Very Compressible Fluid Flow through a Porous Medium with Double Porosity

Pascale Royer, Jean-Louis Auriault

VERY COMPRESSIBLE FLUID FLOW THROUGH A POROUS RIGID MEDIUM WITH DOUBLE POROSITY

PASCALE ROYER*, JEAN L. AURIAULT

Laboratoire "Sols, Solides, Structures", Institut de Mécanique de Grenoble,
Université Joseph Fourier, Institut National Polytechnique de Grenoble,
CNRS URA 1511. BP 53 X 38041 Grenoble cedex, France.
* Presently with: Institute of Geotechnics and Hydrotechnics
of the Technical University of Wrocław, Wybrzeże Wyspiańskiego 27,
50-370 Wrocław, Poland.

Streszczenie: Opisano makroskopowy proces quasi-statycznego przepływu w ośrodku zbudowanym ze sztywnej matrycy, którą tworzą szczeliny i pory. Użyto metody homogenizacji, umożliwiającej uzyskanie makroskopowego opisu, gdy znany jest proces w skali zarówno por, jak i szczelin. Opis makroskopowy zależy od stosunku pomiędzy wielkościami charakterystycznymi dla poszczególnych skal: l/l' i l'/l'', gdzie l, l', l'' są długościami charakterystycznymi por, szczelin i ośrodka makroskopowego. W pracy założono, że $l'/l'' = O((l/l')^2)$. Uzyskany opis makroskopowy tworzą równania bilansu masy i klasyczne prawo Darcy'ego. W wyniku przeprowadzonych badań uzyskano dodatkowy człon określający źródło, które tłumaczy rolę zbiornika ściśliwej cieczy w mikroporach rozpatrywanego ośrodka.

Abstract: We investigate the macroscopic quasi-static description of a porous rigid medium with a double porosity constituted by pores and fractures. We use an homogenization method which allows the macroscopic modelling from the description at the pore and the fracture scales. The macroscopic description is strongly dependent upon the ratios between the different scales l/l' and l'/l'' where l, l', l'' are characteristic lengths for the pores, the fractures and the macroscopic medium, respectively. We investigate here the case: $l'/l'' = O((l/l')^2)$. The macroscopic description which is obtained is a mass balance equation with a classical Darcy's law. The interest of the investigation lies in the source term that highlights the role of a compressible fluid reservoir played by the microporous medium.

Резюме: Описан макроскопический процесс кваси-статического протекания в среде, построенной из жесткой матрицы, которую образуют щели и поры. Использован метод гомогенизации, дающий возможность получения макроскопического описания, когда известен процесс в масштабе как пор, так и щелей. Макроскопическое описание зависит от отношения между величинами, характерными для отдельных масштабов: l/l' и l'/l'', где l, l', l'' являются характерными длинами пор, щелей и макроскопической среды. Было предположено, что $l'/l'' = O((l/l')^2)$. Полученное макроскопическое описание составляют уравнения баланса массы и классический закон Дарси. В результате проведенных исследований получен добавочный член, определяющий источник, который выясняет роль резервуара сжимаемой жидкости в микропорах рассматриваемой среды.

Résumé: On détermine la description macroscopique quasi-statique d'un milieu poreux rigide avec double porosité constitué de pores et de fractures. On utilise une méthode d'homogénéisation, laquelle donne le modèle macroscopique à partir de la description aux échelles des pores et des fractures. La description macroscopique dépend des rapports entre les différentes échelles: l/l' et l'/l'' où l, l', l'' sont les longueurs caractéristiques respectivement pour les pores, les fractures et le

milieu macroscopique. On considère ici le cas: $l'/l'' = O((l/l')^2)$. La description macroscopique obtenue est un bilan de masse avec une loi de Darcy classique. L'intérêt de l'étude réside dans le terme source, lequel met en évidence le rôle de réservoir de fluide compressible joué par le milieu micro-poreux.

