Macroscopic Modelling of Gas Flow through a Dual Porosity Medium

Pascale Royer, Jean-Louis Auriault

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P. Royer
Ecole des Mines, Paris, France
J. L. Auriault
Laboratoire 'Sols, Solides, Structures', UJF, INPG, CNRS, Grenoble, France

ABSTRACT : This work is concerned with modelling the seepage of gas through a rigid fractured medium. It summarises results obtained using the homogenization method for periodic structures. Thereby, unlike the phenomenological approaches, the macroscopic behaviour is deduced from the physics at the microscopic scales, without any prerequisite. The homogenized model is then compared to the model of Warren and Root for a slightly compressible fluid flow in a fractured porous medium. Finally, a correction of pseudo steady state phenomenological models is proposed.

1 INTRODUCTION

A fractured porous medium is a dual porosity medium, i.e., it consists of two interacting porous systems whose permeabilities are very different. One of the two porous structures is associated with the fractures and the other one with the porous matrix. Modelling such an heterogeneous system turns out to be a difficult task. Nonetheless, the internal disorder repetition allows a large-scale continuous description. In other words, physical processes can be described by means of equations with transfer coefficients that are independent of the macroscopic boundary conditions. Two kinds of approaches may be distinguished: i) Directly macroscopic approaches ; ii) Upscaling methods.

The first investigations were on the basis of phenomenological approaches, i.e. directly macroscopic approaches (Barenblatt et al. 1960), (Barenblatt 1963), (Warren & Root 1963). The first model (Barenblatt et al. 1960) shows an important characteristic of dual porosity systems: the interporosity flow, i.e. the fluid exchange between both constitutive media. In these phenomenological models, a pseudo steady state flow is described. They are based on the conjecture that the interporosity flow occurs in response to the fracture-porous matrix difference in pressure.

On the other hand, homogenization techniques allow the determination of an equivalent macroscopic behaviour by upscaling the local description. By definition, this macroscopically equivalent medium behaves " in average " like the initial heterogeneous medium under a given excitation. The condition required for applying these methods is the separation of scales. Homogenization techniques have already proved to be efficient for modelling fluid flow in porous media. In particular, the homogenization method for periodic structures leads to precise descriptions since no macroscopic prerequisite is required. When looking for a macroscopic equivalent description of fluid flow in fractured porous media, three separated scales whose characteristic lengths are very different may be under consideration : the pore scale, the fracture scale and the macroscopic scale. An innovative three scale homogenization method for periodic structures was established in (Auriault & Boutin 1992), (Auriault & Boutin 1992), (Royer & Auriault 1994), (Royer 1994). Pore-flow and fracture-flow are controlled by Stokes equations. This upscaling method allows the influence of the local effects, i.e. at the pore scale, to be conveyed to the macroscopic level.

The goals of this paper are two-fold: i) To present the model of gas flow in fractured porous media derived via homogenization; ii) To compare the homogenized model with the phenomenological models. Since there is no dual porosity phenomenological model for highly compressible fluid flow, this comparison is carried out for a slightly compressible fluid.

In section 2, the description obtained by the three scale homogenization approach is presented. Attention will be focused on the result itself. For details concerning the derivation of this model, the

reader will be referred, when required, to several former papers. Section 3 relates specifically to comparing the homogenized model with the model of Warren and Root for a slightly compressible fluid. Finally, a correction of the interporosity flow term is proposed, which yields a more accurate description for transient regimes.

2 HOMOGENIZATION OF GAS FLOW THROUGH A FRACTURED POROUS MEDIUM

2.1 *General*

To fit the homogenization method for periodic structures to three scales problems, the medium is assumed to be doubly periodic. There is no loss of generality by introducing the assumption of periodicity (Auriault, 1991). No specific internal geometry is at issue for both periods, the work is aimed towards deriving a general macroscopic model. At the pore scale, consider the medium to be Ω-periodic and its characteristic length to be l. The solid and the pores occupy the domains Ω_s and Ω_p, respectively, and their common boundary is Γ (Fig. 1a). A second periodic structure exists at the fracture level, whose period is Ω' and whose characteristic length is l', such that $l'>>l$. The porous matrix and the fractures occupy the domains Ω'_{sp} and Ω'_f, respectively, and their common boundary is Γ' (Fig. 1b). In a given medium, l and l' are defined but the macroscopic characteristic length, l'', must be chosen such that $l''>>l'$. Therefore, the dual porosity medium exhibits two separations of scales instead of one in the single porosity case:

$l'/l''<<1$ between the fracture scale and the macroscopic level,

$l/l'<<1$ between the pore scale and the fracture scale.

