Phase Change during Paper Pressing: Mathematical Modelling by an Upscaling Technique

Jean-Francis Bloch, Jean-Louis Auriault, Pascale Royer

PHASE CHANGE DURING PAPER PRESSING: MATHEMATICAL MODELLING BY AN UPSCALING TECHNIQUE

J.-F. BLOCH(*) - J.-L. AURIAULT(**) – P. ROYER(**)
(*)LGP2/EFPG/INPG/CNRS UMR 5518
(**) 3S/UJF/INPG/CNRS UMR 5521

1. INTRODUCTION.

Nowadays industrial applications increasingly need improved technical and scientific understanding so as to optimise production and competitiveness. Hence, developing empirical, numerical or theoretical models has become crucial. Derivation of mathematical models consists in determining behaviour laws and the corresponding effective parameters. The knowledge of the domains of validity of these models is of great importance. Paper industry would also draw a great benefit from the improved understanding of physical processes provided by such theoretical investigations. As an example, deriving knowledge models of physical processes involved in the new technology of paper pressing at high temperature will allow enhancement of both paper quality and production rate.
This article is devoted to the mathematical modelling of phase change of water into vapour phase during hot paper pressing. The overall behaviour is obtained from the fibre scale by an upscaling technique which is based on asymptotic expansions. The main characteristics of the method are briefly recalled in part 2. Then, the method is used to investigating phase change in porous media in order to derive a model for hot paper pressing.

2. PRESENTATION OF THE METHOD.

Numerous mathematical techniques may be found in literature, that allow derivation of the macroscopic behaviour from the physical description at a finer scale (Kaviany (1)). The essential goal is to reproduce the behaviour of the finely heterogeneous paper at the macroscopic scale at which numerical calculations will be performed. Then, the effective macroscopic parameters are generally obtained by identification with experimental results that are obtained at the macroscopic sample scale. The most frequently used upscaling technique is the volume averaging method. Unfortunately, this method does not give the domain of validity of the macroscopic models. In this work, we use an homogenisation technique, namely the method of homogenisation for periodic structures, that has initially been introduced by Bensoussan and co-authors (2). It permits to obtain the macroscopic laws, their domain of validity, and also the effective parameters without any macroscopic pre-requisite. The main steps of the upscaling procedure are the following: 1) definition of the physical phenomena involved at the microscopic scale; 2) identification of the different scales. In the present case there are two scales whose ratio is measured via a small parameter, ε; 3) writing of the microscopic description in a dimensionless form. The dimensionless numbers arising from this new writing are estimated, which gives rise to the definition of several distinct dimensionless local descriptions; 4) writing of the physical variables in the form of asymptotic expansions in powers of ε; 5) investigation of the local boundary value problems obtained at the successive orders of magnitude of ε.
That yields a catalogue of macroscopic models which are related to the distinct dimensionless local descriptions. The domain of validity of each model is defined by means of the orders of magnitude of the dimensionless numbers. For a specific practical problem (i.e. a given pressing process and a given paper), the corresponding dimensionless numbers can easily be estimated by selecting the appropriate characteristic quantities. The suitable model for describing the process under consideration can then be selected throughout the catalogue.

3. LOCAL EQUATION DESCRIPTION.

The local physical description is based on Ishii's work (1975). We analyse different problems linked to the distinction of dimensionless parameters. These have to be evaluated for the physical studied situation and therefore the macroscopic equivalent model is found when it exists.
The chosen scale is microscopic scale corresponds to the pore one. The source term is not written at this scale. Nevertheless, phase change may occur at the interface between the considered fluids (air/vapour and

water). The following conservation equations are therefore written : mass, momentum, energy. They are completed by the boundary conditions. We assume that the vapour has no contact with the solid, that is equivalent to an annular situation. The skeleton may be considered as rigid or deformable. This modifies only the water velocity at the solid interface.

3.1 - BULK EQUATIONS

Mass conservation : $$\frac{\partial \rho_k}{\partial t} + \nabla.(\rho_k v_k) = 0$$

Where ρ_k and $\mathbf{v}_k$ are density and velocity of k component, respectively.