1. INTRODUCTION

We investigate the macroscopic behaviour of a compressible fluid filtering through a porous matrix with double porosity. One of the porosity systems is constituted of macropores which will be called *fractures*. The second one, of characteristic dimensions much smaller than the first one, is constituted of micropores which will be called *pores*. Therefore, the respective permeabilities K_f and K_p are very different:

$$K_f \gg K_p.$$

This kind of porous structure is of importance in many domains of application, for instance, in petroleum engineering for fractured gas-filled reservoirs or mining engineering for the coal-gas system behaviour. In spite of their more complicated structures than those presented here, the reasoning constitutes a first approach to the understanding of their macroscopic behaviour.

We are going to determine this one by passing from the microscopic scale to the macroscopic one by means of the method of asymptotic developments [1]. The formalism used was described in [2]. If there is separation of scales, medium with random thin structure or with periodic one has similar macroscopic behaviour. We assume also, and the generalization is still valid, that the medium is doubly periodic. At the pore scale, the medium is periodic with the period Ω and the

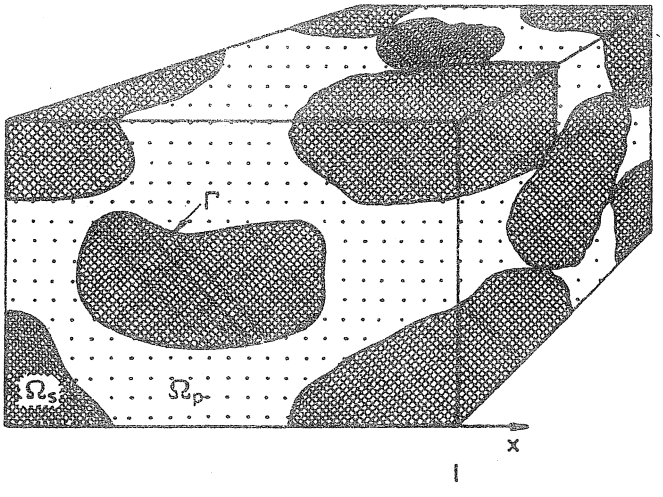

Fig. 1. Microporous medium periodic cell

characteristic length is l. Ω_s is the volume occupied by the solid, while Ω_p is the volume occupied by the pores. Γ is the boundary between them (figure 1). At the fracture scale, the medium has a second porosity with a characteristic length $l' \gg l$, and is periodic with the period Ω'. Ω'_{sp} and Ω'_f are the volumes occupied by the pore matrix and the fractures, respectively (figure 2). The pores and the fractures are assumed to be connected. Finally, $l'' \gg l' \gg l$ denotes the characteristic length of the macroscopic medium. l'' is the gas reservoir, the coal mining seam or the laboratory sample size.

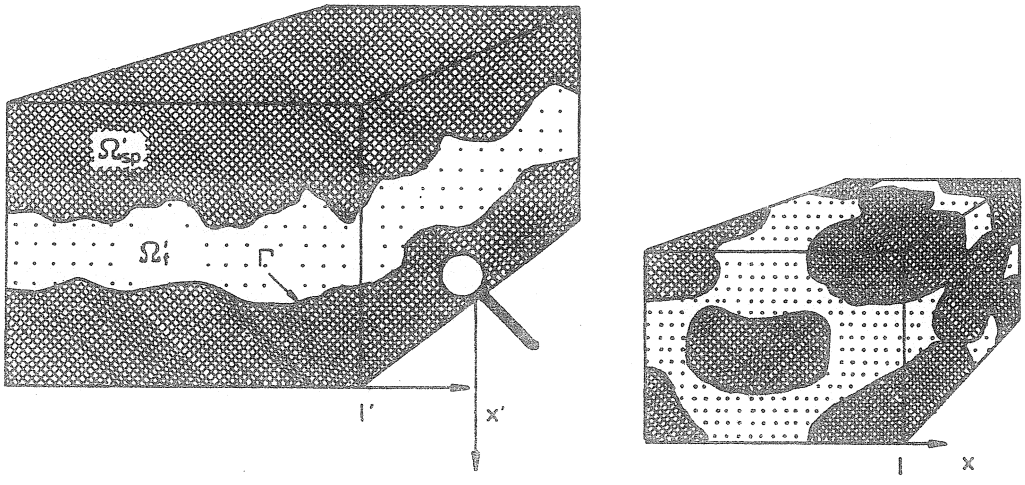

Fig. 2. Fractured medium periodic cell

Such a medium has already been studied [3], [4] in the case of an incompressible fluid and of a deformable matrix. As it has been noted, the macroscopic behaviour is hardly depending on the separations between the three scales which are present in our problem. Similarly as in [3], we limit the study to the case:

$$\frac{l'}{l''} = O\left(\frac{l}{l'}\right)^2 = O(\varepsilon^2).$$

Herein, the homogenization process takes up again, step by step in a new context. In other respects, the flow of a compressible fluid through a deformable porous matrix with a single porosity has been studied in [5]. It is shown there that for slow flows, the local Reynolds number is negligible, inertial transient terms in Navier–Stokes equations must be small, too, so that the phenomenon may be homogenizable, i.e., there is an equivalent macroscopic medium. In this case, convective, transient and inertial terms can be neglected for the derivation of the macroscopic description at the first order. On the other hand, the local Strouhal number introduced by the mass balance equation must be, for the same reasons, in an order of magnitude ε or less.

This squares with a O(1) macroscopic Strouhal number. We use here entirely these results. Inertial terms are also neglected in Navier–Stokes equations.

In the second part of the paper, local equations that describe the flow and estimations required for homogenization are presented. Homogenization is the subject of the third part. The macroscopic description is obtained in the form of mass balance equation with a classical Darcy's law.

2. THE PROBLEM FORMULATION

2.1. EQUATIONS

We consider the small perturbation of an equilibrium state. In the case of a slow flow we have the following equations, with: $\alpha = p$ in the pores and $\alpha = f$ in the fractures.

Momentum balance

λ and μ are viscosities, v is the velocity and P denotes the pressure. According to the previous assumptions, inertial terms have been neglected:

$$\mu \Delta \vec{v}_\alpha + (\lambda + \mu) \vec{\nabla}(\vec{\nabla} \cdot \vec{v}_\alpha) - \vec{\nabla} P_\alpha = \rho_\alpha \left[\frac{\partial \vec{v}_\alpha}{\partial t} + (\vec{v}_\alpha \cdot \vec{\nabla}) \vec{v}_\alpha \right] = \vec{0}. \tag{1}$$

Mass balance

$$\frac{\partial \rho_\alpha}{\partial t} + \vec{\nabla}(\rho_\alpha \vec{v}_\alpha) = 0. \tag{2}$$

Gas state equation

We assume that the fluid is linear.

$$\rho_\alpha = A P_\alpha. \tag{3}$$

Boundary conditions

On Γ:

$$\vec{v}_p/_\Gamma = \vec{0}. \tag{4}$$

On Γ':

$$\vec{v}_j/_{\Gamma'} = \langle \vec{v}_p \rangle_\Omega \quad \text{where } \langle \vec{v}_p \rangle_\Omega = \frac{1}{|\Omega|} \int_{\Omega_p} \vec{v}_p d\Omega, \tag{5}$$

$$P_f = P_p. \tag{6}$$

2.2. DRIVING FORCE

The driving force, which causes the flow, is a macroscopic pressure gradient. Its order of magnitude is:

$$\vec{\nabla} P_a = O\left(\frac{P}{l''}\right)$$

where P is a difference of pressures applied to the macroscopic boundaries. According to (1), the pressure force is balanced by viscous forces:

$$\vec{\nabla} P_a = O(\mu \Delta \vec{v}_a).$$

We deduce the following orders of magnitude:

$$\frac{P}{l''} = O\left(\mu \frac{v_f}{l'^2}\right) = O\left(\mu \frac{v_p}{l^2}\right). \tag{7}$$

