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# Non-Abelian adiabatic geometric transformations in a cold Strontium gas

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Topology, geometry, and gauge fields play key roles in quantum physics as exemplified by fundamental phenomena such as the Aharonov-Bohm effect, the integer quantum Hall effect, the spin Hall, and topological insulators. The concept of topological protection has also become a salient ingredient in many schemes for quantum information processing and fault-tolerant quantum computation. The physical properties of such systems crucially depend on the symmetry group of the underlying holonomy. We study here a laser-cooled gas of strontium atoms coupled to laser fields through a 4-level resonant tripod scheme. By cycling the relative phases of the tripod beams, we realize non-Abelian  $SU(2)$  geometrical transformations acting on the dark-states of the system and demonstrate their non-Abelian character. We also reveal how the gauge field imprinted on the atoms impact their internal state dynamics. It leads to a new thermometry method based on the interferometric displacement of atoms in the tripod beams.

*Introduction.* In 1984, M. V. Berry published the remarkable discovery that cyclic parallel transport of quantum states causes the appearance of geometrical phases factors [1]. His discovery, along with precursor works [2, 3], unified seemingly different phenomena within the framework of gauge theories [4, 5]. This seminal work was rapidly generalized to non-adiabatic and noncyclic evolutions [5] and, most saliently for our concern here, to degenerate states by F. Wilczek and A. Zee [6]. In this case, the underlying symmetry of the degenerate subspace leads to a non-Abelian gauge field structure. These early works on topology in quantum physics have opened up tremendous interest in condensed matter [7–11] and more recently in ultracold gases [12–20] and photonic devices [21–23].

Moreover, it has been noted that geometrical qubits are resilient to certain noises, making them potential candidates for fault-tolerant quantum computing [24–27]. So far, beside some recent proposals [28, 29], experimental implementations have been performed for a 2-qubit gate on NV-centers in diamond [30] and for a non-Abelian single qubit gate in superconducting circuits [31]. These experiments were performed following a non-adiabatic protocol allowing for high-speed manipulation [29, 32, 33]. Recently, coherent control of ultracold spin-1 atoms confined in optical dipole traps was used to study the geometric phases associated with singular loops in a quantum system [34]. If non-adiabatic manipulation are promising methods for quantum computing, they prevent the study of external dynamic of quantum system in a non-abelian gauge field, where non-trivial coupling occurs between the internal qubit state and the center-of-mass wave function of the particle.

We report here on non-Abelian adiabatic geometric transformations implemented on a noninteracting cold

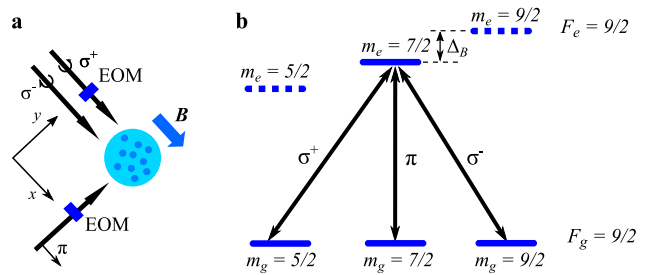


Figure 1. **Tripod scheme.** **a**, laser beams polarizations and propagation directions along a magnetic bias field  $\mathbf{B}$ . Two EOMs are used to sweep the two independent relative phases of the laser beams. **b**, bare energy level structure of the tripod scheme, implemented on the intercombination line of Strontium-87. The magnetic bias field shifts consecutive excited levels by  $\Delta_B \simeq 760\Gamma = 5.7$  MHz. It allows each tripod polarized laser beam to selectively address one of the magnetic transitions marked by the black arrows.

fermionic gas of Strontium-87 atoms by using a 4-level resonant tripod scheme set on the  $^1S_0, F_g = 9/2 \rightarrow ^3P_1, F_e = 9/2$  intercombination line at  $\lambda = 689$  nm (linewidth:  $\Gamma = 2\pi \times 7.5$  kHz). About  $10^5$  atoms are loaded in a crossed optical dipole trap, optically pumped in the stretched Zeeman state  $|F_g = 9/2, m_g = 9/2\rangle$  and Doppler-cooled down to temperatures  $T \sim 0.5 \mu K$  [35, 36], see Methods. A magnetic bias field isolates a particular tripod scheme in the excited and ground Zeeman substate manifolds, see Fig. 1. Three coplanar coupling laser beams are set on resonance with their common excited state  $|e\rangle = |F_e = 9/2, m_e = 7/2\rangle$ . Their relative phases can be varied using two electro-optic modulators (EOM), to operate the non-Abelian transformation in the

system.

