;$ and the strong force is related to the electromagnetic force, by means of the expression: $F_{S} / F_{E}=\alpha_{W} / \alpha_{E}$; and the gravitational force, $F_{G}$, is related to the electromagnetic force, by means of the expression: $F_{G} / F_{E}=\alpha_{G} / \alpha_{E}$.

