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Mimetic gravity as DHOST theories

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Abstract

We show that theories of mimetic gravity can be viewed as degenerate higher-order scalar-tensor (DHOST) theories that admit an extra local (gauge) symmetry in addition to the usual diffeomorphism invariance. We reformulate and classify mimetic theories in this perspective. Using the effective theory of dark energy, recently extended to include DHOST theories, we then investigate the linear perturbations about a homogeneous and isotropic background for all mimetic theories. We also include matter, in the form of a k -essence scalar field, and we derive the quadratic action for linear perturbations in this case.

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1 Introduction

Mimetic Matter was introduced by Chamseddine and Mukhanov in [1] as a model of modified gravity that mimics cold dark matter [2–5]. The original proposal was then extended to inflation, dark energy and also theories with non-singular cosmological and black hole solutions [6–10]. Mimetic theories have also been studied in [11–14]. More specifically, their linear stability has been considered in [15–21]. See also e.g. [22] for a review on mimetic gravity and [23–27] for earlier related works.

The goal of this paper is to revisit mimetic gravity in the context of Degenerate Higher-Order Scalar-Tensor (DHOST) theories, introduced in [28] and further explored in [29–33], as summarized in e.g. [34] (see also [35–37] for a study of the screening mechanism in these theories). DHOST theories are scalar-tensor theories that encompass Horndeski [38] and so-called Beyond Horndeski (or GLPV) theories [39, 40] (another particular subclass of DHOST theories was also found earlier in [41] via disformal transformations of Einstein gravity). Despite the presence of second derivatives of the scalar field in the Lagrangian and higher order Euler-Lagrange equations, DHOST theories contain at most three degrees of freedom (one scalar and two tensorial modes), because their Lagrangian is degenerate [28].

Mimetic theories can be reformulated as scalar-tensor theories with second derivatives of the scalar field in their Lagrangians. Moreover, the conformal (or, more generally, the disformal) symmetry that characterizes the resulting scalar-tensor Lagrangian guarantees

that the latter is degenerate, thus implying that they form a subclass of DHOST theories, as already pointed out in [30]. When mimetic theories are restricted to a quadratic or cubic dependence on the second derivatives of the scalar field, one can use the full classification of quadratic and cubic DHOST theories obtained in [33] and identify the subclasses that contain these mimetic theories. Mimetic theories with a quartic or higher dependence on second derivatives of the scalar field provide examples of DHOST theories that are not included in the classification of [33].

In the present work, we draw upon our recent analysis of DHOST theories [42], based on the effective approach to dark energy developed in [43–45], to study linear perturbations in theories of mimetic gravity using their DHOST formulation. Our calculations are consistent with those presented in [46], based on the Lagrange multiplier formulation.

The paper is structured as follows. In Section 2, we start with a brief introduction to mimetic gravity, presenting two equivalent formulations: the DHOST formulation, which we exploit in this work, and the Lagrange multiplier formulation, which has often been used in the literature. We also show that mimetic theories, with a quadratic or cubic dependence on second derivatives of the scalar field, belong to specific subclasses of DHOST theories. We then turn to the study of linear cosmological perturbations and first review the effective theory of dark energy, in Section 3. Concentrating on the DHOST formulation of mimetic gravity, we obtain the quadratic action for linear cosmological perturbations. For completeness, we briefly present the study of linear perturbations within this alternative formulation in Section 5. We summarize our work and conclude in the final section. A few technical details are summarized in the Appendix.

2 Mimetic gravity: short review and classification

In this section, we first give a brief review of mimetic gravity before constructing the general mimetic gravity action which propagates at most three degrees of freedom.

2.1 Non-invertible disformal transformation

Mimetic gravity is a scalar-tensor theory defined by a general action of the form

$$S[\tilde{g}_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \mathcal{L}(\phi, \partial_\mu \phi, \nabla_\mu \nabla_\nu \phi; g_{\mu\nu}), \quad (2.1)$$

where the variation must be taken with respect to ϕ and the auxiliary metric $\tilde{g}_{\mu\nu}$, related to $g_{\mu\nu}$ via a *non-invertible* disformal transformation,

$$g_{\mu\nu} = \tilde{A}(\phi, \tilde{X}) \tilde{g}_{\mu\nu} + \tilde{B}(\phi, \tilde{X}) \partial_\mu \phi \partial_\nu \phi, \quad \tilde{X} \equiv \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi. \quad (2.2)$$

Note that in the original model of mimetic dark matter, the Lagrangian in (2.1) depends only on $g_{\mu\nu}$ and not explicitly on ϕ , since the Lagrangian is the Einstein-Hilbert term for $g_{\mu\nu}$, $\mathcal{L} = R$. In the following, we will use the notation $\phi_\mu \equiv \partial_\mu \phi$ and $\phi_{\mu\nu} \equiv \nabla_\mu \phi_\nu$.

The fact that the disformal transformation is non-invertible implies that the functions \tilde{A} and \tilde{B} are not arbitrary but are related according to [47]

$$\frac{\tilde{A}(\phi, \tilde{X})}{\tilde{X}} + \tilde{B}(\phi, \tilde{X}) = h(\phi), \quad (2.3)$$

where $h(\phi)$ is an arbitrary function of ϕ . Due to the non-invertibility condition (2.3), the metric $\tilde{g}_{\mu\nu}$ cannot be fully determined from $g_{\mu\nu}$ and ϕ . Indeed, the metric $g_{\mu\nu}$ is left invariant under the local transformations

$$\delta\phi = 0 \quad \text{and} \quad \delta\tilde{g}_{\mu\nu} = \varepsilon \left(\tilde{A}_{,\tilde{X}} \tilde{g}_{\mu\nu} + \tilde{B}_{,\tilde{X}} \phi_\mu \phi_\nu \right) \quad (2.4)$$

of the field ϕ and the metric $\tilde{g}_{\mu\nu}$, where ε is an arbitrary function and $\tilde{A}_{,\tilde{X}}$ (or $\tilde{B}_{,\tilde{X}}$) denotes the derivative of \tilde{A} (or \tilde{B}) with respect to the variable \tilde{X} . As a consequence, (2.4) defines a local invariance of the mimetic action. In general, the finite version of the infinitesimal transformation (2.4) can only be defined implicitly. For $\tilde{A} = -\tilde{X}$ and $\tilde{B} = 0$, as in the original mimetic theory [1], the finite transformation is an arbitrary conformal transformation, $\tilde{g}_{\mu\nu} \rightarrow C(x^\rho) \tilde{g}_{\mu\nu}$. See [48] for a recent study of a large class of conformally invariant scalar-tensor theories, which partially overlaps with the mimetic theories studied here (as discussed in detail in the appendix of [46]).

From a Hamiltonian point of view, the existence of this extra symmetry leads to the presence of a first-class constraint that generates the above gauge transformations (2.4). This constraint adds to the usual Hamiltonian and momentum constraints associated with diffeomorphism invariance. This has been explicitly shown in a simple case in [30]. As a consequence, mimetic gravity is necessarily a degenerate theory in the sense defined in [28] and thus belongs to the family of DHOST theories.

However, contrary to most DHOST theories, the first class primary constraint in mimetic gravity does not lead to a secondary constraint, which is necessary to remove the Ostrogradsky ghost, at least in the context of classical mechanics [49, 50] or field theory with multiple scalars [51]. For this reason, it was unclear whether the scalar mode in mimetic gravity is healthy or not [52–54].

