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## Sound Absorption of Dry Porous Media with Single and Double Porosity

C. Boutin<sup>1</sup>, P. Royer<sup>2</sup>, J.L. Auriault<sup>2</sup>

### Abstract

This paper is devoted to the modeling of sound absorption of a porous medium saturated by a gas. Homogenization is used to perform the analysis of acoustics in such a medium: the macroscopic equations are obtained from the pore-scale description. Since acoustic wavelengths are far larger than the usual pore-size, this theory is suitable.

The first part of the paper deals with single-porosity materials. As for the second part, double-porosity media, i.e. the case for which also the grains are porous, are under consideration. Both descriptions involve the effects of gas viscosity, inertial forces and heat transfer.

### Introduction and Physical Background

Let us focus on acoustic absorption of porous media such as porous road surfacings, that are, especially in France, commonly used in road engineering. These materials, that present an open porosity ( $n = 0.25$ ), and a large intrinsic permeability ( $k = 10^{-9} \text{ m}^2$ ), are well known for their efficiency to decrease traffic noise [Bar, Delanne, 1993]. This phenomenon is due to the fact that when a given acoustic wave has reached the pervious surface, the air has already been pushed within the pores, so that the wave has only been partially reflected. Therefore, the description of absorption requires the analysis of the physics of gas flow in the pores.

Since the level of acoustic pressure is very weak, the skeleton may be assumed to be perfectly rigid. Furthermore, in acoustics motions are very small perturbations. Thus, the governing equations at the pore-scale are linearized. The thermal gas diffusivity is larger than the solid diffusivity. Consequently, the solid

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can be considered as remaining at the thermal equilibrium, at the first approximation.

The pore-scale equations are given below for harmonic pulsations. For simplicity, the term  $e^{i\omega t}$  is systematically omitted. Variables indexed with  $^e$  are connected to the equilibrium values.  $\mu$ ,  $\zeta$ ,  $\nu$  are the shear, the volume and the kinematic viscosities of the gas, respectively ( $\nu = \mu / \rho^e$ ).  $\lambda$ ,  $C_p$  and  $\alpha$  are the gas thermal conductivity, the specific heat and the diffusivity, respectively ( $\alpha = \lambda / \rho^e C_p$ ).

#### In the gas volume

*Mass balance*  $i\omega\rho + \rho^e \operatorname{div}(\mathbf{v}) = 0$   $\mathbf{v}$  velocity,  $\rho$  density variation.

*State equation*  $p = P^e (\rho/\rho^e + T/T^e)$   $T$ ,  $p$  temperature and pressure variations.

*Navier-Stokes*  $-\operatorname{grad}(p) + \{(\mu + \zeta) \operatorname{grad}(\operatorname{div}(\mathbf{v})) + \mu \Delta(\mathbf{v})\} = \rho^e i\omega\mathbf{v}$

*Heat balance*  $\operatorname{div}(\lambda \operatorname{grad}(T)) = i\omega(\rho^e C_p T - p)$

#### At the gas-solid interface

*Adherence condition*  $\mathbf{v} = 0$

*Temperature continuity*  $T = 0$

Gas motion is governed by the linearized Navier-Stokes equations. On the one hand, it depends on whether inertial transient terms (high frequencies) or viscous terms (low frequencies) are preponderant. The dimensional analysis shows that the determining parameter is the ratio of the viscous-skin thickness ( $\sqrt{\mu / \rho^e \omega}$ ) to the pore-size.

On the other hand, the gas is compressed under the pressure-gradient influence. This compression occurs either isothermally for slow perturbations (low frequencies) or adiabatically for fast perturbations (high frequencies). From the dimensional analysis, it turns out that heat transfer in the gas depends upon the ratio of the thermal-skin thickness ( $\sqrt{\lambda / \rho^e C_p \omega}$ ) to the pore-size.

In the case of air and porous surfacing excited at medium acoustic frequencies ( $\sim 1$  kHz), thermal and viscous layers are both of the order of magnitude of the pore-size.

