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# THE THREE CHARACTERISTIC BEHAVIOURS OF DUAL-POROSITY MEDIA

Pascale Royer\*, Jean-Louis Auriault\*\* & Claude Boutin\*\*\*

\*Laboratoire de Mécanique et Génie Civil, (CNRS, Université Montpellier II), CC048 Place Eugène Bataillon, 34095 Montpellier cedex 5, France. (Pascale.Royer@lmgc.univ-montp2.fr)

\*\*Laboratoire 3S-R (CNRS, INPG, UJF), BP 53 38041 Grenoble Cedex 9, France.

\*\*\* Laboratoire Géomatériaux (ENTPE, CNRS), Rue Maurice Audin, 69518 Vaulx-en-Velin, Cedex, France.

*Summary* Homogenisation of consolidation, transient fluid flow and solute transport in dual-porosity media has highlighted the existence of three characteristic macroscopic behaviours. The present paper is aimed towards presenting the original result that consists of relating each type of behaviour to a specific relationship between two characteristic times. The result is valid for any phenomenon.

## INTRODUCTION

The focus of the present study is on the fundamentals of fluid flow and transport in deformable dual-porosity media, i.e. in media that consist of two interacting porous systems of distinctly different fluid transfer properties. In the sequel, we consider the case of a medium that is composed of a network of connected macropores and blocks of microporous matrix. Homogenisation of various phenomena (consolidation, compressible fluid flow, solute transport) has led to the identification of three characteristic behaviours for dual-porosity media [1], [2],[3], [4], [5]. These three behaviours include a dual and a single porosity description, and an intermediate behaviour which we refer as behaviour with reservoir effects. With this latter, the whole dual-porosity medium is represented by an equivalent single-porosity medium. In contrast with a single-porosity behaviour, the porosity of the entire dual-porosity medium is accounted for. For consolidation [1], [2] and compressible fluid flow [4], we have shown that the three models are obtained for three distinct relationships between the scales ratios  $\alpha = l'/l$  and  $\varepsilon = l/L$ , where  $l'$ ,  $l$  and  $L$  denote the micropore size, the macropore size and the macroscopic size, respectively. For non-reactive solute transport [5], only models with reservoir effects are obtained, which means that the experimentally observed dual-porosity effects cannot be described in the framework of an equivalent continuum macroscopic description. The objective of the present work is to determine a clear interpretation of the existence of these three behaviours, by relating each of them with a characteristic condition. For this purpose, we consider the results obtained for thermal conduction in composite media and we show that the domains of validity of the three behaviours can be translated into orders of magnitude of characteristic time ratios. Application of the corresponding rule for dual-porosity media provides a clear interpretation of the three characteristic behaviours.

## ANALYSIS OF THE RESULTS FOR THERMAL CONDUCTION IN COMPOSITE MEDIA

The objective is to analyse the domains of validity of the distinct models obtained by homogenisation for thermal conduction in composite media. For upscaling the local description, we use the homogenisation method for periodic structures [6], with the methodology introduced in [7]. By starting from the description over the period, this approach allows derivation of the distinct macroscopic models together with their domains of validity. These latter are defined in terms of orders of magnitude of dimensionless numbers.

### Local description

We consider a periodic composite medium of period  $\Omega$ . The dimension of the period  $l$  is small compared to the characteristic macroscopic length  $L$ :  $\varepsilon = l/L \ll 1$ . The period is composed of two constituents, which we distinguish by subscripts 1 and 2, and which occupy domains  $\Omega_1$  and  $\Omega_2$ , respectively. The boundary between both constituents is denoted by  $\Gamma$ . Within the period, thermal conduction is described by the following set of equations:

$$\begin{cases} \rho_\gamma c_\gamma \frac{\partial T_\gamma}{\partial t} - \vec{\nabla} \cdot (\bar{\lambda}_\gamma \vec{\nabla} T_\gamma) = 0 & \text{in } \Omega_\gamma \ (\gamma = 1, 2), \\ T_1 = T_2 & \text{on } \Gamma, \\ (\bar{\lambda}_1 \vec{\nabla} T_1) \cdot \vec{n} = (\bar{\lambda}_2 \vec{\nabla} T_2) \cdot \vec{n} & \text{on } \Gamma. \end{cases}$$

We assume that constituent 1 is connected and much more conductive than constituent 2, and that both thermal capacities are of same order of magnitude, which in the framework of homogenisation is expressed by:  $\lambda_{2c}/\lambda_{1c} < \mathcal{O}(\varepsilon^0)$ ; and  $\rho_{2c} c_{2c}/\rho_{1c} c_{1c} = \mathcal{O}(\varepsilon^0)$ . By definition, a quantity  $Q$  is said to be  $\mathcal{O}(\varepsilon^q)$  when  $\varepsilon^{q+1} \ll Q \ll \varepsilon^{q-1}$ .

