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Design and Tuning of fixed structure PID Controllers A Survey

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Abstract

This paper provides a survey of methods proposed in the literature for the design of fixed structure Proportional Integral Derivative (PID) controllers. The design of single input single output (SISO), Cascade and multiple input multiple output (MIMO) PID controllers is covered. Both linear and non-linear approaches are reviewed.

1. Introduction

When future historians write the history of Engineering in the twentieth century, they certainly indicate that PID controllers were the most popular controllers of the century. Many thousands of Instrument and Control Engineers world-wide are using such controllers in their daily work. According to a survey held in 1977, 34 out of 37 listed industrial analogue controllers were of the PID type. [Seraji, 1983]. The same is true till today and well over ninety percent of existing control loops involve PID controllers [Koivo, 1994; Unar, 1995]. These controllers will well remain dominant in the next century because of their remarkable effectiveness, simplicity of implementation and broad applicability. Although these controllers became commercially available in the 1930s [Ziegler, 1975], interest in their design remains very high even today. Early PID controllers were pneumatic and gained widespread industrial acceptance during the 1940s. Their electronic counterparts entered the market in the 1950s. Over the past thirty years, an enormous amount of effort has been expended in designing these controllers. Hundreds of research papers, a number of M.Phil./Ph.D. theses [e.g. McClusky, 1989; Othmann, 1989; Garcia, 1992; Zhuang, 1992; Barnes, 1994; etc.] and books [e.g. Åström and Hägglund, 1988; Åström, 1995 etc.] have been written on this subject during the period.

Despite these advancements and the popularity of this approach, the design of PID controllers is still a challenge for engineers and researchers. Since the 1940s, many methods for tuning single-loop and multi-loop/multivariable PID controllers have been proposed but every method has some limitations. The purpose of writing this paper is to provide a review of the main methods proposed in literature for the design of a fixed structure PID controller. The paper is organised as follows. Section 2 describes basic concepts of a PID controller. Section 3 presents a review of main methods for the design of a single loop PID controller. In section 4 some other methods of SISO PID design are reviewed briefly. Section 5 gives a survey of methods for cascade PID controller design. Section 6 and 7 cover the design of multi-loop/multivariable PID controllers and section 8 presents some conclusions.

2. PID Controller

The transfer function of a PID controller has the following form:

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s \quad (1)$$

where K_p , K_i and K_d are the proportional, integral and derivative gains respectively. $U(s)$ is the controller output and $E(s)$ is the error signal. Another equivalent form can be obtained as

$$\frac{U(s)}{E(s)} = K_c \cdot \left[1 + sT_d + \frac{T_i}{s} \right] \quad (2)$$

where $T_i = \frac{K_p}{K_i}$ and $T_d = \frac{K_d}{K_p}$ are known as the integral and derivative time constant respectively and $K_c \approx K_p$ is called the controller gain. Some controllers, specially older models have a proportional band setting instead of a controller gain.

$$\text{i.e. } K_c = \frac{100}{PB}$$

where PB is the proportional mode and is defined as, “the percentage error that results in a 100% change in controller output” [Johnson, 1993]. Thus equation (2) can also be written as

$$\frac{U(s)}{E(s)} = \frac{100}{PB} \left[1 + sT_d + \frac{T_i}{s} \right] \quad (3)$$

Although the transfer functions given in equation (1) and (2) are very popular they are impractical. The derivative term causes the gain to increase without bound as frequency goes up. This gives a controller transfer function which is unrealisable in exact form. Practical PID controllers limit this high frequency gain with a first-order lag element (low pass filter). The time constant ϵ of this filter is usually set to about one - tenth of the derivative term. [Dorf, 1991]. Hence, the transfer function of a practical PID controller is given by

$$\frac{U(s)}{E(s)} = \frac{K_p s^2 + K_d s + 1}{s(\epsilon K_d s + 1)} \quad (4)$$

or

$$\frac{U(s)}{E(s)} = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + T_d s}{1 + \epsilon s} \right) \quad (5)$$

Many other structures are also available. [e.g. Gerry, 1988; Kompass, 1989 etc.]

