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# **Forecasting the Volatility of the Chinese Gold Market by ARCH Family Models and extension to Stable Models**

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# **Abstract**

Gold plays an important role as a precious metal with portfolio diversification; also it is an underlying asset in which volatility is an important factor for pricing option. The aim of this paper is to examine which autoregressive conditional heteroscedasticity model has the best forecast accuracy applied to Chinese gold prices. It seems that the Student's *t* distribution characterizes better the heavy-tailed returns than the Gaussian distribution. Assets with higher kurtosis are better predicted by a GARCH model with Student's distribution while assets with lower kurtosis are better forecasted by using an EGARCH model. Moreover, stochastic models such as Stable processes appear as good candidates to take heavy-tailed data into account. The authors attempt to model and forecast the volatility of the gold prices at the Shanghai Gold Exchange (SGE) during 2002–2016, using various models from the ARCH family. The analysis covers from October 30, 2002 to December 7, 2015 and from December 8, 2015 to May 6, 2016 as in-sample and out-of-sample sets respectively. The results have been estimated with MAE, MAPE and RMSE as the measures of performance.

**Keywords**: Forecasting, Return, Volatility, Gold Market, ARCH, GARCH, GARCH-M, IGARCH, NGARCH, EGARCH, PARCH, NPARCH, TARCH, Student's t distribution, Symmetric Stable models, *H*-self-similar processes

*JEL Classification:* C53, C58, G17

#### **1. Introduction**

Gold is a precious metal which forms an integral part of many investment portfolios; it has similar characteristics to money (see Solt and Swanson, 1981). Ciner (2001) indicates that gold plays an important role as a precious metal with portfolio diversification properties: holding gold leads to a more balanced portfolio. Lawrence (2003) finds that gold returns are less correlated with returns on equities and bonds than other assets. Many researchers and professionals believe that gold plays the role of a hedge against inflation. Sjaastad and Scacciavillani (1996) claim that gold is a refuge value against inflation. Central banks and international financial institutions hold a large proportion of gold for their diversification capacity, and gold is considered as an insurance against market crises. According to Smith (2002), when the economic environment becomes more uncertain, gold plays a role as a safe haven. A number of authors have reported the fact that gold reflects the value of the US dollar, because a depreciation in the dollar may increase interest in gold, Tully and Lucey (2007) report that a lower US dollar makes gold less expensive for other international investors to buy dollar-denominated gold.

The Chinese assets market is one of the emerging assets markets which offer an opportunity for international diversification. Since the 1990s, reforms in regulations as well as in the attitudes of regulators have rendered the stock market more efficient. However, it would seem that in the Chinese assets market, anomalies persist (see Mookerjee and Yu, 1999; Mitchell and Ong, 2006). This article studies the volatility of gold prices in China through gold prices at the Shanghai Gold Exchange (SGE). As mentioned in Hoang et al. (2015), it is important to use local gold prices when studying the Chinese gold market since it is still forbidden for Chinese investors to trade gold abroad without authorization.

It has become more important for financial institutes to consider the volatility of a financial asset. The movements are usually measured by the volatility and can be seen as the risk of the asset. One of the problems with modelling the volatility is that it is non-stationary: there are periods with low movements and then periods with high movements. The first model that assumed that volatility is not constant is ARCH (autoregressive Conditional Heteroscedasticity Model) by Engle in 1982. The ARCH model has been transformed and developed into more sophisticated models, such as GARCH, EGARCH, PARCH and TARCH. The questions to ask are: Do the new models capture the volatility better? If one

model better captures the volatility, would it lead to a more efficient forecast accuracy? Currently, results of research into the model's performance are conflicting and confusing. These observations lead the author to determine how well these different models perform in terms of forecasting volatility and will be assessed based on the forecasts they produce. This study focuses on the gold prices at the Shanghai Gold Exchange (SGE); it contributes to the existing finance literature by investigating the Chinese gold market during the recent period. In addition, this work opens another suitable way to model heavy-tailed data, such as returns. Heavy-tailed data can be modelled very well by  $\alpha$ -Stable distributions because of the high variability of the data.

The structure of this article is as follows. Section 2 provides a brief overview of the existing literature. Data and methodology are described in Section 3 which firstly recalls the different ARCH Family models. Then, construction and properties of Symmetric α-Stable processes are presented; the aim is to show the adequacy of this model in this context. Next, Section 4 displays the results of this study with the numerical comparison between Student and Gaussian distribution.

# **2. Literature Review**

Engle's (1982) study was the first to distinguish between unconditional and conditional variance. According to the ARCH models, the variance of the current error term is a function of the previous time period's error terms. Bollerslev (1986) points out that the ARCH model has a long lag in the conditional variance equation. To avoid problems of negative variance parameter estimates, his solution was to generalize the ARCH model. Nelson (1991) proposed the EGARCH model to account for the presence of asymmetric effects. Poon and Granger (2003) noted that models that allowed for asymmetric effects are able to provide better forecasts owing to the negative relationship between volatility and shocks.

