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► **To cite this version:**

Saker Sabkha, Christian de Peretti. On the performances of Dynamic Conditional Correlation models in the Sovereign CDS market and the corresponding bond market. *Financial and Economic Systems*, WORLD SCIENTIFIC (EUROPE), pp.187-212, 2022, 10.1142/9781786349507_0008 . hal-01710398

HAL Id: hal-01710398

<https://hal.science/hal-01710398>

Submitted on 15 Feb 2018

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On the performances of Dynamic Conditional Correlation models in the Sovereign CDS market and the corresponding bond market

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Abstract

The study of an efficient financial assets' modeling method is still an open hot issue especially during recent crises. Using credit risk data from 33 worldwide countries, this paper investigates the performance of 9 Dynamic Conditional Correlation models taking into account different properties of financial markets (long memory behavior, asymmetry and/or leverage effects...). This comparative study is based on the results of several multivariate diagnostic tests. Findings show that no model outperforms the others in all situations, though, the straightforward DCC-GARCH model seems to provide the most relevant estimator parameters. Yet, the innovations distributions assumption significantly impacts the statistical fit of the model. Our work is useful for financial markets' participants so as to making decision in terms of arbitrage, hedging or speculation.

JEL Classification

G11, G12, F02, C58

Keywords

DCC-class models, Time-varying correlation, Multivariate diagnostic tests, Sovereign credit market.

1 Introduction

The Mathematical formalization of stock market fluctuations has been developed for the first time by Bachelier (1900) who mathematized the speculative values' evolution as a Brownian-motion (Walter, 2013). The probabilistic analysis of the variation of assets' prices proposed by Bachelier (1900) seems not to be valid in the long-run and discords with the efficient-market assumptions. In fact, The random walk equation that Bachelier (1900) obtained from this model is based on the hypothesis that the instantaneous and nonconstant volatility is independent, linear and follow a Gaussian marginal distribution¹. De facto, this model presents some shortcomings: Empirical studies easily show that financial times series exhibit kurtosis coefficient in excess rejecting the distribution under normality. Yet, assets' prices in levels are obviously non-stationary and an over-time non-linear dependence is clearly observed between financial returns.

In order to consider for the non-linearity, the non-stationarity and the extreme fluctuations of stock prices, a new class of financial models is proposed by Engle (1982) called Autoregressive Conditional Heteroscedasticity (ARCH). A generalization of this ARCH model (GARCH) is suggested by Bollerslev (1986) enabling for more flexibility and simplicity in the lag structure and the estimation process². Several extensions and restrictive versions of GARCH type have followed allowing to take into account further financial markets' properties and specifications (Engle and Bollerslev, 1986; Engle et al., 1987; Nelson, 1991; Higgins and Bera, 1992; Glosten et al., 1993; Ding et al., 1993; Zakoian, 1994; Sentana, 1995; Hentschel, 1995; Baillie et al., 1996; Bollerslev and Mikkelsen, 1996; Tse, 1998; Klüppelberg et al., 2004; Davidson, 2004).

Numerous applications of the univariate GARCH models to financial data have been done by researchers as well as by finance practitioners. Even though they have been a good toll for financial managers in risk assessment, these volatilities models are inadequate when it comes to estimate a portfolio risk with several and different underlying assets. Because of the interdependence between stock market variables, the estimation of correlations and covariance matrix turns out to be a requisite to financial markets' investors, so they can pursue optimal risk allocation and hedging policies. Thus, portfolio optimization motivates the work of Bollerslev et al. (1988) who propose the first multivariate framework for Generalized Autoregressive Conditional Heteroscedastic models (MV-GARCH). Although it is considered as a considerable theoretical contribution, empirical application of this MV-GARCH - as it is initially proposed - is problematic due to the high-dimension of the parameters and the number of necessary restrictions.

In order to circumvent this estimation complexity, Engle (2002) introduce a new class of multivariate models called Dynamic Conditional Correlation (DCC, hereafter) allowing for correlations to vary over time. This model keeps up with the flexibility of the univariate GARCH models and yet simplifies the parametrization and the empirical estimation. Another DCC-GARCH model is suggested by Tse and Tsui (2002) whereby the correlation computing is also done in two steps: a first step for estimating the univariate GARCH model for each time series and a second step for conditional covariance (see Engle and Sheppard (2001) for further details about technical difference between the two previous models). Engle and Kelly (2012) also propose a time-varying correlation model considering more financial data specifications known as a dynamic equicorrelation model (DECO). In fact, the DCC-DECO model assumes that whenever the time is, correlations' pairwise across markets are equals basing on an arbitrary estimation of the covariance matrix (Engle and Kelly, 2012; Cai et al., 2016)³.

¹These are the fractal properties of the random walk and Brownian-motion paths. For more details, see Walter (2013);

²A simple GARCH(1,1) could be an alternative for a long lag length of an ARCH model.

³Cai et al. (2016) empirically compare the reliability of dynamic conditional correlation models using both

Based on the aforementioned considerations, DCC models constitute a reliable tool for estimating interconnections between several assets, markets and/or countries. It is obvious thus, that this class of model is used in previous research to investigate financial phenomena: contagion, risk spillover (Lee, 2006; Cai et al., 2016) or co-movement (Coudert and Gex, 2010), given the considerable number of reasons why correlation is an important variable in financial and economic studies.

This paper aims to extend previous works - comparing the performance of GARCH models - to the multivariate dimension⁴. An extensive set of linear and non-linear DCC-GARCH models are estimated: 9 volatility model specifications are represented considering the clustering volatility, the volatility persistence, the dependence between the mean and the conditional variance, the asymmetry of the error distribution, the leverage effect, the long memory property, the slow hyperbolic rate of decay of the volatility persistence, the long-run mean reverting property and the fractionally integrated specifications. These models (GARCH-DCC, IGARCH-DCC, EGARCH-DCC, APARCH-DCC, GJR-GARCH-DCC, FIGARCH-DCC, FIEGARCH-DCC, FIAPARCH-DCC, HYGARCH-DCC) are estimated using both Engle (2002) and Tse and Tsui (2002) propositions. Yet, results of the models are presented for both Gaussian and Student marginal distribution.

Owing to the noteworthy part played by the credit market during the recent two financial crises, the interest of the scientific community to bonds and to credit derivatives (CDS, CDO, ABX...) is ever-increasing. Modeling multivariate volatilities of credit assets is a high-priority concern among policy makers, portfolio managers and hedge funds. That's why, our studied models are applied to credit risk data (sovereign CDS spreads and the underlying bond spreads) over a relatively long period covering the last two financial crises spanning from January 2006 until April 2014.

Our paper contributes to the extant literature in several ways. First, while there exists a considerable number of studies using the DCC model to estimate multivariate volatilities, the literature comparing the empirical performance of different Dynamic Conditional Correlation models is still not developed, especially when it comes to use the credit assets as an application reference. As far as we are concerned, there is no study comparing 9 dynamic conditional correlation GARCH models for the credit market. Second, the assessment of the interconnectedness between credit markets in an international context puts in forward the fact that countries from different regions present different features. This suggests to improve the standard method abundantly applied to comovement studies by using rather flexible approaches with an individual readjustment for each country and each set of data. Third, studying the multivariate modeling framework in the credit markets presents accurate results since it improves the predictive ability of financial tools in terms of future evolutions in sovereign borrowing conditions.

The rest of the paper is organized as follows: Sections 2 and 3 give some background informations about literature on the comparison of GARCH performances and discuss the methodology approach used in this paper. Section 4 is dedicated to sample description and the descriptive statistics. Empirical results and discussion are covered in section 5. Section 6 concludes the paper and outlines possible directions for future work.

conventional DCC and DECO estimates.

⁴We extend the work of Fantazzini (2011) on the univariate GARCH models and we transmit the same idea to the multivariate volatility models

2 Literature review

Unlike volatility models⁵, not much studies has been conducted to assess the performance of the multivariate GARCH-type models. The few works dealing with the time-varying correlation are only interested in straighten out the model's shortcomings by developing some extensive forms taking into account more financial data features.

Evaluating the conditional covariance of 100 companies' assets from the S&P 500 and the Dow Jones American indices, Engle and Sheppard (2001) develop a framework where the time-varying correlation converges easily and presents strong estimation. Authors propose a simplified method in which correlation coefficients are estimated through two steps⁶ and give some theoretical and empirical evidences of the estimator consistency. In a more theoretical framework, Degiannakis and Xekalaki (2004) summarize the existing literature on ARCH processes and present the most useful multivariate Autoregressive Conditional Heteroscedasticity models, their respective properties and the different estimating techniques.