Navier stokes equation : $$\rho_k \left[\frac{\partial v_k}{\partial t} + v_k . \nabla v_k \right] = \mu_k (\Delta v_k + \text{grad}(\text{div} v_k)) - \text{grad} p_k$$

Where p_k and μ_k are the pressure and the viscosity.

Energy conservation : $$\rho_k C_{pk} \frac{DT_k}{Dt} - T_k \beta \frac{Dp_k}{Dt} = \nabla.(\lambda \nabla T_k)$$

Where Cp is the specific heat , T is the temperature, λ is the thermal conductibility and $\beta = -\frac{1}{\rho}\left[\frac{\partial \rho}{\partial T}\right]_p$ the volume dilatation rate. For a perfect gas, $\beta = 1/T$ and for a liquid β is negligible.

State equation for fluids : $p_k = A_k \rho_k$

3.2 - SOLID - LIQUIDE INTERFACE Γ_{ws}

* velocity continuity : $\mathbf{V_W} = \mathbf{u'_S}$ (or = **0** if the skeleton is assumed as rigid)

where $\mathbf{V_W}$ is the wetting phase velocity and $\mathbf{u'_S}$ represents the partial derivative of the solid displacement.

* Temperature jump : $T_W = T_S$

* Energy conservation: $\lambda_w \, \mathbf{grad}\, T_w . \mathbf{N}_w = \lambda_s \, \mathbf{grad}\, T_s . \mathbf{N}_s$ where $\mathbf{N}_{(w,s)}$ is the unit normal vector.

3.3 - LIQUID – GAZ INTERFACE Γ_{WA}

a) **Mass** : $\rho_a (\mathbf{v}_a - \mathbf{v}_i).N_a + \rho_w (\mathbf{v}_w - \mathbf{v}_i).N_w = 0$, where $\mathbf{v}_i$ is the interface velocity.

b) **Momentum equilibrium** : $$(p_w - p_{sat}(T_i)) = -2H_{wa}\gamma\left[\frac{\rho_w}{\rho_w - \rho_a}\right] + \frac{1}{2}\left[\rho_w N_w .(v_w - v_i)\right]^2 \left[\frac{1}{\rho_a} - \frac{1}{\rho_w}\right]$$

where p_{sat}, H_{wa} and γ are the saturated vapor pressure, the liquid-gas interface curvature and the superficial tension, respectively.

c) **Energy** : $$0 = \left[\overset{0}{m}_w\right]\left[(C_{pw}T_w - C_{pa}T_a) + \left[\frac{1}{2}\mathbf{v}_w^2 - \mathbf{v}_w . \mathbf{v}_p\right] - \left[\frac{1}{2}\mathbf{v}_a^2 - \mathbf{v}_a . \mathbf{v}_p\right]\right] + (\mathbf{N}_w . \mathbf{g}_w + \mathbf{N}_a . \mathbf{g}_a)$$

where $\mathbf{v}_p = (\mathbf{v}_i.\mathbf{n}_k)\mathbf{n}_k + \mathbf{v}^t$, $\overset{0}{m}_k = \rho_k (\mathbf{v}_k - \mathbf{v}_i).\mathbf{N}_k$, g_k,, is the free Gibbs energy $g_k = u_k - T_k s_k + (p_k/\rho_k)$, u_k is the mass internal energy and s_k is the mass entropy.

d) **Thermal condition** : $T_{wi} = T_{ai} = T_i$

e) **Energy condition** : $$p_a - p_{sat}(T_i) = -2H_{wa}\gamma\left[\frac{\rho_a}{\rho_w - \rho_a}\right] + \frac{1}{2}\left[\rho_w N_w .(v_w - v_i)\right]^2 \left[\frac{1}{\rho_w} - \frac{1}{\rho_a}\right]$$

4 – FORMAL EQUATIONS

4.1 - Introduction of dimensionless quantities.

For each physical value Ψ, we may define the associated dimensionless quantity : $\Psi = \Psi_R \, . \, \Psi^*$ where Ψ_R is the physical reference value and Ψ^* the dimensionless variable.

4.2 – Numerical values of reference variables

Reference values presented in the above table are used here to obtain dimensionless variables.

