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Julien Bel, Christian Marinoni

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The cosmic aberration drift: proposal for a real-time detection of our acceleration through space

Julien Bel and Christian Marinoni

Aix Marseille Univ, Université de Toulon, CNRS, CPT, Marseille, France.

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Our proper acceleration with respect to the Cosmic Microwave Background results in a real-time change of the angular position of distant extragalactic sources. The cosmological component of this aberration drift signal, the non-inertial motion generated by the large-scale distribution of matter, can in principle be detected by future high-precision astrometric experiments. It will provide interesting consistency tests of the standard model of cosmology, set independent constraints on the amplitude of the Hubble constant and the linear growth rate of cosmic structures, and be instrumental in searching for evidence of new physics beyond the standard model. We present the formalism of this novel cosmological test, discuss the physics to which it is sensitive and show simulated forecasts of the accuracy with which it can be implemented.

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According to the standard model of cosmology there are no special positions nor orientations in the sky. At the same time, the universe exhibits a preferred (comoving) state of motion, well represented by the reference frame of the cosmic microwave background (CMB). The most substantial deviation from theoretical expectations, a $\sim 10^{-3}$ dipolar modulation of the CMB temperature map, is conventionally attributed to our own *peculiar* motion with respect to the CMB rest frame: we fail to be comoving observers by about 600 km s^{-1}! [1].

Despite various supporting evidence [2–5], we still lack CMB-independent measures of our speed and the ultimate proof that it is gravitationally induced by large-scale matter fluctuations. The analysis of the peculiar velocity field of galaxies — a tracer of the large-scale distribution of mass in the local universe — is a promising but challenging approach in this direction [6]. Current results are not conclusive and the convergence to the CMB dipole still disputed in light of not yet fully understood systematics [7–12]. Several attempts to measure our motion by searching for dipole modulations in high-redshift radio galaxy samples are similarly affected by observational uncertainties [13] and mostly yield velocity amplitudes larger ($> 3\sigma$) than expected [14–17].

We propose to determine the nature and the amplitude of our peculiar velocity, specifically the velocity of the Local Group center (LG) with respect to the CMB, by constraining the rate at which it changes in time, *i.e.*, by measuring our peculiar acceleration.[1] This motion adds to – and must be disentangled from – those generated by local non-cosmological dynamical processes: the acceleration of the terrestrial observer around the Sun, of the Sun around the Milky Way center [18, 19], and this last around the LG (see this paper). A dimensional argument suggests that, in a uniform universe, the physical peculiar velocity $\boldsymbol{\beta}$ of a test particle decays in time roughly as the inverse of the cosmic scale factor $a(t)$, i.e. $\dot{\beta} \sim H\beta$ where H is the Hubble parameter. We thus predict a cosmological aberration drift (CAD) effect — a time-dependent change of the angular position of distant (non-Galactic) sources — of order $\delta\theta \sim \dot{\beta}dt$, where dt is the time lag between observations. Assuming that $\beta \sim 622$ km s^{-1} in the Galactic direction $l = 272°, b = 28°$ [20] is the velocity of the LG with respect to the CMB, i.e. of the fluid element to which cosmological perturbation theory results can be consistently applied, a change in aberration of roughly 0.4 μas, in ten years, per each cosmic source and in some sky directions is predicted. This figure is of the same order of parallactic effects [21, 22] that can in principle be extracted from all-sky, high-precision astrometric observations spaced by several years [23], and calls for further investigation. Notably a detailed assessment of the gravitational contribution of large-scale inhomogeneities to the CAD signal, as well as of the cosmological information that can be inferred from our own peculiar acceleration.