### Transient macroscopic models and domains of validity

Homogenisation of the above local description leads to three first-order transient macroscopic models. They are obtained by considering the order of magnitude  $L^2 \rho_{1c} c_{1c}/\lambda_{1c} t_c = \mathcal{O}(\varepsilon^0)$ , which characterizes a transient macroscopic regime, where  $t_c$  denotes the characteristic time of temperature variations. Each model is then derived by considering a specific order of magnitude for the conductivity ratio  $\mathbb{R}_\lambda = \lambda_{2c}/\lambda_{1c}$ . For each model, its domain of validity is thus given by the

order  $L^2 \rho_{1c} c_{1c} / \lambda_{1c} t_c = \mathcal{O}(\varepsilon^0)$ , together with the corresponding order of  $\mathbb{R}_\lambda = \lambda_{2c} / \lambda_{1c}$ . The three models read:

- **Model with resevoir effects (I):**  $\mathbb{R}_\lambda = \mathcal{O}(\varepsilon)$   

$$C_T^{\text{eff}} \frac{\partial T}{\partial t} - \vec{\nabla} \cdot (\tilde{\lambda}_1^{\text{eff}} \vec{\nabla} T) = 0$$
- **Dual-conductivity model (II):**  $\mathbb{R}_\lambda = \mathcal{O}(\varepsilon^2)$   

$$C_1^{\text{eff}} \frac{\partial T_1}{\partial t} - \vec{\nabla} \cdot (\tilde{\lambda}_1^{\text{eff}} \vec{\nabla} T_1) = - \langle \rho_2 c_2 \frac{\partial T_2}{\partial t} \rangle_{\Omega_2}$$
- **Single-conductivity model (III):**  $\mathbb{R}_\lambda \leq \mathcal{O}(\varepsilon^3)$   

$$C_1^{\text{eff}} \frac{\partial T_1}{\partial t} - \vec{\nabla} \cdot (\tilde{\lambda}_1^{\text{eff}} \vec{\nabla} T_1) = 0$$

In model I,  $C_T^{\text{eff}}$  is the volume average of thermal capacities of both constituents, while the partial thermal capacity  $C_1^{\text{eff}}$  in models II and III only accounts for constituent 1. In the three models, the effective conductivity is  $\tilde{\lambda}_1^{\text{eff}}$  which accounts only for the contribution of constituent 1. The derivation of model II is detailed in [8]. It is a two-temperature-field model ( $T_1 \neq T_2$ ), in which both temperature fields satisfy an expression of the following type that characterizes the presence of memory effects:

$$T_2 = T_1 - h(t) * \frac{\partial T_1}{\partial t} = T_1 - \int_{-\infty}^t h(t - \tau) \frac{\partial T_1}{\partial \tau} d\tau.$$

### Interpretation of the domains of validity

It can be demonstrated that the domains of validity of the three models are fully defined by orders of magnitude of the ratio  $t_{2i}/t_{1L}$ , where  $t_{2i} = l^2 \rho_{2c} c_{2c} / \lambda_{2c}$  and  $t_{1L} = L^2 \rho_{1c} c_{1c} / \lambda_{1c}$ . We thus obtain the following result:

$$\left\{ \begin{array}{l} \text{Model with resevoir effects:} \quad \frac{t_{2i}}{t_{1L}} = \mathcal{O}(\varepsilon) \Rightarrow t_{2i} \ll t_{1L}; \\ \text{Dual-conductivity model:} \quad \frac{t_{2i}}{t_{1L}} = \mathcal{O}(\varepsilon^0) \Rightarrow t_{2i} \approx t_{1L}; \\ \text{Single conductivity model:} \quad \frac{t_{2i}}{t_{1L}} \geq \mathcal{O}(\varepsilon^{-1}) \Rightarrow t_{2i} \gg t_{1L}. \end{array} \right. \quad (1)$$

## CHARACTERISTIC CONDITIONS FOR THE THREE BEHAVIOURS IN DUAL-POROSITY MEDIA

By similarity with (1), we consider the ratio  $t_{mi}/t_{pL}$ , so as to analyse the results in dual-porosity media, where  $t_{mi}$  and  $t_{pL}$  represent the local characteristic time in the matrix and the macroscopic characteristic time in the macropores, respectively. For any transient phenomenon and provided that both macropores and micropores are connected, we obtain:

$$\left\{ \begin{array}{l} \frac{t_{mi}}{t_{pL}} < \mathcal{O}(\varepsilon^0) : \quad \text{Reservoir effect;} \\ \frac{t_{mi}}{t_{pL}} = \mathcal{O}(\varepsilon^0) : \quad \text{Dual porosity;} \\ \frac{t_{mi}}{t_{pL}} > \mathcal{O}(\varepsilon^0) : \quad \text{Single porosity.} \end{array} \right.$$

For consolidation and compressible fluid flow, we get  $t_{mi}/t_{pL} = \mathcal{O}(\varepsilon^2 \alpha^{-2})$ , which explains the domains of validity obtained in the initial works. For solute transport, the above result allows to identify the specific conditions under which dual-porosity effects may occur, although this case cannot be described by means of an equivalent continuum description.

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