For any value of the amplitude and phase of the lasers beams, the effective Hilbert space defined by the four coupled bare levels contains two degenerate dark-states  $|D_1\rangle$  and  $|D_2\rangle$  which do not couple to  $|e\rangle$  and are thus protected from spontaneous emission decay by quantum interference. For equal Rabi transition frequencies,

$$\begin{aligned} |D_1\rangle &= \frac{e^{-i\Phi_{13}(\mathbf{r})}|1\rangle - e^{-i\Phi_{23}(\mathbf{r})}|2\rangle}{\sqrt{2}} \\ |D_2\rangle &= \frac{e^{-i\Phi_{13}(\mathbf{r})}|1\rangle + e^{-i\Phi_{23}(\mathbf{r})}|2\rangle - 2|3\rangle}{\sqrt{6}}, \end{aligned} \quad (1)$$

where  $|i\rangle \equiv |m_g = i + 3/2\rangle$  ( $i = 1, 2, 3$ ).  $\Phi_{ij} = \Phi_i - \Phi_j$ , where the space-dependent laser phases read  $\Phi_i(\mathbf{r}) = \mathbf{k}_i \cdot \mathbf{r} + \vartheta_i$ .  $\mathbf{k}_i$  is the wavevector of the beam coupling state  $|i\rangle$  to  $|e\rangle$  and  $\vartheta_i$  its phase at origin. The two independent offset phases tuned by the EOM shown in Fig. 1a are  $\phi_i = \vartheta_i - \vartheta_3$  ( $i = 1, 2$ ).

In a first set of experiments, we probe and quantify the thermal decoherence of the dark-states due to the finite temperature of our atomic sample. In a second set of experiments, we analyze the non-Abelian character of a cyclic sequence of geometric transformations within the dark-state manifold by changing their relative order. In all experiments, we monitor the subsequent manipulation and evolution of the atomic system in the dark-state manifold by measuring the bare ground-state populations with a nuclear spin-sensitive shadow imaging technique, see Methods.

*Thermal decoherence.* Starting from state  $|3\rangle$ , we prepare the atoms in dark-state  $|D_2\rangle$  after a suitable adiabatic laser ignition sequence, see Methods. We assume that the atoms do not move significantly during the time duration of this sequence, see Fig. 2. Following [37, 38], the subsequent evolution of the atoms is described by the Hamiltonian

$$H = \frac{1}{2M}(\hat{\mathbf{p}}\mathbb{1} - \mathbf{A})^2 + W \quad (2)$$

where  $\hat{\mathbf{p}} = -i\hbar\nabla$  is the momentum operator,  $\mathbb{1}$  is the identity operator in the internal dark-state subspace,  $M$  the atom mass,  $\mathbf{A}$  the geometrical vector potential with matrix entries  $\mathbf{A}_{jk} = i\hbar\langle D_j | \nabla D_k \rangle$  and  $W$  the geometrical scalar potential with matrix entries

$$W_{jk} = \frac{\hbar^2 \langle \nabla D_j | \nabla D_k \rangle - \mathbf{A}_{jk}^2}{2M}. \quad (3)$$

With our laser beams geometry,  $\mathbf{A}$ ,  $\mathbf{A}^2$ , and  $W$  have the same matrix form, and are uniform and time-independent, see Methods. Since  $\mathbf{A}$  is Abelian here, we can look for states in the form  $|\psi\rangle \otimes |\mathbf{p}\rangle$  where  $\mathbf{p} = M\mathbf{v}$  is the initial momentum of the atoms and  $|\psi\rangle$  some combination of dark-states. Denoting by  $P_0(\mathbf{v})$  the initial atomic velocity distribution, we find that the population