Using daily 45 stock returns from the DAX and the FTSE, Hafner and Franses (2003) propose a generalized form of the DCC model that controls for heterogeneity. Results show that the GDCC has a superior performance and better fits financial data. One of the DCC issues discussed by Billio et al. (2006), is the restrictive hypothesis of mutual dynamics between different class of assets. Authors propose a flexible Dynamic Conditional Correlation model (fDCC) where each group of assets is allowed to have a different dynamics. By applying the fDCC to three sectors from the Italian stock market, they show that this model provides a better portfolio allocation with a lesser risk and a greater yield. Furthermore, Billio and Caporin (2009) generalize the DCC and the fDCC model and propose a Quadratic Flexible DCC model allowing for interaction between assets' correlations.

Based on a monte carlo simulation, Hafner and Reznikova (2012) study the performance of the estimators of the DCC and the consistent DCC models. Authors find some estimation bias in large sample size and show that the shrinkage technique, as an alternative to the maximum likelihood, can be a solution. Through a theoretical study, Caporin and McAleer (2013) studies the estimating and forecasting limits of the time-varying covariance and correlation models. Among the ten discussed shortcomings, the authors argue that the variance misspecification - due to some financial markets properties (leverage and asymmetry effects) - leads to biased correlation estimator parameters. This research line is followed by Boudt et al. (2013) who propose an extensive form of the DCC model (BIP-cDCC) to overcome the forecasting issue. Application of this new class of models to the exchange rate returns of the euro and the yen against the dollar show that the BIP-cDCC outperforms the standard DCC in terms of forecasting.

In the same context, Aielli (2013) discusses a particular limit of the DCC model and argue that the second stage estimator is biased leading to inappropriate conclusions. Application to simulated and real data show that the alternative model (cDCC) is more consistent with slightly better forecasting performances.

More recently, García de la Fuente et al. (2014) propose an extension of the DCC model that takes into account the non-normality features of financial time series while modeling DCC models. A Bayesian DCC with multivariate skew-slash distribution is applied to simulated data and real data sets from three worldwide stock market indices (CAC40, DAX, Nikkei). Authors show that even though this new class of model allows a better capture of the slight skewness and the high kurtosis characterizing stock returns, it doesn't outperform the standard normal-distributed DCC model. Moreover, Cai et al. (2016) compare the estimators' accuracy

⁵See Bollerslev et al. (1992) for a complete theoretical and empirical survey on the use of univariate ARCH processes in financial studies.

⁶More details about the two stages estimation method are given in the next section.

of the dynamic conditional correlation and the dynamic equicorrelation (DECO)⁷ MGARCH on 9 stock markets from East Asia and the USA during the Global financial crisis. Results of the DECO model are consistent and confirms those of the standard DCC.

3 Methodology

Even though they have been abundantly developed and examined in the theoretical and empirical literature, several correlation models are so sophisticated and over-parameterized with a complicated optimization making them very few used by practitioners. Unlike these models, the DCC nonlinear model proposed by Engle (2002) and Tse and Tsui (2002) is a more reliable estimator of correlation with a limited number of parameters to estimate and an easiest estimation method. The flexibility of this new class of model lies on the fact that the estimation needs to go through two-steps process: the conditional volatility of each time series is estimated from a traditional univariate GARCH model in the first step, then in the second step, the multivariate conditional volatility is estimated using the Quasi-Maximum Likelihood function.

3.1 Marginal volatility processes: univariate ARCH-type models

As an extension of the ARCH model proposed by Engle (1982), Bollerslev (1986) propose a generalized ARCH model given by:

$$x_{i,t} = \mu_{i,t} + a_{i,t} \quad / \quad a_{i,t} = \sigma_{i,t}\varepsilon_{i,t}, \quad \varepsilon_{i,t}/\mathcal{F}_{t-1} \rightsquigarrow \mathcal{N}(0, 1);$$

$$\sigma_{i,t}^2 = E(x_{i,t}/\mathcal{F}_{t-1}) = \alpha_{i,0} + \sum_{k=1}^q \alpha_{i,k}a_{i,t-k}^2 + \sum_{h=1}^p \beta_{i,h}\sigma_{i,t-h}^2. \quad (1)$$

with x_t is a time series, i is a given country from the sample and μ_t and σ_t are respectively conditional mean and conditional volatility. To satisfy the positive-definite condition, some restrictions are imposed: $p \geq 0$, $q > 0$, $\alpha_k \geq 0$ for $k = 1 \dots q$, $\beta_h \geq 0$ and $\alpha_0 \geq 0$ for $h = 1 \dots p$.

For sake of simplicity and suitability, only models with process orders (p and q) equal to 1 are estimated. In fact, the simplest GARCH (1,1) specification is the most useful and fitted for financial time series (Bollerslev, 1986; Wei et al., 2010).

The GARCH(1,1) process, as proposed by Bollerslev (1986) is given by the following formula:

$$\sigma_{i,t}^2 = \alpha_{i,0} + \alpha_{i,1} + \alpha_{i,1}a_{i,t-1}^2 + \beta_{i,1}\sigma_{i,t-1}^2. \quad (2)$$

Furthermore to the previous model restrictions, α_1 and β_1 parameters must satisfy the condition of $\alpha_1 + \beta_1 < 1$ to comply with the stationarity in the broad sense.

A more restrictive version of the GARCH (1,1) is proposed by Engle and Bollerslev (1986) where the equivalent of the unit root in the mean is included in the variance so we can handle for the stationarity of the variance. The Integrated GARCH (1,1) takes into account the persistence of conditional volatilities⁸. The main difference with the GARCH(1,1) is that the IGARCH require the parameters α_1 and β_1 to respect the equality of $\alpha_1 + \beta_1 = 1$. Thus, the IGARCH (1,1)⁹ can be written as follows:

$$\sigma_{i,t}^2 = \alpha_{i,0} + \alpha_{i,1}a_{i,t-1}^2 + (1 - \alpha_{i,1})\sigma_{i,t-1}^2. \quad (3)$$

⁷The DECO-MGARCH model, as proposed by Engle and Kelly (2012), supposes that the time-varying conditional correlation is equal between markets (or assets) whatever the time is.

⁸Today's shocks on a financial asset (future contracts for example) have a significant impact on the conditional volatility several periods in the future.

⁹The IGARCH(1,1) is equivalent to the Exponentially Weighted Moving Average (EWMA) model developed by Morgan (1996).

Besides the aforementioned linear models, there exist some nonlinear GARCH-class of models taking into account the other financial markets properties. The Exponential GARCH, as proposed by Nelson (1991), is one of these models that accounts for the leverage effect and the asymmetry of the error distribution. While the nonnegativity of linear GARCH model is ensured by several parameters restrictions, the EGARCH model proposes another formulation allowing for a positive volatility without any restrictive constraints. The EGARCH (1,1) is expressed as follows:

$$\ln(\sigma_{i,t}^2) = \alpha_{i,0} + \alpha_{i,1-1}\ln(\sigma_{i,t}^2) + \beta_{i,1}g(\varepsilon_{i,t-1}) \quad (4)$$

, where $g(\varepsilon_t) = \theta\varepsilon_t + \gamma[|\varepsilon_t| - E(|\varepsilon_t|)]$.

The asymmetric relation between assets' fluctuation and volatility changes is depicted by the θ and γ representing respectively the the sign and the magnitude of ε_t .

Glosten et al. (1993) propose a model that allows the sign and the amplitude of the innovations (ε_t) to affect separately the conditional volatility. The asymmetric leverage effect¹⁰ is represented in the following formulation of the GJR-GARCH(1,1)¹¹ model:

$$\sigma_{i,t}^2 = \alpha_{i,0} + \alpha_{i,1}a_{i,t-1}^2 + \gamma_i I_{i,t-1}a_{i,t-1}^2 + \beta_{i,1}\sigma_{i,t-1}^2. \quad (5)$$

with I_t is a dummy variable equals to 0 when a_t is positive and 1 otherwise.

The first model accounting for the long-range persistence of financial assets variance is developed by Ding et al. (1993). This asymmetric power ARCH model allows the volatility to be long-memory¹². The APARCH (1,1) model is:

$$\sigma_{i,t}^2 = \alpha_{i,0} + \alpha_{i,1}(|a_{i,t-1}| - \gamma_i a_{i,t-1})^\delta + \beta_{i,1}\sigma_{i,t-1}^\delta. \quad (6)$$

where δ depicts the Box-Cox power transformation of the conditional volatility (σ_t) and satisfies the condition of $\delta \geq 0$.