The goal of PID controller design is to determine a set of gains (K_p , K_d , K_i) or (K_c , T_i , T_d) of the structures given above to meet a given set of closed loop system performance requirements. These gains are to be calculated in such a way that the transient response, disturbance rejection and steady-state error specifications are met. Moreover, the robust performance should also be ensured. In practice, it is not possible to achieve all of these goals. For example, if the controller is adjusted to provide better transient response (i.e. minimum overshoot and minimum settling time) to a set point change, it usually results in a sluggish response for disturbances. On the other hand, if control system is made robust by choosing conservative values for the controller gains, this choice may result in sluggish responses to load and set point changes. Many researchers have tried to overcome these difficulties and to

obtain a global design of a PID controller. As a result, many methods for tuning single-loop and multi-loop PID controllers are available in literature. Some of these methods are reviewed in the following sections.

3. Design methods for SISO systems:

This section covers a review of linear SISO PID controller design methods

3.1 Ziegler-Nichols methods:

The Ziegler-Nichols (Z-N) design methods are the most popular heuristic methods used in process control for determining the parameters of a PID controller. Although these methods date back to early 40s for analogue controllers, they are still appropriate for modern digital control systems.

It was the year 1942, when the first method of Ziegler and Nichols was published which is known as the *continuous cycling method*. In this method, the controller gain is increased until a sustained oscillation takes place at gain K_u . If the corresponding period of oscillation be T_u , then the parameters of the PID controller, as suggested by Ziegler and Nichols, can be calculated using Table 1(a). In their second paper, published in 1943, Ziegler and Nichols proposed an alternative method, commonly known as the *process reaction curve method*. In this method, the open loop unit step response of the plant is measured which usually has the form shown in figure (1). The response is approximated by straight lines, with τ_d , T and K indicated as shown. This response can be approximated by the unit step response of a first-order plus time delay (FOPTD) model given in equation (6). Then the rules of Ziegler and Nichols are as shown in Table 1(b).

$$G(s) = \frac{K e^{-\tau_d s}}{Ts + 1} \quad (6)$$

There are two major disadvantages of the Z-N methods: [Friedland, B.; 1996] The resulting closed loop system is often oscillatory than desirable. Another disadvantage is that it is based entirely on the gain and frequency at which the uncompensated system would oscillate in a proportional feedback loop, hence it may not be valid for plants in which the excess of poles over zeros is only two and hence which may have root loci that do not cross the imaginary axis. Many researchers

have tried to overcome these disadvantages of the Z-N methods. For example, Hang et al [1991] suggest some heuristic refinements to the Ziegler-Nichols formula which give some improvements in the performance of a PID controller. The normalised dead time and the normalised gain of the process are utilised to modify the parameters initially set by the Z-N formula. They have also introduced a weighting factor in the proportional term to reduce the overshoot.

Table 1(a): Z-N (Continuous Cycling Method) tuning rules

Controller	Parameters		
	K_c	T_i	T_d
P	$0.5K_u$		
PI	$0.45K_u$	$\frac{1}{1.2}T_u$	
PID	$0.6K_u$	$0.5T_u$	$\frac{1}{8}T_u$

Table 1 (b): Z-N (Process Reaction Curve Method) tuning rules

Controller	Parameters		
	K_c	T_i	T_d
P	$\frac{T}{K\tau_d}$		
PI	$\frac{0.9T}{K\tau_d}$	$\frac{\tau_d}{0.3}$	
PID	$\frac{1.2T}{K\tau_d}$	$2\tau_d$	$0.5\tau_d$

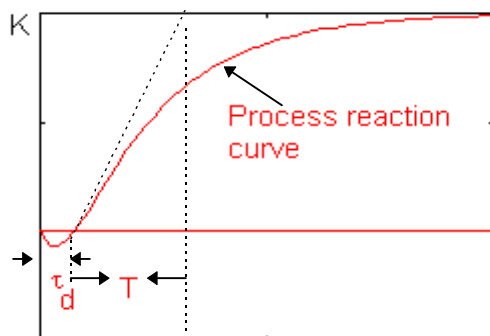


Figure 1: Open loop step response of a

FOPTD model.