#### Single-Porosity Medium

The homogenization method is applied to derive the macroscopic acoustic behavior. This technique is now commonly used to solve multiple-scale problems. The existence of an elementary representative volume (ERV) and the separation of scales are the fundamental assumptions that enables us to define a continuum which is equivalent to the heterogeneous medium. It means that if  $l$  and  $L$  are the characteristic size of the ERV and the macroscopic size - here the acoustic wavelength - the following condition must be fulfilled:  $l/L = \epsilon \ll 1$ .

Basically, the homogenization technique is an asymptotic method which is on the basis of the small parameter  $\epsilon$  [Sanchez-Palencia, 1980]. The macroscopic description is given by the first significant order: it is an approximation of the order of  $\epsilon$  of the real behavior.

Due to the lack of space, the developments of this method for solving our problem is not presented here, and we straightly go to the results. Considering the fact that both thermal and viscous layers are of the order of the pore-size, the harmonic linear acoustics of the porous surfacing is governed by the following equations, where  $P$ ,  $V$  and  $\langle \rho \rangle$  are the perturbations of the pressure, the average velocity and the density, respectively, and where  $\gamma$  is the specific heat ratio:

$$\rho^s \operatorname{div} (V) + i\omega n \langle \rho \rangle = 0$$

$$V = -K (f/f_c) \operatorname{grad} (P) / \mu$$

$$\langle \rho \rangle / \rho^s = [\gamma - (\gamma - 1) G (f/f_c)] P / (\gamma P^s)$$

$$f_c = n \mu / \rho^s k$$

$$f_c = \lambda / \rho^s C_p R^2$$

(R: "size" of the pores)

The first equation expresses the bulk mass balance. The second one links the velocity to the air pressure-gradient. It is a dynamic Darcy's law. The tensor  $K$  is complex-valued and depends upon the dimensionless frequency  $f/f_c$ . As for the porous medium saturated by liquid, it can be shown that:

- at low frequencies, i.e. when the viscous effects dominate,  $K$  tends to the real intrinsic permeability  $k$ ;

- at high frequencies, i.e. when the inertial effects dominate,  $K$  is a pure imaginary number with:  $(f/f_c) K/k \rightarrow -i/\tau$ , where  $\tau$  is the "tortuosity" and shows the "added mass".

The last equation corresponds to the "macroscopic state equation". Through the dimensionless frequency  $f/f_c$ , the scalar complex-valued function  $G$  describes the dependence of the thermal exchanges upon the frequency. It can be proved that at low frequencies  $G \rightarrow 0$ , and that at high frequencies  $G \rightarrow 1$ . It leads to the classical isothermal and adiabatical compressibilities, respectively.

Notice that this description includes two dissipative effects: one is due to the viscosity, and the other one is connected to the thermal non-equilibrium.

Sound-absorption properties of porous surfacings are derived from this description. Since the skeleton is assumed to be rigid, acoustic waves are of the  $P_2$  Biot's type. The complex-valued celerity  $C(\omega)$  is obtained by eliminating the velocity and the density between the constitutive equations. Denoting the sound velocity in air by  $C_s = \sqrt{\gamma P^s / \rho^s}$ , we get (in the case of isotropy):

$$C(\omega) = C_s \{(i K/k) (f/f_c) / [\gamma - (\gamma - 1) G]\}^{1/2}$$

At low frequencies ( $f \ll f_c$ ), we get  $C(\omega) \approx [C_s / \sqrt{\gamma}] \sqrt{i f/f_c}$ . Waves are diffusive and their characteristics depend on the intrinsic permeability, the viscosity and the isothermal gas compressibility. These properties correspond to a weak acoustic impedance of the medium, that leads to a large sound absorption.

At high frequencies ( $f \gg f_c$ ), we get  $C(\omega) \approx C_s / \sqrt{\gamma}$ . Waves are propagative. The celerity is linked to the adiabatical gas compressibility and the gas density is corrected by the "tortuosity". This introduces a weak impedance contrast between the outer air and the porous medium.

At medium frequencies, the wave is attenuated by both viscous and thermal effects.

