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Reduction of Stiffness and Mass Matrices

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JUST as it is often necessary to reduce the size of the stiffness matrix in statical structural analysis, the simultaneous reduction of the nondiagonal mass matrix for natural mode analysis may also be required. The basis for one such reduction technique may follow the procedure used in Ref. 1 for the stiffness matrix, namely, the elimination of coordinates at which no forces are applied.

Arrange the structural equations $\{F\} = [K]\{x\}$ so that after partitioning in the form

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} A & B \\ B' & C \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

the forces F_2 are to be zero. The two resulting equations yield

$$F_1 = (A - BC^{-1}B')x_1$$

from which the reduced stiffness matrix is seen to be

$$K_1 = A - BC^{-1}B'$$

The foregoing amounts to a coordinate transformation $x = Tx_1$ or

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} I \\ -C^{-1}B' \end{bmatrix} \{x_1\}$$

If the structure energies are written $T = \frac{1}{2}\dot{x}'M\dot{x}$ and $V = \frac{1}{2}x'Kx$ and the foregoing transformation is employed, the result is

$$T = \frac{1}{2}\dot{x}_1'T'MT\dot{x}_1$$

$$V = \frac{1}{2}x_1'T'KTx_1$$

The reduced stiffness matrix is seen to be $K_1 = T'KT$ and the reduced mass matrix $M_1 = T'MT$. Then with

$$[M] = \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{B}' & \bar{C} \end{bmatrix}$$

the reduced mass matrix becomes

$$M_1 = \bar{A} - \bar{B}C^{-1}B' - (C^{-1}B')'(\bar{B}' - \bar{C}C^{-1}B')$$

In the case of the reduced stiffness matrix, none of the structural complexity is lost since all elements of the original stiffness matrix contribute. However, in the reduced mass matrix, combinations of stiffness and mass elements appear. The result is that the eigenvalue-eigenvector problem is closely but not exactly preserved. Some comparative results are reported in Ref. 2 for beam vibrations.

References

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² Archer, J. S., "Consistent mass matrix for distributed mass systems," *Proc. Am. Soc. Civil Engrs.* **89**, 161-178 (August 1963).

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