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## Model reduction for studying a Bike Sharing System as a closed queuing network

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### Abstract

The users of a B2C Bike Sharing System are satisfied if they find always free bikes and free docks at the stations. Modeling such systems is challenging because they are large-scaled and characterized by their stochastic dynamic. To study the performance of a BSS, we rely on a queuing model solved by the Entropy Maximization. In this paper, we study the possibility of aggregating a number of stations in a single virtual station to reduce the complexity of the model and its resolution. Experiments are performed to qualify impacts of the model reduction on the accuracy of the obtained results.

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**Keywords:** Bike Sharing System; Performance Evaluation; Closed Queuing Network; Repetitive Service Random Destination; Entropy Maximization; Model reduction

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### 1. Introduction

The Business-to-Customer Bike Sharing System (BSS) is a successful scheme of green transportation. Since its first introduction in Amsterdam in 1965, it has been spread out into many cities. A BSS consists of stations distributed throughout the city. Every station has a docking capacity. Users pick up a bike from any station and drop it back to another. The bike imbalance at stations is one of the main concerns in such systems. In fact, the attractiveness of stations differs according to their location and to the time window of the day. For instance, stations close to the touristic zones are more likely to be full of bikes in the morning than the same stations in the evening.

To ensure an appropriate balance of bikes among stations, the BSS operators monitor and control the system as a whole by redistributing bikes (by trucks) among stations for example. Such regulation operations dispatch appropriate proportions of bikes among stations, mainly in short-term basis (every day for instance). Other control actions, such as fleet sizing or capacity sizing, are operated for longer time horizons.

Recently, some research papers have been interested in BSSs seeking for performance improvement. However, this turns to be tough because modeling a BSS is challenging. A BSSs parameters are stochastic. This is the case for

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instance of the users inter-arrival time, the choice of destination and the duration of trips [1]. There is a vast amount of literature tackling the redistribution of bikes problem using deterministic mathematical programming models [1,2]. Interested readers may refer to [3] for a survey of such techniques. The deterministic models remain nonetheless a restriction of the reality. Other research works used stochastic models including simplifications such as considering an infinite capacity of the stations [4–6]. It would be interesting to have a model of the BSS gathering several aspects simultaneously: the randomness of parameters, the limited capacity of the stations, the routing of the blocked users to the neighboring stations due to the saturation of the destination stations. This is the goal we are following in our research activities. The paper begins with a brief presentation of a queuing network model under a Repetitive Service Random Destination Blocking for a BSS [7]. This model was formerly developed in [8]. This model is stochastic and represents more faithfully a BSS since it considers the limited capacity of stations and it permits to model the movement of users rejected by full stations. The resolution for such model, i.e. finding the probabilities of the different states of the network, is time consuming. Our aim in this work is to discuss a possible reduction of the complexity of the resolution by aggregating several stations into one virtual station. We compare the accuracy of reduced models regarding the relative deviation of the calculated performance indicators and the reduced resolution time. The rest of this paper is organized as follows. Section 2 discusses briefly the state of the art. In section 3, we present the stochastic model for the BSS. The resolution technique, i.e. the Entropy Maximization, is briefly presented in section 4. Afterwards, the methodology of reducing the model and the case study are presented in section 5. This approximation is evaluated in terms of the accuracy of the values of the availabilities of bikes and docks and in term of gained resolution time. Finally, the numerical results are revealed before concluding and opening further research works.

## 2. State of the art

In [4], a new stochastic model for the vehicle sharing system was introduced by using the closed queuing networks theory. The aim of this work was to find the optimal fleet size that should be deployed to have the most profitable system. Literally, the advantage is that the model has a product form solution. In fact, the product form solution defines the states probability of the network based on the individual queues' states probability. The used resolution technique is called the mean value analysis method. It has a very interesting complexity; namely  $O(ML)$  with  $M$  the number of nodes and  $L$  the number of jobs in the closed queuing network.

In [9], authors use the mean field analysis technique, originating from the statistical physics [10]. This technique is interesting because it allows to deal with large scale systems where a product form solution is not reachable. The technique is applied to a Markovian model of a BSS to provide, then, the asymptotic steady-state queue length in the system as it gets large (number of stations and fleet size). Authors use the term homogeneous system for the BSS they study to get the proper conditions to apply this technique. The stations are studied as a single cluster where parameters of the stations are identical. So, the arrival rate of users to pick up a bike, the capacity of the stations and the travel time are identical for all stations. The objective was to reduce the number of the saturated or starving stations. To improve the global performance of the system, the optimal fleet size was found, recommendations for consumers incentives were proposed and bikes redistribution are studied.