3.2 Cohen and Coon (C-C) Method:

In 1953 Cohen and Coon developed design relations for a FOPTD model of equation (6) to provide closed loop responses with a decay ratio of 1/4. The main advantage of the method is its simplicity but the main disadvantage is that the responses are judged to be too oscillatory. The relations are given below:

$$K_c = \frac{T}{K\tau_d} \left[\frac{16T + 3\tau_d}{12T} \right]$$

$$T_i = \tau_d \left[\frac{32 + 6\left(\frac{\tau_d}{T}\right)}{13 + 8\left(\frac{\tau_d}{T}\right)} \right] \quad (7)$$

$$T_d = \frac{4\tau_d}{11 + 2\left(\frac{\tau_d}{T}\right)}$$

3.3 Design Relations based on Integral Error criteria

A powerful approach to develop PID controller design relations is based on a performance index that considers the entire closed loop response. Some popular performance indices are the Integral of the absolute value of the error (IAE), integral of the square error (ISE), integral of time-multiplied square error (ITAE) and the integral of the time weighted absolute error ((ITAE). The optimal settings are obtained by minimising these integrals. It has been shown that the ITAE provides better responses as compared to other criteria (Ogata, 1990). A number of researchers [e.g. Lopez et al, 1967; Murill, 1967; Rovira et al, 1969; Sood et al, 1973; Fertik, 1975 etc.] have derived design relations based on the first-order or second-order plus time delay models by minimising a performance index. A powerful method is developed by Nishikawa et al [1984] who introduce the concept of the weighted ISE as a performance index that is minimised to obtain the optimum settings of a PID controller. The weighting factor is an exponential function of time. The method is very useful in the sense that it is applicable to various kinds of processes such as the self regulating processes, non-self regulating processes and the processes with long dead time. In the recent

years, Zhuang and Atherton [1993] have made remarkable contributions in obtaining optimum settings of a SISO PID controller. They have developed new formulae and a menu driven MATLAB program for determining the parameters of a PID controller for a FOPTD model of equation (6). The Zhuang and Atherton method provides much better performance as compared to many other methods. A limitation of the method is that the plant transfer function must be available and this, ofcourse, is often not accurately known.

3.4 Åström-Hägglund method

The Åström-Hägglund (A-H) method [1984] can be described in terms of the gain margin or phase margin and is based on the idea that a point on the Nyquist curve of the plant can be moved to another position by choosing appropriate PI or PID parameters. They derived the following formulae for computing the parameters of a PID controller.

$$K_c = K_u \cos \Phi$$

$$T_d = \frac{\tan \Phi + \sqrt{\frac{4}{\mu} + \tan^2 \Phi}}{2\omega_c} \quad (8)$$

$$T_i \approx \mu T_d$$

where Φ is the phase margin and μ is a constant.

The main feature of this method is that it does not require knowledge of the plant transfer function but may not perform well in the case of systems having large dead times.

3.5 Internal Model Control

Internal Model Control is a comparatively new design strategy that was introduced in early 1980s. Morari et al [1984], Rivera et al [1986], Chien et al [1990] and Chia et al [1991] have derived PID parameters using the IMC approach. Some of the results obtained by Morari et al are given in Table 2, where η is a design parameter. This is a model based approach that provides a better control performance and offers the following advantages: [Rivera et al, 1986]

- (i) It explicitly takes in to account model uncertainty.
- (ii) It allows the designer to trade off control system performance against control system robustness to produce changes

and modelling errors.

This approach was extended by Zhu and Saucier [1992] for the design of digital PID controllers.