A more flexible class of GARCH models is proposed by Baillie et al. (1996) who introduce a new feature of the unit root for the variance. In fact, the fractionally integrated GARCH model (FIGARCH) highlights the fact that - unlike stationary processes where the persistence of volatility shocks is finite - in unit root processes, the impact of lagged errors occurs at a slow hyperbolic rate of decay. The FIGARCH model allows, thus, to capture the long memory in financial volatility with a complete flexibility regarding the persistence degree. In fact, the FIGARCH (1,d,1) formulation depends on fractional integration parameter (d) as follows:

$$\sigma_{i,t}^2 = \alpha_{i,0} + [1 - (1 - \beta_i(L))^{-1}(1 - \phi(L))(1 - L)^d]a_{i,t}^2 + \beta\sigma_{i,t-1}^2. \quad (7)$$

with $0 < d < 1$. When $d=1$, the FIGARCH (1,d,1) is equivalent to an IGARCH (1,1) where the persistence of conditional variance is supposed to be complete, while when $d=0$, it is rather equivalent to a GARCH (1,1) and no volatility persistence is taken into consideration. L is the lag operator and $(1 - L)^d$ is the financial fractional differencing operator.

Other ARCH formulations are extended to the fractionally integrated GARCH, including asymmetric leverage effect presented in the EGARCH model. Bollerslev and Mikkelsen (1996) propose a new class of model combining characteristics of the FIGARCH and the EGARCH models so-called FIEGARCH (p,d,q). Financial assets' volatilities are, thus, better explained

¹⁰Positive and negative financial shocks revamp asymmetrically the variance. Furthermore, bad news (shocks) generate greater volatility than good news.

¹¹The volatility's different reactions to signs and sizes of past innovations are also suggested in the Threshold Heteroskedastic model (TGARCH) of Zakoian (1994). The major difference is that in the TGARCH model the conditional standard deviation (σ_t) is considered rather than the conditional variance (σ_t^2).

¹²The autocorrelation function of time series returns decreases gradually.

and depicted by a mean-reverting fractionally integrated process. The FIEGARCH(1,d,1) model is written as follows:

$$\ln(\sigma_{i,t}^2) = \alpha_{i,0} + \phi(L)^{-1}(1-L)^{-d}[1 + \psi(L)]g(\varepsilon_{i,t-1}). \quad (8)$$

where $\phi(L)$ and $\psi(L)$ are lag polynomials, and - as in the EGARCH(1,1)¹³ - $g(\varepsilon_t)$ is a quantization function of information flows such as:

$$g(\varepsilon_t) = \theta\varepsilon_t + \gamma[|\varepsilon_t| - E(|\varepsilon_t|)].$$

An extension of the conventional fractionally integrated GARCH model is proposed by Tse (1998) so-called FIAPARCH (1,d,1). The new approach combines the long-range dependencies feature and the asymmetric impact of lagged positive and negative shocks on future volatilities in one fractionally integrated model. The FIAPARCH (1,d,1) is written as follows:

$$\sigma_{i,t}^\delta = \alpha_{i,0}(1-\beta)^{-1} + [1 - (1-\beta(L))^{-1}\phi(L)(1-L)^d](|a_{i,t} - \gamma a_{i,t}|)^\delta. \quad (9)$$

More recently, another linear GARCH model, called hyperbolic GARCH (HYGARCH) is proposed by Davidson (2004) who argues that the impact of lagged errors on the conditional variance discloses near-epoch dependence feature. The main contribution of this model is that the fractional integration parameter is negative (-d) instead of positive and that d increases rather when it approaches zero¹⁴. The statistical properties included in the HYGARCH make it the most successful and used approach by financial practitioners in modeling time series volatilities. The HYGARCH (1,d,1) is defined under the following formulation:

$$\sigma_{i,t}^2 = \alpha_{i,0} + [1 - (1-\beta(L))^{-1}(1-\phi(L))][1 + \alpha((1-L)^d - 1)]a_{i,t}^2. \quad (10)$$

As mentioned before, The 9 GARCH-class models, above cited, are estimated in the first step through a univariate process to capture the apparent conditional univariate standard deviations. Then, a multivariate process is estimated in the second step so we can have the conditional bivariate volatilities (conditional correlations).

3.2 Joint volatility processes: multivariate ARCH-type models

In the second step, the multivariate specification is introduced to each one of the GARCH-class models in order to estimate the bivariate dynamic conditional correlations. Following the propositions of Engle (2002) and Tse and Tsui (2002), the DCC model is defined as a time varying structure:

$$Var(x_t/\mathcal{F}_{t-1}) = \sigma_t^2 = \Omega_t = D_t H_t D_t. \quad (11)$$

where \mathcal{F}_{t-1} is the information set available at the time (t-1). D_t is a diagonal matrix $N \times N$ such as $D_t = diag\{\sigma_{ii,t}\} = diag(\sigma_{11,t} \dots \sigma_{NN,t})$, $\sigma_{NN,t}$ is the conditional standard deviation obtained from the univariate models. H_t is the time-varying correlation matrix of the standardized residuals ε_t (of x_t).

Engle (2002) and Tse and Tsui (2002) propose two DCC models presenting some similarities and divergences. For Engle (2002),

$$H_t = diag(q_{11,t}^{-1/2} \dots q_{NN,t}^{-1/2})Q_t diag(q_{11,t}^{-1/2} \dots q_{NN,t}^{-1/2}). \quad (12)$$

¹³When the memory parameter, d=0, the FIEGARCH formulation is equivalent to the conventional EGARCH (1,1) (FIEGARCH (1,0,1) \simeq EGARCH (1,1)).

¹⁴When d of the HYGARCH is positive, it is considered as a unit root process.

where $\overline{q_{ij}} = \overline{\rho_{ij}}$ ($\overline{\rho_{ij}}$ is the unconditional correlation between $\varepsilon_{i,t}$ et $\varepsilon_{j,t}$). Q_t is symmetric covariance $N \times N$ matrix that can be written as follows:

$$Q_t = \overline{Q_{ij}}(1 - \alpha - \beta) + \alpha(\varepsilon_{t-1}\varepsilon'_{t-1}) + \beta Q_{ij,t-1}. \quad (13)$$

with \overline{Q} is a symmetric time invariant unconditional correlation matrix. α et β parameters are positive and respect the constraint of $\alpha + \beta < 1$. The bivariate dynamic conditional correlation coefficient of Engle (2002) is, thus, defined as:

$$\rho_{12,t} = \frac{\overline{q_{12}}(1 - \alpha - \beta) + \alpha(\varepsilon_{1,t-1}\varepsilon_{2,t-1}) + \beta q_{12,t-1}}{\sqrt{(\overline{q_{11}}(1 - \alpha - \beta) + \alpha\varepsilon_{11,t-1}^2 + \beta q_{11,t-1})(\overline{q_{22}}(1 - \alpha - \beta) + \alpha\varepsilon_{22,t-1}^2 + \beta q_{22,t-1})}}. \quad (14)$$

For Tse and Tsui (2002), $H_t = \{\rho_{ij}, t\}$ is assumed to follow a univariate GARCH process and is obtained from the recursion of:

$$H_t = (1 - \theta_1 - \theta_2)H + \theta_1 H_{t-1} + \theta_2 \Xi_{t-1}. \quad (15)$$

where the parameters θ_1 and θ_2 are supposed to satisfy the non-negativity constraint and the inequality $\theta_1 + \theta_2 \leq 1$. H is a symmetric time invariant $N \times N$ with $\rho_{ii} = 1$ (a unit diagonal element). Ξ_{t-1} is the correlation matrix of the lagged standardized residuals ε_t such as $\varepsilon_{i,t} = \frac{a_t}{\sigma_{i,t}}$. Since Ξ_{t-1} depends on the lagged residuals, and analogously to the x_{t-1}^2 in the GARCH (1,1) representation, Ξ_{t-1} can be specified by the following formula:

$$\Xi_{ij,t-1} = \frac{\sum_{m=1}^M \varepsilon_{i,t-m}\varepsilon_{j,t-m}}{\sqrt{(\sum_{m=1}^M \varepsilon_{i,t-m}^2)(\sum_{m=1}^M \varepsilon_{j,t-m}^2)}}, 1 \leq i < j \leq N \leq M. \quad (16)$$

$N \leq M$ is a necessary condition for ψ_{t-1} and H_t to be nonnegative definite. Thus, in the bivariate case, the conditional correlation coefficient of Tse and Tsui (2002) is defined as:

$$\rho_{12,t} = (1 - \theta_1 - \theta_2)\rho_{12} + \theta_1\rho_{12,t-1} + \theta_2 \frac{\sum_{s=1}^S \varepsilon_{1,t-s}\varepsilon_{2,t-s}}{\sqrt{(\sum_{s=1}^S \varepsilon_{1,t-s}^2)(\sum_{s=1}^S \varepsilon_{2,t-s}^2)}}. \quad (17)$$

In this paper, dynamic conditional correlation coefficients are estimated using both models proposed by Engle (2002) and Tse and Tsui (2002): For each of the estimated model, time series are assumed to follow an AR(1) process with a normal and t-student marginal distributions. Models' performances comparison is made upon the value of the Akaike information criterion (AIC) and some multivariate diagnostic tests: Vector normality test and Hosking's Multivariate Portmanteau Statistics on standardized residuals (and on squared standardized residuals) are used to investigate the reliability of the multivariate GARCH specifications.