2.3. SPACE VARIABLES

Three characteristic length, i.e., l, l', l'', allow the introduction of three dimensionless variables:

$$\frac{\vec{x}''}{l}, \frac{\vec{x}''}{l'}, \frac{\vec{x}''}{l''}$$

which describe the pores, the fractures and the macroscopic medium, respectively. The ratios of the variables are very small. Every physical quantity is also a function of these three variables. We use preferably three physical variables which are as follows:

$$\vec{x} = O\left(\frac{l''}{l}\right)\vec{x}'' \quad \text{pore describing variable},$$

$$\vec{x}' = O\left(\frac{l''}{l'}\right)\vec{x}'' \quad \text{fracture describing variable},$$

$$\vec{x}'' \quad \text{macroscopic variable}.$$

2.4. TIME SCALES AND TIME VARIABLES

The separation of space scales leads to a separation of time scales. In order to highlight them, let us consider two reference displacement (fictious) fields, u_p and u_f for the pores and the fractures, respectively, and let us assume that:

$$\frac{u_p}{u_f} = O(1).$$

We define also the characteristic times by:

$$\vec{v}_p = \frac{\vec{u}_p}{T_p}, \quad \vec{v}_f = \frac{\vec{u}_f}{T_f}. \tag{8}$$

Then, estimation of (7) leads to:

$$\frac{T_f}{T_p} = O\left(\frac{l^2}{l'^2}\right). \tag{9}$$

From the time t, two dimensionless time variables can also be defined:

$$\frac{t}{T_f} \quad \text{for the fractures,}$$

$$\frac{t}{T_p} \quad \text{for the pores.}$$

Again, because of their separation, every quantity is a function of these two times, too. We will use sooner the following variables:

$$t \quad \text{for the fractures,}$$

$$T = O\left(\frac{T_f}{T_p}\right)t \quad \text{for the pores.}$$

Let us note that the previous choices determine an essential role of the variables x'' and t. Therefore we have adopted a macroscopic point of view for the description of the medium [2]. Consequently, dimensionless numbers will be estimated by means of the characteristic length l'' and the characteristic time T_f.

3. DERIVATION OF THE MACROSCOPIC DESCRIPTION

We investigate the following case:

$$\frac{l'}{l''} = O\left(\frac{l}{l'}\right)^2 = O(\varepsilon^2).$$

3.1. SPACE VARIABLES

According to the relationships between the characteristic length, they are itemized as follows:

$\vec{x} = \varepsilon^{-3}\vec{x}''$ is the pore space variable,
$\vec{x}' = \varepsilon^{-2}\vec{x}''$ is the fracture space variable,
$\vec{x}''$ is the macroscopic space variable.

3.2. TIME VARIABLES

t is the fractured medium time variable,
$T = \varepsilon^2 t$ is the microporous medium time variable.

3.3. DIMENSIONLESS NUMBERS

Equations (1)–(6) require some dimensionless numbers. According to momentum balance equation (1), we can introduce:

$$Q_\alpha = \frac{|\vec{\nabla} P_\alpha|}{|\mu \Delta \vec{v}_\alpha|}.$$

Estimation of (7) and use of the characteristic length l'' allow us to obtain:

$$Q = Q_f = Q_p = O(\varepsilon^{-4}).$$

The mass balance equation (2) enables introduction of the Strouhal number:

$$S_\alpha = \frac{\left|\frac{\partial \rho_\alpha}{\partial t}\right|}{|\vec{\nabla}(\rho_\alpha \vec{v}_\alpha)|}.$$

According to the results of [5], we investigate the case where:

$$S = S_f = S_p = O(1).$$

Taking into account these estimations, the equations describing the flow are written in the dimensionless normalized form:

In the pores

$$\varepsilon^4 \mu \Delta \vec{v}_p + \varepsilon^4 (\lambda + \mu) \vec{\nabla}(\vec{\nabla} \cdot \vec{v}_p) - \vec{\nabla} P_p = \vec{0}, \tag{10}$$

$$\frac{\partial \rho_p}{\partial t} + \vec{\nabla} \cdot (\rho_p \vec{v}_p) = 0, \tag{11}$$

$$\rho_p = A P_p, \tag{12}$$

$$\vec{v}_p / _\Gamma = \vec{0}. \tag{13}$$

In the fractures

$$\varepsilon^4 \mu \Delta \vec{v}_f + \varepsilon^4 (\lambda + \mu) \vec{\nabla}(\vec{\nabla} \cdot \vec{v}_f) - \vec{\nabla} P_f = \vec{0}, \tag{14}$$