THE AMPLITUDE OF THE COSMOLOGICAL ABERRATION DRIFT

Consider an inhomogeneous universe characterised by the (longitudinal gauge) metric

$$ds^2 = e^{2\Phi(t,\mathbf{x})}dt^2 - a^2(t)e^{-2\Psi(t,\mathbf{x})}\delta_{jk}dx^j dx^k \qquad (1)$$

where Φ and Ψ are the Newtonian and curvature scalar potentials and where the smooth underlying background has flat spatial hypersurfaces. An observer S', in (timelike) geodesic motion with respect to the comoving observer S, has four-velocity $\frac{dx^\mu}{d\tau} = \gamma e^{-\Phi}(1, \dot{\mathbf{x}})$, where τ is the proper time, $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz boost factor, and where $\boldsymbol{\beta} \equiv e^{-(\Phi+\Psi)}a\dot{\mathbf{x}}$ is the physical peculiar velocity ($\dot{} \equiv d/dt$). The aberration angle θ' between the observer's velocity and the photon arrival direction

[1] The observer is in gravitational free-fall and would, of course, measure no acceleration on a local accelerometer.

relates to the un-aberrated angle θ seen by an observer at rest as $\cos\theta' = (\beta + \cos\theta)/(1 + \beta\cos\theta)$ or, equivalently, $\sin\theta' = \gamma^{-1}\sin\theta/(1 + \beta\cos\theta)$. The CAD when $\dot{\boldsymbol{\beta}}$ is parallel to $\boldsymbol{\beta}$, the case of cosmological interest (see eq. 4), is[2]

$$\frac{d\theta'}{d\tau} \approx -\sin\theta'\dot{\beta} + \dot{\theta} \tag{2}$$

where $\dot{} \equiv d/dt$.

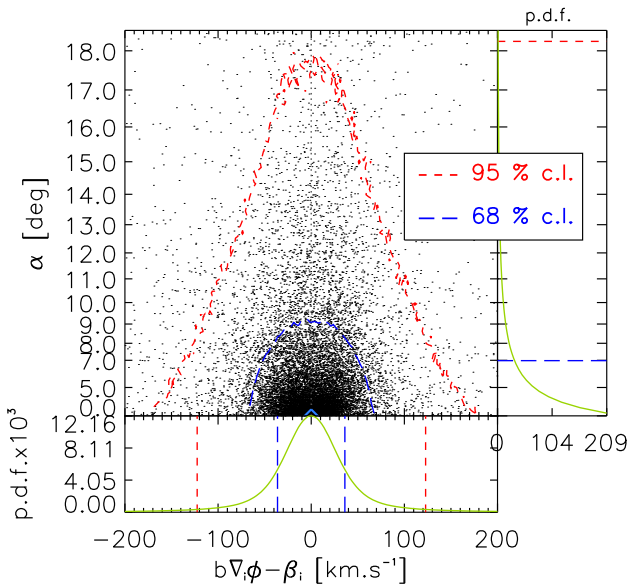

FIG. 1. *Main panel:* the angle α quantifies the misalignment (in degrees) between the gradient of the gravitational potential field $\nabla\Phi$ and the direction of the peculiar velocity vector of LG-like systems identified in the DEMNUni N-body simulation [24]. *Right panel:* the probability distribution function (PDF) of α is displayed. The linearly reconstructed velocity vector for 68% of the LG-like systems is expected to be misaligned by at most 7°. *Bottom panel:* PDF of the deviations between the amplitude of the (cartesian) peculiar velocity components β_i of simulated LG-like systems and the theoretically predicted linear amplitude $b\nabla_i\Phi$, where $b = 2f/3H_0\Omega_{m0}\mu$ (Planck ΛCDM cosmology is assumed).

The acceleration is accounted for by the equation of motion expected in the Newtonian approximation (large sub-horizon scales) for perfect fluids with no anisotropic stress ($\Phi = \Psi$) and negligible pressure [3] *i.e.* $\dot{\boldsymbol{\beta}} = -H\boldsymbol{\beta} - \nabla\Phi$. The the amplitude of the gravitational potential follows from combining the linear continuity equation $\dot{\delta} + \nabla\cdot\boldsymbol{\beta} = 0$ for matter overdensity fluctuations δ, and the Poisson equation $\Delta\Phi = 4\pi G_N\mu\bar{\rho}\delta$ where G_N is the Newton gravitational constant, and μ, the effective gravitational constant, incorporates potential departure from Einstein gravity on large scales. By assuming the existence of a factorized solution $\delta(\mathbf{x}, t) = D_+(t)\delta(\mathbf{x})$, i.e. by neglecting decaying modes in the evolution of the overdensity field, as well as rotational modes in the velocity field (compatibly with the assumption that the underlying perturbed metric has no vector contributions), one finds that the gradient of the gravitational potential is proportional to the peculiar velocity