If the first condition is not checked the homogenization cannot be applied because the macroscopic scale and the fracture scale are not separated. If the second one is not checked, it means that the medium is a two scale medium.

Thus, it is assumed that both conditions of separation of scales are satisfied, so that the medium is a three scale medium and the homogenization theory can be applied.

The macroscopic behaviour of such a medium under fluid flow depends upon the relative order of magnitude between both scale ratios. The largest coupling effects between the pore-flow and the fracture-flow appear when the scales are equally separated (Auriault & Boutin 1992), (Auriault & Boutin 1992), (Royer & Auriault 1994), (Royer 1994), i.e. when

$$\frac{l}{l'} = O(\frac{l'}{l''}) = O(\varepsilon) .$$

This is the case under consideration hereafter for the investigation of gas flow in a rigid fractured medium.

2.2 *Local description*

In the pores (Ω_p) and in the fractures (Ω'_f), gas flow is expressed by Navier-Stokes equations. For slow flows, inertial and transient terms of Navier-Stokes equations can be neglected (Auriault et al. 1990). For the sake of simplicity, assume the fluid to be linear. Thus, the gas state equation is a linear relationship between the fluid pressure and the fluid density. Let us assume the system to be initially at rest: fluid velocity is zero-valued and pressure and density are constant (P_0 and ρ_0, respectively). Thus, the problem is governed by the following set of equations with $k=p$ in the pores (Ω_p) and $k=f$ in the fractures (Ω'_f). Fluid pressure and density are $P_0+P'_k$ and $\rho_0+\rho_k$, respectively, where P_k and ρ_k are increments.

$$\mu\Delta\vec{v}_k + (\lambda+\mu)\vec{\nabla}(\vec{\nabla}.\vec{v}_k) - \vec{\nabla}P_k = \vec{0} \qquad (1)$$

$$\frac{\partial\rho_k}{\partial t} + \vec{\nabla}.((\rho_0+\rho_k)\vec{v}_k) = 0 \qquad (2)$$

$$\rho_k = AP_k \quad where\ A = \frac{\rho_0}{P_0}\ is\ a\ constant \qquad (3)$$

$$\vec{v}_p = \vec{0} \quad on \quad \Gamma \qquad (4)$$

$$\vec{v}_f.\vec{n}' = <\vec{v}_p>_\Omega.\vec{n}' = (\frac{1}{|\Omega|}\int_{\Omega_p}\vec{v}_p\, d\Omega).\vec{n}' \quad on\ \Gamma' \qquad (5)$$

$$P_f = P_p \quad on\ \Gamma' \qquad (6)$$

2.3 *Non-dimensional numbers*

Details concerning the estimation of the following non-dimensional numbers are given in (Royer & Auriault 1994), (Royer 1994).

The momentum balance equation (1) introduces:

$$Q_k = \frac{|\vec{\nabla}P_k|}{|\mu\Delta\vec{v}_k + (\lambda+\mu)\vec{\nabla}(\vec{\nabla}.\vec{v}_k)|}, \quad k=p,f.$$