Table 2: IMC based PID controller parameters

Model	K_c	T_i	T_d
$\frac{K}{s}$	$\frac{1}{K\eta}$	-----	-----
$\frac{K}{Ts+1}$	$\frac{1}{K\eta}$	T	-----
$\frac{K}{s(Ts+1)}$	$\frac{1}{K\eta}$	-----	T
$\frac{Ke^{-\tau_d s}}{Ts+1}$	$\frac{2\left(\frac{T}{\tau_d}\right)+1}{K\left[2\left(\frac{T}{\eta}\right)+1\right]}$	$\frac{\tau_d}{2}+1$	$\frac{1}{2\left(\frac{T}{\tau_d}\right)+1}$
$\frac{K}{(T_1s+1)(T_2s+1)}$	$\frac{T_1+T_2}{K\eta}$	T_1+T_2	$\frac{T_1T_2}{T_1+T_2}$
$\frac{K}{T^2s^2+2\zeta Ts+1}$	$\frac{2\zeta T}{K\eta}$	$2\zeta T$	$\frac{T}{2\zeta}$

3.6 Ho-Hang-Cao Method

Recently W.K. Ho, C.C. Hang and L.S. Cao [1995] have developed a useful method for the design of PI/PID controllers based on gain and phase margin specifications. In this method, some simple approximations have been introduced in to the calculations so that neither numerical methods nor graphical methods need to be used. The authors have chosen the FOPTD model of equation (6) for the design of a PI controller and the second order plus time delay model of equation (9) for the design of a PID controller.

$$G(s) = \frac{Ke^{-\tau_d s}}{(T_1s+1)(T_2s+1)} \quad (9)$$

For the FOPTD model, the authors have derived the following PI settings in terms of the gain margin A and phase margin Φ .

$$K_c = \frac{w_p T}{AK}$$

$$T_i = \left(2w_p - \frac{4w_p^2 \tau_d}{\pi} + \frac{1}{T} \right)^{-1} \quad (10)$$

where

$$w_p = \frac{A\Phi + \frac{1}{2}\pi A(A-1)}{(A^2 - 1)\tau_d} \quad (11)$$

and for the second-order plus time delay model (equation (9)), the authors have derived the following PID settings:

$$K_c = \frac{w_p T_1}{AK}$$

$$T_i = \left(2w_p - \frac{4w_p^2 \tau_d}{\pi} + \frac{1}{T_1} \right)^{-1} \quad (12)$$

$$T_d = T_2$$

The performance achieved by this method is comparable to that achieved by the IMC approach.

3.7 Other Methods

The methods described in the previous section are the most popular methods but many other useful methods are also available in literature. It is not possible for the authors to cover all the methods, however, some of these are given below:

Yuwana and Seborg [1982] propose a very useful method for on-line identification and controller tuning based on a single closed loop test. The method is shown to be more powerful and easier to apply than the Z-N approach and many other tuning rules and is further improved by Jutan and Rodriguez [1984], Lee [1989] and Chen [1989]. A comparative study of these four methods can be found in reference [Taiwo, 1993].

McGregor et al [1975] and Issermann [1980] have presented methods for the design of PID controllers based on identification with arbitrary model structure, followed by numerical optimisation of the control parameters.

Some researchers [e.g. Athans, 1971; Williamson et al, 1971; Parker, 1972; Calovic and Cuk, 1974 etc.] have proposed the design

methods by the use of optimal linear quadratic theory.

Wining et al [1977] introduce the so called *sensitivity function approach* to tune the parameters of a PID controller. The approach does not require the model of the controlled system nor does it require information about the process model order. Mannes and Murray-Smith [1987] have successfully applied this technique for the tuning of helicopter flight control systems. Kocijan et al [1995] have extended the approach for the design of MIMO PID controllers.

Murray-Smith [1971] proposes a complex frequency response method for the design of PID controllers.

Phillips and Parr [1984] propose a procedure for designing robust PID and PID predictor digital controller.

Hägglund and Åström [1985] and Persson [1993] propose methods based on Dominant pole design.

Eitelberg [1987] develops a method that covers both the regulating and tracking characteristics of the controller.

Appukattan and Ramer [1991] propose a pole assignment method using a PID output feedback.

Minhu [1992] gives an optimal method for discrete PID controller.

Shing-Gwo wu and Chen [1988] use Pulse width modulation for the analysis and design of PID controller.

Barnes et al [1993] provide a frequency domain design method based on a least squares fit of the actual to the desired open loop Nyquist plot.

Stuckman and Stuckman [1993] propose an iterative global design for systems without models.