2.2 Mimetic Lagrangians

In the following, we are interested in Lagrangians of the form

$$\mathcal{L} = f_2(\phi, X)R + f_3(\phi, X)G^{\mu\nu}\phi_{\mu\nu} + \mathcal{L}_\phi(\phi, \phi_\mu, \phi_{\mu\nu}), \quad (2.5)$$

where the only Riemann-dependent terms are proportional to the scalar curvature R and the Einstein tensor $G_{\mu\nu}$ (associated with the metric $g_{\mu\nu}$) to ensure that the metric carries only two tensor degrees of freedom (see e.g. discussion in [33] and also [55]). Furthermore, the relation (2.3) implies immediately that X depends only on ϕ , since

$$X \equiv g^{\mu\nu}\phi_\mu\phi_\nu = \tilde{A}^{-1} \left(\tilde{g}^{\mu\nu} - \frac{\tilde{B}}{\tilde{A} + \tilde{B}\tilde{X}} \tilde{\nabla}^\mu\phi \tilde{\nabla}^\nu\phi \right) \phi_\mu\phi_\nu = \frac{\tilde{X}}{\tilde{A} + \tilde{B}\tilde{X}} = \frac{1}{h(\phi)}. \quad (2.6)$$

Hence, the functions f_2 and f_3 depend on ϕ only. As a consequence, the term proportional to f_3 can always be transformed, via an integration by parts, into [30]

$$f_3(\phi)G^{\mu\nu}\phi_{\mu\nu} = \frac{f_{3,\phi}}{2h}R + f_{3,\phi}[\phi_{\mu\nu}\phi^{\mu\nu} - (\phi_\mu^\mu)^2] - \frac{f_{3,\phi\phi}}{h}\phi_\mu^\mu - \frac{h_{,\phi}}{2h^3}f_{3,\phi\phi} + \nabla_\mu J^\mu, \quad (2.7)$$

where the explicit expression of J^μ is irrelevant here. For this reason, one can take $f_3 = 0$ in (2.5) without loss of generality (up to a redefinition of f_2 and \mathcal{L}_ϕ).

In [33], only terms that are at most cubic in second derivatives of the scalar field have been considered. Generalizing the classification of degenerate theories to higher powers of second derivatives of ϕ would probably be tedious. Interestingly, mimetic gravity provides us naturally with a particular class of DHOST theories that involves arbitrary functions of second derivatives of ϕ .

As shown in App. A.1 and A.2, one can restrict eq. (2.3) to the case $h(\phi) = -1$ (for configurations with time-like gradient) and $\tilde{B} = 0$ without loss of generality, assuming that matter is coupled to the metric $g_{\mu\nu}$. Therefore, for simplicity we restrict to the non-invertible conformal transformation

$$g_{\mu\nu} = -\tilde{X} \tilde{g}_{\mu\nu} \quad (2.8)$$

in the action. This implies the condition

$$X = g^{\mu\nu} \phi_\mu \phi_\nu = -1. \quad (2.9)$$

Let us now discuss the term \mathcal{L}_ϕ , involving second derivatives of the scalar field. This term can be viewed as a scalar constructed by contracting powers of the matrix

$$[\phi]_\nu^\mu \equiv \phi^\mu{}_\nu, \quad (2.10)$$

with the vector field ϕ^μ or with the metric $g_{\mu\nu}$. Hence, it can be expressed as a function of ϕ and of the two sets of scalar quantities $\vartheta_n \equiv \phi^\mu \phi^\nu [\phi]_{\mu\nu}^n$ and

$$\chi_n \equiv g^{\mu\nu} [\phi]_{\mu\nu}^n, \quad (2.11)$$

where n is an integer. However, as $X = -1$, we have

$$2 \phi^{\mu\nu} \phi_\nu = \nabla^\mu X = 0, \quad (2.12)$$

which implies immediately that $\vartheta_n = 0$ for any $n \geq 1$. As a consequence, \mathcal{L}_ϕ is a function of ϕ and χ_n only, and the mimetic action can be reduced, without loss of generality, to the form

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} [f_2(\phi)R + \mathcal{L}_\phi(\phi, \chi_1, \dots, \chi_n)], \quad (2.13)$$

where n is arbitrary and where the variation is taken with respect to $\tilde{g}_{\mu\nu}$ related to $g_{\mu\nu}$ by eq. (2.8).

2.3 DHOST formulation

To illustrate the DHOST formulation of mimetic gravity, it is convenient to work with a concrete model. For simplicity, we first restrict to the quadratic case and consider

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} [f_2(\phi)R + a_1(\phi)L_1^{(2)} + a_2(\phi)L_2^{(2)}], \quad (2.14)$$

where $a_1(\phi)$ and $a_2(\phi)$ are arbitrary functions of ϕ in front of the quadratic terms $L_1^{(2)} = \chi_2$ and $L_2^{(2)} = \chi_1^2$, and we have used the first two elementary quadratic Lagrangians among the five $L_A^{(2)}$ ($A = 1, \dots, 5$) introduced in [28], i.e.,

$$\begin{aligned} L_1^{(2)} &= \phi_{\mu\nu} \phi^{\mu\nu}, & L_2^{(2)} &= (\square\phi)^2, & L_3^{(2)} &= (\square\phi) \phi^\mu \phi_{\mu\nu} \phi^\nu, \\ L_4^{(2)} &= \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu, & L_5^{(2)} &= (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2. \end{aligned} \quad (2.15)$$

Upon the change of metric (2.8) (see [30] for details about the transformations of the quadratic action under general conformal-disformal transformations), the action (2.14) can be rewritten, after some integrations by parts, as a quadratic DHOST action of the form

$$S[\tilde{g}_{\mu\nu}, \phi] = \int d^4x \sqrt{-\tilde{g}} \left[\tilde{f}_0(\phi, \tilde{X}) + \tilde{f}_1(\phi, \tilde{X}) \tilde{\square} \phi + \tilde{f}_2(\phi, \tilde{X}) \tilde{R} + \sum_{A=1}^5 \tilde{a}_A(\phi, \tilde{X}) \tilde{L}_A^{(2)} \right], \quad (2.16)$$

with

$$\begin{aligned} \tilde{f}_0 &= 3\tilde{X}^2 f_{2,\phi\phi}(\phi), & \tilde{f}_1 &= 3\tilde{X} f_{2,\phi}(\phi), & \tilde{f}_2 &= -\tilde{X} f_2(\phi), & \tilde{a}_1 &= a_1(\phi), & \tilde{a}_2 &= a_2(\phi), \\ \tilde{a}_3 &= \frac{2}{\tilde{X}} [a_1(\phi) + 2a_2(\phi)], & \tilde{a}_4 &= -\frac{2}{\tilde{X}} [3f_2(\phi) + a_1(\phi)], & \tilde{a}_5 &= \frac{2}{\tilde{X}^2} [a_1(\phi) + 2a_2(\phi)]. \end{aligned} \quad (2.17)$$

If $f_2 = 0$, one can easily check that the above theories belong to the subclass IIIa (or M-I), as defined in [30] (or in [31]). If $f_2 \neq 0$ but $a_1 + a_2 = 0$, then one gets theories belonging to the subclass Ia (or N-I). In particular, this is the case of the original mimetic theory. Finally, in the generic case where $f_2 \neq 0$ and $a_1 + a_2 \neq 0$, these theories are in the class II. More precisely, they belong to subclass IIa (or N-III) if $f_2 \neq a_1$ and to subclass IIb (or N-IV) if $f_2 = a_1$.