4 Data

The models detailed above are estimated using daily price changes data from the credit market of 33 countries worldwide belonging to four different economic status (low economic growth, developed countries, newly industrialized countries and emerging countries). DCC models are estimated on both credit spreads of sovereign CDS market and the corresponding government bond market, on a period spanning from January, 2nd 2006 until April, 3th 2014. CDS spreads and bond yields are extracted from Bloomberg® and Thomson Reuters®.

During the studied period (2006-2014), the world economy has been affected by two major crises - namely the Global Financial Crisis and the European Debt Crisis - and the state borrowing costs greatly fluctuated whatever the country's economic level is or its geographical

position. Thus, evaluating and comparing the performances of multivariate volatility models is particularly relevant during this period.

For sake of homogeneity, bond yields are transformed into spreads regarding the risk-free interest rate. Several studies interest in the issue of the appropriate choice of this riskless rate. A first strand of the literature suggests the use of the US treasury yield (Longstaff et al., 2005) or the swap rate of the same maturity (Blanco et al., 2005). Another strand promotes the use of the bond rate issued by the country considered to be the least risky of the zone (government bond yield curves of Deutsch Bundesbank, the Federal Reserve Board and the bank of England) (Norden and Weber, 2009; Gilchrist and Mojon, 2016). In this paper, the 5 year German federal government bond is chosen as a reference rate for European countries, and the United States sovereign bonds for American and Asian countries. In order to not reduce our sample, for Germany, euro-area generic bond is used as a benchmark yield, and the US Treasury Zero-Coupon Yield Curve for the United States.

Descriptive statistics are provided in Table 1. CDS and Bond spreads exhibit similar statistical properties. The average daily spreads are almost always positive (except for Germany, USA and Japan where the Bond spreads' mean is negative). Negative credit spreads can be explained by the fact that during recessions markets participants are willing to pay government for riskless investments, making bonds of some countries (Germany among others) issued with negative yields (Dolvin, 2012). This phenomenon can be caused, likewise, by a temporary liquidity factors (Beber et al., 2009; Bhanot and Guo, 2011). Another explanation is that these negative spreads are due to a mis-choice of the risk-free rate. The high levels of standard deviations show that our data is widely spread and not centered around the mean.

The Jarque-Bera test confirms the non-normality of the data distribution at the 1% significance level. These findings are reaffirmed but the significant excess kurtosis and the positive skewness coefficients. As expected, the Augmented Dickey-Fuller test confirms the null hypothesis of the unit-root presence. Both CDS spreads and bond spreads are non-stationary. The ARCH-LM test and the Q statistics of Ljung-Box both confirm the presence of ARCH effect and reject the null hypothesis of no serial correlation in the CDS and Bond spreads.

5 Empirical results

For each of the 33 countries composing our sample, 9 DCC models are estimated following Engle (2002) and Tse and Tsui (2002). Results of the conditional multivariate volatility models are not reported here due to space limits but are available upon request. Table 2 reports diagnostic checking tests for different models of some chosen countries.

Table 2: Dynamic Conditional Correlation models: Some multivariate diagnostic tests

		Austria						Spain						Brazil						Russia					
Countries		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)	
Distribution	Information Criterion	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student
		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Numerical convergence		9.287	8.691	9.286	8.691	12.504	11.619	12.515	11.619	13.747	12.400	13.746	12.400	13.024	12.245	13.015	12.248								
Vector normality test		2140.6 ***	2161.1 ***	2139.6 ***	2144.4 ***	5753.6 ***	8313.3 ***	7212.2 ***	9465.4 ***	2.1 ***	21419.0 ***	21323.0 ***	21419.0 ***	4535.7 ***	4581.7 ***	4530.4 ***	4517.2 ***								
Hosking's Multivariate		61.845 ***	59.946 ***	63.639 ***	62.915 ***	68.982 ***	70.408 ***	72.748 ***	69.354 ***	45.116 ***	44.475 ***	44.273 ***	44.475 ***	78.465 ***	74.480 ***	80.472 ***	80.112 ***								
Portmanteau Statistics on		86.663 ***	85.059 ***	87.524 ***	87.110 ***	82.492 ***	83.633 ***	86.052 ***	82.740 ***	64.472 ***	65.230 ***	65.548 ***	65.230 ***	98.707 ***	95.172 ***	100.348 ***	99.892 ***								
standardized residuals		124.682 ***	123.461 ***	125.525 ***	125.545 ***	127.578 ***	130.453 ***	130.133 ***	127.668 ***	94.396 ***	95.395 ***	94.824 ***	95.395 ***	140.639 ***	135.520 ***	143.794 ***	142.658 ***								
Multivariate diagnostic tests		238.177 **	235.964 **	237.767 **	237.607 **	242.304 **	248.300 **	246.113 **	244.775 **	241.293 **	248.345 **	246.638 **	248.345 **	256.031 **	250.422 **	257.080 **	255.635 **								
Hosking's Multivariate		8.369	7.766	7.797	7.753	3.727	3.259	3.413	3.320	2.993	3.279	2.932	3.279	16.136	16.159	16.202	16.168								
Portmanteau Statistics on		17.068	16.507	16.645	16.547	7.079	6.298	6.211	6.211	8.461	8.651	8.278	8.651	20.862	20.570	20.901	20.876								
squared standardized residuals		36.90	36.38	36.36	36.20	59.705	66.107	55.626	63.492	13.875	14.310	13.675	14.310	51.302	48.761	51.590	51.754								
residuals		126.34	123.56	123.34	122.81	122.81	96.601	100.240	91.824	100.938	136.426	140.120	131.720	140.120	85.666	81.152	86.598								
The Hosking's Portmanteau test is the multivariate version of the Box-Pierce Q statistic under the null hypothesis of no serial correlation. *, **, and *** denotes the rejection of the null hypothesis at the 1%, 5% and 10% level of significance.																									
		Belgium						Italy						Turkey						Hungary					
Countries		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)	
Distribution	Information Criterion	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student
		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Numerical convergence		10.583	9.784	10.584	9.785	11.246	10.929	11.267	10.938	15.182	14.983	15.182	14.983	14.062	13.623	14.061	13.623								
Vector normality test		1135.0 ***	1136.0 ***	1133.0 ***	1132.0 ***	1197.7 ***	1233.0 ***	1212.0 ***	1233.6 ***	932.1 ***	946.9 ***	954.6 ***	958.7 ***	2896.3 ***	3035.3 ***	2801.6 ***	3017.4 ***								
Hosking's Multivariate		98.674 ***	98.440 ***	99.201 ***	99.748 ***	94.736 ***	99.781 ***	104.750 ***	105.510 ***	126.473 ***	126.153 ***	128.036 ***	127.658 ***	113.192 ***	116.127 ***	117.225 ***	119.705 ***								
Portmanteau Statistics on		116.302 ***	116.103 ***	116.392 ***	117.075 ***	110.940 ***	116.331 ***	121.276 ***	122.166 ***	143.777 ***	143.222 ***	144.961 ***	144.575 ***	132.288 ***	134.514 ***	136.604 ***	138.121 ***								
standardized residuals		155.775 ***	155.309 ***	156.018 ***	156.766 ***	181.443 ***	186.748 ***	189.093 ***	190.494 ***	188.842 ***	187.851 ***	188.940 ***	187.851 ***	177.086 ***	178.060 ***	181.455 ***	181.455 ***								
Multivariate diagnostic tests		291.862 ***	290.224 ***	292.699 ***	293.245 ***	356.740 ***	362.413 ***	365.798 ***	367.116 ***	296.192 ***	295.874 ***	297.636 ***	297.450 ***	276.710 ***	283.619 ***	281.020 ***	286.444 ***								
Hosking's Multivariate		16.140	16.245	15.955	16.078	25.044	25.071	24.480	24.573	28.142 *	27.884 *	27.096 *	27.172 *	33.234 **	35.206 **	37.697 **	40.530 **								
Portmanteau Statistics on		27.907	27.846	27.768	27.804	48.637	49.465	48.904	49.286	42.102	41.900	40.992	41.116	52.620 *	54.559 *	58.334 *	57.760 *								
squared standardized residuals		88.67	87.89	88.99	89.12	97.378 **	99.019 *	97.311 *	98.042 *	87.236	86.173	85.678	84.763	73.289	74.412	79.721	77.375								
residuals		238.40 **	235.51 **	238.64 **	238.27 **	240.069 **	243.570 **	238.583 **	239.994 **	165.690	164.886	163.999	163.452	139.906	133.078	141.468	136.202								
The Hosking's Portmanteau test is the multivariate version of the Box-Pierce Q statistic under the null hypothesis of no serial correlation. *, **, and *** denotes the rejection of the null hypothesis at the 1%, 5% and 10% level of significance.																									
		Finland						Portugal						Latvia						Romania					
Countries		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)	
Distribution	Information Criterion	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student
		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Numerical convergence		7.616	6.571	7.616	6.572	13.240	12.837	13.270	12.848	13.220	10.878	13.215	10.855	13.126	12.310	13.124	12.308								
Vector normality test		5279.9 ***	5265.7 ***	5275.2 ***	5268.7 ***	222.7 ***	229.2 ***	233.70 ***	233.62 ***	50128.0 ***	39111.0 ***	49566.0 ***	17828.0 ***	5289.4 ***	5272.2 ***	5284.6 ***	5648.0 ***								
Hosking's Multivariate		37.736 ***	37.098 ***	37.887 ***	37.800 ***	109.444 ***	112.479 ***	127.546 ***	116.353 ***	56.330 ***	42.397 ***	53.599 ***	50.762 ***	63.228 ***	62.052 ***	62.877 ***	60.605 ***								
Portmanteau Statistics on		51.113 *	50.277 *	51.145 *	51.032 *	137.712 ***	140.620 ***	154.723 ***	143.215 ***	111.421 ***	72.175 ***	105.111 ***	91.847 ***	165.047 ***	162.170 ***	163.004 ***	162.048 ***								
standardized residuals		109.511 **	108.398 **	109.439 **	109.610 **	178.501 **	181.290 **	193.954 **	183.167 **	339.245 **	364.768 **	329.854 **	341.145 **	245.127 **	240.489 **	243.095 **	239.530 **								
Multivariate diagnostic tests		228.053 *	227.045 *	228.321 *	228.242 *	301.254 **	304.544 **	319.241 **	306.360 **	475.888 **	436.216 **	455.480 **	513.120 **	483.721 **	474.735 **	480.532 **	471.585 **								
Hosking's Multivariate		1.442	1.442	1.444	1.449	8.083	7.947	8.958	8.080	2.784 **	0.294 **	2.722	1.668	13.692	13.772	13.676	13.676								
Portmanteau Statistics on		3.113	3.104	3.112	3.105	17.593	17.500	18.907	17.646	3.532	3.434	3.532	3.434	69.457 **	72.134 **	66.303 **	80.338 **								
squared standardized residuals		7.601	75.791	75.365	75.041	118.285 **	120.495 **	124.375 **	121.804 **	1143.380	1430.520	1152.630 ***	1621.130 ***	108.545 **	111.592 **	105.417 **	119.874 **								
residuals		162.616	161.413	161.360	160.936	212.402	212.600	212.535	211.250	1146.000 **	1155.620 **	1622.060 **	169.150	171.709	166.092	179.283	179.283								
The Hosking's Portmanteau test is the multivariate version of the Box-Pierce Q statistic under the null hypothesis of no serial correlation. *, **, and *** denotes the rejection of the null hypothesis at the 1%, 5% and 10% level of significance.																									

