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# Analyzing a Bipolar Decision Structure through Qualitative Decision Theory

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## Abstract

In this paper we study the link between a bipolar decision structure called BLF (bipolar leveled framework) and the qualitative decision theory based on possibility theory. A BLF defines the set of possible decision principles that may be used in order to evaluate the admissibility of a given candidate. A decision principle is a rule that relates some observations about the candidate to a given goal that the selection of this candidate may achieve or miss. The decision principles are ordered according to the importance of the goal they support. Oppositions to decision principles are also described in the BLF under the form of observations that contradict the realization of the decision principle. In order to show that this rich and visual framework is well founded we show how the notions defined in the BLF can be translated in terms of qualitative decision theory.

## 1 Introduction

One of the main challenges in the design of an intelligent system is to produce a tool able to help people to make decisions. As recalled in [13] decision analysis is a process requiring first to formulate the decision goals, then to identify the attributes that characterize the potential alternatives and then decide. A standard way [10] to make this decision is to use a utility function that evaluates the quality of each decision hence that helps to select the one that has the best utility. This utility function should take into account uncertainty and the multi-criteria aspects of the problem.

In the context of optimization under uncertainty, maximizing utility is equivalently seen as minimizing cost, there are three classical criteria [9]: *expected cost minimization*, *risk minimization* and *chance constraints*. *Expected cost minimization* is the most popular approach, it consists in choosing the decision that minimizes the mathematical expectation of the cost. In the discrete case, the mathematical expectation is the sum of the possible costs weighted with their respective probabilities. Considering the decision process that minimizes the mathematical expectation, if we repeat this process on a large enough set of worlds then it will generally tend to have a cost

approaching this expectation. However, applying this method on only one situation can lead to have an effective cost far from the expectation. The two other approaches do not aggregate probability and utility, they manage them separately. *Risk minimization* [9] consists in minimizing the probability that a cost is greater than a threshold and *Chance constraints* approach [5] minimizes the cost that can appear with a given probability level. Those approaches have been extended to the case of possibility distributions and this in a quantitative context. This paper extends the latter approach (optimizing the utility under a given uncertainty threshold) to the case of bipolar qualitative decision problems. Moreover, our proposal shows that *chance constraints* approach is related to defeasible reasoning.

In the decision domain and more generally when dealing with uncertainty, it is convenient to have both a compact and exception tolerant representation of information. It is the case for default rules [11, 12] that allow us to express general principles concisely (i.e., without making explicit all the possible exceptions). They also allow us to derive a conclusion in presence of incomplete knowledge with the ability to revise it when more precise information is known (specificity principle [12]). In our context, the use of default rules is convenient to express general decision principles that may admit exceptions. Moreover, [12] has established that a default rule “generally if a then b” corresponds to a translation in terms of constraints under a possibility distribution such that the worlds where  $a \wedge b$  holds are strictly more possible than those where  $a \wedge \neg b$  holds. This translation together with the use of utility levels will allow us to relate our framework to qualitative decision theory.

In this paper we study a new representation framework for decision making, called Bipolar Leveled Framework (BLF), which was first introduced in [1]. Indeed, it is often the case that people evaluate possible alternatives considering positive and negative aspects separately [4]. The BLF is a bipolar structure that enables the decision maker to *visualize the attributes and goals that are involved in the decision problem, together with their links and their importance levels*. The structure is bipolar in the sense that the goals are either positive (i.e., it is desirable to achieve that goal) or negative (i.e., it leads to a dreaded situation). Informally, a BLF may be viewed as a kind of qualitative utility function with some extra features: 1) the defeasible links between attributes and goals are made explicit into what we call “decision principles”, 2) an opposition to a decision principle, called “inhibitor”, is represented by an arc directed towards it, 3) the importance levels of decision principles are represented by the height of their position in the structure. The aim of this paper is to establish the link between BLFs and utility/uncertainty measures. More precisely, we give a possibilistic reading of a BLF and conversely show how to build a BLF from a standard qualitative decision problem.

Like a utility function, the BLF is a declarative representation with which a set of decisions should be justified. The BLF can be established either by one person or by a group of people in order to achieve a collaborative consensual decision. Moreover as it is often the case in practice, the aim of the decision with a BLF is not necessarily to select a *best* candidate (e.g., having a highest expected utility) but rather to ensure a choice of a *convenient* one, since maybe the best candidate is not convenient at all (if all the candidates are bad). In this paper, the notion of convenience is captured by the introduction of “admissibility statuses”. Then the use of a BLF focuses on deciding whether a candidate is “admissible” or not: this is well adapted for “take it or leave it” decisions when decision makers can accept more than one candidate or when the flow of candidates is continuous over time and they want to stop the selection process at the first acceptable candidate.

## 2 BLF: a structure encoding decision criteria

We consider a set  $\mathcal{C}$  of candidates<sup>1</sup> about which some information is available and two languages  $\mathcal{L}_F$  (a propositional language based on a vocabulary  $\mathcal{V}_F$ ) representing information about some features that are believed to hold for a candidate and  $\mathcal{L}_G$  (another propositional language based on a distinct vocabulary  $\mathcal{V}_G$ ) representing information about the achievement of some goals when a candidate is selected. In the propositional languages used here, the logical connectors “or”, “and”, “not” are denoted respectively by  $\vee$ ,  $\wedge$ , and  $\neg$ . A *literal* is a propositional symbol  $x$  or its negation  $\neg x$ , the set of literals of  $\mathcal{L}_G$  are denoted by  $LIT_G$ . Classical inference, logical equivalence and contradiction are denoted respectively by  $\models$ ,  $\equiv$ ,  $\perp$ . We propose two distinct languages in order to clearly differentiate beliefs (coming from observations) from desires (goals to be achieved when selecting a candidate). In the following we denote by  $K$  a set of formulas representing features that are believed to hold: hence  $K \subseteq \mathcal{L}_F$  is the available information. Using the inference operator  $\models$ , the fact that a formula  $\varphi \in \mathcal{L}_F$  holds<sup>2</sup> in  $K$  is written  $K \models \varphi$ .