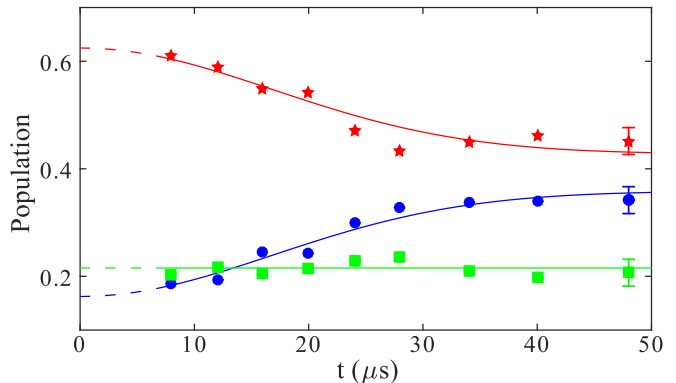


Figure 2. **Ballistic expansion.** Time evolution of the bare state populations after the tripod ignition sequence (duration  $t_0 \simeq 8 \mu\text{s}$ ) is completed and laser beams have reached equal Rabi frequencies  $\Omega = 2\pi \times 250 \text{ kHz}$ . The blue circles, the green squares, and the red stars correspond to the populations  $\bar{P}_1$ ,  $\bar{P}_2$ , and  $\bar{P}_3$  with  $|i\rangle \equiv |m_g = i + 3/2\rangle$  ( $i = 1, 2, 3$ ), respectively. Solid lines: theoretical predictions given by equations. 5. The temperature  $T$ , the initial and the final populations of each spin state are the fit parameters. The dashed lines, at early times, extrapolate the fits into the time window  $t_0$ . We get a temperature  $T = 0.5(1) \mu\text{K}$  meaning that the atoms do not move significantly during the dark-state preparation sequence since  $\bar{v}t_0/\lambda \simeq 0.08$ .

of state  $|2\rangle$  remains constant while the two others display an out-of-phase oscillatory behavior at a velocity-dependent frequency  $\omega_v = \frac{2}{3}[k(v_x - v_y) + 2\omega_R]$ :

$$\begin{aligned} P_1(\mathbf{v}, t) &= \frac{5P_0(\mathbf{v})}{12} \left( 1 - \frac{3}{5} \cos \omega_v t \right), \\ P_2(\mathbf{v}, t) &= \frac{P_0(\mathbf{v})}{6}, \\ P_3(\mathbf{v}, t) &= \frac{5P_0(\mathbf{v})}{12} \left( 1 + \frac{3}{5} \cos \omega_v t \right), \end{aligned} \quad (4)$$

where  $\omega_R = \hbar k^2/(2M)$  is the atomic recoil frequency and  $k = 2\pi/\lambda$  is the laser wavenumber. The frequency component proportional to  $k(v_x - v_y)$  comes from the momentum-dependent coupling term  $\mathbf{A} \cdot \hat{\mathbf{p}}/M$  in equation 2 (Doppler effect) whereas the other frequency component proportional to  $\omega_R$  comes from the scalar term  $\mathbf{A}^2/2M + W$ .

Averaging over the Maxwellian velocity distribution of the atoms, the bare state populations of the thermal gas read

$$\begin{aligned} \bar{P}_1(t) &= \frac{5}{12} - \frac{1}{4} \cos \left( \frac{4}{3} \omega_R t \right) \exp \left[ -\frac{4}{9} (k\bar{v}t)^2 \right], \\ \bar{P}_2(t) &= \frac{1}{6}, \\ \bar{P}_3(t) &= \frac{5}{12} + \frac{1}{4} \cos \left( \frac{4}{3} \omega_R t \right) \exp \left[ -\frac{4}{9} (k\bar{v}t)^2 \right], \end{aligned} \quad (5)$$

where  $\bar{v} = \sqrt{k_B T/M}$  is the thermal velocity of the gas at temperature  $T$ . We see that  $\bar{P}_1$  and  $\bar{P}_3$  converge to the same value at long times. Because of the orientation of the laser beams, this means that the thermal average breaks the tripod scheme into a  $\Lambda$ -scheme coupled to the two circularly-polarized beams and a single leg coupled to the linearly-polarized beam. As a consequence, quantum coherence partially survives the thermal average.

Our experimental results confirm this behavior even if  $\bar{P}_1$  and  $\bar{P}_3$  do not merge perfectly, see Fig. 2. This discrepancy can be lifted by introducing a 10% imbalance between the Rabi transition frequencies in our calculation. The population difference  $\bar{P}_3 - \bar{P}_1$  measures in fact the Fourier transform of the velocity distribution along the diagonal direction  $\hat{\mathbf{x}} - \hat{\mathbf{y}}$ . It decays with a Gaussian envelope characterized by the time constant  $\tau = 3/(2k\bar{v})$ . This interferometric thermometry is similar to some spectroscopic ones such as recoil-induced resonance [39, 40] or stimulated two-photon transition [41, 42]. From our measurements, we get  $T = 0.5(1) \mu\text{K}$ ,  $\tau \simeq 24 \mu\text{s}$  and  $\bar{v} \simeq 6.9 \text{ mm/s}$ .