Variable	Unit	Water	Air	Fibre
μ	Pa . s	10^{-3}	19.10^{-6}	
ρ	kg. m^{-3}	10^{3}	1.23	$1{,}5.10^{3}$
ν	m^2 . s^{-1}	10^{-6}	$1{,}5.10^{-5}$	
T	s	10^{-3}	10^{-3}	10^{-3}
P	Pa	10^{5} à 10^{6}	10^{5} à 10^{6}	10^{5} à 10^{6}
V	m . s^{-1}	10^{-2}	10^{-2}	
T	K	293 à 373	293 à 373	293 à 373
λ	J .m^{-1} .s^{-1} .K^{-1}	0.602	0.026	0.33
C_p	J . kg^{-1} . K^{-1}	$4.18.10^{3}$	10^{3}	$1{,}33.10^{3}$
α	m^2 . s^{-1}	$1{,}4.10^{-7}$	$2{,}1.10^{-5}$	$1{,}6.10^{-7}$
γ	N . m^{-1}	$72{,}75.10^{3}$ (293)	$58{,}8.10^{3}$ (373)	
K	Pa	2.10^{9}	10^{5}	
A	kg . m^{-1} . s^{-2}	2.10^{6}	$8{,}1.10^{4}$	

Table 1 : Reference values

We may now estimate the order of magnitude of these figures in respect of ε.

$\mu_a = O(\varepsilon \, . \, \mu_w)$; $\rho_a = O(\varepsilon \, . \, \rho_w)$; $\nu_a = O(\nu_w)$; $T_a = O(t_w)$; $p_a = O(p_w)$; $T_a = O(T_w)$;
$\lambda_a = O(\varepsilon \, . \, \lambda_w)$; $C_{pa} = O(C_{pw})$; $\alpha_a = O(\varepsilon^{-1} . \alpha_w)$; $K_a = O(\varepsilon^2 \, . \, K_w)$; $A_a = O(\varepsilon \, . \, A_w)$

Table 2 :Reference values evaluation versus ε with $\varepsilon = O(10^{-2})$

4.3 – Formal equations

We may introduce in the preceding equation each dimensionless variables. Nevertheless in order to simplify the notation, the symbols * and R are now omitted. Furthermore, dimensionless parameters constituted by given reference values are introduced.

(1) – Mass Conservation : $S_t \frac{\partial \rho_k}{\partial t} + \nabla.(\rho_k v_k) = 0$ where $St = \frac{\left|\frac{\partial \rho_\alpha}{\partial t}\right|}{\left|div(\rho_\alpha v_\alpha)\right|} \# \frac{l_r}{V_r \, t_r}$ is the Strouhal number. It has been demonstrated that the situation is homogenisable only in the case : $St \leq O(\varepsilon)$ (AURIAULT - 1990).

(2) – Momentum conservation : $\rho_k \left[S_t \frac{\partial v_k}{\partial t} + v_k . \nabla v_k \right] = \frac{1}{R_e} \mu_k (\Delta v_k + \text{grad}(\text{div} v_k)) - E_u \, \text{grad} P_k$

where $E_u = \frac{\left|\mathbf{grad}\ P_k\right|}{\left\|\rho_k \mathbf{v}_k \mathbf{.grad}\ \mathbf{v}_k\right\|} \# O\left[\frac{p}{\rho V^2}\right]$ and $R_e = \frac{\left|\rho_k v_k . \text{grad} v_k\right|}{\left|\mu_k \left[\text{grad}(\text{div}(v_k)) + \Delta v_k\right]\right|} \# O\left[\frac{\rho l V}{\mu}\right]$ are the Euler and Reynolds numbers, respectively.