$$\nabla\Phi = -\frac{3H\Omega_m}{2}\frac{\mu}{f}\boldsymbol{\beta} \tag{3}$$

where Ω_m is the time-dependent reduced matter density of the universe and $f = d\ln D_+/d\ln a$ is the linear growth rate of mass fluctuations. We thus conclude that, at leading order, proper motions evolve as

$$\dot{\boldsymbol{\beta}} = -H\boldsymbol{\beta}\left(1 - \frac{3\Omega_m}{2}\frac{\mu}{f}\right). \tag{4}$$

Once reconstructed by averaging over sufficiently large spatial scales, the net bulk velocity $\boldsymbol{\beta}$ of distant extragalactic sources, such as quasars, is expected to approach zero [25]. The frame defined by these distant objects thus embodies the comoving system with respect to which the LG is in relative acceleration according to eq. (4).

SOURCES OF NOISE

The term $\dot{\theta}$ in eq. (2) encodes contributions from the intrinsic proper motion of sources and time dependent lensing effects. This last effect, being of order $H\theta_L$ (where θ_L is the typical deflection of a distant source, $\sim$ 1arcmin) is negligible, of order $0.004\,\mu$as yrs^{-1} for distant galaxies. Proper motions, in the most unlucky configuration, *i.e.* when generated by transverse velocity as high as ~ 1000 km s^{-1}, are of order $\dot{\theta} = 0.2(Gpc/d_A)$ μas yrs^{-1} where d_A is the angular diameter distance [26].

[2] The exact amplitude is

$$\frac{d\theta'}{d\tau} = \frac{-\sin\theta\left[\mathring{\beta} + \mathring{\gamma}\gamma^{-1}(\beta + \cos\theta)\right] + \mathring{\theta}(1 + \beta\cos\theta)}{[1 + \beta\cos\theta]^2}$$

where $\mathring{}$ indicates a derivative with respect to physical time of the comoving observer S ($\mathring{} \equiv e^{-\Phi}\frac{d}{dt}$).

[3] The acceleration of a time-like observer, in our case the LG center, can be evaluated exactly by solving the geodesic equation of motion for massive test particles. We obtain

$$\mathring{\boldsymbol{\beta}} = -\mathcal{H}\boldsymbol{\beta}(1 - \beta^2) - \left[\nabla\Phi - \boldsymbol{\beta}\cdot\nabla(\Phi + \Psi)\boldsymbol{\beta} + \beta^2\nabla\Psi\right]$$

where $\nabla \equiv a^{-1}e^{\Psi}\frac{\partial}{\partial\mathbf{x}}$ is the physical (spatial) gradient, and where $\mathcal{H} \equiv e^{-\Phi}\left(H - \frac{\partial\Psi}{\partial t}\right)$ is the local Hubble factor, the log-derivative of the local scale factor ($a\,e^{-\Psi}$) with respect to the physical time of the comoving observer S ($dT = e^{\Phi}dt$).

Being statistically uncorrelated on large scales, as well as redshift-dependent, they can be suppressed by averaging data over the azimuthal (un-aberrated) angle ϕ, the same averaging procedure which filters out random instrumental noise and systematic effects in identifying the positions of individual sources.