*Non-Abelian transformations.* We now investigate the geometric non-Abelian unitary operator  $U$  acting on the dark-state manifold when the relative phases of the tripod beams are adiabatically swept along some closed loop  $\mathcal{C}$  in parameter space. For a pinned atom ( $M \rightarrow \infty$ ),  $U$  is given by the loop integral along  $\mathcal{C}$  of the  $2 \times 2$  Mead-Berry 1-form  $\omega \equiv [\omega_{jk}] \equiv [i\hbar \langle D_j | dD_k \rangle]$

$$U = \mathfrak{P} \exp \left( \frac{i}{\hbar} \oint_{\mathcal{C}} \omega \right), \quad (6)$$

where  $\mathfrak{P}$  is the path-ordering operator [6].

As before, the system is initially prepared in dark-state  $|D_2\rangle$ . Then, starting from the origin, the phase loop is cycled counterclockwise, see Fig. 3a. Each segment is linearly swept in  $\Delta t = 4 \mu\text{s}$  and the phase excursion is  $\phi_0$ . The total duration  $3\Delta t$  of the loop is thus less than the thermal decoherence time  $\tau$  discussed above. In Fig. 3b, we plot the bare state populations measured right after the phase loop as a function of  $\phi_0$  and their comparison with our theoretical predictions for a pinned atom and for atoms at finite temperature, see Methods. The very good agreement with the finite temperature case shows that we capture most of the mechanisms at play and that the adiabatic approximation is indeed valid. Note that the mismatch with a pinned atoms decreases with increasing  $\phi_0$ . This is because the thermal decoherence is quenched by the increasing geometrical coupling among the dark-states when the sweep rate  $\phi_0/\Delta t > k\bar{v}, \omega_R$ . As a further approximation, we now disregard thermal decoherence and consider that the system after the phase loop is described by a pure quantum state  $|\psi_{out}\rangle = \sum_{j=1,2} d_j |D_j\rangle$  [43]. As shown in Figs. 3b and 3c, one can easily extract the dark-state populations and the absolute value of the azimuthal angle  $\varphi = \text{Arg}(d_2) - \text{Arg}(d_1)$  from the measured bare state populations, see Methods. When