The BLF is a structure that contains two kinds of information: decision principles and inhibitors. A decision principle can be viewed as a defeasible reason enabling to reach a conclusion about the achievement of a goal. More precisely, a decision principle is a pair  $(\varphi, g)$ , it represents the default rule meaning that “if the formula  $\varphi$  is believed to hold for a candidate then the goal  $g$  is a priori believed to be achieved by selecting this candidate”:

**Definition 1 (decision principle (DP))** A decision principle  $p$  is a pair  $(\varphi, g) \in \mathcal{L}_F \times LIT_G$ , where  $\varphi$  is the reason and  $g$  the conclusion of  $p$ .  $\mathcal{P}$  denotes the set of decision principles.

Depending on whether the achievement of its goal is wished or dreaded, a decision principle may have either a positive or a negative polarity. Moreover some decision principles are more important than others because their goal is more important. The decision principles are totally ordered accordingly.

**Definition 2 (polarity and importance)** A function  $pol : \mathcal{V}_G \rightarrow \{\oplus, \ominus\}$  gives the polarity of a goal  $g \in \mathcal{V}_G$ , this function is extended to goal literals by  $pol(\neg g) = -pol(g)$  with  $-\oplus = \ominus$  and  $-\ominus = \oplus$ . Decision principles are polarized accordingly:  $pol(\varphi, g) = pol(g)$ . The set of positive and negative goals are abbreviated  $\oplus$  and  $\ominus$  respectively:  $\oplus = \{g \in LIT_G : pol(g) = \oplus\}$  and  $\ominus = \{g \in LIT_G : pol(g) = \ominus\}$ .

$LIT_G$  is totally ordered by the relation  $\preceq$  (“less or equally important than”). Decision principles are ordered accordingly:  $(\varphi, g) \preceq (\psi, g')$  iff  $g \preceq g'$ .

The polarities and the relative importances of the goals in  $\mathcal{V}_G$  are supposed to be given by the decision maker, *e.g.*, he may want to avoid to select an expensive hotel (hence “expensive hotel” can be a negative goal), while selecting a hotel where it is possible to swim can be a positive goal, moreover he may give more importance to swim than to pay less.

A decision principle  $(\varphi, g)$  is a defeasible piece of information because sometimes there may exist some reason  $\varphi'$  to believe that it does not apply in the situation, this reason is called an *inhibitor*.

The fact that  $\varphi'$  inhibits a decision principle  $(\varphi, g)$  is interpreted as follows: “when the decision maker only knows  $\varphi \wedge \varphi'$  then he is no longer certain that  $g$  is achieved”. In that case, the inhibition is represented with an arc towards the decision principle. The decision principles and their inhibitors are supposed to be given by the decision maker. An interpretation in terms of possibility theory is described in Section 3.

We are now in position to define the structure BLF.

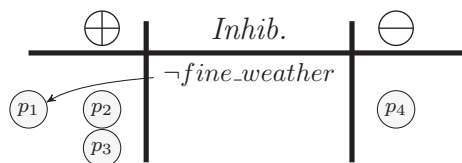
<sup>1</sup>Candidates are also called alternatives in the literature.

<sup>2</sup>The agent’s knowledge  $K$  being considered to be certain, we write “ $\varphi$  holds” instead of “ $\varphi$  is believed to hold”.

**Definition 3 (BLF)** Given a set of goals  $\mathcal{V}_G$ , a BLF is a triplet  $(\mathcal{P}, \mathcal{R}, \text{pol}, \preceq)$  where  $\mathcal{P}$  is a set of decision principles ordered<sup>3</sup> accordingly to their goals by  $\preceq$  and with a polarity built on  $\text{pol}$  as defined in Definition 2,  $\mathcal{R} \subseteq (\mathcal{L}_F \times \mathcal{P})$  is an inhibition relation.

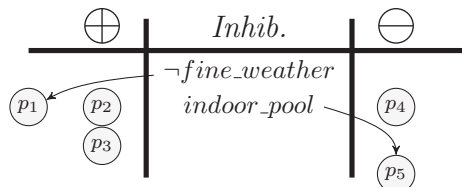
The four elements of the BLF are supposed to be available prior to the decision and to be settled for future decisions as if it was a kind of utility function. A graphical representation of a BLF is given below, it is a tripartite graph represented in three columns, the DPs with a positive level are situated on the left column, the inhibitors are in the middle, and the DPs with a negative polarity are situated on the right. The more important (positive and negative) DPs are in the higher part of the graph, equally important DPs are drawn at the same horizontal level. By convention the highest positive level is at the top left of the figure and the lowest negative level is at the top right. The height of the inhibitors is not significant only their existence is used.

**Example 1** Let us imagine an agent who wants to find an inexpensive hotel in which he can swim. This agent would also be happy to have free drinks but it is less important for him. It means that the set of possible goals is  $\mathcal{V}_G = \{\text{swim}, \text{free\_drinks}, \text{expensive}\}$ , with  $\text{pol}(\text{swim}) = \text{pol}(\text{free\_drinks}) = \oplus$  and  $\text{pol}(\text{expensive}) = \ominus$  and  $\text{swim} \simeq \text{expensive} \prec \text{free\_drinks}$ . The possible pieces of information concern the following attributes:  $\mathcal{V}_F = \{\text{pool}, \text{indoor\_pool}, \text{open\_bar}, \text{four\_star}, \text{fine\_weather}\}$ . The agent considers the following principles:  $\mathcal{P} = \{p_1 = (\text{pool}, \text{swim}), p_2 = (\text{indoor\_pool}, \text{swim}), p_3 = (\text{open\_bar}, \text{free\_drinks}), p_4 = (\text{four\_star}, \text{expensive})\}$ . When the weather is not fine then the fact that there is a pool is not sufficient to ensure that the agent can swim, it means that there is an inhibition on  $p_1$  by  $\neg \text{fine\_weather}$ , i.e.  $\mathcal{R} = \{(\neg \text{fine\_weather}, p_1)\}$ :



Note that  $\neg \text{fine\_weather}$  only inhibits  $p_1$ , even if the conclusion of  $p_1$  is the same as the conclusion of  $p_2$  because with an indoor pool, bad weather is not a problem for swimming.

