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Enhanced MILP approach for maximizing the robustness of paced transfer lines with uncertain parallel tasks and fixed number of machines

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\textbf{Keywords:} transfer lines, uncertainty, robust optimization, stability radius, optimality-based cuts, MILP

\textbf{Context}

We address a complex combinatorial optimization problem arising in the design of transfer lines and aiming to handle uncertain production tasks, whose processing time is likely to vary. The studied transfer line has a fixed number of machines, where each of them is equipped by blocks. Each block is dedicated to execute several production tasks simultaneously. The number of tasks per block is limited, and the tasks are subject to the following non-strict precedence constraints: if task $i$ precedes task $j$, then $j$ can be assigned to the same block as $i$ or to one of the succeeding blocks. The block time is determined by the longest processing time among the tasks assigned to it, and the machine load by the sum of the block time over all the blocks allocated to the machine. Since the considered transfer lines are paced, the load of any machine cannot be greater than a given duration referred to as the cycle time. Thus, the addressed design problem is to assign all the tasks to blocks, and allocate these latter to the machines, determining inside them the order of blocks activation, subject to the mentioned above constraints so as to stand the uncertainty of task processing time as much as possible.

The transfer lines are designed for a long-term period of usage and have no possibility for reconfiguration due to its significant cost. Therefore, the classic stochastic approach is not applicable for handling the uncertain task processing times, because of the absence of reliable historical data on their variability for establishing the corresponding adequate probability distributions. Only a subset of tasks, whose processing time is subject to be uncertain without any additional information, can be identified by experts before the line design stage. As a consequence, for evaluating the designed line configuration in such a situation, a new appropriate approach is needed.

As it was shown in our previous work \cite{2}, dedicated to a similar industrial situation, but for the case of assembly lines, an approach, based on the concept of stability radius, can be considered as a suitable alternative to stochastic one. Thus, usually measured in the two classic metrics $l_1$ and $l_\infty$, the stability radius can be interpreted as the maximal amplitude of variations for uncertain task processing times, which can be supported by the line configuration without violating its admissibility. The definition of stability radius follows well the principle of robustness, since the greater its value, the greater the ability of the intended line configuration to resist against uncertainty. Consequently, seeking a line configuration with the greatest stability radius becomes a natural solution for the presented context.
Proposed approach

In order to solve exactly the mentioned above problem, we firstly construct its compact mixed-integer linear programming (MILP) formulation for each metric of stability radius, $l_1$ and $l_\infty$, and then investigate some useful problem properties permitting to reduce the search space by generating new specific cuts. Namely, based especially on the precedence constraints and the nominal task processing times, it is possible to construct for each task $j$ a set $Q_j$ of blocks able to contain $j$ and for each machine $p$ a set $U_p$ of blocks, known beforehand to be empty. Computing $Q_j$ and $U_p$ in a pre-processing manner reinforces the initial MILP model by taking into account the following constraints:

$$x_{jk} = 0, \quad \forall j \in V, \forall k \notin Q_j,$$
$$y_k = 0, \quad \forall p \in W, \forall k \in U_p,$$

where $V$ is the set of tasks, $W$ is the set of machines, $x_{jk}$ equals 1, if task $j$ is assigned to block $k$ and 0 otherwise, and $y_k$ is 1, if at least one task is assigned to block $k$, 0 otherwise.

The particularity of the studied problem consists in the fact that, based on the same mentioned above problem properties, the availability of a new best feasible solution allows for adjusting the sets $Q_j$ and $U_p$ and, as a consequence, to generate new tighter constraints:

$$x_{jk} = 0, \quad \forall j \in V, \forall k \in Q_j^{l-1} \setminus Q_j^l,$$
$$y_k = 0, \quad \forall p \in W, \forall k \in U_p^{l-1} \setminus U_p^l,$$

where $Q_j^l$ and $U_p^l$ are respectively the states of $Q_j$ and $U_p$ after the detection of the $l$-th new best feasible solution (here, $Q_j^0 = Q_j$ and $U_p^0 = U_p$). The latter constraints can be considered as *optimality-based cuts*, since they do not remove all optimal solutions of the initial MILP, but are capable to exclude from consideration some feasible ones.

The presentation of this work will be dedicated to presenting in details the concept of stability radius, the used problem properties and the realization of optimality-based cuts generation within the framework of commercial solvers such as IBM ILOG CPLEX, for which an initial feasible solution will be provided by an appropriate heuristic method, described in [1]. The performance of the proposed solution technique and the corresponding computational results on the devoted problem instances will be discussed. Potential improvements and suggestions for future work will be given as well.

References