The above calculation can be generalized to the case of a non-invertible disformal transformation, given by eq. (2.2) with eq. (2.3), using the results of [30]. Starting from the action (2.14), one obtains an action of the form (2.16), where now \tilde{f}_2 is an arbitrary function and

$$\begin{aligned} \tilde{a}_1 &= \tilde{c}_1(\phi) \left(\frac{\tilde{f}_2(\phi, \tilde{X})}{\tilde{X}} \right)^3 + \frac{\tilde{f}_2(\phi, \tilde{X})}{\tilde{X}}, & \tilde{a}_2 &= \tilde{c}_2(\phi) \left(\frac{\tilde{f}_2(\phi, \tilde{X})}{\tilde{X}} \right)^3 - \frac{\tilde{f}_2(\phi, \tilde{X})}{\tilde{X}}, \\ \tilde{a}_3 &= \frac{4\tilde{c}_2 \tilde{f}_2^2 \left(3\tilde{X} \tilde{f}_{2,X} - 2\tilde{f}_2 \right) - 2\tilde{c}_1 \tilde{f}_2^2 \left(\tilde{f}_2 - 2\tilde{X} \tilde{f}_{2,X} \right) + 2\tilde{X}^2 \left(\tilde{f}_2 - 2\tilde{X} \tilde{f}_{2,X} \right)}{\tilde{X}^4}, \\ \tilde{a}_4 &= -\frac{2\tilde{c}_1 \tilde{f}_2^3}{\tilde{X}^4} - \frac{4\tilde{f}_{2,\tilde{X}}}{\tilde{X}} + \frac{8\tilde{f}_{2,\tilde{X}}^2}{\tilde{f}_2}, \\ \tilde{a}_5 &= \frac{2\tilde{c}_1 \tilde{f}_2^2 \left(3\tilde{f}_2^2 + 6\tilde{X}^2 \tilde{f}_{2,X}^2 - 8\tilde{f}_2 \tilde{X} \tilde{f}_{2,X} \right) + 4\tilde{c}_2 \tilde{f}_2^2 \left(2\tilde{f}_2 - 3\tilde{X} \tilde{f}_{2,X} \right)^2 - 2\tilde{X}^2 \left(\tilde{f}_2 - 2\tilde{X} \tilde{f}_{2,X} \right)^2}{\tilde{f}_2 \tilde{X}^5}, \end{aligned} \quad (2.18)$$

with $\tilde{c}_1(\phi)$ and $\tilde{c}_2(\phi)$ two independent functions of ϕ only. For mimetic theories in the subclass Ia, one must impose the additional condition that $\tilde{c}_2 = -\tilde{c}_1$. One can check that the above functions satisfy the degeneracy conditions given in [28]. The remaining functions, \tilde{f}_0 and \tilde{f}_1 , have rather complicated expressions in terms of \tilde{f}_2 and we do not give them explicitly here.

We can also extend our discussion to cubic (or even higher) theories adding to (2.14) all the terms that are cubic in second derivatives, i.e.,

$$b_1(\phi) \chi_1^3 + b_2(\phi) \chi_1 \chi_2 + b_3(\phi) \chi_3. \quad (2.19)$$

These correspond to the first three elementary cubic Lagrangians $L_1^{(3)}$, $L_2^{(3)}$ and $L_3^{(3)}$ introduced in [33]. Performing the non-invertible conformal transformation (2.8) leads to a minimal cubic DHOST for the metric $\tilde{g}_{\mu\nu}$, whose Lagrangian is

$$\sum_{A=1}^{10} \tilde{b}_A(\phi, \tilde{X}) \tilde{L}_A^{(3)}, \quad (2.20)$$

with

$$\begin{aligned} \tilde{b}_1 &= -\frac{b_1}{\tilde{X}^3}, & \tilde{b}_2 &= -\frac{b_2}{\tilde{X}^3}, & \tilde{b}_3 &= -\frac{b_3}{\tilde{X}^3}, & \tilde{b}_4 &= -\frac{6b_1 + 2b_2}{\tilde{X}^4}, \\ \tilde{b}_5 &= \frac{2b_2}{\tilde{X}^4}, & \tilde{b}_6 &= \frac{2b_2 - 3b_3}{\tilde{X}^4}, & \tilde{b}_7 &= \frac{3b_3}{\tilde{X}^4}, & \tilde{b}_8 &= \frac{3b_3 + 4b_2}{\tilde{X}^5}, \\ \tilde{b}_9 &= -3\frac{4b_1 + 2b_2 + b_3}{\tilde{X}^5}, & \tilde{b}_{10} &= -2\frac{4b_1 + 2b_2 + b_3}{\tilde{X}^6}. \end{aligned} \quad (2.21)$$

According to the classification of cubic DHOST theories in [33], these theories either belong to class ${}^3\text{M-I}$, if $9b_1 + 2b_2 \neq 0$, or to class ${}^3\text{M-III}$, if $9b_1 + 2b_2 = 0$, and are compatible with the quadratic theories (2.17).

2.4 Lagrange multiplier formulation

The mimetic action (2.1) can also be reformulated as an action for the metric $g_{\mu\nu}$ instead of $\tilde{g}_{\mu\nu}$ as follows [52]

$$S'[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} [\mathcal{L}(\phi, \phi_\mu, \phi_{\mu\nu}; g_{\mu\nu}) + \lambda(X + 1)], \quad (2.22)$$

where λ enforces the mimetic constraint $X = -1$, given in eq. (2.9). If we express the action as in (2.13), this action takes the form

$$S'[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} [f_2(\phi)R + \mathcal{L}_\phi(\phi, \chi_1, \dots, \chi_n) + \lambda(X + 1)]. \quad (2.23)$$

Note that there is no X dependence in this Lagrangian, except in the term proportional to λ . If one assumes an explicit dependence of f_2 and \mathcal{L}_ϕ on X (as in e.g. [12, 14]), as well as on $\vartheta_n = \phi^\mu \phi^\nu [\phi]_{\mu\nu}^n$, then the equations of motion turn out to be equivalent to those obtained from the Lagrangian (2.23) where X is replaced by -1 and all the ϑ_n by zero.

Let us illustrate this point with a simple Lagrangian of the form

$$\int d^4x \sqrt{-g} [f_2(\phi, X)R + \mathcal{L}_\phi(\phi, X, \vartheta_1) + \lambda(X + 1)], \quad (2.24)$$

where we have introduced an explicit dependence on X and $\vartheta_1 = \phi^\mu \phi_{\mu\nu} \phi^\nu$. This action leads to the mimetic constraint $X = -1$ and to the metric equation of motion

$$\begin{aligned} f_2 G_{\mu\nu} &= \left(\frac{1}{2} \mathcal{L}_\phi - \square f_2 \right) g_{\mu\nu} + \nabla_\mu \nabla_\nu f_2 - \mathcal{L}_{\phi, \vartheta_1} X_\mu \phi_\nu \\ &\quad - [\lambda + f_{2,X} R + \mathcal{L}_{\phi, X} + \nabla_\alpha (\phi^\alpha \mathcal{L}_{\phi, \vartheta_1})] \phi_\mu \phi_\nu. \end{aligned} \quad (2.25)$$

The equation for the scalar field ϕ can be obtained from the Bianchi identity. The trace of the above equation enables us to express the Lagrange multiplier λ in terms of the metric and the scalar field, namely

$$\lambda = 2\mathcal{L}_\phi - 3\Box f_2 + f_2 R - f_{2,X}R - \mathcal{L}_{\phi,X} - \nabla_\alpha(\phi^\alpha \mathcal{L}_{\phi,\vartheta_1}), \quad (2.26)$$

where we have used the mimetic constraint and also $X_\mu = 0$. Substituting this expression back into (2.25) leads to the traceless metric equation

$$f_2 G_{\mu\nu} = \left(\frac{1}{2}\mathcal{L}_\phi - \Box f_2 \right) g_{\mu\nu} + \nabla_\mu \nabla_\nu f_2 + (2\mathcal{L}_\phi - 3\Box f_2 + f_2 R)\phi_\mu \phi_\nu. \quad (2.27)$$

This is exactly the same equation as the one obtained from (2.24) with $X = -1$ and $\vartheta_1 = 0$ directly in the Lagrangian. This conclusion remains valid when \mathcal{L}_ϕ is an arbitrary function of the variables ϑ_n .