Later, authors in [11] use the same resolution method modeling different stations. Two main models with blocking mechanisms for the BSS were proposed. Product form solutions are derived under specific assumptions. These models are asymptotically (both large number of bikes and stations) studied and approximations on the performance indicators of saturated and empty stations were considered. Moreover, the fleet size and the capacities of the station were approximately determined to meet the expected demand of users. However, the model does not consider the routing of the rejected users by a full station to the neighboring stations.

These works intended to use methods capable of treating the real scale systems which have a large number of bikes and stations. This is done neglecting some other aspects of the reality. Representing faithfully the whole dynamics of these systems is a tough problem. Keeping in mind these observations, in this paper we intend to reduce the complexity of the resolution algorithm of the model of a BSS while quantifying the impact of simplification hypotheses. This model and its resolution are presented in [8]; we would not develop them here due to space limitation. This model comes one more step closer to the real system in that it supports the limited capacity stations and takes into account the users rejection to the neighboring stations. Nevertheless, the complexity of resolution technique is such that making

large experimentation campaigns is practically impossible (about 53 hours for one complete resolution of a network of 20 stations and of 440 bikes). The idea we followed is to look for aggregating the "rest" of a network in a virtual station while keeping only some studied (critical for instance) stations. The results would allow decision-makers to find out their trade-off between the precision of the model and the time they are ready to use for resolution. To do so, we applied the suggested approach on a hypothetic balanced system where (i) the studied stations have equal docking capacity, (ii) their inter arrival rate of users is identical, and (iii) the routing between the stations follows the same pattern.

### 3. Model of Bike Sharing System

The BSS is modeled by a closed queuing network with limited capacity stations. We adopt the Repetitive Service Random Destination [7] as the blocking mechanism. This means that if a new job looks for entering a full queue, the blocking mechanism impeaches it. The blocked job returns to the upstream node to undergo another service time before being routed again. The jobs circulating in the queuing network are the bikes. Hereafter, the model is defined in detail, see Fig. 1. Three classes of queues are defined:

- A Single Server (SS) queue. It is a limited capacity queue representing the real bike station. Its capacity is the number of docks of the real station. Bike users come to the station to pick up a bike. This demand is random. The service process in SS is exactly the demand process of bikes by users. As such, when the station is not empty, the bike departs from the station at the same rate of the bikes demand of users. When the station is empty, the users wanting to pick up a bike are lost.
- A Multiple Server (MSB) queue. A MSB queue is put at the upstream of every SS. The number of the parallel servers in such node is  $L$  (the total number of bikes in the network) and the service time is small, and negligible, compared to the service time of the other type of nodes (let say 1min). A MSB queue receives the rejected bikes due to the full waiting space of SS. The rejected bikes undergo repeatedly a service time in MSB until a waiting space in SS becomes available or they are routed to the neighboring stations.
- A Multiple Server (MS) queue. It models the trip time. The bike is bearing a user from a departure station to a destination station by occupying a server in the MS queue. The number of servers equals the bike fleet size,  $L$ . So even when all the bikes take one unique route, they can be held by this queue.

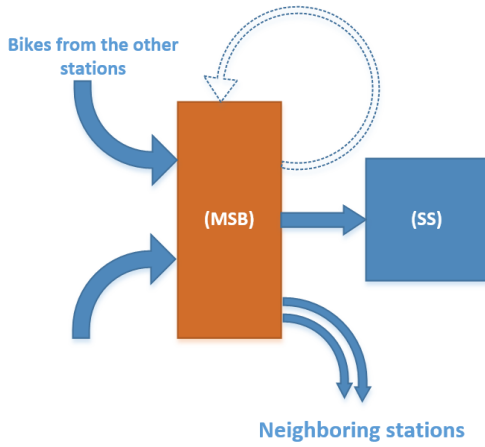
Fig. 1 (a) represents the model of a bike station combining a MSB and a SS. The bikes entering SS pass first by MSB and are then routed towards mainly SS. However, there is a small probability to route them towards the neighboring stations. Whenever a user riding a bike comes to SS two situations are possible: either the user finds a free dock or not. In the first case, the bike is attached and the trip ends. In the second case, the bike is turned back to the MSB. This last process of blocking is repeated several times when the bike intends to enter SS and finds no available dock in it. We can interpret the time spent in MSB as a short waiting time till a dock is freed when the blocking mechanism occurs several times. Otherwise, it can be seen as a short time spent before taking decision to go to one of the neighboring stations (recall the 1min servicing time of MSB). Fig. 1 (b) illustrates a model of two bike stations, extended from the original model developed by [4]. We distinguish two different routes going from every station: one to represent the trip to the other station and the second to represent the trip from the station to itself. We also distinguish the routing (arrows in orange) towards the neighboring stations.