The EULER number may be written as : Eu = Q / Re = O (ε^{-1}) / Re

With : $Q = \frac{|\text{grad } P_k|}{|\mu_k [\Delta v_k + \text{grad}(\text{div}(v_k))]|} = \frac{|P.l|}{|\mu.v|} = \frac{|L|}{|l|} = O(\varepsilon^{-1})$ as $\frac{|P|}{|L|} = O\left[\frac{\mu.v}{l^2}\right]$

(3) – Energy conservation :

$$\left[S_t \frac{\partial \rho_k h_k}{\partial t} + V_k . \nabla(\rho_k h_k)\right] = -\frac{1}{P_e} \nabla . g_k + E_u * E_c * \left[S_t \frac{\partial p_k}{\partial t} + V_k . \nabla p_k\right] + S_o q_k + \frac{E_c}{R_e} \tau_k : \nabla V_k$$

where h_k represents the mass enthalpy. $E_c = \frac{|v_k . \text{grad} p_k|}{|\rho_k v_k . \text{grad}(h_k)|} . \frac{|\rho_k v_k . \text{grad}(v_k)|}{|\text{grad}(p_k)|} \# = O\left[\frac{V^2}{\Delta h_{k0}}\right]$ and

$P_e = \frac{|\rho_k v_k . \text{grad}(h_k)|}{|\nabla . g_k|} \# O\left[\frac{l \rho V \Delta h}{\lambda \Delta T}\right]$ are the Eckert and Peclet numbers, respectively. The source term

$S_o = \frac{|q_k|}{|\nabla . \mathbf{g}_k|}$ will be considered as zero valued.

(4) – Solid – liquid interface

-4 velocity continuity at the 'interface : $\mathbf{v_w} = \mathbf{u'_s}$

-5 Thermal condition : $T_W = T_S$

The Biot number may be introduced here : $B_i = \frac{|h(T_\alpha - T_\beta)|}{|\lambda_\alpha \text{grad}(T_\alpha) . N|} \# O\left[\frac{|h(T_\alpha - T_\beta) l|}{|\lambda_\alpha T_\alpha|}\right] = 0$

The Biot number B is zero valued here as there are a finite flux and a negligible thermal resistance. Hence the temperatures (T_α and T_β) corresponding to each interface side are equal.

-4 Energy : $C_{ws} \lambda_w \text{ grad } T_w . N_w = \lambda_s \text{ grad } T_s . N_s .$

The parameter $C_{ws} = \frac{|\lambda_w \text{grad}(T_w) . N_w|}{|\lambda_s \text{grad}(T_s) . N_s|} \# O\left[\frac{\lambda_w T_w}{\lambda_s T_s}\right] = O\left[\frac{\lambda_w}{\lambda_s}\right]$ is of the order of magnitude of 1 as the thermal conductibilities for both fibre and water are almost equal.

(5) - Boundary conditions Liquid - Gas :

- Mass conservation : $CL_{wa1} \rho_a (\mathbf{v_a} - \mathbf{v_i}) . N_a + \rho_w (\mathbf{v_w} - \mathbf{v_i}) . N_w = 0$

where $CL_{wa1} = \frac{|\rho_a (v_a - v_i) . N_a|}{|\rho_w (v_w - v_i) . N_w|} \# O\left[\frac{\rho_a}{\rho_w} . \frac{\Delta V_a^r}{\Delta V_w^r}\right] = O\left[\frac{\rho_a}{\rho_w}\right]$, if the velocities are assumed equal.

- Momentum equilibrium :

$$\frac{1}{N_\gamma}(p_w - p_{sat}(T_i)) = -2 H_{wa} \gamma \left[\frac{\rho_w}{\rho_w - N_\rho \rho_a}\right] + \frac{1}{2} CL_{wa4} [\rho_w N_w . (v_w - v_i)]^2 \left[\frac{1}{N_\rho \rho_a} - \frac{1}{\rho_w}\right]$$

where $N_\rho = \frac{|\rho_{a0}|}{|\rho_{w0}|} \# = O(\varepsilon)$, $N_\gamma = \frac{|H^r_{wa} \gamma|}{|\Delta P_0|}$ and $CL_{wa4} = \frac{\left|\left[\overset{o}{m}_w\right]^2\right|}{|H_{wa} \gamma \rho_w|}$ are the density ratio, surface tension number, the phase change number, respectively. The surface tension number N_γ will be considered at the order of magnitude of 1.