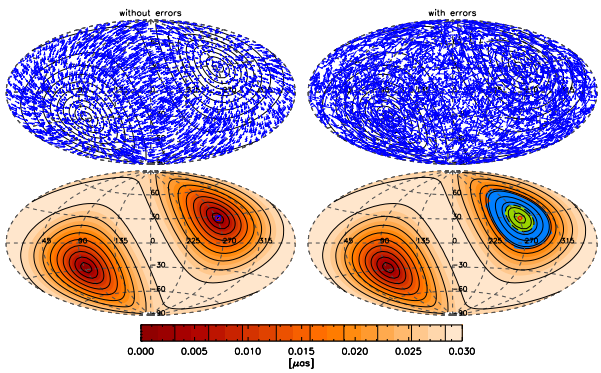

FIG. 2. *Top:* all-sky isotropic distribution of a Monte Carlo simulated sample of distant extragalactic objects. On the left panel, 2-dimensional blue vectors show the (out of scale) CAD signal expected for the LG moving towards the apex of the CMB temperature dipole, while on the right panel a random, and dominant, error component, illustrating astrometric imprecisions is added. *Bottom:* we simulate the CAD signal reconstructed from a sample of $2\cdot10^6$ sources with an EoM astrometric accuracy on proper motions of $\sigma = 0.6$ and $1.4\,\mu$as yr^{-1} respectively. The red color scale shows the amplitude of the signal (the red diamond represents the simulated direction of the observer's motion), while the green/blue regions display the solid angle within which 68% of the reconstructed apex directions lie. The imprecision in the dipole position is estimated using 10000 Monte Carlo realisations and compared to the analytical predictions given in the text (thick black lines).

To extract the cosmological component of the signal one needs also to subtract the local astrophysical motions of the terrestrial observer around the Sun, of the Sun around the Milky Way center, and of this last with respect to the LG frame. While the first two effects are measurable [27–29] and can be disentangled (their apex is in a different direction with respect to the CAD), the motion of our own galaxy within LG potential must be modelled. We assume that the LG is a spherical overdensity that decoupled from the cosmological expansion and that its gravitational potential is essentially provided by M31, its largest mass ($M = (1.33 \pm 0.18) \times 10^{12} M_\odot$), towards which the MW – at a distance $\mathbf{r} = (0.770 \pm 0.040)\hat{\mathbf{r}}$ Mpc – is moving with a relative velocity $\boldsymbol{\beta}_{MW/LG} = (-135 \pm 20)\hat{\mathbf{r}}$ km s^{-1}. We obtain

$$\dot{\boldsymbol{\beta}} = -H[\boldsymbol{\beta}_{MW/LG} + \Omega_m/2(1+\delta_r)H_0\mathbf{r}] = (3\pm48)\hat{\mathbf{r}}\ km\ s^{-2} \tag{5}$$

where $\delta_r = 16.1 \pm 3.9$ is the average LG spherical overdensity at the MW position.

The two body system occupies a favorable region in configuration space, that where the Hubble expansion is exactly cancelled by the internal system gravity. Although this model can be further refined, the contamination is expected to be one order of magnitude smaller than the cosmological signal we are looking for (cfr. eq. (4)).

We finally investigate the accuracy to which LG-like systems satisfy the predicted linear relation (3) between peculiar velocity $\boldsymbol{\beta}$ and $\nabla\Phi$ describes the actual dynamics of the LG center. To this purpose, we use the $z = 0$ output of the DEMNUni [24], an N-body simulation of the distribution of dark matter in the standard Λ Cold Dark Matter (ΛCDM) universe. The simulation contains $N_{cdm} = 2048^3$ cold dark matter particles within a cubic periodic universe of comoving size $L = 2000\ h^{-1}$Mpc. The mass resolution, $m_p = 8.27 \times 10^{10} h^{-1} M_\odot$, which is an order of magnitude smaller than the mass of the Milky Way, and the large comoving volume make this simulation particularly well suited for analysing the galaxy clustering properties (density, velocity and acceleration fields) in the nonlinear regime. Following [30, 31], we define LG-like systems in the DEMNUni mock catalogs as regions with (top-hat) smoothed overdensity $0 < \delta_R < 0.5$, where $R = 4h^{-1}$Mpc. Figure 1 shows that the peculiar velocity of LG-like systems ($\boldsymbol{\beta}$) is proportional to the Newtonian gravitational force ($\nabla\Phi$) with a scatter which is small enough in both amplitude (< 50 km s^{-1}) and direction ($< 7°$) at 68% *c.l.* (see also [10]).