**Example 2** Let us consider the same BLF in which the decision maker in addition has expressed that  $\neg \text{swim}$  has a negative utility but has lower importance than swim and free\_drinks,  $\text{swim} \simeq \text{expensive} \prec \text{free\_drinks} \prec \neg \text{swim}$ . He has also added a DP saying that if the weather is not fine he will not swim  $p_5 = (\neg \text{fine\_weather}, \neg \text{swim})$ , this DP is inhibited by indoor\_pool:



In this new example, we can see that it is different to have a decision principle with the negation of a goal like  $\neg \text{swim}$  in  $p_5 = (\neg \text{fine\_weather}, \neg \text{swim})$  from having an inhibition  $\neg \text{fine\_weather}$  towards a DP with the goal swim, here  $p_1 = (\text{pool}, \text{swim})$ . In Example 1, the utility of  $\neg \text{swim}$  was not considered while here  $\neg \text{swim}$  has a disutility (at a lower level).

<sup>3</sup>The equivalence relation associated to  $\preceq$  is denoted  $\simeq$  ( $x \simeq y \Leftrightarrow x \preceq y$  and  $y \preceq x$ ) and the strict order is denoted  $\prec$  ( $x \prec y \Leftrightarrow x \preceq y$  and not  $y \preceq x$ ).

In the following, the BLF  $(\mathcal{P}, \mathcal{R}, pod, \preceq)$  is set and we show how it can be used for analyzing the acceptability of a candidate. First, we present the available information and the notion of instantiated BLF, called valid-BLF.

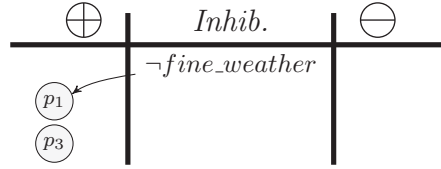
Given a candidate  $c \in \mathcal{C}$ , we consider that the knowledge of the decision maker about  $c$  has been gathered in a knowledge base  $K_c$  with  $K_c \subseteq \mathcal{L}_F$ . Given a formula  $\varphi$  describing a configuration of features ( $\varphi \in \mathcal{L}_F$ ), the decision maker can have three kinds of knowledge about  $c$ :  $\varphi$  holds for candidate  $c$  (i.e.,  $K_c \models \varphi$ ), or not ( $K_c \models \neg\varphi$ ) or the feature  $\varphi$  is unknown for  $c$  ( $K_c \not\models \varphi$  and  $K_c \not\models \neg\varphi$ ). When there is no ambiguity about the candidate  $c$ ,  $K_c$  is denoted  $K$ .

**Definition 4 ( $K$ -valid-BLF)** Given a base  $K$ , a  $K$ -valid-BLF is a quadruplet  $(\mathcal{P}_K, \mathcal{R}_K, pol, \preceq)$  where

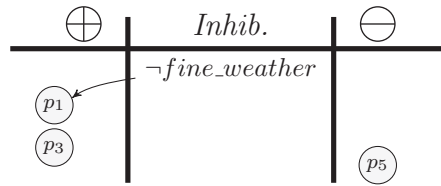
- $\mathcal{P}_K = \{(\varphi, g) \in \mathcal{P}, s.t. K \models \varphi\}$  is the set of DPs in  $\mathcal{P}$  whose reason  $\varphi$  holds in  $K$ , called valid-DPs.
- $\mathcal{R}_K = \{(\varphi, p) \in \mathcal{R}, s.t. K \models \varphi\}$  is the set of valid inhibitions according to  $K$ .

When there is no ambiguity, we simply use “valid-BLF” instead of “ $K$ -valid-BLF”. The validity of a DP only depends on the fact whether the features that constitute its reason  $\varphi$  hold or not, it does not depend on its goal  $g$  since the link between the reasons and the goal is given in the BLF (hence it is no longer questionable).

**Example 1 (continued):** The agent has information about a hotel situated in a place where the weather will not be fine ( $\neg fine\_weather$ ) and that has a pool (reason of  $p_1$ ) and an open bar (reason of  $p_3$ ). The valid-BLF corresponding to what is known about this hotel is:



**Example 2 (continued):** With the same information ( $\neg fine\_weather$ , pool, open\_bar), the valid-BLF corresponding to what is known about the hotel in the case of the BLF of Example 2 is the following:



Now in the valid-BLF the principles that are not inhibited are the ones that are going to be trusted. A goal in  $\mathcal{V}_G$  is said to be “realized” if there is a valid-DP that is not inhibited by any valid-inhibitor.

**Definition 5 (realized goal)** Let  $g$  be a goal in  $LIT_G$ ,  $g$  is realized w.r.t. a  $K$ -valid-BLF  $(\mathcal{P}_K, \mathcal{R}_K, pol, \preceq)$  iff  $\exists(\varphi, g) \in \mathcal{P}_K$  and  $(\varphi, g)$  not inhibited w.r.t.  $\mathcal{R}_K$ .

The set of realized goals is denoted  $\mathbf{R}$ , the positive and negative realized goals are denoted by  $\mathbf{R}^\oplus = \mathbf{R} \cap \oplus$  and  $\mathbf{R}^\ominus = \mathbf{R} \cap \ominus$  respectively.