- Energy :

$$0 = \left[\overset{0}{m}_w \right] \left(C_{pw} T_w - C_{pa} T_a \right) + E_c \left[\left[\frac{1}{2} \mathbf{v}_w^2 - \mathbf{v}_w . \mathbf{v}_p \right] - \left[\frac{1}{2} \mathbf{v}_a^2 - \mathbf{v}_a . \mathbf{v}_p \right] \right] + CL_{wa9} \left(\mathbf{N}_w . \mathbf{g}_w + N_\lambda \mathbf{N}_a . \mathbf{g}_a \right)$$

where $CL_{wa9} = \frac{|g_w . n_w|}{|(\rho_w (v_w - v_i) . n_w)(C_{pw} T_w)|} \# O\left[\frac{\lambda_w}{l_r \rho_w V_r C_{p_w}} \right] = O\left(P_e^{-1}\right)$

We assume that $v_p = O(v_r)$. If $v_p = O(\varepsilon . v_r)$, it is then necessary to introduce the scale factor ε in front of the scalar product. Nevertheless, as the Eckert number is weak, this modification has no influence.

- Thermal condition : $T_{wi} = T_{ai} = T_i$

-5 Energy condition :

$$\frac{1}{N_\gamma} \left(p_a - p_{sat}(T_i) \right) = -2 H_{wa} \gamma \left[\frac{\rho_a}{\frac{1}{N_\rho} \rho_w - \rho_a} \right] + \frac{1}{2} CL_{wa4} \left[\rho_w \mathbf{N}_w . (\mathbf{v}_w - \mathbf{v}_i) \right]^2 \left[\frac{1}{\rho_w} - \frac{1}{N_\rho \rho_a} \right]$$

where $N_\rho = \frac{|\rho_a|}{|\rho_w|} \# O(\varepsilon)$, $N_\gamma = \frac{|H_{wa} \gamma|}{|\Delta P|}$, $CL_{wa4} = \frac{\left| \left[\overset{o}{\mathbf{m}}_w \right]^2 \right|}{|H_{wa} \gamma \rho_w|}$ are the density ratio, the surface tension number and the phase change number, respectively.

The following table recapitulates the order of magnitude for the different dimensionless parameters for both gas and water, considering that the fluid velocities are of the same order of magnitude.

DIMENSIONLESS NUMBERS	FLUID
S_t (STROUHAL)	$S_t \le O(\varepsilon)$
Re (REYNOLDS)	$R_e \le O(\varepsilon)$
Eu (EULER)	$E_u \le O(\varepsilon^{-2})$
C_{ws}	$C_{ws} = O(1)$
CL_{wa1}	
N_ρ	
CL_{wa9}	

Table 3 : Evaluation of dimensionless parameters in order of magnitude of ε.

The Eckert, Peclet and CL_{wa4} numbers will be considered as variable parameters for the following presentation. Furthermore, it is interesting to note that :

$$E_{ca} = O\left[\frac{V_a^2}{C_{pa} T_a} \right] = O\left[\frac{V_w^2}{C_{pw} T_w} \right] = O(E_{cw}) \quad \text{and} \quad P_{ea} = O\left[\frac{\rho_a C_{pa} V_a l_r}{\lambda_a} \right] = O\left[\frac{\rho_w C_{pw} V_w l_r}{\lambda_w} \right] = O(P_{ew})$$

(6) – Synthesis of formal equations and dimensionless parameters.

From the preceding equations and the dimensionless parameter evaluations, the following formal equations may be rewritten where k represents each present phase :

1 – Mass conservation : $\nabla.(\rho_k v_k) = 0$

2 – Momentum equilibrium : $0 = \mu_k(\Delta v_k + \mathrm{grad}(\mathrm{div}\, v_k)) - \varepsilon^{-1}\mathrm{grad}P_k$

3 – Energy conservation : $P_e V_k . \nabla(\rho_k h_k) = -\nabla . g_k + (E_u . P_e . E_c) V_k . \nabla p_k$

4 – Solid-liquid interface :

a) Velocity continuity : $v_w = u'_s$

b) Thermal condition : $T_w = T_s$

c) Energy : $\lambda_w \, \mathbf{grad}\, T_w . \mathbf{N}_w = \lambda_s \, \mathbf{grad}\, T_s . \mathbf{N}_s$