## 4. The resolution method

### 4.1. Network characteristics

The model of the BSS is a closed network having an arbitrary topology under RS-RD blocking mechanism. It consists of  $M$  (First Come First Served) nodes with general inter-arrival time and service-time distributions. These

a



b

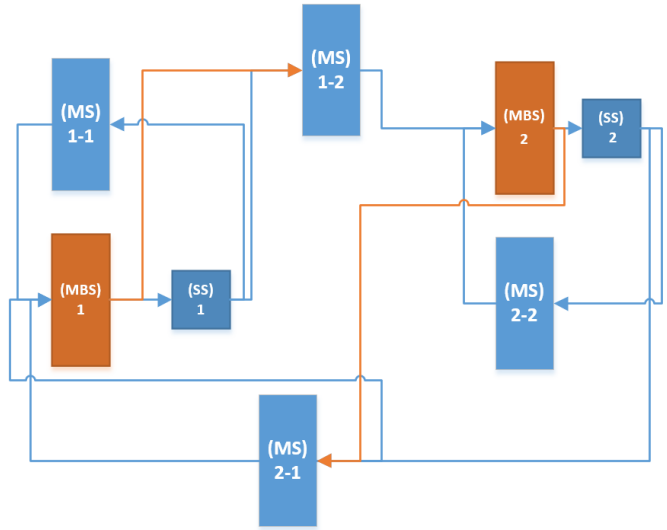


Fig. 1: (a) a queuing model for the station; (b) model for a system of two stations.

nodes can either be with a limited capacity (i.e.  $N$ ) single server ( $G/G/1/N$ ) or a multiple servers with infinite capacity ( $G/G/L$ ). For a given fleet size  $L$ , the state space of the network can be represented by:

$$S = \left\{ (n_1, \dots, n_M) : \sum_{i=1}^M n_i = L, 0 \leq n_i \leq L_i, i = 1, \dots, M \right\} \quad (1)$$

where  $L_i$  is the virtual capacity of the node  $i$ .

#### 4.2. Performance of the BSS

Solving a queuing network model means finding out the probability of the states in  $S$ . Then, the joint probability of having a certain queue length in a queue  $i$  (noted  $n_i$ ) can be calculated. The availability of bikes was first proposed in [4] and a very similar performance indicator measuring the availability of both bikes and docks was proposed in [12]. Hereafter, we define the following performance indicators:

- Availability of bikes in a station  $i$ :

$$Ab_i = 1 - p_i(n_i = 0) \quad (2)$$

- Availability of docks in a station  $i$ :

$$Ad_i = 1 - p_i(n_i = N_i) \quad (3)$$

- General availability of a station  $i$ . We define a single indicator aggregating the two aforementioned availabilities of a station:

$$A_i = a_i \cdot Ab_i + b_i \cdot Ad_i \quad (4)$$

where  $a_i$  and  $b_i$  are the weights attributed to each of the availabilities with  $(a_i + b_i) = 1$ .