**Example 3** *In Example 1, the conclusion of  $p_3$  is the only realized goal while in Example 2 both conclusions of  $p_3$  and  $p_5$  are realized. To summarize, the first valid-BLF has one positive realized goal, while the second valid-BLF has a positive and a negative realized goal.*

Note that we have seen two ways to encode a situation: in the first one there is no explicit mentioning of a disutility (*i.e.*, utility to not reach the goal of swimming) while a DP with the negation of the goal swimming is elaborated in the second case. In the first case a positive conclusion may be drawn while in the second case the disutility is taken into account in terms of a negative counterpoint to the other positive aspect (the presence of an open bar).

In Section 4.2 we will explain how to use a BLF in order to make a decision. The decision consists in saying whether or not a candidate is admissible based on the goals that are realized in its corresponding valid-BLF.

### 3 Interpreting BLF principles as possibilistic default rules

In this section we show how the levels of the decision principles of a BLF can be computed when they are viewed as defeasible rules in a possibilistic setting, and we explain how inhibitions can be defined according to this view.

#### 3.1 Background on possibility theory and defaults

The use of possibility theory as a basis for qualitative decision theory was introduced by Dubois and Prade (see [7]). The idea is to define the expected pay-off  $u(x)$  of a situation  $x$ . In this theory, it is supposed that there exists a linear ordering over the situations which gives a preference relation  $\preceq$  over situations s.t.  $x \preceq y$  iff  $u(x) \geq u(y)$ . When situations are not precisely known, the belief state about what is the actual situation is represented by a possibility distribution  $\pi$ . The theory of possibility was introduced by Zadeh [16] and was further developed by Dubois and Prade in [6]. It is well adapted for representing partial ignorance and it is qualitative in the sense that a possibility distribution  $\pi$  defined on a set of situations  $X$  takes its values on a valuation scale  $V$  where max, min and order-reversing operations are defined. However, it is usual to use some numbers for representing this scale without losing the qualitative aspect (since the exact values of the numbers are not meaningful, it is only their order in the scale that is taken into account). The usual convention is to set  $\sup_V = 1$  and  $\inf_V = 0$ . Writing  $\pi(x) \leq \pi(x')$  means that it is at least as plausible for  $x'$  to be the actual situation as for  $x$  to be it.  $\pi(x) = 0$  means that it is impossible that  $x$  is the actual situation, while  $\pi(x) = 1$  means that  $x$  being the actual situation is seen as unsurprising or normal. The state of total ignorance is represented by a possibility distribution where any situation is totally possible ( $\forall x, \pi(x) = 1$ ). The conditional possibility measure denoted  $\Pi(\varphi|\psi)$  is the possibility that  $\varphi$  holds in the worlds where  $\psi$  holds. It is related to the conditional possibility distribution as follows:  $\Pi(\varphi|\psi) = \max_{\omega \models \psi} \pi(\omega|\varphi)$ .

In [8], two possibilistic utility measures are described and justified: the optimistic and pessimistic ones. The authors show that the utility of a decision  $d$  can be evaluated by combining the plausibilities  $\pi(x)$  of the states  $x$  in which  $d$  is made and the utility  $u(d(x))$  of the possible resulting state  $d(x)$  after  $d$ , where  $u(d(x))$  represents the satisfaction to be in the precise situation  $d(x)$  (it is equal to the membership degree to the fuzzy set of preferred situations). The pessimistic criterion has been first introduced by Whalen [14] and leads to a pessimistic utility level of a decision  $d$  defined as follows:  $u_{pes}(d) = \inf_{x \in X} \max(1 - \pi(x), u(d(x)))$ . The optimistic criterion has been first proposed by Yager [15] and is defined by:  $u_{op}(d) = \sup_{x \in X} \min(\pi(x), u(d(x)))$ .

In possibilistic decision theory, the scales for possibilities and utilities are the same, hence, commensurable. In our proposal the commensurability of the two scales is not required: we do



not aggregate possibilities and utilities, we rather use a kind of chance constraint approach in which they are dealt with separately.

We also need to recall some basics about handling defeasible rules in a possibilistic setting. Indeed a decision principle (DP) represents a defeasible reason to believe that some goal is achieved. It is a compact way to express a general rule without mentioning every exception to it. In a BLF the exceptions to a decision principle will be inhibitors. The notion of “valid-BLF” is what makes a BLF suitable for reasoning with incomplete information since what is known validates some DP and inhibitors while if there is no information about some reasons of a particular DP or about an inhibitor then it will not be present in the valid-BLF. Hence it is possible to reason with a BLF even if we have an incomplete description of the world. This is why BLF decision principles are closely related to default rules.

A default rule  $a \rightsquigarrow b$  translates, in the possibility theory framework, into the constraint  $\Pi(a \wedge b) > \Pi(a \wedge \neg b)$  which expresses that having  $b$  true is strictly more possible than having it false when  $a$  is true [12]. Here  $\Pi$  is a possibility measure that evaluates how unsurprising a formula is, where  $\Pi(\varphi) = 0$  means that  $\varphi$  is bound to be false. The necessity measure is its dual defined by  $N(\varphi) = 1 - \Pi(\neg\varphi)$ :  $N(\varphi) = 1$  means that  $\varphi$  is bound to be true. The definition of  $N$  from a possibility distribution  $\pi$  is given by:  $N(\varphi) = \min_{\omega \models \neg\varphi} (1 - \pi(\omega))$ , it expresses that a formula is all the more necessary as its counter models are less plausible. Note that the constraint  $\Pi(a \wedge b) > \Pi(a \wedge \neg b)$  is equivalent to  $N(a \wedge b) > 0$ . Hence, if we know  $a$  and we search for a conclusion which satisfies the constraint  $N() > 0$  then a solution is  $b$ . In this sense, decision principles are related to chance constraints in quantitative optimization problem.