5 - Liquid – gas interface

a) Mass : $\varepsilon \rho_a (v_a - v_i).N_a + \rho_w (v_w - v_i).N_w = 0$

b) Momentum :

$$p_w - p_{sat}(T_i) = -2H_{wa}\gamma\left[\frac{\rho_w}{\rho_w - \varepsilon\rho_a}\right] + \frac{1}{2}CL_{wa4}\left[\rho_w N_w .(v_w - v_i)\right]^2\left[\frac{1}{\varepsilon\rho_a} - \frac{1}{\rho_w}\right]$$

c) Energy :

$$0 = \left[\overset{0}{m}_w\right]\left[(C_{pw}T_w - C_{pa}T_a) + E_c\left[\frac{1}{2}\mathbf{v}_w^2 - \mathbf{v}_w . \mathbf{v}_p\right] - \left[\frac{1}{2}\mathbf{v}_a^2 - \mathbf{v}_a . \mathbf{v}_p\right]\right] + P_e^{-1}(\mathbf{N}_w . \mathbf{g}_w + N_\lambda \mathbf{N}_a . \mathbf{g}_a)$$

d) Thermal condition : $T_{wi} = T_{ai} = T_i$

e) Chemistry : $p_a - p_{sat}(T_i) = -2H_{wa}\gamma\left[\frac{\varepsilon\rho_a}{\rho_w - \varepsilon\rho_a}\right] + \frac{1}{2}CL_{wa4}\left[\rho_w N_w .(v_w - v_i)\right]^2\left[\frac{1}{\rho_w} - \frac{1}{\varepsilon\rho_a}\right]$

f) State equation : $P_k = A_k \rho_k$

5. APPLICATION TO HOT PAPER PRESSING.

5.1 - Problem presentation.

Hot paper pressing is a new technology that brings up the following main technical problems: The temperature of the heated roll may reach temperatures as high as 300 °C. As a result, water phase change is particularly important near the heated roll. Thus, the vapour pressure becomes considerably high, which gives rise to an enhancement of the phenomenon of water expulsion. This phenomenon is favourable to the process but must be fully predicted in order to optimise the technology; The main technical problem occurs at the outlet of the press. In this region, as the applied pressure is considerably reduced, a new violent phase change occurs that may induce a damage of the paper. In this study, we derive mathematical models that allow a better understanding of these problems. Therefore, use of these models would help to improve and optimise this new technology. The mathematical modelling technique can also be used so as to improve the performance of any other technical process that involves porous media.

As presented above, 3 dimensionless independent parameters (phase change, Peclet and Eckert parameters) govern the choice of the macroscopic equation to be selected. We will analyse here only the thermal problem, assuming the mechanical one to be solved. Furthermore, we investigate the following cases : $CL_{wa4} = O(\varepsilon^3$ or ε^2 or ε or 1), $Pe = O(\varepsilon^2$ or ε or 1) and $Ec \le O(\varepsilon^4)$.

5.2 Homogenisation results.

I : $CL_{wa4} = O(1)$

This situation is not homogenisable, that is to say that no equivalent macroscopic description exists. It is therefore necessary to notice that the experimental results can not be extrapolate to another situation, in which the boundary condition type are modified.

II - CL_{wa4} = O (ε^2)

A : Pe = O (ε^2)

In this case, diffusion is the only effect that is obtained at the macroscopic scale. The given effective coefficient presented here is the same for all the II cases.

$$\nabla_X.\left(\lambda^{eff}\nabla_X T\right)=0$$

with $$\lambda_{ij}^{eff}=\frac{1}{|\Omega|}\int_{\Omega_w}\left(\lambda_{w_{ik}}\left(\delta_{jk}+\frac{\partial\chi_{w\,j}}{\partial y_k}\right)\right)d\Omega$$

B : Pe = O (ε)

A convective term appears. We may remark that only the wetting phase appears here.

$$<\mathbf{V}_w>_{\Omega_w}.\nabla_x\left(C_{pw}\rho_w T\right)=-\nabla_x.\left(\lambda^{eff}\nabla_x T\right)$$