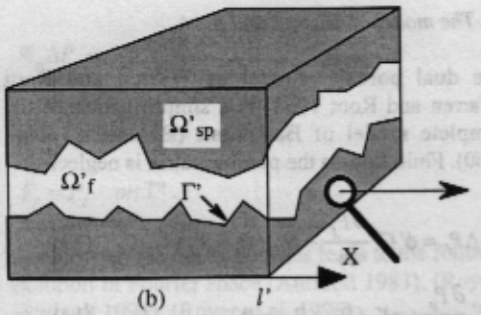


Figure 1: a, fracture scale period, b pore scale period

It can be shown that the homogenization method can be applied if:

$$Q = Q_p = Q_f = O((\frac{l'}{l''})^2) = O(\varepsilon^2) .$$

The Strouhal number is defined from the mass balance equation (2):

$$S_k = \frac{\left|\frac{\partial \rho_k}{\partial t}\right|}{\left|\vec{\nabla}.((\rho_0 + \rho_k)\vec{v}_k)\right|}, \quad k = p, f .$$

The case of interest is that where $S_f = O(1)$. For equally separated scales we get $S_p = O(1)$.

Thus, when cast in non-dimensional form, the governing equations are written as follows, where all quantities are now non-dimensional quantities:

$$\varepsilon^2[\mu\Delta\vec{v}_k + (\lambda+\mu)\vec{\nabla}(\vec{\nabla}.\vec{v}_k)] - \vec{\nabla}P_k = \vec{0} \qquad (7)$$

$$\frac{\partial \rho_k}{\partial t} + \vec{\nabla}.((\rho_0 + \rho_k)\vec{v}_k) = 0 \qquad (8)$$

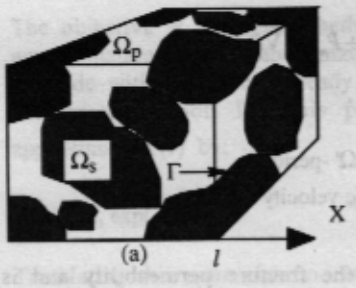
$$\rho_k = AP_k \quad where\ A = \frac{\rho_0}{P_0}\ is\ a\ constant \qquad (9)$$

$$\vec{v}_p = \vec{0} \quad on \quad \Gamma' \qquad (10)$$

$$\vec{v}_f.\vec{n}' = <\vec{v}_p>_\Omega.\vec{n}' = (\frac{1}{|\Omega|}\int_{\Omega_p}\vec{v}_p\, d\Omega).\vec{n}' \quad on\ \Gamma' \qquad (11)$$

$$P_f = P_p \quad on\ \Gamma' \qquad (12)$$

The upscaling process may now be performed. Density, pressure and velocity fields are now looked for in the form of asymptotic expansions in power of ε. Once these expansions have been introduced in the non-dimensional set of equations (7-12), the successive boundary value problems at the different orders of ε must be resolved so as to derive the macroscopic description. Application of homogenization theory to this problem and calculus are detailed in (Royer & Auriault 1994), (Royer 1994). For a general introduction to the homogenization method for periodic structures, the reader is referred to (Auriault 1991).



2.4 *Scaled-up model*

The derived macroscopic behaviour is:

$$\phi'\frac{\partial P_f}{\partial t} + \phi\frac{\partial < P_p >_{eff}}{\partial t} - \vec{\nabla}.[(P_0 + P_f)\vec{\tilde{K}}_f\vec{\nabla}P_f] = 0 \qquad (13)$$

where

$\phi' = \frac{|\Omega'_f|}{|\Omega'|}$ is the fracture porosity,

$\phi = \frac{|\Omega_p|}{|\Omega|}$ is the pore porosity.

The term $\phi\frac{\partial < P_f >_\phi}{\partial t}$ shows the fluid exchange between both porous systems. $< P_f >_\phi$ is defined by:

$$< P_p >_{eff} = \frac{1}{|\Omega'|}\int_{\Omega_{sp}} P_p\, d\Omega ,$$

where P_p is defined by the following boundary value problem over Ω'_{sp}:

$$\phi \frac{\partial P_p}{\partial t} - \vec{\nabla}.[(P_0 + P_p)\vec{K}_p \vec{\nabla} P_p] = 0$$

$$P_p = P_f \quad on\ \Gamma',$$

in which P_p is Ω'-periodic.

The macroscopic velocity is:

$$\vec{V} = -\vec{K}_f \vec{\nabla} P_f$$

where $\vec{K}_f$ is the fracture permeability and is determined from the geometry of Ω'_f.