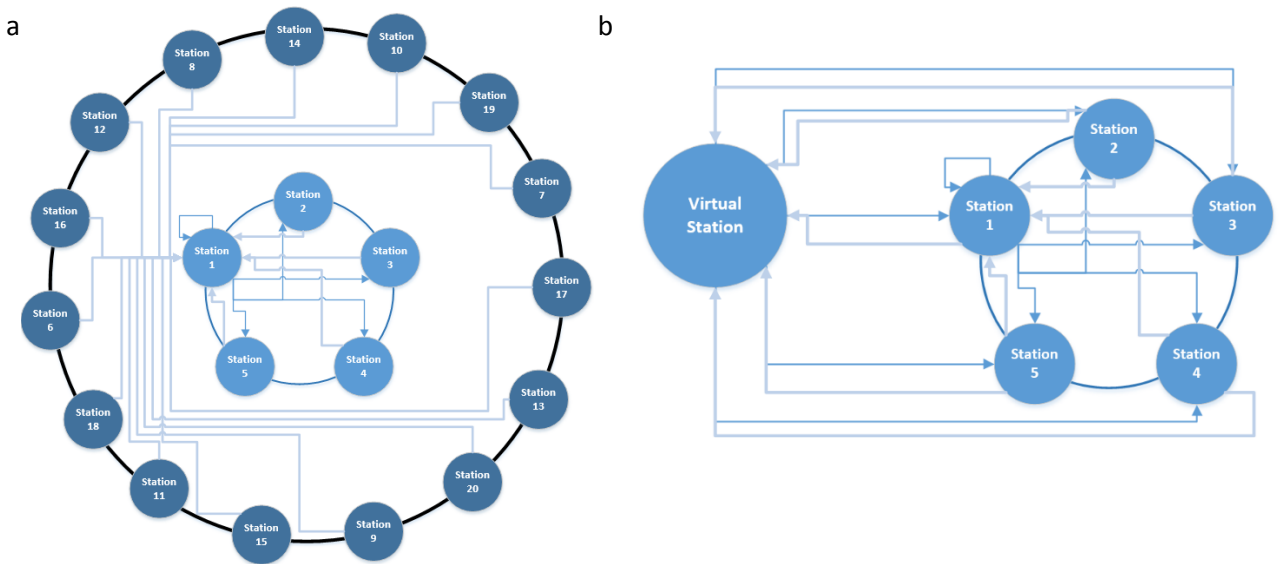


Fig. 2: (a) BSS of 20 stations; (b) the reduced BSS.

### 4.3. Resolution by the Entropy Maximization

The presented network in section 2 has quite general criteria as its queues have general time distributions and it can support limited capacity queues. Furthermore, it is an arbitrary network. For this reasons, there are no simple analytical resolution methods for this class of networks. The resolution method used here was introduced in [13] and [14]. The Maximization of the Entropy function of Shannon of the probability distribution of the states under the normalization and some marginal constraints [13] leads to an approximate product-form solution of the open network. For this maximization the Lagrange multipliers method is used. In the second phase of the resolution, the Lagrange coefficients of the solution of the open network are used to approximate a solution for the closed queuing network by adding conditions on the conservation of the flow and the conservation of the fixed population in the closed network. Then a convolution type algorithm is applied to evaluate the probability of the states of the closed network. Finally, a correction of the Lagrange coefficients is done and the second phase is repeated until the convergence of the ratio of the rate of the effective inter-arrival time (without rejection) of bikes to the throughput in all the nodes of the network. This resolution algorithm is quite time consuming. In fact, there are iterations in the first and second phases depending on the convergence speed. For the case of the convolution algorithm which is one step in the second phase it has a time complexity of  $O(M^2L^2)$ .

This complexity tends to be a tough obstacle for studying large systems. The target is to look for its simplification by using one virtual station and to qualify the loose of accuracy through this process.

## 5. The model reduction methodology and the case study

We are about to study a balanced system of 20 stations. A system is considered to be balanced if and only if all the stations have the same capacity of docking, the same inter-arrival rate of users and similar routings. We are interested in evaluating the performance of a group of 5 inner stations located on the inner circle in Fig. 2(a). This represents those stations located at the very center of a city for instance. The exterior stations are located on the exterior ring. They are equidistant from the center. For clarity reasons, Fig. 2(a) presents only the routes from and to the Station 1. The key idea is to aggregate the exterior stations, one after another, into a unique virtual station (see Fig. 2(b)); i.e. it will contain step by step more and more real stations. By doing so, the number of real stations to study is reduced progressively and so is the resolution time.

We observe the efficiency of these successive models comparing them with the original model (20 stations model) which is taken as the reference. The performance indicators assess the mean availability of bikes and docks of the 5 inner stations.