3 Effective approach to cosmological perturbations

Since mimetic theories can be formulated in different ways, there are various but equivalent approaches to study cosmological perturbations in mimetic gravity. For our purpose, it will be convenient to use the unifying formulation given by the effective approach developed in [43–45] and extended to higher-order scalar-tensor theories in [42].

Let us start by briefly reviewing this approach for general DHOST theories [42]. We will use the ADM parametrization for the metric:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (3.1)$$

where N is the lapse function, N^i the shift vector, h_{ij} the three dimensional metric and the components of the extrinsic curvature tensor K_{ij} are given by

$$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - D_i N_j - D_j N_i), \quad (3.2)$$

where D_i denotes the covariant derivative compatible with h_{ij} .

We work in the so-called unitary gauge where the constant time hypersurfaces coincide with the uniform scalar field hypersurfaces. Assuming that the evolution of ϕ is monotonic, without loss of generality, we choose $\phi(t) = t$ on the background solution. In this gauge, we can expand any action in terms of the metric and matter fluctuations up to quadratic order. Adopting the notation of [42] for the effective description of DHOST theories, the gravitational part of the action expanded around a flat FLRW metric $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$ can always be written in the form

$$\begin{aligned} S_{\text{ETofDE}} = \int d^3x dt \sqrt{h} \frac{M^2}{2} \left\{ \delta K_{ij} \delta K^{ij} - \left(1 + \frac{2}{3}\alpha_L \right) \delta K^2 + (1 + \alpha_T)^{(3)}R \right. \\ \left. + H^2 \alpha_K \delta N^2 + 4H \alpha_B \delta K \delta N + (1 + \alpha_H)^{(3)}R \delta N + 4\beta_1 \delta K \delta \dot{N} + \beta_2 \delta \dot{N}^2 + \frac{\beta_3}{a^2} (\partial_i \delta N)^2 \right\}, \end{aligned} \quad (3.3)$$

where δN and δK_{ij} are respectively the perturbations of the lapse and of the extrinsic curvature, δK is the trace of δK_{ij} and $^{(3)}R$ is the 3-dimensional Ricci scalar.

In [42] it was shown that the effective parameters introduced above satisfy either of the degeneracy conditions

$$\mathcal{C}_I : \quad \alpha_L = 0, \quad \beta_2 = -6\beta_1^2, \quad \beta_3 = -2\beta_1 [2(1 + \alpha_H) + \beta_1(1 + \alpha_T)], \quad (3.4)$$

$$\mathcal{C}_{II} : \quad \beta_1 = -(1 + \alpha_L) \frac{1 + \alpha_H}{1 + \alpha_T}, \quad \beta_2 = -6(1 + \alpha_L) \frac{(1 + \alpha_H)^2}{(1 + \alpha_T)^2}, \quad \beta_3 = 2 \frac{(1 + \alpha_H)^2}{1 + \alpha_T}, \quad (3.5)$$

depending on the class of the DHOST theory considered (see Table 1 in App. B1 of [42]). As a consequence, this restricts by three the number of independent parameters.

In the following, we use this approach to study cosmological perturbations of mimetic theories, first in Sec. 4, using the DHOST formulation of mimetic actions and then in the Lagrange multiplier formulation (2.23) in Sec. 5. The two formulations are equivalent and lead to the same results, as expected.

For the analysis of linear perturbations, we will also include matter in the action, described in terms of a scalar field ψ with a Lagrangian depending on its first derivatives [56],

$$S_m = \int d^4x \sqrt{-g} P(Y) = \int d^4x \sqrt{-\tilde{g}} \tilde{X}^2 P(\tilde{Y}), \quad (3.6)$$

where

$$Y \equiv g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi = -\frac{1}{\tilde{X}} \tilde{g}^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \equiv \tilde{Y}. \quad (3.7)$$

We have written the matter action explicitly in terms of the physical metric $g_{\mu\nu}$ and in terms of the auxiliary metric $\tilde{g}_{\mu\nu}$, since we will need both expressions to analyse linear perturbations in the DHOST formulation and in the Lagrange multiplier formulation.

4 Perturbations in the DHOST formulation

In this section, we study linear perturbations of mimetic theories in their DHOST formulation. In the first subsection we will characterize the effective formulation of all mimetic DHOST and thus we will consider the general non-invertible transformation (2.2). It is sufficient to use the simplest non-invertible transformation (2.8) to study the linear perturbations of all mimetic theories, which we do in the subsequent subsection.

4.1 Effective description

In the unitary gauge, using (2.6), one obtains

$$\frac{\dot{\phi}^2(t)}{N^2} = -X = -\frac{1}{h(t)}, \quad (4.1)$$

which implies that N has no spatial dependence. Moreover, one can always redefine the time and the scalar field such that $\phi = t$ and $N = 1$ without loss of generality, see also App. A.1. Furthermore, according to the definition of χ_n given in (2.11), we find, in unitary gauge,

$$\chi_n = (-1)^n \text{tr}(K^n) \equiv (-1)^n K_{i_1}^{i_2} K_{i_2}^{i_3} \cdots K_{i_{n-1}}^{i_n} K_{i_n}^{i_1}, \quad (4.2)$$

so that the action (2.13) reduces to

$$S = \int d^4x \sqrt{h} \left[f_2 {}^{(3)}R - 2f_{2,\phi} K + f_2 (K_{ij} K^{ij} - K^2) + \mathcal{L}_\phi(t, -K; \dots, (-1)^p \text{tr}(K^p)) \right], \quad (4.3)$$

where we have used the Gauss-Codazzi relation¹ and integrated by parts its last term.

We need to expand this action up to quadratic order in the perturbations. The expansion of \mathcal{L}_ϕ is given by

$$\mathcal{L}_\phi = \bar{\mathcal{L}}_\phi(t) + c_0(t) \delta K + c_1(t) \delta K_i^j \delta K_j^i + c_2(t) (\delta K)^2 + \mathcal{O}(\delta K_{ij}^3), \quad (4.5)$$

where $\bar{\mathcal{L}}_\phi(t)$ denotes \mathcal{L}_ϕ evaluated on the background. In the second term, c_0 is defined by

$$c_0(t) \equiv \sum_{n=1} (-1)^n n H^{n-1} \mathcal{L}_{\phi,n}, \quad (4.6)$$

where $\mathcal{L}_{\phi,n}$ stands for the derivative of \mathcal{L}_ϕ with respect to χ_n evaluated on the background. The coefficients c_1 and c_2 in the third and fourth terms of (4.5) are given by combinations of the first and second derivatives of \mathcal{L}_ϕ with respect to χ_n , respectively

$$c_1(t) \equiv \frac{1}{2} \sum_{n=1} (-1)^n n(n-1) H^{n-2} \mathcal{L}_{\phi,n}, \quad (4.7)$$

$$c_2(t) \equiv \frac{1}{2} \sum_{(n,m)} nm H^{n+m-2} \mathcal{L}_{\phi,nm}, \quad (4.8)$$

where the right-hand sides are evaluated on the background. Notice that in the simple case (2.14), these expressions reduce to $c_1 = a_1$ and $c_2 = a_2$.