Countries		Denmark						Ireland						Turkey						Croatia					
Dynamic correlation models (DCC-CJR(L,1))		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)					
Distribution		Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student				
Numerical convergence		Yes	Yes	Yes	No	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes				
Information Criterion		AIK	8,135	7,026	8,135	13,450.0 ***	13,450.0 ***	12,784	11,683	-	11,662	15,143	14,974	15,141	14,973	763.5 ***	763.5 ***	14,672.0 ***	14,349.0 ***	14,681.0 ***	14,726.0 ***				
Vector normality test		AIK	5	25,641	25,703	134,500 ***	134,500 ***	114,831 ***	88,525 ***	-	102,185 ***	123,968 ***	123,918 ***	126,657 ***	126,171 ***	52,276 ***	52,144 ***	146,720 ***	143,449.0 ***	146,810 ***	147,260 ***				
Hosking's Multivariate		5	3,794	35,855	35,794	142,999 ***	108,844 ***	142,999 ***	108,844 ***	-	128,761 ***	143,495 ***	143,279 ***	145,848 ***	145,352 ***	67,483 ***	67,058 ***	113,752 ***	113,536 ***	114,435 ***	112,570 ***				
Portmanteau Statistics on		10	74,185	74,153	74,185	171,544 ***	132,305 ***	171,544 ***	132,305 ***	-	155,772 ***	190,138 ***	189,811 ***	191,219 ***	190,880 ***	113,752 ***	113,536 ***	208,437 ***	208,437 ***	208,111 ***	206,004 ***				
standardized residuals		20	249,156 ***	249,397 ***	249,156 ***	287,530 ***	237,662 ***	287,530 ***	237,662 ***	-	276,710 ***	299,412 ***	299,739 ***	302,245 ***	302,171 ***	207,720	208,437 ***	4,684	4,684	4,684	4,684				
Hosking's Multivariate		5	0,295	0,295	0,295	1,382	0,693	1,382	0,693	-	1,204	26,298	26,254	26,254	26,377	4,991	4,991	5,868	5,697	5,868	5,613				
Portmanteau Statistics on		10	0,716	0,714	0,716	2,776	1,495	2,776	1,495	-	2,695	42,967	42,769	42,834	42,519	5,868	5,697	11,046	10,788	10,883	10,873				
squared standardized		20	1,343	1,341	1,343	7,631	3,723	7,631	3,723	-	7,352	88,262	87,891	88,935	88,190	23,518	23,034	23,518	23,034	23,229	23,309				
residuals		50	36,188	36,083	36,188	19,991	14,756	19,991	14,756	-	31,482	169,364	169,313	170,099	169,461	23,518	23,034	23,518	23,034	23,229	23,309				
The Hosking's Portmanteau test is the multivariate version of the Box-Pierce Q statistic under the null hypothesis of no serial correlation. *, ** and *** denotes the rejection of the null hypothesis at the 1%, 5% and 10% level of significance.																									
Countries		Netherlands						Italy						China						Latvia					
Dynamic correlation models (DCC-APARCH (1,1))		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)					
Distribution		Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student				
Numerical convergence		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes				
Information Criterion		AIK	8,012	7,113	8,013	4,354.7 ***	4,333.8 ***	4,354.7 ***	4,333.8 ***	4,317.0 ***	4,317.0 ***	11,179	10,887	11,210	10,897	11,175	11,580	11,175	11,580	11,175	13,221				
Vector normality test		AIK	5	40,982 ***	42,966 ***	43,226 ***	43,338 ***	44,129 ***	44,129 ***	90,634 ***	97,912 ***	101,122 ***	101,122 ***	123,533 ***	123,533 ***	480,573 ***	480,573 ***	480,573 ***	480,573 ***	480,573 ***	480,573 ***	480,573 ***			
Hosking's Multivariate		10	56,147 ***	57,923 ***	57,923 ***	105,809 ***	105,809 ***	105,103 ***	105,103 ***	173,437 ***	181,126 ***	181,126 ***	181,126 ***	184,057 ***	184,057 ***	184,948 ***	146,242 ***	146,242 ***	146,242 ***	146,242 ***	146,242 ***	146,242 ***			
Portmanteau Statistics on		20	236,249 ***	238,154 ***	238,154 ***	239,801 ***	240,149 ***	240,149 ***	240,149 ***	342,986 ***	351,243 ***	351,243 ***	351,243 ***	355,381 ***	355,381 ***	355,381 ***	259,628 ***	259,628 ***	259,628 ***	259,628 ***	259,628 ***	259,628 ***			
standardized residuals		50	9,836	9,828	9,828	9,876	9,876	9,876	9,876	37,925 ***	38,503 ***	38,503 ***	38,698 ***	38,698 ***	38,698 ***	15,132	15,124	15,124	15,124	15,124	15,124	15,124			
Hosking's Multivariate		5	13,329	13,279	13,279	13,375	13,249	13,249	13,249	60,599 ***	62,305 ***	62,305 ***	62,375 ***	62,375 ***	62,665 ***	19,740	19,725	19,725	19,725	19,725	19,725	19,725			
Portmanteau Statistics on		10	26,266	26,254	26,254	26,515	26,279	26,279	26,279	100,213 ***	102,322 ***	102,322 ***	101,702 ***	102,055 ***	102,055 ***	54,857	54,833	54,833	54,833	54,833	54,833	54,833			
squared standardized		20	90,096	89,942	89,942	90,505	89,943	89,943	89,943	222,544	225,654	225,654	224,032	224,598	224,598	127,619	127,561	127,561	127,561	127,561	127,561	127,561			
residuals		50	90,096	89,942	89,942	90,505	89,943	89,943	89,943	222,544	225,654	225,654	224,032	224,598	224,598	127,619	127,561	127,561	127,561	127,561	127,561	127,561			
The Hosking's Portmanteau test is the multivariate version of the Box-Pierce Q statistic under the null hypothesis of no serial correlation. *, ** and *** denotes the rejection of the null hypothesis at the 1%, 5% and 10% level of significance.																									
Countries		Japan						Portugal						Brazil						Czech					
Dynamic correlation models (DCC-FIGARCH (1,1))		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)		DCC (Engle)		DCC (Tse and Tsui)					
Distribution		Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student	Gauss	Student				
Numerical convergence		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes				
Information Criterion		AIK	23,208	22,428	23,207	22,428	22,428	22,428	22,428	13,358	12,935	13,352	12,930	14,814	13,624	14,814	13,644	11,475	11,023	11,476	11,026				
Vector normality test		AIK	5	195,419 ***	193,301 ***	189,269 ***	189,269 ***	189,269 ***	189,269 ***	7088 ***	6812 ***	7082 ***	7082 ***	16,037 ***	16,037 ***	16,037 ***	16,037 ***	16,037 ***	16,037 ***	16,037 ***	16,037 ***				
Hosking's Multivariate		10	231,197 ***	229,332 ***	229,332 ***	229,332 ***	229,332 ***	229,332 ***	229,332 ***	160,892 ***	160,892 ***	160,892 ***	160,892 ***	160,892 ***	160,892 ***	160,892 ***	160,892 ***	160,892 ***	160,892 ***	160,892 ***	160,892 ***				
Portmanteau Statistics on		20	916,837 ***	899,809 ***	899,809 ***	899,809 ***	899,809 ***	899,809 ***	899,809 ***	1,914,667 ***	1,914,667 ***	1,914,667 ***	1,914,667 ***	1,914,667 ***	1,914,667 ***	1,914,667 ***	1,914,667 ***	1,914,667 ***	1,914,667 ***	1,914,667 ***	1,914,667 ***				
standardized residuals		50	416,201 ***	413,917 ***	413,917 ***	413,917 ***	413,917 ***	413,917 ***	413,917 ***	410,050 ***	410,050 ***	410,050 ***	410,050 ***	410,050 ***	410,050 ***	410,050 ***	410,050 ***	410,050 ***	410,050 ***	410,050 ***	410,050 ***				
Hosking's Multivariate		5	93,443 ***	99,667 ***	99,667 ***	99,667 ***	99,667 ***	99,667 ***	99,667 ***	10,281	10,344	10,449	10,635	10,635	10,635	5,286	5,286	5,286	5,286	5,286	5,286				
Portmanteau Statistics on		10	108,210 ***	114,823 ***	114,823 ***	114,823 ***	114,823 ***	114,823 ***	114,823 ***	20,934	21,087	21,635	21,912	21,912	21,912	7,523	7,523	7,523	7,523	7,523	7,523				
squared standardized		20	177,133 ***	184,612 ***	183,667 ***	183,667 ***	183,667 ***	183,667 ***	183,667 ***	102,146 ***	102,289 ***	103,635 ***	104,681 ***	104,681 ***	104,681 ***	24,064	24,064	24,064	24,064	24,064	24,064				
residuals		50	308,321 ***	315,740 ***	314,098 ***	314,098 ***	314,098 ***	314,098 ***	314,098 ***	217,324	217,543	218,916	223,059	223,059	223,059	148,552	148,552	148,552	148,552	148,552	148,552				
The Hosking's Portmanteau test is the multivariate version of the Box-Pierce Q statistic under the null hypothesis of no serial correlation. *, ** and *** denotes the rejection of the null hypothesis at the 1%, 5% and 10% level of significance.																									