### 3.2 From possibilistic default rules to constraints on inhibitions and on goal importances in a BLF

Let us first denote  $u(g) \in [0, 1]$  the satisfaction degree associated with a goal  $g$  which is an evaluation of how “happy” the decision maker is when the goal  $g$  is achieved ( $u$  is also used to express the utility of a world with  $u(\omega) = \max_{g: \omega \models g} u(g)$ ). The importance of a DP is only defined from the importance of its goal (see Definition 2) hence can be based on the satisfaction degree associated to it. The DP  $(\varphi, g)$  itself is viewed as a default rule  $\varphi \rightsquigarrow g$  and induces some constraints on the possibility measure  $\Pi$  as recalled in Section 3.1.

#### Definition 6 ( $\Pi$ -DP)

Given a possibility measure  $\Pi$ , a  $\Pi$ -DP  $p = (\varphi, g)$  is s.t.  $\Pi(\varphi \wedge g) > \Pi(\varphi \wedge \neg g)$ .

We should now define the inhibition relation corresponding to this interpretation of decision principles. The case that a formula  $\varphi'$  inhibits a DP  $(\varphi, g)$  is translated by the fact that the default rule  $\varphi \rightsquigarrow g$  no longer holds in presence of  $\varphi \wedge \varphi'$ , i.e.,  $N(\varphi \wedge \varphi' \wedge g) = 0$ . Note that, in the selection process, taking into account only the default rules that hold is equivalent to taking into account only the non-inhibited DP. It is in agreement with the notion of realized goal since realized goals should appear in the conclusion of a non-inhibited default rule, in other words, a realized goal is supported by a default rule that holds.

**Definition 7 ( $\Pi$  – inhibition)** Given a possibility measure  $\Pi$ , a relation  $\mathcal{R} \subseteq \mathcal{L}_F \times (\mathcal{L}_F, LIT_G)$  is a  $\Pi$  – inhibition iff for any  $(\varphi', (\varphi, g)) \in \mathcal{R}$ ,

- $\varphi \wedge \varphi' \not\models \perp$  and
- $\Pi(\varphi \wedge \varphi' \wedge \neg g) \geq \Pi(\varphi \wedge \varphi' \wedge g)$ .

### 3.3 Example

In this section we illustrate the construction of a BLF in a context where there are solely three positive goals associated to the decision problem. Note that the process is the same in the case of several positive and negative goals. This process is only presented for explanation purposes, in an application context the BLF is supposed to be directly given by the decision maker.

Let us consider that the language  $\mathcal{L}_F$  contains only three features,  $p$  (for *pool*),  $ip$  (for *indoor pool*),  $fw$  (for *fine weather*), describing possible observations that may be made about a hotel. Note that we suppose that the hotel has at most one swimming-pool either indoor or outdoor, hence  $\neg ip$  means *outdoor pool only* and  $ip$  means *indoor pool only*. We can dispose of a possibility distribution over the possible worlds (here the possible hotels). There are eight possible worlds in which the goal of swimming is either achievable or not. We distinguish three sub goals: *swim outside under sunshine* ( $s1$ ), *swim without being cold when I get out of the water* ( $s2$ ) and *swim anyhow* ( $s3$ ) with the respective utilities  $u(s1) = 1$ ,  $u(s2) = 0.8$ ,  $u(s3) = 0.2$ , meaning that a world where  $s1$  (resp.  $s2$  and  $s3$ ) is achieved has a utility of at least 1 (resp. at least 0.8 and at least 0.2):

World $\omega$	Description	Achieved goals	Utility $u(\omega)$
$\omega_1$	$(p, ip, fw)$	$\{s2, s3\}$	0.8
$\omega_2$	$(p, ip, \neg fw)$	$\{s2, s3\}$	0.8
$\omega_3$	$(p, \neg ip, fw)$	$\{s1, s2, s3\}$	1
$\omega_4$	$(p, \neg ip, \neg fw)$	$\{s3\}$	0.2
$\omega_5$	$(\neg p, ip, fw)$	impossible	/
$\omega_6$	$(\neg p, ip, \neg fw)$	impossible	/
$\omega_7$	$(\neg p, \neg ip, fw)$	$\emptyset$	0
$\omega_8$	$(\neg p, \neg ip, \neg fw)$	$\emptyset$	0

Table 1: Possible worlds with the achieved goals

Note that the worlds  $\omega_5$  and  $\omega_6$  are not possible since it would mean that the hotel has both no pool and an indoor pool. The possibility distribution  $\pi$  over the possible worlds should be in accordance with the following constraints on the possibility measure  $\Pi$  (that are supposed to be given):  $\Pi(ip|p) = 0.5$ ,  $\Pi(\neg ip|p) = 1$ ,  $\Pi(p) = 1$ ,  $\Pi(\neg p) = 0.8$ ,  $\Pi(fw) = 1$ ,  $\Pi(\neg fw) = 0.6$ .

From this information about the preferences and the uncertainty of the worlds we are going to define a BLF. In the first step, we build the decision principles with reasons  $\varphi$  containing only one known feature. In order to do so, we need to compute a possibility distribution  $\pi$  over each world under the knowledge of each feature given the constraints on  $\Pi$ . For instance, if we know that  $p$  holds, *i.e.*, *the hotel has a pool* then the most normal world is  $\omega_3$  ( $\pi(\omega_3) = 1$  and  $\omega_3$  concerns a hotel that *has a pool not in-door* and situated in a place where *the weather is fine*), it is possible but less normal ( $\pi(\omega_4) = 0.6$ ) that *the hotel is in a place where the weather is not fine* ( $\omega_4$ ), the hotel may have *an indoor pool* but it is less natural ( $\pi(\omega_1|p) = 0.5$  and  $\pi(\omega_2|p) = 0.5$ ), the hotels corresponding to  $\omega_7$  and  $\omega_8$  are impossible given that the hotel has a pool. The possibility distribution is entirely given in Table 2.

From Table 1 we can identify 3 importance levels in the BLF since there are 3 different degrees of satisfaction 1, 0.8, 0.2. For each level, we can build the BLF decision principles using Definition 6.