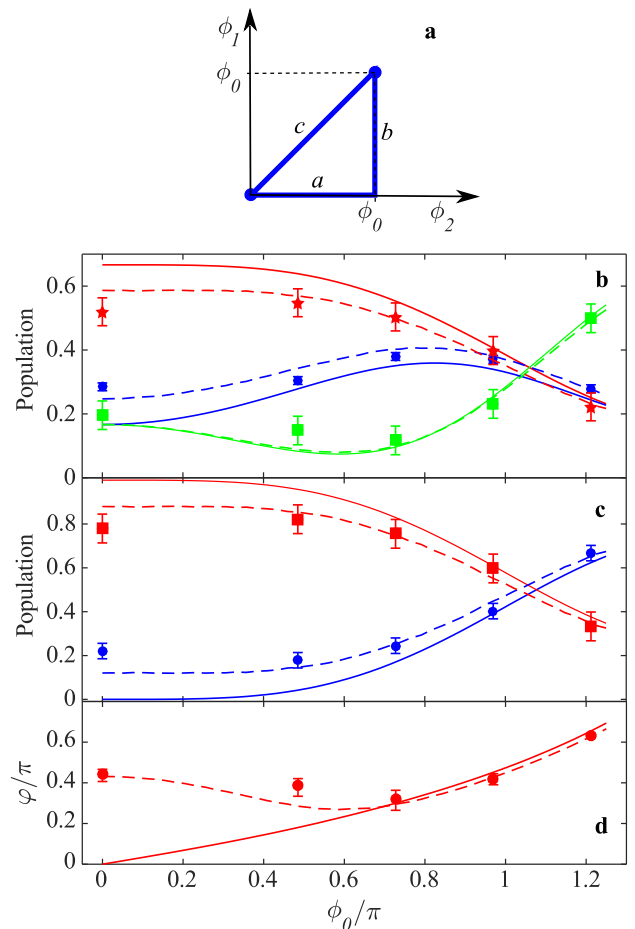


Figure 3. **Geometric gate operation.** **a**, phase loop in parameter space ( $\phi_2, \phi_1$ ).  $\phi_i$  ( $i = 1, 2$ ) are the two independent offset phases tuned by the EOM shown in Fig. 1a. We have performed two counterclockwise cycles: the first one is  $a \rightarrow b \rightarrow c$  and starts from the origin, the second one is  $c \rightarrow a \rightarrow b$  and starts from the upper corner. The loop is completed in  $12 \mu\text{s}$  and its excursion is  $\phi_0$ . **b**, measured bare state populations  $\bar{P}_1$  (blue circles),  $\bar{P}_2$  (green squares) and  $\bar{P}_3$  (red stars) as a function of  $\phi_0$  for the first cycle. The Rabi frequencies are  $\Omega = 2\pi \times 450 \text{ kHz}$  and  $T = 0.5 \mu\text{K}$ . Dark state reconstruction as a function of  $\phi_0$ . **c**, population of  $|D_1\rangle$  (blue circles) and  $|D_2\rangle$  (red squares). **d**, azimuthal phase  $\varphi$ . The solid and dashed curves in **b**, **c**, and **d** are the theoretical predictions for a pinned atom and for a gas at temperature  $T$  respectively.

$\phi_0 \gtrsim 0.7\pi$ , the values for  $\varphi$  match well with the prediction for a pinned atom confirming the quenching of thermal decoherence. At  $\phi_0 = \pi$ , the two dark-state populations are almost equal. In the language of the Bloch sphere representation, this corresponds to a rotation of the initial south pole state  $|D_2\rangle$  to the equatorial plane.

We now reconstruct the full geometric unitary operator  $U$  for  $\phi_0 = \pi$ . Up to an unobservable global phase, we

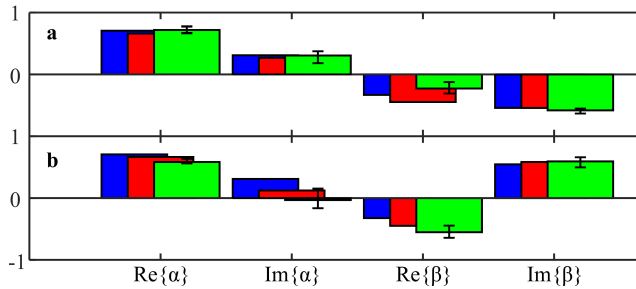


Figure 4. **Unitary operators reconstruction.** **a**, elements of operator  $U$  given by equation 7 for the phase loop  $a \rightarrow b \rightarrow c$  at  $\phi_0 = \pi$ , see Fig. 3a. The blue, red, and green bars correspond to a pinned atom, a gas at temperature  $T = 0.5 \mu\text{K}$ , and the experimental data respectively. **b**, same as **a** but for the phase loop  $c \rightarrow a \rightarrow b$ .

write:

$$U = \begin{bmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{bmatrix} \quad (7)$$

with  $|\alpha|^2 + |\beta|^2 = 1$ . The previous dark-state reconstruction, done after the phase loop is applied on  $|D_2\rangle$ , gives access to  $|\alpha|$ ,  $|\beta|$  and  $\text{Arg}(\alpha) - \text{Arg}(\beta)$ , see Methods. To obtain  $\text{Arg}(\alpha)$  and  $\text{Arg}(\beta)$  and fully determine  $U$ , we start from a linear combination of dark-states  $|D_1\rangle$  and  $|D_2\rangle$ , perform the phase loop and process the new data. The results are shown in Fig. 4a and compared to the theoretical predictions for a pinned atom and a gas at finite temperature. The good agreement with our data validates the expected small impact of temperature for  $\phi_0 = \pi$ .

*Probing non-Abelianity.* With the previous phase loop protocol, we have  $U = U_c U_b U_a$  where  $a$ ,  $b$ , and  $c$  label the edges of the loop, see Fig. 3a. To illustrate the non-commutative nature of the transformation group, we will cycle the phase loop counterclockwise starting from the upper corner. We then reconstruct the corresponding unitary operator  $U' = U_b U_a U_c$  like done for  $U$ . The results are depicted in Fig. 4b and show that  $U$  and  $U'$  are indeed different, though unitarily related, confirming the sensitivity of these geometric transformations to path ordering. The Frobenius distance between the two unitaries is  $D = \sqrt{2 - |\text{Tr}(U^\dagger U')|} = 1.27(25)$  and is in agreement with the theoretical result for a pinned atom ( $D = 1.09$ ) and for a finite-temperature gas ( $D = 1.14$ ). These values have to be contrasted with the maximum possible Frobenius distance  $D = 2$ .