DETECTION OF THE CAD SIGNAL AND COSMOLOGICAL FORECASTS

The characteristic sinusoidal modulation of the cosmological component of the CAD signal, independent from frequency and distance, and, even more distinctively, the fact that its apex is predicted to be aligned in direction with the CMB temperature dipole, are essential features for gauging the reliability of real-time measurements of LG acceleration and disentangle it from other signals. The induced CAD signal, being a 2D, curl-free, vector field, can be orthogonally decomposed on the sphere as $d\vec{\theta}' = \sum V_{lm} \boldsymbol{\Psi}_{lm}$ where $\boldsymbol{\Psi}_{lm} = r\nabla Y_{lm}$ and where Y_{lm} are the spherical harmonics. The $l = 1$ multipole of the divergence of the CAD vector field $\Theta = \nabla\cdot d\vec{\theta}' = \sum \Theta_{lm} Y_{lm}$ is the dipole vector $\vec{B}$ whose Cartesian coordinates, defined through $\Theta_{1,1} = -\frac{1}{\sqrt{2}(B_x - iB_y)}$, $\Theta_{10} = B_z$ $\Theta_{1,-1} = \frac{1}{\sqrt{2}(B_x + iB_y)}$ provide the apex of the observer's motion ($l = \arccos\left[\frac{Sg(B_y)B_z}{\sqrt{B_x^2+B_y^2}}\right] + \Pi(-B_y)\pi$, $b = \arccos B_z/B$, where Sg and Π are the sign and Heaviside step functions respectively. We predict that, in the limit in which astrometric errors dominate over statistical ones, the apex is recovered within the solid angle $\Delta_\Omega = -\frac{3\pi}{N}\left(\frac{\sigma}{|\dot{\beta}|}\right)^2 \ln(1 -$

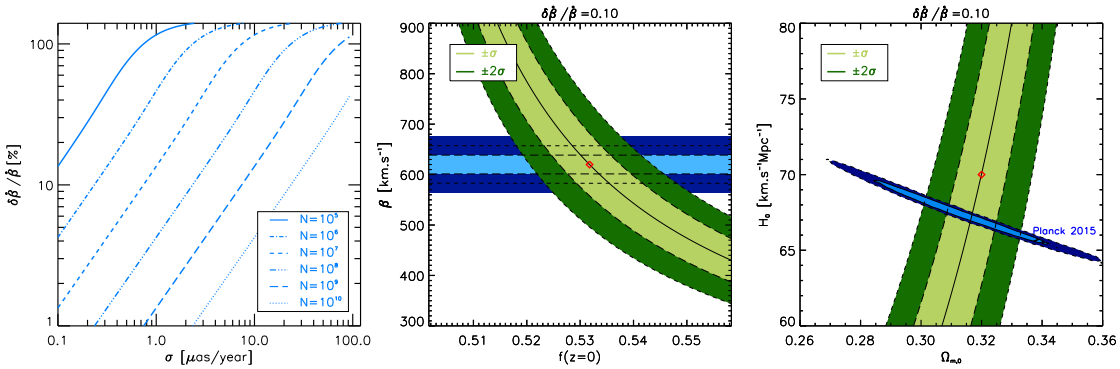

FIG. 3. *Left:* relative precision in the estimates of the CAD signal as a function of the EoM astrometric error on proper motions (x-axis) and of the number of cosmological sources (as shown in the inset). We assume the ΛCDM cosmological model of Planck-2015. *Center:* marginalised likelihood $\mathcal{L}$ in the parameter space defined by the peculiar velocity β and the linear growth rate f at present time, and by the Hubble constant H_0 and the present-day reduced matter density Ω_{m0} (*right*). Confidence levels are drawn for $-2\ln\mathcal{L}/\mathcal{L}_{max} = 2.3$ and 6.17.

p) where p is the desired confidence level, σ the end-of-mission (EoM) proper motion error and N the number of distant sources. The above analytical predictions is confirmed by Montecarlo simulations of the CAD signal as shown in FIG. 2. The relative error on the amplitude of the CAD signal, instead, is $\sqrt{\frac{3}{2N}}|\dot{\beta}|^{-1}\sigma$ and its scaling as a function of the sample size is shown in FIG. 3.