$$\frac{\partial \rho_f}{\partial t} + \vec{\nabla} \cdot (\rho_f \vec{v}_f) = 0, \tag{15}$$

$$\rho_f = A P_f, \tag{16}$$

$$\vec{v}_f / _{\Gamma'} = \langle \vec{v}_p \rangle_\Omega, \tag{17}$$

$$P_f = P_p \quad \text{on } \Gamma'. \tag{18}$$

3.4. METHOD

Due to the existence of the small parameter ε, we can search for every dimensionless unknown function ϕ_α in the form [3]:

$$\phi_\alpha = \phi_\alpha^0(\vec{x}, \vec{x}', \vec{x}'', t, T) + \varepsilon\phi_\alpha^1(\vec{x}, \vec{x}', \vec{x}'', t, T) + \varepsilon^2\phi_\alpha^2(\vec{x}, \vec{x}', x'', t, T) + \dots \quad (19)$$

in which ϕ_α^i are of an order of 1, ϕ_p^i are Ω-periodic, ϕ_f^i are Ω'-periodic and:

$$\vec{x} = \varepsilon^{-3}\vec{x}'',$$

$$\vec{x}' = \varepsilon^{-2}\vec{x}'',$$

$$T = \varepsilon^2 t.$$

Now the method consists in the introduction of all unknown quantities to the dimensionless equations in the form of such an expansion.

3.5. HOMOGENIZATION OF THE PROBLEM GIVEN

According to (19), pressure, volumetric mass and velocity fields are written in the form of asymptotic expansions. Because of their independence in terms of the microporous medium space variable and of the order of magnitude of the separation of scale between l and l'', the fields relative to the fractures are written in the form of expansions in ε^2 [3]. Once more, the estimation of (7) gives us:

$$\frac{V_p}{v_f} = O\left(\frac{l^2}{l'^2}\right) = O(\varepsilon^2). \quad (20)$$

We then deduce that the expansion of the pore velocity begins in ε^2. The unknown quantities are therefore written as follows:

$$P_p = P_p^0(\vec{x}, \vec{x}', \vec{x}'', t, T) + \varepsilon P_p^1(\vec{x}, \vec{x}', \vec{x}'', t, T) + \varepsilon^2 P_p^2(\vec{x}, \vec{x}', \vec{x}'', t, T) + \dots$$

$$\rho_p = \rho_p^0(\vec{x}, \vec{x}', \vec{x}'', t, T) + \varepsilon\rho_p^1(\vec{x}, \vec{x}', \vec{x}'', t, T) + \varepsilon^2\rho_p^2(\vec{x}, \vec{x}', \vec{x}'', t, T) + \dots$$

$$\vec{v}_p = \varepsilon^2\vec{v}_p^2(\vec{x}, \vec{x}', \vec{x}'', t, T) + \varepsilon^3\vec{v}_p^3(\vec{x}, \vec{x}', \vec{x}'', t, T) + \dots$$

$$P_f = P_f^0(\vec{x}', \vec{x}'', t, T) + \varepsilon^2 P_f^2(\vec{x}', \vec{x}'', t, T) + \dots$$

$$\rho_f = \rho_f^0(\vec{x}', \vec{x}'', t, T) + \varepsilon^2\rho_f^2(\vec{x}', \vec{x}'', t, T) + \dots$$

$$\vec{v}_f = \vec{v}_f^0(\vec{x}', \vec{x}'', t, T) + \varepsilon^2\vec{v}_f^2(\vec{x}', \vec{x}'', t, T) + \dots$$

We have now to introduce these expansions in the dimensionless equations with taking into account the fact that:

$$\frac{\partial}{\partial x_i''} \quad \text{is written} \quad \frac{\partial}{\partial x_i''} + \varepsilon^{-2}\frac{\partial}{\partial x_i'} + \varepsilon^{-3}\frac{\partial}{\partial x_i},$$

$$\frac{\partial}{\partial t} \quad \text{is written} \quad \frac{\partial}{\partial t} + \varepsilon^2\frac{\partial}{\partial T}.$$

In the pores

The first problem: Equation (10) at the order of ε^{-3} and equation (12) at the order of ε^0.