Let us show how to compute the DPs that are the most important (corresponding to a goal of utility 1): only the world  $\omega_3$  has a utility of 1 (achievement of goal  $s1$ ). Moreover Table 1 contains all the formulas  $\varphi$  that can be built from the literals of  $\mathcal{L}_F$  (only conjunctions are considered). Looking at the first line (*i.e.*,  $\varphi = p$ ) we check if  $\pi(\omega_3|p) > \pi(\omega'|p)$ ,  $\forall \omega' \neq \omega_3$ , here it is the case

$\varphi$	$\omega$ $\pi(\omega \varphi)$	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$	$\omega_7$	$\omega_8$
$p$		0.5	0.5	1	0.6		
$\neg p$						1	0.6
$ip$		1	0.6				
$\neg ip$				1	0.6	1	0.6
$fw$		0.5		1		0.8	
$\neg fw$			0.5		1		0.8
$p \wedge \neg ip$				1	0.6		
$p \wedge fw$		0.5		1			
$p \wedge \neg fw$			0.5		1		
$\neg p \wedge \neg ip$						1	0.6
$\neg p \wedge fw$						1	
$\neg p \wedge \neg fw$							1
$ip \wedge fw$		1					
$ip \wedge \neg fw$			1				
$\neg ip \wedge fw$				1		0.8	
$\neg ip \wedge \neg fw$					1		0.8
$p \wedge \neg ip \wedge fw$				1			
$p \wedge \neg ip \wedge \neg fw$					1		

Table 2: Possibility distribution of worlds given  $\varphi$

since  $\pi(\omega_3|p) = 1$  and the most possible world different from  $\omega_3$  is  $\omega_4$  with  $\pi(\omega_4|p) = 0.6$  in this world  $\omega_4$   $s1$  is not achieved. Hence  $\Pi(p \wedge s1) > \Pi(p \wedge \neg s1)$ . Thus we create the  $\Pi$ -DP :  $(p, s1)$  with the greatest importance.  $s1$  is not achievable in any other world than  $\omega_3$  hence we have 2 possible inhibitors, namely  $ip$  and  $\neg fw$ .

Now, the goal  $s1$  (with utility 1) could also be explained by  $\neg ip$  which also holds in  $\omega_3$ . We set  $\varphi = \neg ip$  and check if  $\Pi(\neg ip \wedge s1)$  is strictly greater than  $\Pi(\neg ip \wedge \neg s1)$ , however they are equal. Hence  $(\neg ip, s1)$  is not a  $\Pi$ -DP. With the same method, the reader can check that  $(fw, s1)$  can be created as a  $\Pi$ -DP with the greatest importance, it has two inhibitors  $\neg p$  and  $ip$ .

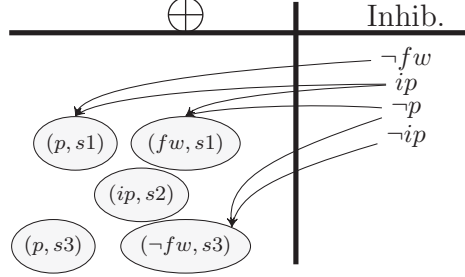
We do the same for the utility level 0.8: the goal  $s2$  is true for the world  $\omega_1, \omega_2$  and  $\omega_3$  hence we obtain two  $\Pi$ -DPs:  $(p, s2)$  and  $(fw, s2)$ . They admit the same inhibitors than  $(p, s1)$  and  $(fw, s1)$ . However it is not necessary to create them since the goal  $s1$  is better than  $s2$ . But considering a utility level of 0.8 leads us to create  $(ip, s2)$  which does not have any inhibitor.

For the last level of utility, the level of  $s3$ , we can define the  $\Pi$ -DP  $(p, s3)$  with no inhibitor (unlike the  $\Pi$ -DP  $(p, s1)$  that has the same reason but two inhibitions). In other words if I only know that  $p$  holds, I have at least a satisfaction of 0.2 and at most of 1. We have also the  $\Pi$ -DP  $(\neg fw, s3)$  inhibited by  $\neg ip$  and  $\neg p$ . The BLF obtained is described below:

This section has brought two main results:

- Each concept of a BLF can be explained w.r.t. possibility distributions and utilities, giving a uni-vocal interpretation to the whole BLF. This could enable a large community of people to use this formalism.
- From a possibility distribution on worlds and utility measures on some goals, we have shown how to build the associated BLF.

The non-inhibited DPs assure that it is strictly more possible to achieve their goal than to fail



them, this is why the decision process described in the next section is based only on the realized goals (*i.e.*, goals that appear in a non-inhibited DP).

## 4 Deciding w.r.t. a BLF

In Section 4.2, we are going to define admissibility notions of candidates based on their corresponding valid BLF. These admissibility notions are useful in the context of “take or leave” decisions where the aim is not to compare all the candidates. It is particularly adapted when we do not have knowledge about the whole set of possible candidates. Nevertheless even if a BLF is not built for comparing candidates directly, it enables us to assign them admissibility statuses which can be compared. In Section 4.3, we are going to study the link between these admissibility statuses and the multi-criteria decision rules of [3] recalled in Section 4.1.

### 4.1 Comparing candidates

We first have to define the levels of goals. We start by attributing levels to the goals starting from the least important ones that are assigned a level 1 and stepping by one each time the importance grows.