Countries				Slovenia				Ireland				China				Ukraine					
Dynamic correlation models (DCC-FIEGARCH (1,1))				DCC (Tse and Tsui)				DCC (Engle)				DCC (Tse and Tsui)				DCC (Engle)					
Distribution				Gauss		Student		Gauss		Student		Gauss		Student		Gauss		Student			
Numerical convergence				Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes		
Information Criterion				AIK	13,374	11,656	13,400	11,634	12,156	11,249	12,196	11,556	11,467	13,735	11,770	13,735	11,770	13,735	11,770		
Vector normality test				Chi ² (4)	9783.5	16826.0	9781.2	13371.0	15618.0	15724.0	14888.0	6685.6	6686.5	21194.0	21236.0	21236.0	21236.0	21236.0	21236.0	21236.0	
Hosking's Multivariate				5	70,015	66,408	71,311	65,536	111,050	109,099	105,710	81,820	81,808	82,089	172,860	173,449	172,860	173,449	172,860	173,449	
Portmanteau Statistics on				10	105,848	100,770	105,643	98,580	154,133	152,679	149,444	146,488	107,736	108,177	107,727	277,286	278,032	277,587	278,032	277,587	278,032
standardized residuals				20	199,377	195,761	193,670	199,797	198,749	198,330	198,699	199,174	159,310	159,641	159,310	333,756	334,083	333,756	334,083	333,756	334,083
Multivariate diagnostic checking				50	367,991	366,017	364,747	391,678	393,563	394,530	406,354	412,436	283,577	283,995	283,577	468,567	468,720	468,567	468,720	468,567	468,720
Hosking's Multivariate				5	3,155	8,017	3,271	3,589	2,123	2,114	1,967	10,799	10,798	10,837	23,514	23,514	23,514	23,514	23,514	23,514	
Portmanteau Statistics on				10	5,505	10,079	5,821	5,690	3,422	3,395	3,093	2,739	20,751	20,746	20,853	39,781	40,109	39,781	40,109	39,781	40,109
squared standardized residuals				20	38,728	28,761	42,306	26,739	21,292	21,344	22,509	45,062	45,057	45,057	70,474	70,761	70,474	70,761	70,474	70,761	
Hosking's Multivariate				50	67,809	52,945	72,633	112,481	136,346	137,025	139,518	143,043	128,913	129,811	128,913	175,215	175,567	175,215	175,567	175,215	175,567

The Hosking's Portmanteau test is the multivariate version of the Box-Pierce Q statistic under the null hypothesis of no serial correlation. *, **, and *** denotes the rejection of the null hypothesis at the 1%, 5% and 10% level of significance.

