Compressible Fluid Flow in Heterogeneous Media with Interface Flow Resistance

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COMPRESSIBLE FLUID FLOW IN HETEROGENEOUS MEDIA WITH INTERFACE FLOW RESISTANCE

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1. General

This work is concerned with modelling compressible fluid flow in a rigid porous composite medium, in the presence of an interface flow resistance. The temperature is assumed to be constant. The equivalent macroscopic model is obtained by upscaling the description at the heterogeneity scale.

Consider a composite medium which is made of two porous solids with a fine periodic structure. Solids 1 and 2, of permeabilities k_1 and k_2 and porosities ϕ_1 and ϕ_2, occupy simply connected domains Ω_1 and Ω_2, respectively. Γ denotes their common boundary. It is assumed that a flow resistance is acting on the interface Γ. The period, of dimension $O(l)$, is small compared to the macroscopic characteristic length L : $\frac{l}{L}=\varepsilon<<1$. Fluid flow in the composite medium at the local scale is governed by the following equations:

(1) $$\bar{\nabla}.\left[k_1 P_1 \bar{\nabla} P_1\right]=\phi_1 \frac{\partial P_1}{\partial t} \qquad \text{in } \Omega_1$$

(2) $$\bar{\nabla}.\left[k_2 P_2 \bar{\nabla} P_2\right]=\phi_2 \frac{\partial P_2}{\partial t} \qquad \text{in } \Omega_2$$

(3) $$(k_1 P_1 \bar{\nabla} P_1).\bar{n}=(k_2 P_2 \bar{\nabla} P_2).\bar{n} \qquad \text{on } \Gamma$$

(4) $$(k_1 P_1 \bar{\nabla} P_1).\bar{n}_1=\frac{h}{2}(P_2^2-P_1^2) \qquad \text{on } \Gamma$$

P is the pressure, $\bar{n}_1$ is the outward normal to Ω_1 and $h>0$ is the interface flow resistance.

2. Upscaling Method

The goal of this work is to derive a macroscopic model which is equivalent to (1-4). The method is as follows: a. Make the local description dimensionless; b. Evaluate the dimensionless numbers with respect to the scale separation parameter ε; c. Use a perturbation method - the multiple scale expansion method- to obtain the macroscopic model.

The set (1-4) introduces four dimensionless numbers: the permeability ratio A, the Biot's number B, the transient number T and the pressure ratio C

$$A=\frac{k_2}{k_1}, \qquad B=\frac{\left|h(P_2^2-P_1^2)\right|}{\left|(k_1 P_1 \bar{\nabla} P_1).\bar{n}_1\right|}=\frac{hl}{k_1}, \qquad T=\frac{\left|\phi_1 \frac{\partial P_1}{\partial t}\right|}{\left|\bar{\nabla}.\left[k_1 P_1 \bar{\nabla} P_1\right]\right|}=\frac{\phi_1 C l^2}{k_1 \tau P}, \qquad C=\frac{P_1}{P_2}$$

For a system undergoing transient excitations, the condition on T for which an equivalent macroscopic model can be derived is $T=O(\varepsilon^2)$, [1]. Let us limit the study to cases where permeabilities and pressures are of the same order of magnitude: $A=O(1)$, $C=O(1)$. Concerning B, the cases of interest are

$$B = O(\varepsilon^q), q = -1, 0, 1, 2, 3$$

These cases yield different macroscopic models. Clearly, A and B are well determined for a given composite medium. Therefore, their values with respect to ε depend on the macroscopic length L which is related to the excitation. As a consequence, a given medium shows different macroscopic models, depending on the excitation.

The pressure fields may now be looked for in the form of expansions in powers of ε. The macroscopic models are deduced from the existence condition of this asymptotic expansion.

3. Macroscopic models

Note that the set (1-4) is strongly non-linear. However, the upscaling is conducted as in [2] and [3], where linear heat transfer and linear pollutant diffusion are investigated, respectively. The different macroscopic models are summarised in Figure 1.

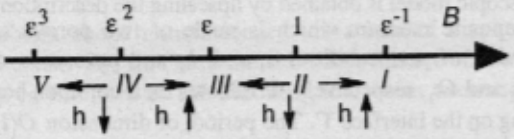

Figure 1. The different macroscopic models

Horizontal arrows show the sole continuous paths between the different models, which are made possible by either increasing or decreasing h. Models *I*, *II* and *III* are one-pressure-field models in the form

$$\bar{\nabla}.\left[K^{\alpha} P \bar{\nabla} P\right] = <\phi> \frac{\partial P}{\partial t}, \alpha = I, II, III$$

$K^{\alpha}, \alpha = I, II, III$, are three different effective permeabilities. They are similar to the effective permeabilities obtained from the linear seepage problem, see [2], [3] where heat and diffusion problems are solved. K^{I} is the effective permeability in absence of contact resistance ($h = \infty$). K^{II} is h-dependent. $K^{III} = K^{1} + K^{2} \neq K^{I}$ is independent of h. $<\phi>$ is the volume average of the porosity.

Models *IV* and *V* are two-pressure-field models. Model *IV* is as follows

$$\bar{\nabla}.\left[K^{1} P_1 \bar{\nabla} P_1\right] = \phi_1 \frac{\partial P_1}{\partial t} + \frac{H}{2}(P_2^2 - P_1^2)$$

$$\bar{\nabla}.\left[K^{2} P_2 \bar{\nabla} P_2\right] = \phi_2 \frac{\partial P_2}{\partial t} - \frac{H}{2}(P_2^2 - P_1^2)$$

H is the average of the interface flow resistance. Model *V* is derived from model *IV* by taking $H = 0$.

4. References

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