Voda and Landua [1995] present an auto-calibration method for PI and PID controllers based on the knowledge of a point of the plant frequency characteristics. The tuning rules are inspired from the symmetrical optimum principles introduced by Kessler.

The design of a fixed structure PID controller for SISO systems is still a hot topic of research, although the new generation controllers usually employ techniques of adaptive/Self tuning and/or Intelligent control.

4. Design of Non-linear PID controllers

Many physical systems are significantly non-linear within the normal operating range or contain some non-linearity. Unfortunately the design of non-linear PID

controllers for non-linear plants have received much less attention and only a few papers have been published so far on this important topic of research. Taylor and Åström [1986] have used the relay autotuning method to design a non-linear PID controller that consists of a non-linearity in series with a linear PID controller. Nanka and Atherton [1990] have applied the approach of the M-circle for the design of a non-linear PID controller. Atherton et al [1993] have presented various strategies for automatic tuning of non-linear PID controllers. These strategies are suitable for non-linear systems where only the gain characteristics or both the gain and phase are affected by the input amplitudes. This is a new field of research and many advances are expected in near future.

5. Design of cascade PID controllers

A cascade control scheme is a multi-loop control scheme commonly used in chemical process control. It is extremely useful when processes are difficult to control due to large disturbances and load changes. In a cascade controller scheme, there are two controllers and two control loops (primary and secondary). Despite the simple structure of the scheme, very little has been reported in the literature on the design of cascade PID controllers.

Brita [1977] and Jha et al [1993] have developed methods and a software package for the optimal design of cascade controllers using error integral criteria as the objective function, which is to be minimised. Hang et al [1994] have proposed a very useful autotuning method based on relay feedback control. In this method, the secondary loop is put on relay feedback first and a P or PI controller is auto tuned using established tuning rules. The primary loop is then placed on relay feedback with the secondary loop closed and the ultimate parameters obtained are then used to auto tune the PI/PID controller for the primary loop. Zhuang and Atherton [1994a] have also developed a very useful method for determining optimum settings automatically. Their method often results in a good closed loop performance with small overshoot and short settling time.

6. Design of MIMO PID controllers

The design of MIMO systems is a real challenge to control system designers because of its complex interactive nature. A MIMO PID controller may be designed either on a single loop basis or by considering the

multivariable system as an entity. In section 6.1 we shall review some most dominant methods for the design of MIMO systems.

6.1 Generalised Ziegler-Nichols Method

In 1971, Neiderlinski extended the Continuous cycling method to multiple input multiple output (MIMO) system. This method is generally known as the *Generalised Ziegler-Nichols method* and is briefly described below:

- (i) Choose n weighting factors c_i ($i = 1, \dots, n$) for the relative control quality of the n controlled variables.
- (ii) Using the best input-output pairing configuration, bring the P-controlled system to the stability limit keeping the following relations between the loop gains:

$$\frac{K_{c,i} G_{ii}(0)}{K_{c,i+1} G_{i+1,i+1}(0)} = \frac{c_i}{c_{i+1}}, \quad i=1, \dots, n-1$$

(13)

where $K_{c,i}$ is the gain of the P controller in the i -th loop and G_{ii} are the diagonal process gains.

- (iii) Determine the critical frequency w_c from the oscillation period T_u and the critical controller gain $K_{u,i}$ for the given system when the oscillation just commences with $K_{c,i}$ at $K_{u,i}$
- (iv) Determine the controller parameters by the Generalised Ziegler-Nichols formulas listed in Table 3 where the choice of the coefficients α_i depends on the ratios

$$\alpha_i = \frac{w_c}{w_{i,c}}$$

where $w_{i,c}$ is the critical frequency of G_{ii} .

- (v) Check whether the control quality is satisfactory. If not, change α_i appropriately and return to step 2.

The disadvantages of the method are the same as those of the Z-N methods of SISO systems. Moreover, it is very difficult (and sometimes impossible) to obtain sustained oscillation with the introduction of proportional control.