Countries				Sweden				Greece				Turkey				Poland			
Dynamic correlation models (DCC-FIAPARCH (1,1))				DCC (Tse and Tsui)				DCC (Engle)				DCC (Tse and Tsui)				DCC (Engle)			
Distribution				Gauss		Student		Gauss		Student		Gauss		Student		Gauss		Student	
Numerical convergence				Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Information Criterion				AIK	7,655	6,776	7,655	6,776	16,643	15,064	16,653	15,106	14,942	15,102	14,940	11,713	11,529	11,713	11,529
Vector normality test				Chi ² (4)	3008.4	3011.0	3013.6	3013.6	10907.0	10794.0	10968.0	11434.0	705.6	709.8	699.0	1022.1	1020.1	1012.1	1015.6
Hosking's Multivariate				5	41,755	41,481	41,669	41,180	126,337	105,698	121,428	102,031	127,922	124,841	127,282	63,961	63,852	63,549	63,762
Portmanteau Statistics on				10	69,052	68,606	69,017	68,496	208,221	185,072	204,933	186,068	145,336	147,710	147,506	79,417	79,470	78,609	78,901
standardized residuals				20	128,905	128,602	128,925	128,483	307,803	283,329	303,129	281,371	194,792	194,434	195,815	195,780	141,211	140,899	141,712
Multivariate diagnostic tests				50	251,374	250,921	250,706	249,869	595,438	563,512	589,652	574,956	307,092	306,915	309,249	229,279	228,690	231,321	231,248
Hosking's Multivariate				5	4,641	4,624	4,631	4,617	143,552	108,661	144,335	94,906	24,019	23,732	14,496	14,248	13,587	13,377	
Portmanteau Statistics on				10	10,698	10,673	10,687	10,654	157,071	120,461	156,861	103,713	38,084	37,646	37,933	37,600	24,282	23,987	23,154
squared standardized residuals				20	17,434	17,398	17,382	17,308	171,928	133,787	171,287	115,507	86,548	85,375	86,787	85,823	48,170	47,643	46,828
Hosking's Multivariate				50	52,454	52,327	52,407	52,256	296,042	241,469	290,054	203,759	167,972	166,812	169,010	167,838	154,229	153,020	152,916

The Hosking's Portmanteau test is the multivariate version of the Box-Pierce Q statistic under the null hypothesis of no serial correlation. *, **, and *** denotes the rejection of the null hypothesis at the 1%, 5% and 10% level of significance.

Countries				Finland				Ireland				Norway				Lithuania			
Dynamic correlation models (DCC-HYGARCH (1,1))				DCC (Tse and Tsui)				DCC (Engle)				DCC (Tse and Tsui)				DCC (Engle)			
Distribution				Gauss		Student		Gauss		Student		Gauss		Student		Gauss		Student	
Numerical convergence				Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Information Criterion				AIK	25,774	24,192	25,772	24,192	12,561	11,547	-	11,513	22,029	20,812	22,027	20,812	14,859	13,542	14,854
Vector normality test				Chi ² (4)	3866.4	3865.4	3865.4	3865.4	12755.0	13258.0	-	12907.0	4084.5	4626.1	4190.7	6055.9	6097.6	6179.9	6031.3
Hosking's Multivariate				5	67,495	67,545	67,545	67,553	110,461	112,673	-	108,010	111,048	88,968	19,378	38,882	61,640	61,527	61,586
Portmanteau Statistics on				10	89,029	89,074	89,074	89,081	145,623	147,495	-	143,181	161,022	80,469	56,278	80,404	98,653	98,647	98,794
standardized residuals				20	153,155	153,200	153,200	153,204	172,657	173,584	-	168,293	215,205	112,314	100,070	112,206	132,728	132,576	132,892
Multivariate diagnostic checking				50	298,260	298,334	298,239	298,249	287,436	287,895	-	284,058	423,003	281,108	224,093	280,985	244,347	243,969	245,918
Hosking's Multivariate				5	398,057	398,331	398,141	398,334	0,989	0,851	-	0,899	574,787	597,806	67,623	597,806	25,106	24,934	25,114
Portmanteau Statistics on				10	631,940	632,431	632,486	632,486	1,574	1,504	-	1,674	867,268	886,829	110,580	887,559	167,567	164,611	170,734
squared standardized residuals				20	1111,300	1111,960	1112,485	1112,030	6,082	5,997	-	6,367	1280,460	1163,640	213,166	1164,660	182,094	178,979	185,472
Hosking's Multivariate				50	2360,940	2361,960	2361,270	2362,030	16,259	16,184	-	18,971	2470,900	1923,520	398,211	1925,200	234,936	232,898	239,021

The Hosking's Portmanteau test is the multivariate version of the Box-Pierce Q statistic under the null hypothesis of no serial correlation. *, **, and *** denotes the rejection of the null hypothesis at the 1%, 5% and 10% level of significance.

DCC-FIGARCH, DCC-FIAPARCH and DCC-HYGARCH are the most difficult models to estimate with miss-convergences in at least 14 among the 33 countries. At the opposite, the models that great perform as to strong numerical convergence and computing-time delay are the DCC-GARCH, the DCC-IGARCH and the fractional model DCC-FIEGARCH. Among fractionally integrated models, the DCC-FIAPARCH is the most miss-specified model with a non-convergence in 18 countries.

Taking into account different financial markets' properties (long memory behavior, shock persistence and asymmetric leverage effects...) does not necessarily improve the models performances. The complexity of the model's statistical specifications can not be justified in all cases leading to inconsistent and inaccurate estimator parameters. However, the opposite may be true since the more the model is over-parametrized, the more its computation and its convergence are complicated.

The Akaike information criterion (AIK) doesn't allow us to select only one optimal model. AIK results of the studied models are mitigated across each country of the sample. The only conclusion that can be drawn, is that the information criterion is almost always lower for the student marginal distribution. Without knowing the facts, allowing the data to follow a student rather than a Gaussian distribution law make a better fit of dynamic conditional correlation models. This confirms the heavy tails and the slightly skewed data distributions suggested by the Jarque-Bera preliminary test.

The vector normality tests reject the null hypothesis of the linearity of combinations between innovations of all models and for all countries under study. This means that combinations between CDS and bond spreads have too many extreme values (significant excess kurtosis) and a fat-tailed distribution. The assumption of a multivariate student marginal distribution of our time series is once again preferred.

Results of the diagnostic tests on the standardized residuals show that there is an obvious misspecification in the conditional mean equations of all the estimated models (The multivariate Q-statistics is almost always significant at 5% whatever the number of lags is). One explanation is that 1 is not the optimal order number of AR(.) suggesting a more persistent (less quickly) shocks' integration in the credit markets. The autoregressive terms can be allowed to vary from zero to three so we can better control for the serial correlation of CDS and bond spreads times series.

Misspecification are also detected in the conditional multivariate variance of some DCC models. ARCH effects are still present in the squared standardized residuals even after controlling for the heteroscedasticity. The best fitted models, in terms of autocorrelation in the time series, are DCC-GARCH and the DCC-FIEGARCH. The Hoskings' multivariate portmanteau statistics, up to 10th, 20th and 50th order of lags, accept the null hypothesis of no correlation in the sovereign credit assets. However, in the same context, the APARCH doesn't perform good with serial correlations detected in all combinations of innovations for all countries.

The last striking finding is that the Engle's DCC and Tse and Tsui's DCC present very close results and no significant divergence is detected. Focusing on the DCC plots of the Engle and the Tse and Tsui (Fig. 1 and Fig. 2), it is clearly seen that the correlation's evolution dynamic of both estimation techniques is almost the same for all the countries (Except for Japan, Denmark, Norway and China).