**Definition 8 (levels of goals)** *Given a set of goals  $G \subseteq LIT_G$  and the relation  $\preceq$  on  $G$ , the levels of  $G$  are defined by induction:*

- $G_1 = \{g \in G : \nexists g' \in G \text{ s.t. } g' \prec g\}$
- $G_{i+1} = \{g \in G : \nexists g' \in G \setminus (\bigcup_{k=1}^i G_k) \text{ s.t. } g' \prec g\}$

The decision rules defined in [3] by Bonnefon et al. for comparing candidates are recalled below:

**Definition 9 (order of magnitude)** *Given a ranking  $\preceq$  and a set of goals  $G \subseteq LIT_G$  the order of magnitude of  $G$  is  $OM(G) = \max_{g \in G} \{\lambda : g \in G_\lambda\}$  and  $OM(\emptyset) = 0$ .*

**Definition 10 (decision rules of Bonnefon et al.)** *Given two candidates  $c$  and  $c'$  with their associated realized goals  $R$  and  $R'$ , Pareto, Bipolar Possibility and Bipolar Leximin dominance relations (respectively denoted  $\succeq_{Pareto}$ ,  $\succeq_{BiPoss}$  and  $\succeq_{BiLexi}$ ) are defined by:*

- $c \succeq_{Pareto} c'$  iff  $OM(R^\oplus) \geq OM(R'^\oplus)$  and  $OM(R^\ominus) \leq OM(R'^\ominus)$
- $c \succeq_{BiPoss} c'$  iff  $OM(R^\oplus \cup R'^\ominus) \geq OM(R^\ominus \cup R'^\oplus)$

- $c \succeq_{BiLexi} c'$  iff  $|\mathbf{R}_\delta^\oplus| \geq |\mathbf{R}'_\delta^\oplus|$  and  $|\mathbf{R}_\delta^\ominus| \leq |\mathbf{R}'_\delta^\ominus|$   
where  $\delta = \operatorname{argmax}_\lambda \{|\mathbf{R}_\lambda^\oplus| \neq |\mathbf{R}'_\lambda^\oplus| \text{ or } |\mathbf{R}_\lambda^\ominus| \neq |\mathbf{R}'_\lambda^\ominus|\}$

where  $\succeq_r$  stands for “is  $r$ -preferred to”.

As stated in the introduction, the aim of the use of a BLF is more about deciding whether to select a given candidate or not than about comparing two candidates. However we are going to show that our typology is in accordance with the classical bipolar decision rules of Definition 10, in the sense that *e.g.*, a necessarily admissible candidate is preferred or equal to any other candidate w.r.t. Pareto Dominance, Bipolar Possibility relation and Bipolar leximin..

## 4.2 Admissibility statuses of candidates

The admissibility status of a candidate  $c$  is computed from a BLF and a knowledge base  $K_c$  describing what is known about  $c$ , its corresponding  $K_c$ -valid-BLF should be denoted  $\langle \mathcal{P}_{K_c}, \mathcal{R}_{K_c}, \text{pol}, \preceq \rangle$ . However, when there is no ambiguity about the knowledge available,  $\mathcal{P}_{K_c}$  is denoted  $\mathcal{P}$  and  $\mathcal{R}_{K_c}$  is abbreviated to  $\mathcal{R}$ .

**Definition 11 (admissibility status)** *Given a candidate  $c \in \mathcal{C}$ ,  $K_c$  the knowledge about  $c$  and a  $K_c$ -valid-BLF  $\langle \mathcal{P}, \mathcal{R}, \text{pol}, \preceq \rangle$ , we let  $M = OM(\mathbf{R})$ . The status of  $c$  is:*

- necessarily admissible if  $\mathbf{R}_M^\oplus \neq \emptyset$  and  $\mathbf{R}_M^\ominus = \emptyset$
- possibly admissible if  $\mathbf{R}_M^\oplus \neq \emptyset$
- indifferent if  $\mathbf{R} = \emptyset$
- possibly inadmissible if  $\mathbf{R}_M^\ominus \neq \emptyset$
- necessarily inadmissible if  $\mathbf{R}_M^\ominus \neq \emptyset$  and  $\mathbf{R}_M^\oplus = \emptyset$
- controversial if  $\mathbf{R}_M^\oplus \neq \emptyset$  and  $\mathbf{R}_M^\ominus \neq \emptyset$

We respectively denote by  $N_{ad}$ ,  $\Pi_{ad}$ ,  $Id$ ,  $\Pi_{-ad}$ ,  $N_{-ad}$  and  $Ct$  the set of necessarily admissible, possibly admissible, indifferent, possibly inadmissible, necessarily inadmissible and controversial candidates.

In other words, a *necessarily admissible* candidate is supported by positive principles with goals of maximum importance that are realized (*i.e.*, uninhibited) and all the negative goals of the same importance do not hold. A *possibly admissible* candidate has at least one uninhibited positive principle of maximum importance in its favor. An *indifferent candidate*<sup>4</sup> is not concerned by any uninhibited principle (nor positive nor negative), while a *controversial candidate* is both supported and criticized by uninhibited DPs of maximum importance.

**Example 4** *The candidate described by the decision principles given in the valid-BLF of Ex. 1(continued) is necessarily admissible, since at the most important level where there are realized goals (here at level 1) we have a positive realized goal (the DP  $p_3$  concluding `free_drinks`) and no negative realized goal. The agent can admit this hotel because he had not specified that the impossibility to swim is a negative goal. It is also the case for Example 2 since the goal to have `free_drinks` has a higher importance than the negative goal  $\neg\text{swim}$ .*

The definition of admissibility statuses is related to possibility theory [7], where necessarily (resp. possibly) admissible could be understood as it is certain (resp. possible) that the candidate is admissible. The indifferent case is linked to a lack of uninhibited (positive or negative) principles concerning a candidate, thus an impossibility to decide. However it is not related to a standard

<sup>4</sup>Note that the indifference definition uses  $\mathbf{R}$  and not  $\mathbf{R}_M$ .

definition of possibilistic ignorance about the admissibility of a candidate, which rather corresponds to a controversial candidate that is both possibly admissible and inadmissible.