*Conclusion and discussion.* Using a tripod scheme on Strontium-87 atoms, we have implemented adiabatic geometric transformations acting on two degenerate dark-states. This system realizes a universal geometric single-qubit gate. We have studied  $\text{SU}(2)$  transformations associated to laser beams phase loop sequences and shown their non-Abelian character. In contrast to recent works

done in optical lattices [14–20], our system realizes an artificial gauge field in continuous space. Indeed, if the phases of the lasers remain constant in time, a gauge field emerges and governs the center-of-mass dynamics of the spinor-like wave function. Depending on the laser field configuration, different gauge fields can be engineered such as spin-orbit coupling [38, 44], Zitterbewegung [38], magnetic monopole [37] or non-Abelian Aharonov-Bohm effect [44] (see [45] for a review). A generalization to the  $\text{SU}(3)$  symmetry is also discussed in [46]. Some of these schemes might be difficult to implement in optical lattices. Gauge fields generated by optical fields come from a redistribution of photons among the different plane waves modes and involve momenta transfer comparable to the photon recoil. Observing mechanical effects of gauge fields would thus require atomic gases colder than the recoil temperature and thus cooling techniques beyond the mere Doppler cooling done here [47, 48]. However, the gauge field is still driving the internal state dynamics regardless of the temperature of the gas provided the adiabatic condition is fulfilled. This led us to a new interferometric thermometry based on the Fourier transform of the velocity distribution of the gas.

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**Competing financial interests** The authors declare no competing financial interests.

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## METHODS

**Cold sample preparation and implementation of the tripod scheme.** The cold gas is obtained by laser cooling on the  $^1S_0 \rightarrow ^3P_1$  intercombination line at 689 nm (linewidth  $\Gamma = 2\pi \times 7.5$  kHz). Atoms are first laser cooled in a magneto-optical trap and then transferred into an elliptical crossed optical dipole trap at 795 nm (trapping frequencies 150, 70, and 350 Hz) where they are held against gravity. Atoms are then optically pumped in the stretched  $m_g = F_g = 9/2$  magnetic substate and subsequently Doppler cooled in the optical trap using the close  $m_g = F_g = 9/2 \rightarrow m'_e = F'_e = 11/2$  transition, see Fig. 5. The atomic cloud contains about  $10^5$  atoms at a temperature  $T = 0.5 \mu\text{K}$  (recoil energy  $E_R/k_B = 0.2 \mu\text{K}$ ,  $k_B$  is the Boltzmann constant). A magnetic field bias of  $B = 67$  G is applied to lift the degeneracy of the Zeeman excited states. It has essentially no effect on the ground-state levels since their spin is of nuclear origin (Landé factor  $g \sim 0$ ). Because the Zeeman shift between levels in the excited manifold  $F_e = 9/2$  is large, one can isolate a tripod scheme between three ground-state levels and a single excited state, namely  $|e\rangle = |F_e = 9/2, m_e = 7/2\rangle$ , as indicated in Fig. 5. The three laser beams are tuned at resonance and their polarizations are chosen according to the electrical dipole transition selection rules. In practice, the two laser beams with right and left circular polarizations, respectively addressing the  $m_g = 5/2 \rightarrow m_e = 7/2$  and  $m_g = 9/2 \rightarrow m_e = 7/2$  transitions, are co-propagating. The laser beam with linear polarization, aligned with the magnetic bias field, addressing the  $m_g = 7/2 \rightarrow m_e = 7/2$ , is orthogonal to the circularly polarized beams, see Fig. 1b. The plane of the lasers is chosen orthogonal to the direction of gravity. The two independent laser offset phases  $\phi_1$  and  $\phi_2$  (see main text) can be tuned by using two electro-optic modulators.

**Initial dark-state preparation:** Starting with atoms in the  $|m_g = 9/2\rangle$  stretched state, the tripod beams are turned on following two different sequences. The first sequence prepares dark-state  $|D_2\rangle$ , see equation 1. More precisely, we first turn on the two laser beams connecting the empty states  $|m_g = 5/2\rangle$  and  $|m_g = 7/2\rangle$  to the excited state and then adiabatically ramp on the last laser beam. This projects state  $|m_g = 9/2\rangle$  onto  $|D_2\rangle$  with a fidelity of 95%. A different ignition sequence is used to prepare a combination of dark-states  $|D_1\rangle$  and  $|D_2\rangle$ . By turning on sequentially abruptly the left-circular beam, and adiabatically the right-circular beam, we create a coherent (dark) superposition of the state  $|m_g = 5/2\rangle$  and  $|m_g = 9/2\rangle$ . Finally, we turn on abruptly the linearly-polarized beam and we expect to produce the linear combination  $(|D_1\rangle + \sqrt{3}|D_2\rangle)/2$ . In practice, a systematic phase rotation occurs once the last beam is turned on which adds an extra mixing among the dark-states. Performing the bare state population analysis, we find that this initial state corresponds in fact to  $0.6|D_1\rangle + 0.8e^{i0.15\pi}|D_2\rangle$ .