$$\vec{\nabla}_x P_p^0 = \vec{0}, \tag{21}$$

$$\rho_p^0 = A P_p^0. \tag{22}$$

We deduce

$$P_p^0 = P_p^0(\vec{x}', \vec{x}'', t, T), \tag{23}$$

$$\rho_p^0 = \rho_p^0(\vec{x}', \vec{x}'', t, T). \tag{24}$$

The second problem: Equation (10) at the order of ε^{-2} and equation (12) at the order of ε^1.

$$\vec{\nabla}_x P_p^1 + \vec{\nabla}_{x'} P_p^0 = \vec{0}, \tag{25}$$

$$\rho_p^1 = A P_p^1. \tag{26}$$

We deduce

$$\vec{\nabla}_x P_p^1 = \vec{0}, \tag{27}$$

$$\vec{\nabla}_{x'} P_p^0 = \vec{0}. \tag{28}$$

Hence:

$$P_p^0 = P_p^0(\vec{x}'', t, T), \tag{29}$$

$$\rho_p^0 = \rho_p^0(\vec{x}'', t, T), \tag{30}$$

$$P_p^1 = P_p^1(\vec{x}', \vec{x}'', t, T), \tag{31}$$

$$\rho_p^1 = \rho_p^1(\vec{x}', \vec{x}'', t, T). \tag{32}$$

The third problem: Equation (10) at the order of ε^{-1} and equation (12) at the order of ε^2.

$$\vec{\nabla}_x P_p^2 + \vec{\nabla}_{x'} P_p^1 = \vec{0}, \tag{33}$$

$$\rho_p^2 = A P_p^2. \tag{34}$$

In a similar manner, we deduce:

$$P_p^1 = P_p^1(\vec{x}'', t, T), \tag{35}$$

$$\rho_p^1 = \rho_p^1(\vec{x}'', t, T), \tag{36}$$

$$P_p^2 = P_p^2(\vec{x}', \vec{x}'', t, T), \tag{37}$$

$$\rho_p^2 = \rho_p^2(\vec{x}', \vec{x}'', t, T). \tag{38}$$

The fourth problem: Equation (10) at the order of ε^0, equation (11) at the order of ε^{-1} and equation (13) at the order of ε^2.

$$\mu \Delta_x \vec{v}_p^2 + (\lambda + \mu) \vec{\nabla}_x (\vec{\nabla}_x \cdot \vec{v}_p^2) - \vec{\nabla}_x P_p^3 - \vec{\nabla}_{x'} P_p^2 - \vec{\nabla}_{x''} P_p^0 = \vec{0}, \tag{39}$$

$$\vec{\nabla}_x \cdot \vec{v}_p^2 = 0, \tag{40}$$

$$\vec{v}_p^2/_\Gamma = \vec{0}. \tag{41}$$

This is a classical linear boundary problem in terms of the variable x and of the Ω-periodic unknown quantities v_p^2 and P_p^3. The solution is the following:

$$\vec{v}_p^2 = -\tilde{k}_p(\vec{x})(\vec{\nabla}_{x'} P_p^0 + \vec{\nabla}_{x'} P_p^2), \tag{42}$$

$$P_p^3 = \tau_p(\vec{x})(\vec{\nabla}_{x'} P_p^0 + \vec{\nabla}_{x'} P_p^2). \tag{43}$$

The fifth problem: Equation (11) at the order of ε^0.

$$\frac{\partial \rho_p^0}{\partial t} + \vec{\nabla}_{x'} \cdot (\rho_p^0 \vec{v}_p^2) + \vec{\nabla}_x \cdot (\rho_p^0 \vec{v}_p^3 + \rho_p^1 \vec{v}_p^2) = 0. \tag{44}$$

After integration, equation (44) gives a compatibility condition. Taking into account periodicity properties and adherence condition on Γ, we arrive at:

$$\left\langle \frac{\partial \rho_p^0}{\partial t} \right\rangle_\Omega + \vec{\nabla}_{x'} \cdot (\rho_p^0 \langle \vec{v}_p^2 \rangle_\Omega) = 0 \tag{45}$$

in which:

$$\langle \cdot \rangle_\Omega = \frac{1}{|\Omega_{sp}|} \int_{\Omega_p} \cdot \, d\Omega. \tag{46}$$