It turns out that P_p and P_f are such that:

$$P_p = \mathrm{F}(P_f),$$

where F is a non-linear time-dependent functional exhibiting memory effects.

The macroscopic behaviour is strongly influenced by the flow in the pores. It induces memory effects and strong non-linearities. This result highlights how the local effects may affect the macroscopic behaviour.

3 COMPARISON WITH THE MODEL OF WARREN AND ROOT

3.1 *Introduction*

To our knowledge, there is no rigorous phenomenological model for highly compressible fluid flow in fractured porous media. The existing models are derived for slightly compressible fluids. In these models, a pseudo steady state flow is described: it is assumed that the interporosity flow q, i.e. the flux of fluid from matrix to fractures, occurs in response to the fracture-pore difference in pressure:

$$q = sK_p(P_p - P_f)$$

where s is a characteristic coefficient of the fractured rock proportional to the specific surface of the block. In this section, one of these classical models, namely, the model of Warren and Root is compared to the homogenized model. For the purpose of this comparison, the homogenized model is linearised so as to describe the flow of a slightly compressible fluid. The result of this comparison suggests a way to improve the interporosity term in the classical models.

3.2 *The model of Warren and Root*

The dual porosity model of Warren and Root (Warren and Root 1963) is a simplification of the complete model of Barenblatt (Barenblatt et al. 1960). Fluid flow in the porous matrix is neglected:

$$K_f \Delta P_f = \phi' C^* \frac{\partial P_f}{\partial t} - sK_p(P_p - P_f) \qquad (13)$$

$$\phi C^* \frac{\partial P_p}{\partial t} + sK_p(P_p - P_f) = 0 \qquad (14)$$

In Fourier space, the system is written as:

$$K_f \Delta P_f = i\omega\phi' C^* P_f - sK_p(P_p - P_f) \qquad (15)$$

$$i\omega\phi C^* P_p + sK_p(P_p - P_f) = 0 \qquad (16)$$

where ω is the pulsation.

Elimination of P_p between equations (15) and (16) yields:

$$K_f \Delta P_f = i\omega\phi' C^* P_f + \frac{i\omega\phi C^* sK_p}{sK_p + i\omega\phi C^*} P_f \qquad (17)$$

3.3 *Homogenized model for a slightly compressible fluid*

For the comparison to be possible, the model obtained in section 2 must be rewritten for a slightly compressible fluid. The equation of state (3) is replaced by:

$$\rho_o + \rho_k = \rho_o(1 + C^* P_k), \quad k = p, f \qquad (18)$$

where $|C^* P_k| << 1$; and C^* is the compressibility coefficient. Permeabilities are assumed to be constant and isotropic. Then, model (13) is changed to:

$$\phi' C^* \frac{\partial P_f}{\partial t} + \phi C^* \frac{\partial < P_p >_{\sigma f}}{\partial t} - K_f \Delta P_f = 0 \qquad (19)$$

P_p is the solution of the following diffusive equation in Ω'_{sp}:

$$K_p \Delta P_p = \phi C^* \frac{\partial P_p}{\partial t},$$

with the boundary condition:

$$P_p = P_f \quad on\ \Gamma^* .$$

Proceeding by Fourier analysis leads to the following solution in Fourier space (Auriault 1983), (Royer & Auriault 1994), (Royer et al. 1996):

$$P_p = (1-k) P_f$$

where k is complex-valued and ω-dependent.
In Fourier space equation (19) can therefore be written as follows:

$$K_f \Delta P_f = [\phi' C^* + \phi(1-\phi')C^* - \phi C^* < k >_{sff}] i\omega P_f \qquad (20)$$

in which

$$< k >_{sff} = \frac{1}{|\Omega'|} \int_{\Omega'_{sp}} k \, d\Omega .$$

The inverse Fourier transform of equation (20) gives the description for a transient excitation:

$$K_f \Delta P_f = [\phi' C^* + \phi(1-\phi')C^*] \frac{\partial P_f}{\partial t} - \phi C^* \int_{-\infty}^{t} \hat{K}(t-\tau) \frac{\partial^2 P_f}{\partial t^2} d\tau \qquad (21)$$

where $\hat{K}(t)$ is the inverse Fourier transform of $< k >_{sff} / i\omega$ and characterises the memory effects induced by the dual porosity structure of the medium. The convolution product in equation (21) corresponds to the phenomenological interporosity flow. The memory function $\hat{K}(t)$ is similar to that introduced in (Auriault 1983) and taken up again in (Auriault & Royer, 1993) for transient heat transfer in double conductivity composites.