**Spin sensitive imaging system:** The bare state populations in the ground-state are obtained with a nuclear spin-sensitive shadow imaging technique on the  $F_g = 9/2 \rightarrow F'_e = 11/2$  line, see Fig. 5. First we measure the population of state  $|m_g = 9/2\rangle$  with a shadow laser tuned on the closed  $m_g = 9/2 \rightarrow m'_e = 11/2$  transition. Then, using the same atomic ensemble, we measure the population of state  $|m_g = 7/2\rangle$  by tuning the shadow laser on

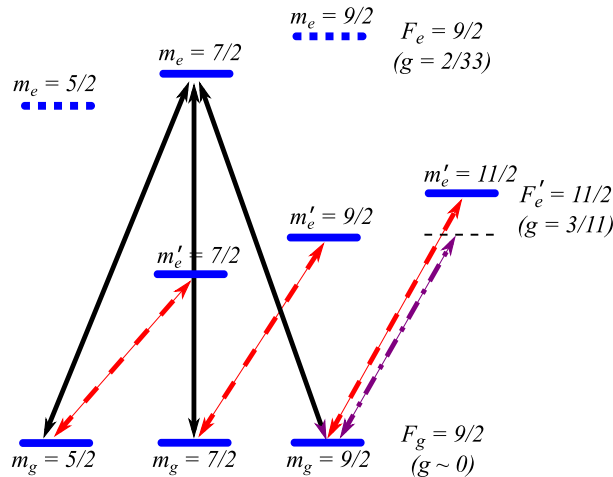


Figure 5. **Energy levels and experimentally relevant transitions.** A magnetic bias field  $B = 67$  G lifts the degeneracy of the different Zeeman manifolds and allows to address each transition individually. The Landé factors  $g$  are indicated for each hyperfine level. The black arrows correspond to the tripod beams (see main text for more details). The dashed red arrows indicate the transitions used for the shadow spin-sensitive imaging system. The dash-dotted purple arrow is the red-detuned cooling transition used in the far-off resonant dipole trap.

the  $m_g = 7/2 \rightarrow m_e' = 9/2$  transition. This transition is open but its large enough Clebsch-Gordan coefficient ( $\sqrt{9/11} \sim 0.9$ ) ensures a good coupling with the shadow laser. The population of state  $|m_g = 5/2\rangle$  is measured in the same way ( $m_g = 5/2 \rightarrow m_e' = 7/2$  open transition, its Clebsch-Gordan coefficient  $\sqrt{36/55} \sim 0.8$  being still large enough). The shadow laser beam shines the atoms during  $40 \mu\text{s}$  with an on-resonance saturation parameter  $I/I_s = 0.5$  (saturation intensity  $I_s = 3 \mu\text{W}/\text{cm}^2$ ). With such values, the average number of ballistic photons scattered per atom is less than one and optical pumping can be safely ignored, ensuring an accurate measurement of the ground-state populations.

**Dark state and unitary matrix reconstruction:** A state in the dark-state manifold takes the form  $|\psi\rangle = \sum_{j=1,2} d_j |D_j\rangle$  with  $|d_1|^2$  and  $|d_2|^2 = 1 - |d_1|^2$  the populations of states  $|D_1\rangle$  and  $|D_2\rangle$  and  $\varphi = \text{Arg}(d_2) - \text{Arg}(d_1)$  the azimuthal angle. Using equation 1, we immediately find

$$|d_2| = \sqrt{\frac{3\bar{P}_3}{2}} \quad \cos \varphi = \frac{\bar{P}_1 - \bar{P}_2}{\sqrt{P_3(2 - 3P_3)}}. \quad (8)$$

Do note that the normalization of  $|\psi\rangle$  restricts the possible values of the  $\bar{P}_i$  summing up to 1. The sign of  $\varphi$  is determined using the prediction of equation 6 for a pinned atom ( $M \rightarrow \infty$ ).

To reconstruct the unitary matrix  $U$ , as expressed in equation 7, we perform the phase loop sequence on two different initial dark-states (their representative points on the Bloch sphere should not be opposite) and perform the dark-state reconstruction for each of them. The two phase terms in  $U$  are reconstructed up to a sign. As for the dark-state reconstruction, we rely on the prediction for a pinned atom to lift this sign ambiguity.