We then have:

$$n \frac{\partial \rho_p^0}{\partial t} + \vec{\nabla}_{x'} \cdot [\rho_p^0 \langle \vec{v}_p^2 \rangle_\Omega] = 0 \tag{47}$$

in which:

$$n = \frac{|\Omega_p|}{|\Omega_{sp}|} \tag{48}$$

is the microporous medium porosity, and with:

$$\langle \vec{v}_p^2 \rangle_\Omega = -\vec{K}_p(\vec{\nabla}_{x'} P_p^0 + \vec{\nabla}_{x'} P_p^2),$$

$$\vec{K}_p = \langle \tilde{k}_p \rangle_\Omega,$$

$$\rho_p^0 = A P_p^0.$$

The fracture scale equivalent behaviour is also as follows:

$$n \frac{\partial P_p^0}{\partial t} - P_p^0 \vec{\nabla}_{x'} \cdot [\vec{K}_p(\vec{\nabla}_{x'} P_p^0 + \vec{\nabla}_{x'} P_p^2)] = 0. \tag{49}$$

This behaviour is available in Ω_{sp}. Equation (49) describes the fluid behaviour at the

microporous massif scale. The effective Darcy's coefficient and the two pressure gradients are x'- and x''-dependent.

In the fractures

The first problem: Equation (14) at the order of ε^{-2}, equation (16) at the order of ε^0 and equation (18) at the order of ε^0.

$$\vec{\nabla}_{x'} P_f^0 = \vec{0}, \tag{50}$$

$$\rho_f^0 = A P_f^0, \tag{51}$$

$$P_f^0 = P_p^0 \quad \text{on } \Gamma'. \tag{52}$$

Equation (50) leads to:

$$P_f^0 = P_f^0(\vec{x}'', t, T). \tag{53}$$

We deduce:

$$\rho_f^0 = \rho_f^0(\vec{x}'', t, T). \tag{54}$$

Finally equation (49) and boundary condition (52) give the following result:

$$P_p^0 = P_f^0 = P^0(\vec{x}'', t, T). \tag{55}$$

The second problem: Equation (14) at the order of ε^{-1}, equation (15) at the order of ε^{-1} and equation (17) at the order of ε^0.

$$\mu \Delta_{x'} \vec{v}_f^0 - \vec{\nabla}_{x'} P^0 - \vec{\nabla}_{x'} P_f^2 = \vec{0}, \tag{56}$$

$$\vec{\nabla}_{x'} \cdot \vec{v}_f^0 = 0, \tag{57}$$

$$\vec{v}_f^0 / _{\Gamma'} = \prec \vec{v}_p^0 \succ_{\Omega} = \vec{0}. \tag{58}$$

Equations (56), (57), (58) constitute the linear boundary problem in terms of x', which determines the following Ω'-periodic quantities:

$$\vec{v}_f^0 = -\vec{k}_f \vec{\nabla}_{x''} P^0, \tag{59}$$

$$P_f^2 = -\tau_f(\vec{x}') \vec{\nabla}_{x''} P^0 + \bar{P}_f^2(\vec{x}'', t, T). \tag{60}$$

The third problem: Equation (15) at the order of ε^0 and equation (17) at the order of ε^2.

$$\frac{\partial \rho^0}{\partial t} + \vec{\nabla}_{x''} \cdot (\rho^0 \vec{v}_f^0) + \vec{\nabla}_{x'} \cdot (\rho^0 \vec{v}_f^2 + \rho_f^2 \vec{v}_f^0) = 0, \tag{61}$$

$$\vec{v}_f^2 / _{\Gamma'} = \prec \vec{v}_p^2 \succ_{\Omega}. \tag{62}$$

Equation (61) gives the compatibility relationship:

$$\prec \frac{\partial \rho^0}{\partial t} \succ_{\Omega'} + \vec{\nabla}_{x''} \cdot [\rho^0 \prec \vec{v}_f^0 \succ_{\Omega'}] + \prec \vec{\nabla}_{x'} \cdot (\rho^0 \vec{v}_f^2) \succ_{\Omega'} = 0. \tag{63}$$