The following proposition<sup>5</sup> concerns the link between the sets of candidates that are necessarily admissible, possibly admissible, controversial, indifferent, possibly inadmissible and necessarily inadmissible :

**Proposition 1 (Inclusion and Duality)**

1.  $N_{ad} = \Pi_{ad} \setminus \Pi_{\neg ad}$  (hence  $N_{ad} \subseteq \Pi_{ad}$ )
2.  $N_{\neg ad} = \Pi_{\neg ad} \setminus \Pi_{ad}$  (hence  $N_{\neg ad} \subseteq \Pi_{\neg ad}$ )
3.  $Ct = \Pi_{ad} \cap \Pi_{\neg ad}$
4.  $Id = \mathcal{C} \setminus (\Pi_{ad} \cup \Pi_{\neg ad})$
5.  $N_{ad} = \mathcal{C} \setminus (\Pi_{\neg ad} \cup Id)$
6.  $N_{\neg ad} = \mathcal{C} \setminus (\Pi_{ad} \cup Id)$ .
7.  $\mathcal{C} = Id \cup \Pi_{ad} \cup \Pi_{\neg ad} = Id \cup Ct \cup N_{ad} \cup N_{\neg ad}$

We define three sets of admissibility:

- $S1 = N_{ad}$ , in this set, the candidates are admissible with no doubt, there are uninhibited principles about the candidates which are all positive.
- Since there are two ways to have doubts about a candidate, namely when she is indifferent ( $Id$ ) or controversial ( $Ct$ ), we define two weaker sets:
  - $S2a = N_{ad} \cup Id$  (i.e.,  $S2a = \mathcal{C} \setminus \Pi_{\neg ad}$ ). In this set, we place candidates of  $S1$  together with those for which no uninhibited principle is available (neither positive nor negative),
  - $S2b = N_{ad} \cup Ct$  (i.e.,  $S2b = \Pi_{ad}$ ). It gathers  $S1$  together with the candidates that are concerned by negative uninhibited principle provided that they are also concerned at least by one positive uninhibited principle.
- $S2a \cup S2b = Id \cup Ct \cup N_{ad}$  (i.e.,  $\Pi_{ad} \cup Id = \mathcal{C} \setminus N_{\neg ad}$ ). It contains also  $S1$ .

### 4.3 From admissibility status to comparison

We show that our admissibility statuses are consistent with the multi-criteria decision rules of [3]. For this purpose we should transform the problem of “take or leave” decision into a problem of classical decision, i.e., it amounts to be able to select the best candidate(s) among a set of candidates. Hence we are going to compare several candidates w.r.t. their admissibility statuses. For this aim, given a BLF, we should compute for each candidate  $c$  its own valid-BLF according to what we know about its features  $K_c$  (see Definition 4).

The following Theorem establishes the well-founded-ness of the admissibility sets  $S1$ ,  $S2a$  and  $S2b$  for the rules *Pareto*, *Biposs* and *BiLexi*. Indeed, these sets represent two levels of preference that are consistent with the classical rules of qualitative decision. It is consistent in the sense that if a candidate has been evaluated as admissible in  $S1$  then a candidate not in this set cannot be strictly preferred to it w.r.t. *Pareto*, *Biposs* or *Bilexi*, it is the same for a candidate in  $S2a$

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<sup>5</sup>All the proofs of this paper can be found in [2].

(and respectively S2b) compared to a candidate outside of this set. Moreover candidates in S2a or S2b cannot be preferred w.r.t. Pareto, Biposs or Bilexi to candidates in S1. Lastly, the sets S2a and S2b are not distinguishable with the classical preference rules since indifferent candidates are equally preferred to controversial ones.

**Theorem 1**

- For all  $c$  in  $Ad$  with  $Ad \in \{S1, S2a, S2b\}$  and for all  $c'$  in  $\mathcal{C} \setminus Ad$ :  $c' \not\prec_r c$ , for all  $r$  in  $\{Pareto, BiPoss, BiLexi\}$
- For all  $c$  in  $S1$  and for all  $c'$  in  $S2a \setminus S1$  or in  $S2b \setminus S1$ :  $c' \not\prec_r c$ , for all  $r$  in  $\{Pareto, BiPoss, BiLexi\}$ .
- $S2a$  and  $S2b$  are indistinguishable w.r.t.  $\{Pareto, BiPoss, BiLexi\}$ .

Note that inside S1, *Pareto*, *Biposs* and *BiLexi* rules can help to refine the selection, *i.e.*, among two necessarily admissible candidates one may be preferred to the other w.r.t. one of those rules.

## 5 Concluding remarks

To sum up, this article has presented a new structure called BLF. A BLF gives a clear definition of candidate admissibility conditions by taking advantage of the efficiency and simplicity of a visual structure. The decision problem about the admissibility of a candidate is defined in terms of the goals that will be achieved by this selection. The set of achieved goals is computed from the decision principles that are applicable for this candidate. Indeed, a decision principle in a BLF relates some attributes to a given goal, it expresses the defeasible knowledge that “selecting a candidate having these attributes can lead to achieve this goal”. An inhibition of a decision principle is an observation that forbids to apply it.

Moreover the decision principles which are given by the designer represent two estimations: on the one hand a confidence level that the decision principle well applies (*i.e.*, is not inhibited by any observation) and on the other hand the importance of its goal. In this paper we focus particularly on the goal levels in order to show the links between a BLF and a symbolic utility measure under uncertainty since we show that a BLF may help to handle the risk (disutility under a threshold of necessity) associated to the decision to select a given candidate.

This study has shown first that it is possible to define a BLF on the basis of possibility distributions about the possible worlds and with utility measures about a given goal. Moreover these results highlights that the BLF has two main benefits:

- The compactness of the representation: The use of default rules is a way to represent compactly information without mentioning exceptions. Note that specificity is embedded in the inhibitions, *e.g.*, it is not necessary to precise that an *indoor\_pool* is a *pool* inside of the DP.
- The handling of incomplete information: It is possible to reason when the value of a given feature is unknown. Moreover, the BLF allows us to discriminate the features that are meaningful w.r.t. the decision, *e.g.*, in the case where we only know that the hotel has a pool we can already make a decision. Note that with more information about the *indoor\_pool*, a more robust decision can be made.

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