**Gauge fields and adiabatic Schrödinger equation:** The time-dependent interaction operator for the resonant tripod scheme, in the rotating-wave approximation, has the following expression:

$$H(t) = \frac{\hbar\Omega(\mathbf{r}, t)}{2} \sum_{i=1}^3 |e\rangle\langle i| + H.c. \quad (9)$$

We assume here that the laser Rabi frequencies coupling the ground-states  $|i\rangle = |m_g = i + 3/2\rangle$  to the excited state  $|e\rangle = |m_e = 7/2\rangle$  have all the same amplitude denoted by  $\Omega$ . The time dependency comes from the cyclic ramping sequence of the two offset laser phases  $\phi_j$  ( $j = 1, 2$ ). Neglecting transitions outside the dark-state manifold (adiabatic approximation), the system is described by a quantum state  $|\psi(\mathbf{r}, t)\rangle = \sum_{j=1,2} \Psi_j(\mathbf{r}, t) |D_j(\mathbf{r}, t)\rangle$ , where  $\Psi_j$  is the wave



function of the centre-of-mass of the atom in an internal state  $|D_j\rangle$ . In this basis, the adiabatic Schrödinger equation for the column vector  $\underline{\Psi} = (\Psi_1, \Psi_2)^T$  reads:

$$i\hbar\dot{\underline{\Psi}} = \left[ \frac{(\hat{\mathbf{p}}\mathbb{1} - \mathbf{A})^2}{2M} + W - \omega_t \right] \underline{\Psi}, \quad (10)$$

where the dot denotes time derivative. The first two terms on the right-hand side describe the dynamics of an atom subjected to the synthetic gauge field. The last term  $\omega_t \equiv [\omega_{jk}] \equiv [i\hbar\langle D_j | \dot{D}_k \rangle]$  is due to the cyclic ramping sequence of the laser phases. Only this term remains for a pinned atom ( $M \rightarrow +\infty$ ), in which case one recovers equation 6. The general expressions of  $\mathbf{A}$  and  $W$  are given in the main text. With equal and constant Rabi frequencies amplitude, and for the orientation of our laser beams, one finds:

$$\mathbf{A} = \frac{2\hbar(\mathbf{k}_2 - \mathbf{k}_1)}{3} \mathcal{M} \quad \frac{\mathbf{A}^2}{2M} = \frac{8E_R}{9} \mathcal{M} \quad W = -\frac{4E_R}{9} \mathcal{M}, \quad (11)$$

where  $E_R = \hbar\omega_R = \hbar^2 k^2 / (2M)$  is the atomic recoil energy and  $\mathbf{k}_j$  is the wavevector of laser beam  $j$  (see main text). As one can see, all these operators have the same matrix form. The matrix  $\mathcal{M}$  reads:

$$\mathcal{M} = \frac{\mathbb{1} + \mathbf{s} \cdot \boldsymbol{\sigma}}{2} = \begin{pmatrix} 3/4 & -\sqrt{3}/4 \\ -\sqrt{3}/4 & 1/4 \end{pmatrix} \quad (12)$$

and satisfies  $\mathcal{M}^2 = \mathcal{M}$ , its unit Bloch vector being  $\mathbf{s} = (-\sqrt{3}/2, 0, 1/2)$ . As a consequence, all these operators can be diagonalized at once by the same transformation and amenable to the simple projector matrix form:

$$\mathcal{M} \rightarrow \mathcal{M}_D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (13)$$

Because of our laser beams geometry, the vector potential  $\mathbf{A}$  is in fact Abelian since its only non-zero matrix component is along  $\mathbf{k}_2 - \mathbf{k}_1$ .

In contrast, the operator  $\omega_t$  has a different matrix form. Indeed, the two offset phases  $\phi_j$  of the lasers (see main text) can be addressed at will. Following [45], we get:

$$\omega_t = \frac{\hbar}{2} \begin{pmatrix} \dot{\phi}_1 + \dot{\phi}_2 & (\dot{\phi}_1 - \dot{\phi}_2)/\sqrt{3} \\ (\dot{\phi}_1 - \dot{\phi}_2)/\sqrt{3} & (\dot{\phi}_1 + \dot{\phi}_2)/3 \end{pmatrix}. \quad (14)$$

In particular, we note that  $\omega_t$  leads to non-Abelian transformations. An immediate consequence is that, for a given phase loop in parameter space, the geometric unitary operator associated with a cycle of phase ramps depends on the starting point of the cycle on the loop. Different starting points lead to different, though unitarily related, non-commuting geometric unitary operators.