The memory function $\hat{K}(t)$ can be represented by an infinite sum of exponential terms:

$$\hat{K}(t) = \sum_{p=1}^{\infty} a_p \exp(-b_p t), \quad b_1 < b_2 < \ldots .$$

The objective is to check whether a long time approximation of the homogenized model would coincide with the pseudo steady state model of Warren and Root. For this purpose, let us approximate $\hat{K}(t)$ by:

$$\hat{K}(t) = a_1 \exp(-b_1 t) .$$

Translating this approximation in Fourier space, equation (20) becomes:

$$K_f \Delta P_f = C^* [\phi' + \phi(1-\phi') - \phi \frac{a_1 i\omega}{b_1 + i\omega}] i\omega P_f \qquad (22)$$

Thus, equation (22) is an approximation of (20) for low pulsations.

3.4 *Comparison with the model of Warren and Root*

Let us now compare both descriptions in Fourier space. The identification of equations (17) and (22) yields:

$$\frac{\phi C^* s K_p}{s K_p + \phi C^* i\omega} \equiv \phi C^* (1 - \phi' - \frac{a_1 i\omega}{b_1 + i\omega}) \qquad (23)$$

This identification is not possible for all values of ω (Auriault & Royer 1993). Thus, the model of Warren and Root cannot be identified to the long time approximation of the homogenized model. This shows that the model of Warren and Root fails to reproduce transient regimes. This is ascribed to the pseudo steady state approximation, i.e. to the form of the interporosity flow term.

Now let us improve the interporosity flow term to make the model of Warren and Root suitable for transient regimes. Consider the convolution term in equation (21):

$$\int_{-\infty}^{t} \hat{K}(t-\tau) \frac{\partial^2 P_f}{\partial t^2} d\tau$$

Integration by parts of this integral gives an infinite sum of time derivatives of P_f. Therefore, model (21) for transient excitations can be written as follows:

$$K_f \Delta P_f = [\phi' C^* + \phi(1-\phi')C^*] \frac{\partial P_f}{\partial t} - d_2 \frac{\partial^2 P_f}{\partial t^2} - \ldots d_n \frac{\partial^n P_f}{\partial t^n} - \ldots \quad (24)$$

Equation (24) suggests to correct the interporosity term by incorporating a time derivative of P_f :

$$q = sK_p(P_p - P_f - \theta \frac{\partial P_f}{\partial t}), \quad (25)$$

where θ is a constant.

It can be shown that using correction (25), the model of Warren and Root can be identified with the long time approximation of the homogenized model (Auriault & Royer 1993), (Royer & Auriault 1994), (Royer 1994).

4 CONCLUSIONS

Through classical phenomenological approaches, transport phenomena in dual porosity media are directly modelled at the macroscopic scale. Thus, the influence of local heterogeneities on the macroscopic behaviour cannot be perfectly disclosed. Furthermore, phenomena such as flow in the pores or storage capacity in the fractures are often simply discarded.

The homogenization method for periodic structures allows the derivation of the macroscopic behaviour from the complete microscopic description. Since there is no macroscopic prerequisite, the influence of the local effects is conveyed to the macroscopic level. Model (13) constitutes the first model of gas flow through a dual porosity medium that takes the strong compressibility of the fluid into account. Pseudo steady state phenomenological models are shown to be rough approximations for transient excitations in the context of slightly compressible fluid flow. Then, it is proved that adding a fracture pressure time derivative in the interporosity flow term overcomes, to some extent, the limitations of the pseudo steady state approximation.

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