Using these results and integrating by parts the term proportional to $c_0(t)$, we can rewrite the quadratic action in terms of the effective parameters introduced in the previous section as

$$S_{\text{ETofDE}} = \int d^3x dt \sqrt{h} \frac{M^2}{2} \left\{ \delta K_{ij} \delta K^{ij} - \left(1 + \frac{2}{3} \alpha_L \right) \delta K^2 + (1 + \alpha_T) {}^{(3)}R \right\}, \quad (4.9)$$

where

$$\frac{M^2}{2} = f_2 + c_1, \quad \alpha_L = -\frac{3c_1 + c_2}{2f_2 + c_1}, \quad \alpha_T = -\frac{c_1}{f_2 + c_1}. \quad (4.10)$$

The parameters that do not appear in this action (because $\delta N = 0$), i.e. α_K , α_B , α_H , β_1 , β_2 and β_3 , simply remain undetermined.

Now, we can express this effective action in terms of the new metric $\tilde{g}_{\mu\nu}$, introduced in (2.2). The transformations of the parameters under general disformal transformations have been given in [42] (see also [57, 58] for earlier work) and we report them for convenience

¹The Gauss-Codazzi relation is

$$R = {}^{(3)}R + K_{\mu\nu} K^{\mu\nu} - K^2 + 2\nabla_\nu (n^\nu K - n^\mu \nabla_\mu n^\nu). \quad (4.4)$$

in App. A.3. As shown in [42], the structure of the action (3.3) is preserved under this transformation of the metric. Therefore, the action takes the form

$$\begin{aligned} \tilde{S}_{\text{ETofDE}} = \int d^3x d\tilde{t} \sqrt{\tilde{h}} \frac{\tilde{M}^2}{2} \left\{ \delta\tilde{K}_{ij} \delta\tilde{K}^{ij} - \left(1 + \frac{2}{3} \tilde{\alpha}_L \right) \delta\tilde{K}^2 + (1 + \tilde{\alpha}_T)^{(3)} \tilde{R} \right. \\ \left. + \tilde{H}^2 \tilde{\alpha}_K \delta\tilde{N}^2 + 4\tilde{H} \tilde{\alpha}_B \delta\tilde{K} \delta\tilde{N} + (1 + \tilde{\alpha}_H)^{(3)} \tilde{R} \delta\tilde{N} + 4\tilde{\beta}_1 \delta\tilde{K} \delta\dot{\tilde{N}} + \tilde{\beta}_2 \delta\dot{\tilde{N}}^2 + \frac{\tilde{\beta}_3}{\tilde{a}^2} (\partial_i \delta\tilde{N})^2 \right\}, \end{aligned} \quad (4.11)$$

where the time-dependent parameters are related to M^2 , α_L , α_T , via the non-invertible disformal transformation (2.2).

In particular, we define

$$\tilde{\alpha}_Y \equiv -\frac{\tilde{X}}{\tilde{A}} \frac{\partial \tilde{A}}{\partial \tilde{X}}, \quad \tilde{\alpha}_D \equiv -\frac{\tilde{B}}{\tilde{B} + \tilde{A}/\tilde{X}}, \quad \tilde{\alpha}_X \equiv -\frac{\tilde{X}^2}{\tilde{A}} \frac{\partial \tilde{B}}{\partial \tilde{X}}, \quad (4.12)$$

with the property that

$$1 + \tilde{\alpha}_X + \tilde{\alpha}_Y = 0, \quad (4.13)$$

which follows from the non-invertibility, see eq. (2.3). One finds (see App. A.3 for details)

$$\tilde{M}^2 = M^2 \tilde{A} \sqrt{1 + \tilde{\alpha}_D}, \quad \tilde{\alpha}_L = \alpha_L, \quad \tilde{\alpha}_T = -1 + \frac{1 + \alpha_T}{1 + \tilde{\alpha}_D}, \quad (4.14)$$

while the other parameters can be expressed as

$$\begin{aligned} \tilde{\alpha}_H &= -1 + \tilde{\alpha}_Y (1 + \tilde{\alpha}_T), \\ \tilde{\beta}_1 &= -\tilde{\alpha}_Y (1 + \tilde{\alpha}_L), \\ \tilde{\beta}_2 &= -6\tilde{\alpha}_Y^2 (1 + \tilde{\alpha}_L), \\ \tilde{\beta}_3 &= 2\tilde{\alpha}_Y^2 (1 + \tilde{\alpha}_T), \end{aligned} \quad (4.15)$$

and

$$\begin{aligned} \tilde{\alpha}_B &= -(1 + \tilde{\alpha}_L) \left(1 + \frac{\dot{\tilde{\alpha}}_Y}{\tilde{H}} \right), \\ \tilde{\alpha}_K &= 6\tilde{\alpha}_B - \frac{6}{\tilde{M}^2 \tilde{a}^3 \tilde{H}^2} \frac{d}{d\tilde{t}} \left(\tilde{M}^2 \tilde{a}^3 \tilde{H} \tilde{\alpha}_Y \tilde{\alpha}_B \right). \end{aligned} \quad (4.16)$$

Note that $\tilde{\alpha}_Y$ can be expressed in terms of $\tilde{\alpha}_T$ and $\tilde{\alpha}_H$ using the first equation in (4.15) and then substituted in the other expressions. As a consequence, from the point of view of the tilde frame, we find that \tilde{M}^2 , $\tilde{\alpha}_L$, $\tilde{\alpha}_T$ and $\tilde{\alpha}_H$ are free functions whereas the functions $\tilde{\beta}_1$, $\tilde{\beta}_2$ and $\tilde{\beta}_3$ are determined, in terms of $\tilde{\alpha}_L$, $\tilde{\alpha}_T$ and $\tilde{\alpha}_H$, by conditions that coincide with the degeneracy conditions \mathcal{C}_{II} , see eq. (3.5). Moreover, $\tilde{\alpha}_K$ and $\tilde{\alpha}_B$ are fixed by the relations (4.16). Therefore, mimetic theories are particular DHOST theories that satisfy \mathcal{C}_{II} and eq. (4.16). For $\tilde{\alpha}_L = 0$ they satisfy both \mathcal{C}_{I} , see eq. (3.4), and \mathcal{C}_{II} . One can check that these conditions are preserved by invertible disformal transformations. See Fig. 1 for a summary on the classification of theories based on linear perturbations.