Table (3): Generalised Z-N rules

Controller	Parameters		
	K_c	T_i	T_d
P	$\alpha_1 K_{u,i}$		
PI	$\alpha_2 K_{u,i}$	$0.8T_u$	
PID	$\alpha_3 K_{u,i}$	$0.5T_u$	$0.12T_u$

where

$$0.5 \leq \alpha_1 \leq \sqrt{0.5}$$

$$0.45 \leq \alpha_2 \leq \sqrt{0.45}$$

$$0.6 \leq \alpha_3 \leq \sqrt{0.6}$$

6.2 Biggest log Modulus (BLT) Method

This method was proposed by Luyben in 1986. It is very popular among chemical engineers and is described below:

(i) Compute the Z-N tuning parameters of the diagonal elements of the process transfer function matrix, as though the diagonal elements represented single input single output (SISO) systems.

(ii) Choose a factor "F" which is usually between 2 and 5.

(iii) Compute the relative gain K_u and the ultimate period T_u for each loop by means of the relationships

$$K_u = \frac{K_{c-ZN}}{F} \text{ and } T = (T_{i-ZN})F \quad (14)$$

where K_{c-ZN} and T_{i-ZN} are the Ziegler-Nichols PI values.

(iv) Calculate the function $W(S)$ over the appropriate frequency range (i.e. (-1,0) point), where

$$W(S) = -1 + |I + G(j\omega)G_c(j\omega)| \quad (15)$$

where G and G_c denote the plant and controller transfer function respectively.

(v) Compute the function $L(j\omega)$ where

$$L(j\omega) = 20 \log \left| \frac{W(s)}{1 + W(s)} \right| \quad (16)$$

(vi) Adjust "F" until the maximum $L^{\max} = 2n$

For $n = 1$, this reduces to the classical +2dB criterion. For a 3×3 system, a L^{\max} of 6 should be used. This method is easy to use and is easily understandable by control engineers. However, it is limited to open-loop stable systems only and requires good knowledge of the process dynamics. This method was later extended by Monica et al [1988] so as to include derivative action in the controller as well as weighted detuning. In 1990 Basualdo and Marchetti further improved the method using an IMC approach.

6.3 Sequential Design Method

Some authors have also applied SISO Z-N tuning rules to design a multi-loop PID controller by a *Sequential Design Method*. For example, Loh et al [1993] have used relay feedback for designing a PI/PID controller by the Sequential Design Method. The main idea is to tune the multivariable system loop by loop, closing each loop once it is tuned, until all loops are tuned. To tune each loop, a relay feedback configuration is set up to determine the ultimate gain and frequency of the corresponding loop. The Z-N PI/PID rules are then computed on the basis of this information.

The performance achieved by this method is usually as better as that achieved by the BLT method.

6.4 Characteristic Locus Design Method

In 1992, Zhuang extended the Åström-Hägglund method for the design of two input two output (TITO) PID controllers and modified the formulae given in equation (8) as follows:

$$T_{dj} = \frac{\tan \Phi + \sqrt{\frac{4}{\mu} + \tan^2 \Phi}}{2w_c}$$

$$K_{ci} = mK_{u,j} \text{Cos} \Phi \quad (17)$$

$$T_{ii} = \mu T_{dj}$$

where j indicates the controller in loop j . Unar [1995] reports that this method is equally suitable for the design of systems with more than two inputs and more than two outputs. This method gives much better response as compared to many other design methods and does not require knowledge of the process dynamics.

6.5 Zhuang and Atherton's Method of Optimisation

This method provides much better transient response as compared to the generalised Z-N, the characteristic loci and other methods. The method is basically developed for TITO systems [Zhuang et al, 1994b] but can be extended for the design of a general MIMO system. A brief description of the method is as follows:

The error signal of a TITO system can be obtained as

$$E(s) = R(s) - C(s) = [I + G(s)G_c(s)]^{-1}R(s)$$

where $R(s)$ and $C(s)$ are 2×1 input and output vectors respectively, $G(s)$ is a 2×2 transfer function matrix and $G_c(s)$ is the controller

transfer function matrix. Now $E(s)$ can be expressed as

$$E(s) = \begin{bmatrix} 1+G_{11}(s) & G_{12}(s)G_{c2}(s) \\ G_{21}(s)G_{c1}(s) & 1+G_{22}(s)G_{c2}(s) \end{bmatrix}^{-1} R(s) \quad (18)$$

we may obtain optimal PID settings by minimising an integral performance criterion that is a function of some elements in equation (18).