This theoretical approach is in agreement with the phenomenological approaches by [Attenborough, 1982] and [Allard et al., 1993]. These results can be used to compute acoustic absorption of porous layers under incident waves. These simulations are qualitatively and quantitatively in close agreement with the measurements performed on porous materials. They show an acoustic absorption that is restricted to frequencies lower than  $f_c$  [Bar, 1993].

## Double-Porosity Medium

Now, let us assume the grains of the skeleton to be themselves porous (with an open porosity  $n'$ ). The role of this second porosity depends on the size of these micropores. When the micropore-size is of the order of the pore-size, there is no distinction between both porosities. Thus, the single-porosity description is still applicable and the whole system porosity must be considered. Now, when the micropores are strongly smaller than the pores, the grain-permeability is very low. The grains appear to be impervious and the single-porosity description is valid here again, and only the largest pores must be under consideration.

It has been proved in [Royer, Auriault, 1994] and [Auriault, Boutin, 1993-1994] that the most interesting case is obtained when the two scale-ratios - of the pore-size to the wavelength and of the micropore-size to the pore-size - are of the same order of magnitude. Under these conditions, it is clear that within the micropores the viscous effects dominate and that the perturbations are "more isothermal" than they are in the pores.

Since three scales are involved, the theoretical treatment of double porosity systems is more complicated. The reader can find these developments in [Royer, Auriault, 1994] and [Auriault, Boutin, 1993-1994]. Finally, the following description is obtained (the subscripts m and p are related to the micropores and to the pores, respectively):

$$\rho^a \operatorname{div} (V) + i\omega [n \langle \rho_p \rangle + n' (1 - n) \langle \langle \rho_m \rangle \rangle] = 0$$

$$V = -K \operatorname{grad} (P) / \mu$$

$$\langle \rho_p \rangle / \rho^a = [\gamma - (\gamma - 1) G] P / \gamma P^a$$

$$\langle \langle \rho_m \rangle \rangle / \rho^a = [\gamma - (\gamma - 1) G_m] [1 - \Pi] P / P^a$$

The difference with the single-porosity is displayed through the air effective density,  $\langle \langle \rho_m \rangle \rangle$ , which is averaged over the micropore-domain of the ERV. It shows coupling effects between the pore and the micropore gas fluxes, that are due to the diffusion of the pore-pressure in the micropores. This phenomenon is expressed through the scalar complex-valued function  $\Pi$ , which depends on the dimensionless frequency  $f / f_0$ , where  $f_0 = (k / k_p) P^a / \mu$  and where  $f_0 \gg f_0$ . It results in a phase shift between pressure and flux. Consequently, for higher frequencies, new dissipation effects appear at the macroscopic scale. Finally, the function  $G_m$  for the micropores is similar to the function  $G$  for the pores.

In conclusion, these results could be used to elaborate new porous materials with enhanced absorption properties and in a larger range of frequencies.

## References

- ALLARD J.F., HERZOG P., LAFARGE D., TAMURA M. (1993) "Recent topics concerning the acoustics of fibrous and porous materials" *Applied Acoustics*, 39, pp 3-21
- ATTENBOROUGH K. (1982) "Acoustical characteristic of rigid fibrous absorbers and granular media" *J. Acoust. Soc. Am.* 73 (3), pp 785-799
- AURIAULT J.L., BOUTIN C. (1993-1994) "Deformable porous media with double porosity. I-II-III" *T.I.P.M.*, 7 pp 63-82; 10 pp 153-169; 14, pp 143-162
- BAR P., DELANNE Y. (1993) *Réduire le bruit Pneumatique-Chaussées*. LCPC. Presses des Ponts et Chaussées.
- ROYER P., AURIAULT J.L. (1994) "Transient quasi-static gas flow through a rigid porous medium with double porosity" *T.I.P.M.*, 17, 33-57
- SANCHEZ-PALENCIA E. (1980) *Nonhomogeneous media and vibration theory*. Lecture note in Physics, 127, Springer-Verlag, Berlin.