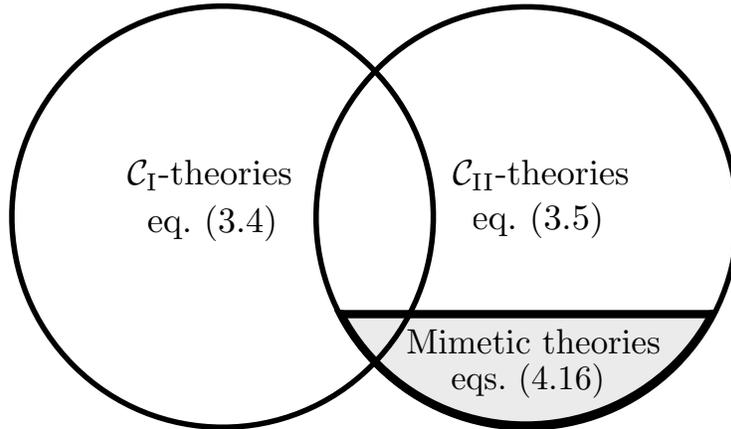


Figure 1: Theories of category \mathcal{C}_I and \mathcal{C}_{II} are respectively characterized by the conditions (3.4) and (3.5) (with tildes on all coefficients). In this figure, they are respectively represented by the left- and right-hand side disc. Note that some theories can be both \mathcal{C}_I and \mathcal{C}_{II} , as shown by the overlapping region between the two discs. Mimetic theories are a subset of theories of category \mathcal{C}_{II} , verifying the conditions on $\tilde{\alpha}_K$ and $\tilde{\alpha}_B$ given by eq. (4.16). They are represented by the gray region. Each category, \mathcal{C}_I , \mathcal{C}_{II} and mimetic, is preserved under invertible disformal transformations.

To illustrate these results, let us consider the simple case (2.8) where $\tilde{A} = -\tilde{X}$ and $\tilde{B} = 0$. In this case, $\tilde{\alpha}_D = 0$ and $\tilde{\alpha}_Y = -1$, and we find that the parameters (4.10) are transformed according to

$$\tilde{M}^2 = -2\tilde{X}(f_2 + c_1), \quad \tilde{\alpha}_L = -\frac{3c_1 + c_2}{2f_2 + c_1}, \quad \tilde{\alpha}_T = -\frac{c_1}{f_2 + c_1}. \quad (4.17)$$

The remaining parameters can be written in terms of the three parameters above by using eq. (4.15) with $\tilde{\alpha}_Y = -1$. Hence, in this special case $\tilde{\alpha}_H$ is no longer independent.

4.2 Linear perturbations

Let us study the linear perturbations in mimetic theories in the presence of matter minimally coupled to $g_{\mu\nu}$ (see eq. (3.6)). To do so, it is sufficient to take a particular DHOST representative by choosing the special case of $\tilde{A} = -\tilde{X}$ and $\tilde{B} = 0$. Therefore, we consider eq. (4.11) with the relations (4.15) and $\tilde{\alpha}_Y = -1$.

Including matter and specializing the action (4.11) to scalar perturbations, defined by

$$\tilde{h}_{ij} = \tilde{a}^2(t)e^{2\tilde{\zeta}}\delta_{ij}, \quad \tilde{N}^i = \delta^{ij}\partial_j\tilde{\chi}, \quad \psi = \psi_0(t) + \delta\psi, \quad (4.18)$$

we obtain the total quadratic action²

$$S^{\text{quad}} = \int d^3x dt \tilde{a}^3 \left\{ \frac{\tilde{M}^2}{2} \left[-6(1 + \tilde{\alpha}_L)\dot{\zeta}^2 + \frac{2}{\tilde{a}^2}(1 + \tilde{\alpha}_T)(\partial_i \zeta)^2 + [4(1 + \tilde{\alpha}_L)\dot{\zeta} - 2P'\dot{\psi}_0\delta\psi] \partial^2 \tilde{\chi} - \frac{2}{3}\tilde{\alpha}_L(\partial^2 \tilde{\chi})^2 \right] - \frac{P'}{c_m^2} \left[\delta\psi^2 - \frac{c_m^2}{\tilde{a}^2}(\partial_i \delta\psi)^2 \right] - 6P'\dot{\psi}_0 \delta\psi \zeta \right\}, \quad (4.19)$$

where we have introduced the sound speed of matter fluctuations [59]

$$c_m^2 \equiv \frac{P'}{P' - 2\dot{\psi}_0^2 P''}, \quad (4.20)$$

and defined

$$\zeta \equiv \tilde{\zeta} - \delta\tilde{N}. \quad (4.21)$$

Moreover, note that $\delta\tilde{N}$ only appears in the combination (4.21). This is due to the conformal invariance of the theory. Indeed, up to linear order, the metric can be written as

$$ds^2 \simeq e^{2\delta\tilde{N}} \left[-dt^2 + 2N_i dx^i dt + \tilde{a}^2 e^{2(\tilde{\zeta} - \delta\tilde{N})} d\vec{x}^2 \right], \quad (4.22)$$

which shows that, for a conformally invariant theory, only the combination given by eq. (4.21) matters.

Following the analysis of [42], we distinguish two cases. For $\tilde{\alpha}_L \neq 0$, variation with respect to $\tilde{\chi}$ yields

$$\partial^2 \tilde{\chi} = \frac{3}{\tilde{\alpha}_L} \left[(1 + \tilde{\alpha}_L)\dot{\zeta} - \frac{P'}{2}\dot{\psi}_0\delta\psi \right], \quad (4.23)$$

which can be plugged back into the action to give

$$S^{\text{quad}} = \int d^3x dt \tilde{a}^3 \left\{ A \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial_i \zeta)^2}{\tilde{a}^2} - P'\dot{\psi}_0\delta\psi\dot{\zeta} \right] - 6P'\dot{\psi}_0\delta\psi\zeta - \frac{P'}{c_m^2} \left[\delta\psi^2 - \frac{c_m^2}{\tilde{a}^2}(\partial_i \delta\psi)^2 \right] + \frac{3}{4} \frac{\tilde{M}^2 P'^2 \dot{\psi}_0^2}{\tilde{\alpha}_L} \delta\psi^2 \right\}, \quad (4.24)$$

with

$$A \equiv \frac{3\tilde{M}^2}{\tilde{\alpha}_L}(1 + \tilde{\alpha}_L), \quad c_s^2 \equiv -\frac{\tilde{\alpha}_L(1 + \tilde{\alpha}_T)}{3(1 + \tilde{\alpha}_L)}. \quad (4.25)$$

At this stage, we remind the reader that the normalization of the quadratic action for tensor modes is fixed by $\tilde{M}^2/8$ and that the speed of propagation of tensors is given by $c_T^2 = 1 + \tilde{\alpha}_T$ in this formulation [44, 45]. Since $(1 + \tilde{\alpha}_L)/\tilde{\alpha}_L > 0$ to avoid that ζ propagates a ghost, the propagation speed squared of scalar fluctuations has a sign opposite to that of tensor fluctuations. This implies an instability either in the scalar or in the tensor sector. In fact, this is true not only for mimetic gravity but for any DHOST theory in the category \mathcal{C}_{II} defined in [42].

²Note that, due to the definition of X , P' has sign opposite to P , so that the matter action has the correct sign in the action (4.19).

For $\tilde{\alpha}_L = 0$, the analysis is different. The variation of the action (4.19) with respect to $\tilde{\chi}$ implies a relation between $\dot{\zeta}$ and $\delta\psi$, i.e.,

$$\delta\psi = \frac{2\dot{\zeta}}{P'\dot{\psi}_0}. \quad (4.26)$$

This can be used to replace $\delta\psi$ in the action (4.24) above, to yield an action for ζ only. This action contains a $\dot{\zeta}^2$ term, which leads to a fourth-order equation of motion. This is consistent with the results of [21, 46], where one can find discussions on their physical interpretation. Notice that, together with the degeneracy condition \mathcal{C}_{II} , the mimetic condition eq. (4.16) is crucial for this result.