The routings between the stations are homogeneous: from an inner station to an exterior station the routing probability is 0.04, from an inner station to another inner station the routing probability is 0.05, from any station to itself the routing probability is 0.2 and finally from an exterior station to any other station (excepting itself) the routing probability is  $0.0421 = (1 - 0.2)/19$ . The capacity of the stations is 30 docks. The routes duration are proportional to the geographical distance and calculated in such a way that the diameter of the inner ring is about 8min of cycling and the diameter of the exterior ring is 24min. All these system parameters are estimated from the parameters of the BSS Velib of Paris. We used the stations very close to the center of Paris, i.e. Notre Dame Cathedral for this study.

The users arrival rate for bike to stations is 13 users per hour. The arrival of users for bikes is considered to be a Poisson process. As the sum of Poisson processes is a Poisson process, we apply this law to aggregate the arrival of users from different station into the virtual station. A similar technique was applied in [15] to merge two types of users in one system. Since the system is balanced, the exterior stations have homogeneous flows of incoming bikes from other stations and homogeneous flows of outgoing bikes to other stations. Necessarily, there are identical filling rate in the stations in the exterior ring. Aggregating the exterior ring stations, the virtual station would have also the same filling rate value. Therefore, the hypothesis of balanced system makes the aggregation of these stations more judicious. The aggregation process is done through the following steps:

- **M0**: 5 inner stations + 15 exterior stations (the reference model).
- **M1**: 5 inner stations + 13 exterior stations + the virtual station aggregating 2 exterior stations.
- ...
- **M13**: 5 inner stations + 1 exterior station + the virtual station aggregating 14 exterior stations.
- **M14**: 5 inner stations + the virtual station aggregating 15 exterior stations.

To use this aggregation process we use these approximations:

- The routing from the virtual station to an inner station is the mean of the routings from the exterior stations of the original model.
- The duration of the trip from the virtual station to an inner station is the mean of the durations of trip from those exterior stations to this particular inner station. In the same manner, the routes from an inner station to exterior stations aggregated in the virtual station are reassembled into one single route.
- The capacity of the virtual station is the sum of the capacity of the aggregated exterior stations.

## 6. Numerical results and discussions

The resolution approach of the case study introduced in section 5 is programmed in Matlab. The whole experiment holds 15 different models. We run all these experiments on Intel Xeon (3.00 GHz) desktops under Windows 7 64 bits. For every model we computed the availability of bikes and docks of stations, and also we collect the resolution time. We consider the mean availability of the inner stations using their general availabilities,  $A_i$ . We considered that the bikes and docks availabilities are of equal importance, i.e.  $a_i = b_i = 0.5, i \in \{1, \dots, 5\}$ , see subsection 4.2.

$$\bar{A} = \frac{1}{5} \sum_{i=1}^5 A_i \quad (5)$$

Fig. 3(a) presents the mean availabilities versus different models,  $M_i$ . Readers may note that the left ordinate axis (purple color y) varies from 0.95 to 0.97. We observe that the mean availability is more and more underestimated while the virtual station contains more and more exterior stations. The same figure presents the percentage of the relative