6.6 Seraji's Method

This method is based on a pole placement design strategy and was proposed by Seraji and Tarokh [1977]. The main steps of the method are as under:

(i) Obtain a state-space representation of the system to be controlled in the following form:

$$\begin{aligned} \dot{x} &= Ax + Bu + Ed \\ y &= Cx \end{aligned} \quad (19)$$

where x is the $n \times 1$ state vector, u is the $m \times 1$ control vector, d is the $v \times 1$ disturbance vector and y is the $L \times 1$ output vector.

(ii) Introduce a vector z of L additional state variables for obtaining the augmented system (AA, BB, CC) where

$$AA = \begin{pmatrix} A & 0 \\ 0 & I \end{pmatrix}, \quad BB = \begin{pmatrix} B \\ 0 \end{pmatrix} \quad \text{and}$$

$CC = \begin{pmatrix} C & 0 \\ 0 & I \end{pmatrix}$ should be controllable and observable.

(iii) Make the augmented system cyclic by initial application of the control law $u = u_c + Kz$, where K is any $m \times L$ matrix of full rank L and u_c is the $m \times 1$ control signal to be designed. Now the new system

$$AA1 = \begin{pmatrix} A & B \cdot K \\ -C & 0 \end{pmatrix} \text{ will be cyclic.}$$

(iv) Derive the transfer function of the cyclic augmented system. This transfer function will always be divisible by S .

(v) Assume that the $m \times L$ controller transfer function has the unity-rank structure

$$k(s) = \left(k_c + \frac{t_i}{s} + t_d s \right) \cdot M \quad (20)$$

where M is an arbitrary $1 \times L$ numerical vector specified by the designer. Determine the characteristic polynomial of the closed loop system. Equate it to the desired closed loop

polynomial and find the parameters of the controller $k(s)$.

(vi) calculate the parameters of the required PID controller for the original system as

$$G_c(s) = \frac{K}{s} + k(s) \quad (21)$$

This is a straightforward and computationally fast method for the design of a linear time invariant multivariable system and provides both desirable steady-state performance and specified closed loop pole positions. However, the method has the following limitations:

1. The controller must be of unity rank structure.
2. The method does not give any indication for the selection of the matrices K and M that play an important role in choosing the optimal settings of the controller.

6.7 Other Methods

In addition to the above methods, many other methods are also available in literature. For example, Paraskevopoulos [1980] proposes a method for the design of linear time invariant multivariable PID controllers that only involve output feedback and imposes no constraints upon the structure of the controller matrices. However, it may not successfully place all the roots of the closed loop characteristic equation at a set of predetermined values.

Cutler and Ramaker [1980] have introduced a numerical design technique that makes use of process knowledge and the desired performance trajectories to calculate the multivariable controller structure. The technique is known as the *Dynamic matrix control* and is mainly suitable for off line design.

Thompson [1982] proposes two methods for the design of multivariable PID controller for unidentified plant. The first method is based on the Z-N tuning rules to give approximate settings for the controller. The second method uses the open loop step response data to estimate the controlled closed loop responses.

Melo and Friedly [1992] have developed a method for on-line identification of all open loop transfer functions of a multivariable system. The method provides a non-parametric, frequency response model that is sufficiently accurate for most control system designs.

Palmor et al [1993] present an algorithm for autotuning of decentralised PID controller which fully generalises the SISO auto tuner to MIMO systems.

7. Conclusion

More than 90% of industrial processes are controlled using PID controllers. For designing and tuning these types of controllers, many methods have been proposed and implemented since 1940s. In this paper, we have surveyed some methods for both linear and non-linear PID controllers. In addition to SISO design methods, cascade and MIMO design methods have also been surveyed. Due to space limitations, this survey is limited to the design of fixed structure PID controllers. A separate paper covering design methods of adaptive/self tuning and Intelligent PID controllers is under preparation and will be presented at a later date.

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