Let us finish with a short discussion on the case $\tilde{\alpha}_L = 0$ without matter. In that case, one must go back to the action (4.19), which reduces to

$$S^{\text{quad}} = \int d^4x \tilde{a}^3 \frac{\tilde{M}^2}{2} \left\{ -6\dot{\zeta}^2 + \frac{2(1 + \tilde{\alpha}_T)}{a^2} (\partial_i \zeta)^2 + 4\dot{\zeta} \Delta \tilde{\chi} \right\}. \quad (4.27)$$

From this action, ζ and $\tilde{\chi}$ satisfy the equations

$$\dot{\zeta} = 0, \quad \frac{d}{dt} \left(\tilde{a}^3 \tilde{M}^2 \tilde{\chi} \right) + \tilde{a} \tilde{M}^2 (1 + \tilde{\alpha}_T) \zeta = 0, \quad (4.28)$$

which shows that ζ is not a dynamical variable. Replacing the second equation into the first one we find a second-order equation for $\tilde{\chi}$,

$$\frac{d}{dt} \left[\frac{1}{\tilde{a} \tilde{M}^2 (1 + \tilde{\alpha}_T)} \frac{d}{dt} \left(\tilde{a}^3 \tilde{M}^2 \tilde{\chi} \right) \right] = 0, \quad (4.29)$$

which involves no gradient. Thus, in the absence of matter $\tilde{\chi}$ behaves as a scalar with a vanishing speed of sound, as it was found in [1, 24, 27] and more generally in [14].

5 Perturbations in the Lagrange multiplier formulation

We now discuss linear perturbations in the Lagrange multiplier formulation, with the action

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{g} [f_2(\phi)R + \mathcal{L}_\phi(\phi, \chi_1, \dots, \chi_n) + \lambda(X + 1)]. \quad (5.1)$$

In unitary gauge, the equation for λ is no longer an equation of motion but reduces to a constraint that fixes the lapse to $N = 1$. Hence, from the beginning, we can set $N = 1$ in the action. The rest of the analysis is identical to what is done at the beginning of Sec. (4.1) and it is straightforward to get eq. (4.9).

Let us now compute the quadratic action for linear perturbations, starting directly from eq. (4.9). We can concentrate on the scalar perturbations and, analogously to what was done in the previous section, we introduce the variables ζ and χ , but this time without a tilde,

$$h_{ij} = a^2(t) e^{2\zeta} \delta_{ij}, \quad N^i = \delta^{ij} \partial_j \chi. \quad (5.2)$$

Substituting into the quadratic action (4.9), we obtain an action for the perturbations ζ and χ ,

$$S^{\text{quad}} = \int d^3x dt a^3 \left\{ \frac{M^2}{2} \left[-6(1 + \alpha_L)\dot{\zeta}^2 + \frac{2}{a^2}(1 + \alpha_T)(\partial_i \zeta)^2 + \left[4(1 + \alpha_L)\dot{\zeta} - \frac{4}{M^2} P' \dot{\psi}_0 \delta\psi \right] \partial^2 \chi - \frac{2}{3} \alpha_L (\partial^2 \chi)^2 \right] - \frac{P'}{c_m^2} \left[\delta\psi^2 - \frac{c_m^2}{a^2} (\partial_i \delta\psi)^2 \right] - 6P' \dot{\psi}_0 \delta\psi \zeta \right\}. \quad (5.3)$$

This is the same action as eq. (4.19) but the tildes are absent from all quantities, not only from ζ . Indeed, this action can be obtained from eq. (4.19) by considering the relation between the metric perturbations in the two frames. From eq. (2.8) one gets

$$N^i = \tilde{N}^i, \quad h_{ij} = \frac{1}{\tilde{N}^2} \tilde{h}_{ij}, \quad (5.4)$$

which yields

$$a = \tilde{a}, \quad \chi = \tilde{\chi}, \quad \zeta = \tilde{\zeta} - \delta\tilde{N}. \quad (5.5)$$

Notice that this last relation is the same as eq. (4.21). From this action, it is obvious that the analysis is completely analogous to the one made in Sec. 4.2. A similar analysis in the formulation with the Lagrange multiplier has been performed in [46].

6 Conclusion

We have studied mimetic theories in the framework of Degenerate Higher-Order Scalar-Tensor (DHOST) theories. Indeed, as explained in Sec. 2, mimetic theories can be viewed as particular DHOST theories characterized by an extra symmetry. In general, this extra symmetry is an invariance under a combination of conformal and disformal transformations of the auxiliary metric used for the variation of the action. From the Hamiltonian point of view, this symmetry gives a constraint in the theory (in addition to the usual Hamiltonian and momentum constraints associated with diffeomorphism invariance), which is first class, in contrast with non-mimetic DHOST theories characterized by a pair of extra second-class constraints.

We have found that, generically (when $f_2 \neq 0$ and $a_1 + a_2 \neq 0$), mimetic theories belong to the class II of DHOST theories. However, some mimetic theories exist in the subclass Ia of DHOST theories (when $a_1 + a_2 = 0$), the subclass that also contains Horndeski theories. For mimetic DHOST theories, the six coefficients of the quadratic terms (i.e. f_2 and the five a_i) are specified in terms of a single function of X , e.g. $f_2(X, \phi)$ and two arbitrary functions of ϕ only ($c_1(\phi)$ and $c_2(\phi)$), which reduce to only one in the subclass Ia), as given by the expressions (2.18).

In the second part of this work, we have investigated the linear perturbations around a FLRW solution in these theories, using the effective theory of dark energy, reviewed in Sec. 3. We have applied this approach to two different formulations of mimetic theories. The first formulation, called ‘‘DHOST’’ formulation, is worked out in Sec. 4. The quadratic action is obtained starting from a generic DHOST theory, using a non-invertible conformal metric transformation, eq. (2.8). The effect of this transformation on the parameters of

the quadratic action was derived in [42] and is reviewed in App. A.3. Using these results, the final quadratic action is given in terms of four independent parameters (instead of 6 independent parameters in the non-mimetic case), with the others fixed by the non-invertible transformation. Interestingly, the quadratic parameters for mimetic theories always satisfy the condition \mathcal{C}_{II} , defined in (3.5). The few mimetic theories that belong to the subclass Ia satisfy both the conditions \mathcal{C}_{I} and \mathcal{C}_{II} . This is illustrated in Fig. 1.

We have studied the quadratic action of mimetic theories in the presence of external matter, which for simplicity we have taken in the form of a scalar field with Lagrangian dependent on its first derivatives. For $\tilde{\alpha}_{\text{L}} \neq 0$ we have found an instability either in the scalar or in the tensor sector. In the case $\tilde{\alpha}_{\text{L}} = 0$, which contains the original mimetic theory [1], we have shown that the dynamics of scalar perturbations can be expressed in terms of an action for ζ only, which is quadratic in $\ddot{\zeta}$. The same conclusion can be reached in the Lagrange multiplier formulation, as shown in Sec. 5 and in Ref. [46].