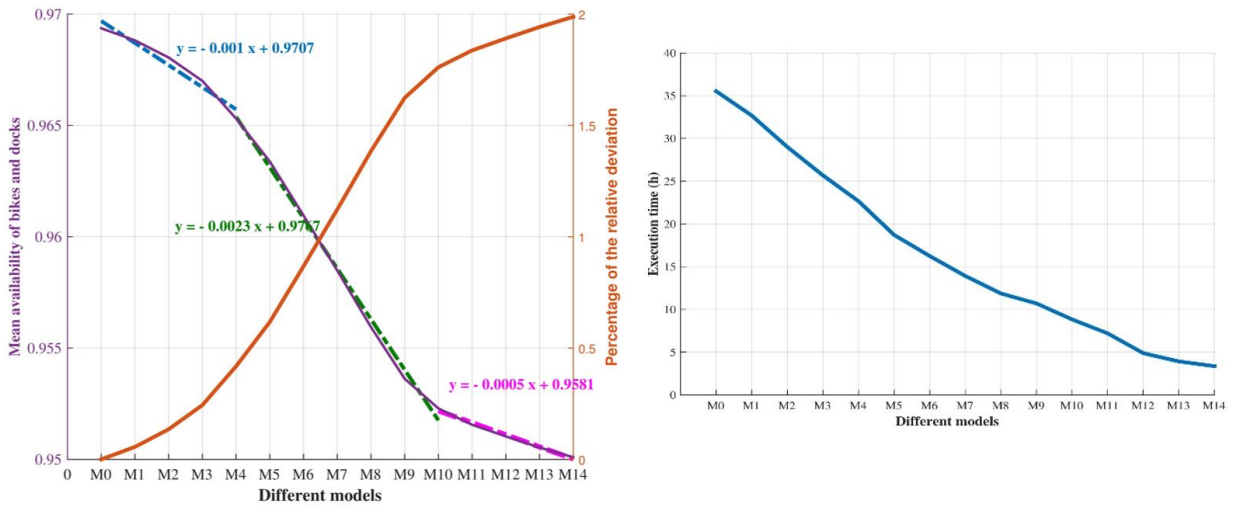


Fig. 3: (a) Performance evaluation; (b) the time execution.

deviation of the mean availabilities of stations relatively to the original model of 20 stations. This is the second ordinate axis on the right (brown color). This indicator is noted  $R_{n.st+v}$ , where n is the number of non-aggregated stations in the virtual station and v is the number of stations aggregated in it, and defined by:

$$R_{n.st+v} = \frac{\bar{A}_{20.st+0} - \bar{A}_{n.st+v}}{\bar{A}_{20.st+0}} * 100 \tag{6}$$

where  $\bar{A}_{20.st+0}$  is the mean availability performance indicator of the original model.

Referring to Fig. 3(a), the relative deviation  $R_{5.st+15}$  of the reduced or aggregated model {5 stations + virtual station aggregating 15 stations} varies at most by only 2% ! This is surprising. However, this conclusion should be moderated due to the hypothesized homogeneity of stations; i.e. the balanced system we use for this study.

It is also possible to simplify the mean availability curve by three linear parts:

- The first part concerns the five models {20.st+0, 18.st+2,..., 15.st+5} where the relative deviation is small, and the variations remains quite reasonable.
- The second part concerns the six models {14.st+6,..., 9.st+11}. This asymptote shows that the quality of the obtained mean availability is quickly degraded.
- For the rest {8.st+12,..., 5.st+15} the relative deviation decreases more slowly; the degradation of results remains more reasonable.

In Fig. 3(b), we show the resolution time of the models. We can notice that the shape of this curve is parabolic which corresponds to the quadratic form of the complexity of the convolution algorithm (see section 4.3) (having the abscissa evolving of 1 unit from one model to another the equation fitting with this curve is:  $y = 0.1088x^2 - 4.0777x + 39.9746$ .) Due to this parabolic shape, the time reduction is quite important during the first steps (i.e. from (20.st+0) to (18.st+2)) but becomes slower for the final steps (i.e. from (6.st+14) to (5.st+15)).

All together, these observations are encouraging showing that the aggregation process, under the aforementioned hypotheses, can be used as a "gross coarse" technique for BSS analysis. The aggregation technique can be used by the system operator at the first steps of the system analysis. For example, a decision maker who wants to have a relative deviation of the performance less than 1% (a reasonable admitted error) should, at best, aggregate 13 stations into the virtual station. In this case, the calculation time is about 16,25h.



## 7. Conclusion and perspectives

In this paper, we present a closed queueing network model supporting a blocking mechanism for the BSS. This model resolution has been developed in a former paper [8]; the focus of the paper was then put on the reduction of the resolution complexity by aggregating those stations which are not of interest into one virtual station. The aggregation is made for a balanced system. This was performed in an iterative way keeping track of the performance indicators found and the computing time.

The study case, a balanced 20 stations, showed a good accuracy under the homogeneity hypothesis: routings and the durations of trips, stations capacity and inter-arrival of users. These results remain encouraging and show a first step of a road-map of simplification of expensive resolution methods. Nevertheless, other questions have to be answered which show further niches:

- How does the suggested aggregation strategy behave for an imbalanced system?
- The model needs to be tested through a set of deeper sensitivity analyses to find out its more influential characteristics in order to fine-tune the aggregation technique (routing probabilities, distances, etc. ).
- What are the most efficient aggregation strategies in large systems ?

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