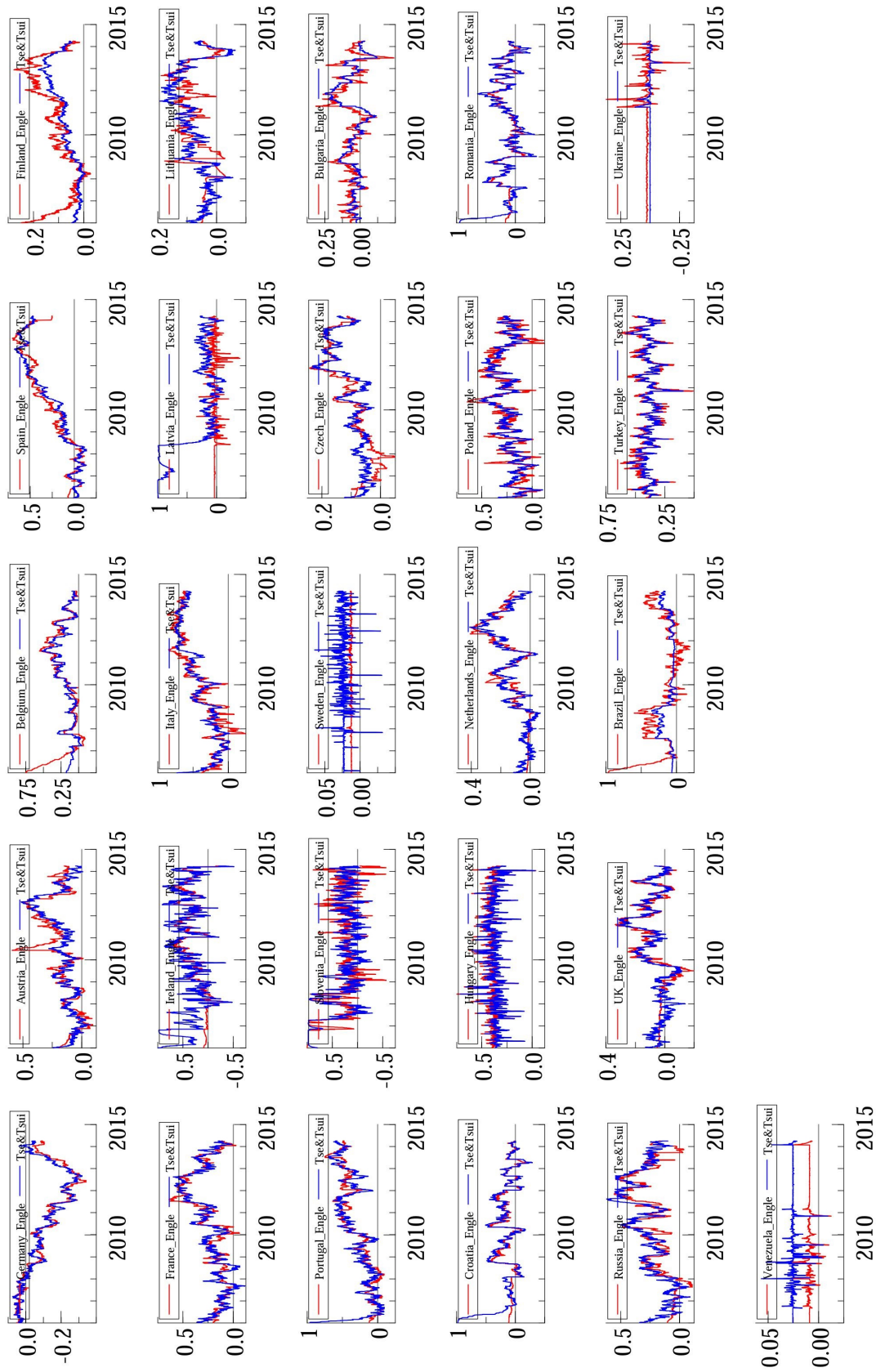


Figure 1: Engle and Tse&Tsui's GARCH(1,1)-DCC correlations plots of some chosen countries

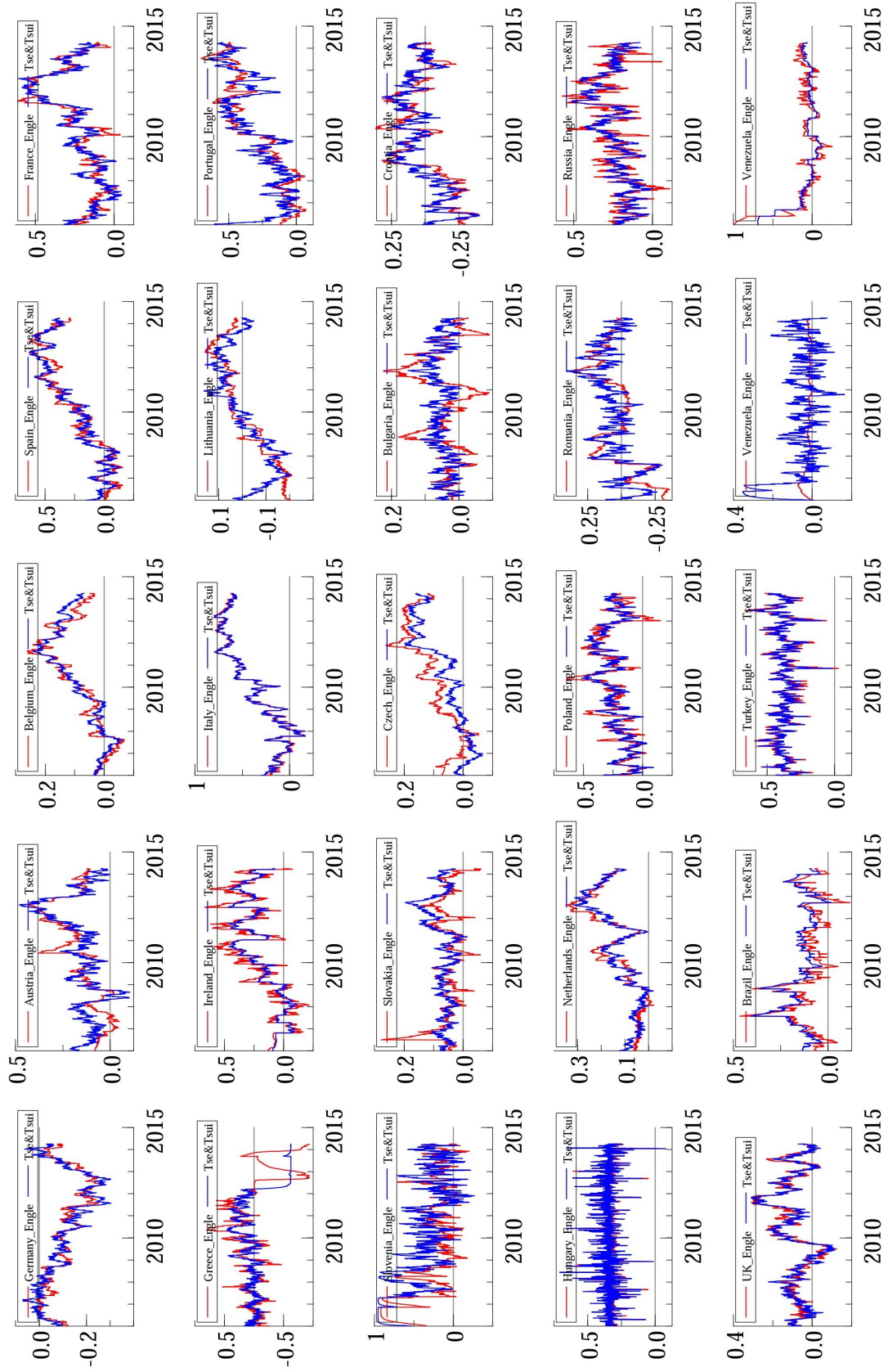


Figure 2: Engle and Tse&Tsui's GARCH(1,1)-DCC correlations plots of some chosen countries

6 Conclusion

This paper aimed to assess the performances of 9 Dynamic Conditional Correlation models. Using sovereign CDS and bond data, DCC-GARCH, DCC-IGARCH, DCC-EGARCH, DCC-GJR, DCC-APARCH, DCC-FIGARCH, DCC-FIEGARCH, DCC-FIAPARCH, DCC-HYGARCH are estimated, allowing to take into account different financial markets' properties such as the leverage effect, the asymmetric reaction to good and bad news and long-range persistence. The performance comparison being made upon the information criteria and several multivariate diagnostic tests, a certain number of conclusions can be drawn.

First, although the CDS and bond data exhibit long memory behavior, shock persistence and asymmetric leverage effects, models taking into account all these specifications does not provide better estimation results. The straightforward DCC-GARCH model seems to provide the most relevant estimator parameters in terms of diagnostic tests. Second, the innovations distributions assumption significantly impacts the statistical fit of the model since the information criterion is almost always lower for models with student marginal distribution compared to models with Gaussian distribution. Third, all models present serial correlation in the standardized residuals indicating a misspecification in the conditional mean equation. This is might be due low order of the AR process and to the persistence of shocks in worldwide credit markets. In addition, a constantly time-varying and different hedging positions should be taken across countries that present dissimilar financial characteristics.

Since modeling correlation is of a paramount importance in assessing diversification risk, in dynamic asset pricing theory and in optimization of portfolio allocation, the economic implication of our findings concerns particularly policymakers, financial practitioners and financial markets participants generally. Even though no model clearly outperforms all the others, our results show that the basic DCC-GARCH and the DCC-FIEGARCH model seems to better fit the studied data. However, results are mitigated and differ from one country to another which implies that no multivariate volatility model should be selected in an arbitrary way. The model selection should rather be based on the particular features of the data used and the countries composing the sample studied.

Our research line can be pursued in several ways. First, a further investigation on the performances of the Dynamic Conditional Correlation models can be done by carrying out a comparative study based of the predictive reliabilities rather than on diagnostic tests as in our case. Second, our study can be generalized to other financial markets, namely the primary equity markets, in order to asses whether the nature of the asset impact the performances of the studied models. Third, since there is a dynamic segmentation in financial markets, it can be interesting to check the robustness of our findings using a different sample from other regions.

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