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A Fields transformations: useful formulae

A.1 Scalar field redefinition

Let us show that we can fix $h(\phi)$ to the constant values ± 1 simply by a field redefinition in (2.3). Indeed, if we assume that there exist a new field $\hat{\phi}$ and a one-variable function F such that $\phi = F(\hat{\phi})$, then the disformal transformation (2.2) becomes

$$g_{\mu\nu} = \hat{A}(\hat{\phi}, \hat{X})\tilde{g}_{\mu\nu} + \hat{B}(\hat{\phi}, \hat{X})\hat{\phi}_{\mu}\hat{\phi}_{\nu}, \quad (\text{A.1})$$

where

$$\hat{X} = \frac{\tilde{X}}{(F')^2}, \quad \hat{A}(\hat{\phi}, \hat{X}) = \tilde{A}[F(\hat{\phi}), (F')^2\hat{X}], \quad \hat{B}(\hat{\phi}, \hat{X}) = (F')^2\tilde{B}[F(\hat{\phi}), (F')^2\hat{X}]. \quad (\text{A.2})$$

Hence, the non-invertibility condition of the disformal transformation (2.3) becomes

$$\frac{\hat{A}(\hat{\phi}, \hat{X})}{\hat{X}} + \hat{B}(\hat{\phi}, \hat{X}) = \hat{h}(\hat{\phi}) \quad \text{with} \quad \hat{h}(\hat{\phi}) \equiv (F')^2 h[F(\hat{\phi})]. \quad (\text{A.3})$$

Now, if we assume that the sign of $h(\phi)$ is constant, we can always find a redefinition of the scalar field which allows to fix the function \hat{h} to ± 1 . Indeed, this is the case if F satisfies the differential equation

$$\frac{dF}{dx} = \frac{1}{\sqrt{|h[F(x)]|}}, \quad (\text{A.4})$$

which can be (at least formally) easily integrated according to

$$F(x) = G^{-1}(x + c), \quad G(y) \equiv \int dy \sqrt{|h(y)|}, \quad (\text{A.5})$$

where c is a constant.

A.2 Disformal transformations of the metric

Let us show that one can fix the functions \tilde{A} and \tilde{B} to $\tilde{A} = \pm\tilde{X}$ and $\tilde{B} = 0$ by a redefinition of the metric $\tilde{g}_{\mu\nu}$ using a disformal transformation. For that, we are looking for an invertible disformal transformation of the metric $\tilde{g}_{\mu\nu}$

$$\tilde{g}_{\mu\nu} = C(\phi, \hat{X})\hat{g}_{\mu\nu} + D(\phi, \hat{X})\phi_\mu\phi_\nu \quad (\text{A.6})$$

such that

$$g_{\mu\nu} = \pm\hat{X}\hat{g}_{\mu\nu}. \quad (\text{A.7})$$

This condition is possible if the functions C and D satisfy the conditions

$$\tilde{A}(\phi, \tilde{X})D(\phi, \hat{X}) + \tilde{B}(\phi, \tilde{X}) = 0, \quad \tilde{A}(\phi, \tilde{X})C(\phi, \hat{X}) = \hat{X}, \quad (\text{A.8})$$

with

$$\tilde{X} = \frac{\hat{X}}{C(\phi, \hat{X}) + \hat{X}D(\phi, \hat{X})}. \quad (\text{A.9})$$

These conditions can be satisfied (at least locally) when the disformal transformation (A.6) is invertible, in which case the relation (A.9) is well-defined.

A.3 Disformal transformation in the effective description

Let us consider the transformation

$$g_{\mu\nu} = \tilde{A}(\phi, \tilde{X})\tilde{g}_{\mu\nu} + \tilde{B}(\phi, \tilde{X})\partial_\mu\phi\partial_\nu\phi. \quad (\text{A.10})$$

Following [42], we review here the effect of this disformal transformation on the parameters M^2 , α_K , α_B , α_T , α_H , α_L , β_1 , β_2 and β_3 . Let us define

$$\tilde{\alpha}_Y \equiv -\frac{\tilde{X}}{\tilde{A}}\frac{\partial\tilde{A}}{\partial\tilde{X}}, \quad \tilde{\alpha}_D \equiv -\frac{\tilde{B}}{\tilde{B} + \tilde{A}/\tilde{X}}, \quad \tilde{\alpha}_X \equiv -\frac{\tilde{X}^2}{\tilde{A}}\frac{\partial\tilde{B}}{\partial\tilde{X}}. \quad (\text{A.11})$$

The effective parameters in the quadratic action derived from S and those associated with \tilde{S} are related via the transformations given in eq. (2.22) of Ref. [42]. One finds

$$M^2 = \frac{\tilde{M}^2}{\tilde{A}\sqrt{1 + \tilde{\alpha}_D}}, \quad \alpha_L = \tilde{\alpha}_L, \quad 1 + \alpha_T = (1 + \tilde{\alpha}_D)(1 + \tilde{\alpha}_T), \quad (\text{A.12})$$

and

$$\begin{aligned}
(1 + \tilde{\alpha}_X + \tilde{\alpha}_Y)(1 + \alpha_H) &= 1 + \tilde{\alpha}_H - \tilde{\alpha}_Y(1 + \tilde{\alpha}_T) , \\
(1 + \tilde{\alpha}_D)(1 + \tilde{\alpha}_X + \tilde{\alpha}_Y)\beta_1 &= \tilde{\beta}_1 + \tilde{\alpha}_Y(1 + \tilde{\alpha}_L) , \\
(1 + \tilde{\alpha}_D)^2(1 + \tilde{\alpha}_X + \tilde{\alpha}_Y)^2\beta_2 &= \tilde{\beta}_2 - 6\tilde{\alpha}_Y(\tilde{\alpha}_Y(1 + \tilde{\alpha}_L) + 2\tilde{\beta}_1) , \\
(1 + \tilde{\alpha}_D)(1 + \tilde{\alpha}_X + \tilde{\alpha}_Y)^2\beta_3 &= \tilde{\beta}_3 + 2\tilde{\alpha}_Y^2(1 + \tilde{\alpha}_T) - 4\tilde{\alpha}_Y(1 + \tilde{\alpha}_H) .
\end{aligned} \tag{A.13}$$

We are interested in a non-invertible transformation (A.10), where $1 + \tilde{\alpha}_X + \tilde{\alpha}_Y = 0$. Therefore, the left-hand side of eq. (A.13) vanishes so that one has

$$\begin{aligned}
1 + \tilde{\alpha}_H &= \tilde{\alpha}_Y(1 + \tilde{\alpha}_T) , \\
\tilde{\beta}_1 &= -\tilde{\alpha}_Y(1 + \tilde{\alpha}_L) , \\
\tilde{\beta}_2 &= -6\tilde{\alpha}_Y^2(1 + \tilde{\alpha}_L) , \\
\tilde{\beta}_3 &= 2\tilde{\alpha}_Y^2(1 + \tilde{\alpha}_T) .
\end{aligned} \tag{A.14}$$

Note that the parameter $\tilde{\alpha}_Y$ can be expressed in terms of $\tilde{\alpha}_H$ and $\tilde{\alpha}_T$ using the first of these equations and replaced in the other three equations, reproducing the degeneracy conditions \mathcal{C}_{II} , eq. (3.5). Therefore, mimetic theories satisfy the conditions \mathcal{C}_{II} .

Using the above relations, from eq. (C.14) of Ref. [42] one also finds, for $1 + \tilde{\alpha}_X + \tilde{\alpha}_Y = 0$,

$$\begin{aligned}
\tilde{\alpha}_B &= -(1 + \tilde{\alpha}_L) \left(1 + \frac{\dot{\tilde{\alpha}}_Y}{\tilde{H}} \right) , \\
\tilde{\alpha}_K &= 6\tilde{\alpha}_B - \frac{6}{\tilde{M}^2 \tilde{a}^3 \tilde{H}^2} \frac{d}{dt} \left(\tilde{M}^2 \tilde{a}^3 \tilde{H} \tilde{\alpha}_Y \tilde{\alpha}_B \right) .
\end{aligned} \tag{A.15}$$

We can thus conclude that mimetic theories are particular DHOST theories which satisfy \mathcal{C}_{II} and the above relations. One can check that mimetic theories are closed under invertible conformal/disformal transformations.

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