It can be written once more using (59):

$$n' \frac{\partial \rho^0}{\partial t} + \vec{\nabla}_{x''} \cdot (\rho^0 \tilde{K}_f \vec{\nabla}_{x''} P^0) + \prec \vec{\nabla}_{x'} \cdot (\rho^0 \vec{v}_f^2) \succ_{\Omega'} = 0 \tag{64}$$

in which:

$$n' = \frac{|\Omega'_f|}{|\Omega'|} \tag{65}$$

is the fracture porosity and:

$$\tilde{K}_f = \prec \tilde{k}_f \succ_{\Omega'} \tag{66}$$

with:

$$\prec . \succ_{\Omega'} = \frac{1}{|\Omega'|} \int_{\Omega'_f} . d\Omega . \tag{67}$$

Using boundary condition (62), we obtain:

$$\prec \vec{\nabla}_{x'} \cdot (\rho^0 \vec{v}_f^2) \succ_{\Omega'} = \frac{1}{|\Omega'|} \int_{\Omega'_f} \vec{\nabla}_{x'} \cdot (\rho^0 \vec{v}_f^2) d\Omega = \frac{1}{|\Omega'|} \int_{\Gamma'} \rho^0 \prec \vec{v}_p^2 \succ_{\Omega} \cdot \vec{n}' dS . \tag{68}$$

After integration of (47) on Ω'_{sp}, we obtain the expression researched:

$$\frac{1}{|\Omega'|} \int_{\Gamma'} \rho^0 \prec \vec{v}_p^2 \succ_{\Omega} \cdot \vec{n}' dS = n(1 - n') \frac{\partial \rho^0}{\partial t} . \tag{69}$$

Now, we are able to write the equivalent macroscopic behaviour. Actually, taking into account the fact that:

$$\rho^0 = AP^0$$

and integrating (69) into (64), we obtain:

$$[n' + (1 - n')n] \frac{\partial P^0}{\partial t} - \vec{\nabla}_{x''} \cdot (P^0 \tilde{K}_f \vec{\nabla}_{x''} P^0) = 0 . \tag{70}$$

Equation (70) describes the fluid behaviour at the macroscopic scale.

4. CONCLUSIONS

The macroscopic description of the porous medium suggests few remarks. The flow of the fluid proceeds according to classical Darcy's law for an incompressible fluid. The fluid flow through the pores is ignored because it is negligible in front of flow through the fractures. This fact, that has already been demonstrated in [6], is currently confirmed. The mass balance (70) is nonlinear; this is a classical result. On the other hand, the source term in (70) is unusual. This term is connected to the fluid

compressibility and calls out the fluid in the pores, which are therefore playing the role of a compressible fluid reservoir within the fractures. Finally, let us remind that the behaviour described is the straight consequence of the separation investigated:

$$\frac{l'}{l''} = O\left(\frac{l^2}{l'^2}\right).$$

The entire study of the compressible fluid flow behaviour needs the analysis of the other possibilities as in [4].

REFERENCES

[1] BENSOUSSAN A., LIONS J.L., PAPANICOLAOU G., *Asymptotic analysis for periodic structures*, North-Holland Publishing Company, Amsterdam 1978.

[2] AURIAULT J.L., *Heterogeneous medium. Is an equivalent macroscopic description possible?* Int. Engng. Sci., 29 (1991), No. 7, pp. 785–795.

[3] AURIAULT J.L., BOUTIN C., *Deformable porous media with double porosity. Quasi-statics I: coupling effects*, TIPM 7 (1992), pp. 63–82.

[4] AURIAULT J.L., BOUTIN C., *Deformable porous media with double porosity. Quasi-statics II: coupling effects*, TIMP (to be published).

[5] AURIAULT J.L., STRZELECKI T., BAUER J., HE S., *Porous deformable media saturated by a very compressible fluid: quasi-statics*, Eur. J. Mech. A/Solids 9, No. 4 (1990), pp. 373–392.

[6] LEVY T., *Filtration in a porous fissured rock: influence of the fissure connexity*, Eur. J. Mech. B/Fluids 9, No. 4 (1990), pp. 309–327.