An ongoing astrometric mission, Gaia, measures proper motions for roughly $5 \cdot 10^5$ QSOs as faint as magnitude $r \sim 20$ with an EoM, magnitude-weighted average proper motion error $\sigma \sim 120$ μas yr^{-1} [32, 33]. Exploiting QSOs count statistics [34], we forecast that the CAD signal could be estimated with 20(/50)% precision if the same sample could be targeted with an EoM proper motion error 0.7(/1.8)μas yr^{-1}. roughly the same accuracy with which the Gaia satellite is targeting the brightest stars of the MW.

Encouragingly, the necessary astrometric precision is within technical reach of proposed ground experiments. The signal could probably be detected by analysing the $\sim 10^{10}$ extragalactic sources targeted by the LSST [35], which will obtain proper-motion measurements of comparable accuracy to those of Gaia at its faint limit ($r < 20$) and smoothly extend the error versus magnitude curve deeper by about 5 mag. The sensitivity, field-of-view and angular resolution of the SKA will result in a large, multi-epoch data base with precise astrometry for $\sim 2 \cdot 10^6$ bright compact radio sources (flux densities above 200 μJy). The forecasted proper motion accuracy of order ~ 1 μas yr^{-1} [36] will allow us to fix the amplitude of the CAD to better then 30% and to locate the apex of the acceleration dipole with a solid angle accuracy $\delta\Omega/4\pi \sim 4.6\%$ (See FIG. 2).

Cosmological constraints expected from the CAD detection with signal-to-noise ratio $S/N = 10$ are shown in FIG. 3. Besides providing CMB-independent evidence supporting the kinematical interpretation of the temperature dipole, the test will allow us to resolve the CMB degeneracy in the estimates of H_0 and Ω_{m0}. Its sensitivity to the matter density and linear growth rate parameters is quite pronounced. The signal is heavily suppressed if $\Omega_{m0} = 0.315$, increasing by nearly a factor of two if this value is lowered by 15%.

Interestingly, the present day value of μ/f predicted by modified gravity models not yet ruled out by current constraints spans a large interval [37, 38]. The resulting CAD signal is characteristically amplified, and if $\mu/f > 18$ could also be detected by Gaia. We remark that local gravity with $\mu/f > \frac{2}{3\Omega_{m,0}}$, which is stronger than that predicted by ΛCDM for $\Omega_{m,0} \approx 0.3$, has a neat observational signature, tilting the CAD dipole in a direction anti-aligned with respect to that of the CMB dipole. More generally, an eventual misalignment between the CAD apex, sensitive to the observer's proper acceleration, and the CMB apex, sensitive to the observer's velocity, would be a clear fingerprint of beyond standard model physics.

CONCLUSIONS

Precision astrometry of solar system bodies was instrumental for testing GR. We argue that precision astrometry (over several years) of distant sources will allow us to understand the gravitational dynamics of the LG with respect to CMB and to figure out what this local dynamics tell us about global properties of the universe, specifically, its current expansion rate H_0 and the linear growth rate history f.

The proposed method populates the still limited class of tests, such as the redshift drift [39–44], designed for monitoring the real-time evolution of the universe trough changes in cosmological observations spaced by several years. Unlike redshift-drift based proposals, the CAD does not necessitate any distance measurement. It can

be implemented using 2D photometric data only, thus exploiting less time-consuming observational techniques and imaging instruments that are more stable, over the years, compared to spectrographs. According to this testing scheme, an ongoing and forecasted astrometric missions such as Gaia could already provide interesting constraints on non-standard theories of gravity, notably un upper bound on the present-day value of μ/f. Beyond Gaia missions, such as LSST or SKA, with specifically tailored time interval between observations, and sufficient astrometric precision for faint sources to the foreseeable level predicted in this study would be enough to probe directly Gpc scales and provide useful cosmological insights.

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