C : Pe = O (1)

We obtain in this case, a change phase term, that completes the convective and diffusive ones. The non wetting phase appears also in the convection.

$$\left(<\mathbf{V}_w>_{\Omega w}.\nabla_x\left(C_{pw}\rho_w T\right)+\varepsilon<\mathbf{V}_a>_{\Omega a}.\nabla_x\left(C_{pa}\rho_a T\right)\right)=-\varepsilon\nabla_x.\left\{\lambda_{eff}^{**}\nabla_x T\right\}+\varepsilon C_{pw}T_{\Gamma_{wa}}\frac{\Gamma_{wa}}{|\Omega|}{}^{0}m_w$$

In this study, we have assumed the temperature continuity at interface. Hence, we have : $T_{\Gamma wa} = T$

With in the II-cases :

$$\lambda_{ij}^{eff}=\frac{1}{|\Omega|}\int_{\Omega_w}\left(\lambda_{w_{ik}}\left(\delta_{jk}+\frac{\partial\chi_{w\,j}}{\partial y_k}\right)\right)d\Omega$$

and

$$\lambda_{eff\;ij}^{**}=\frac{1}{|\Omega|}\int_{\Omega_w}\left(\lambda_{w_{ik}}\left(\delta_{jk}+\frac{\partial\chi_{w\,j}}{\partial y_k}\right)-\chi_i V_{w\,j}^{(0)}\right)d\Omega$$

χ_w is defined by a boundary problem. Nevertheless it is not presented here.

III - $CL_{wa4} = O(\varepsilon)$

A : Pe = O (ε^2)

A diffusive term is found :

$$\nabla_X . (\lambda^{eff} \nabla_X T) = 0$$

B : Pe = O (ε)

Convective and phase change terms appears simultaneously. Moreover each term has the same order of magnitude.

$$< \mathbf{V}_w >_{\Omega_w} . \nabla_x (C_{pw} \rho_w T) = -\nabla_x . (\lambda^{eff} \nabla_x T) + \frac{\Gamma_{wa}}{|\Omega|} \overset{0}{m}_w C_{pw} T_{\Gamma_{wa}}$$

C : Pe = O (1)

Phase change, convection and diffusion terms are present. Non wetting phase appears now in the phase change and the convection terms. However its influence is weak (ε coefficient in front of each term).

$$(< \mathbf{V}_w >_{\Omega_w} . \nabla_x (C_{pw} \rho_w T) + \varepsilon < \mathbf{V}_a >_{\Omega_a} . \nabla_x (C_{pa} \rho_a T)) = -\varepsilon \nabla_x . (\lambda^{**}_{eff} \nabla_x T) + \overset{0}{m}_w (C_{pw} - \varepsilon C_{pa}) T_{\Gamma_{wa}} \frac{\Gamma_{wa}}{|\Omega|}$$

IV : $CL_{wa4} = O(\varepsilon^3)$

The results are equivalent to those without any phase change. Indeed, the introduction of the asymptotic development of the phase change term is introduced at a too high order to take place in the macroscopic equation.

6 - CONCLUSION

The main aspects of the homogenisation were presented. Then, the local physical equations that represent the phase change phenomena were written. These were set in a dimensionless form in order to apply the asymptotic expansion technique. Different thermal equations were obtained at the macroscale depending on dimensionless parameters. It is advisable to evaluate these parameter in the studied situation and to consider the advocated equations. A classical equation structure is found in which a source term is obtained directly in the macroscopic equation. This term does not appears at the microscopic description where the phase change takes place, as this phenomena was describe as an interfacial exchange and not as a source term.

Hence the obtained equation and corresponding parameters are dedicated to each technological situation. Moreover these results are valid for any porous media that correspond to the presented hypotheses.

Bibliography :

(1) Kaviany, M. Principles of Heat Transfer in Porous Media, Mechanical Engineering Series, Springler-Verlag (1991).

(2) Bensoussan, A., Lions, J.L., Papanicolaou, G., Asymptotic Analysis for Periodic Structures, North-Holland Publishing Company, Amsterdam (1978).