Most research on volatility modeling has focused on equity markets and foreign exchange markets, but Sadorsky (2006) pointed out that one type of market cannot be generalized across other markets. It seems that stocks with higher kurtosis were better predicted by using the GARCH model and stocks with lower kurtosis were forecasted better by using EGARCH.

Ederington and Guan (2005) showed that the financial market has a longer memory than explained by the  $GARCH(1,1)$  model. Awrtani and Corradi (2005) found that the asymmetric models perform better than the GARCH(1,1) model. After comparing the different volatility models, Köksal (2009) found that the EGARCH(1,1) performed best. Bracker and Smith (1999) studied the volatility of the copper futures market, and concluded that both the GARCH and EGARCH models best fit this market's volatility. Carvalho et al. (2006) found evidence of asymmetric effects when using the EGARCH model.

However, not all the empirical studies agree with these findings. Ramasamy and Munisamy (2012) found that the GARCH models are better to predict the volatility of exchange rates and the EGARCH model does not improve the quality of the forecasts. Marcucci (2005) found that the GARCH model performs much better than other models in a long time period, but not in a short time period. Zahangir et al. (2013) investigated the ARCH models for forecasting the volatility of equity returns. Their study showed that based on in-sample statistical performance, both the ARCH and PARCH models are considered the best performing model whereas all models except the GARCH and TARCH models are regarded as the best model for the returns series.

# **3. Data and Methodology**

The main motive of this paper is to investigate the use of the ARCH family models for forecasting the volatility of the Shanghai Gold Exchange (SGE) by using daily data. GARCH, EGARCH, PARCH, IGARCH, GARCHM and TARCH models are used for this study.

# **3.1 Data**

The data used is the returns on the Shanghai Gold Exchange (SGE) returns series (Figure 1). In order to calculate these returns, we use data on the daily price. Data on prices spans the period from October 30, 2002 to May 6, 2016, a total of 3,278 observations. The total data set is divided into in-sample samples and out-of-sample samples. The requisite data is obtained from the Factset database. The price taken as the daily price is the last trading price of the day.

Figure 1 shows the gold prices at the Shanghai Gold Exchange (SGE) from October 2002 to May 2016, the crash of 2007 is clearly visible as well as the major long-run rise in the index in 2002-2011. The index is non-stationary, since its mean level is not constant and rises over

time. The daily return on the index appears to be stationary with a constant mean and variance. The outliers, namely very large or very small returns, imply that the returns are nonnormal with fat tails and the distribution may be asymmetric.



**Figure 1: Chines gold market prices and returns (Daily, 2002 – 2016)** 

A positive skewness of 0.11876799 and negative kurtosis of -1.04121692 are revealed in the gold price summary statistics at the Shanghai Gold Exchange (SGE) (Table A1 in the Appendix). At a confidence interval of 99%, the SGE Index is non-normal, in accordance with Jarque-Bera statistics. Therefore, it is necessary to remodel the gold price series into a return series. Plotting autocorrelation and partial autocorrelation of the SGE index designates the series to be non-stationary (Figures A1 and A2 in the Appendix). Applying the Dickey-Fuller test and the Phillips-Perron test to the series confirms this (Table A2, A3 in the Appendix), and suggests that it cannot be used to model volatility.

Movements of stock indices series are not suitable for study as they are usually non-stationary and it is therefore necessary to remodel the daily price into a return series. The SGE index series is converted into a returns series by using the following equation:

$$
R_t = \left(\frac{P_t}{P_{t-1}}\right) - 1\tag{1.1}
$$

The logarithm of the gross return is given by:

$$
Ln(R_t) = ln(\frac{P_t}{P_{t-1}})
$$
\n(1.2)

Where,

 $R_t$  = the rate of return at time t

 $P_t$  = the price at time t

 $P_{t-1}$  = the price just prior to the time t

Statistics on returns are summarized in Table 1, where the SGE index has a 0.04 average daily return and a 0.000208 standard deviation. At -0.05748, the skewness coefficient is negative which is common for most financial time series. The kurtosis value is higher than 3, implying that the returns distribution has fat tails. The ARCH family of models should, therefore, be used to account for these characteristics of the data. When modelling a series of this sort, it must be stationary and the data must be mean-reverting. Therefore, the Dickey-Fuller test is used on the returns series (Table A4 in the Appendix) to confirm that the series is, indeed, stationary. The Phillips-Perron test can also be applied to confirm that the series is stationary and, in addition, for modelling (Table A5 in the Appendix).

#### **Table 1: Summary statistics for returns**



The observed pattern in Figure 1 indicates that GARCH type models are necessary to model return volatility patterns for SGE index returns. The return volatility descriptive statistics presented in Table 1 indicate a Kurtosis at 14.00012. Since the Kurtosis statistic is greater than three, the SGE return series is leptokurtic (Figure2).



**Figure 2: Leptokurtic distribution of SGE return series**

According to recent research, it seems that returns with higher kurtosis were better predicted by using GARCH with student's distribution and returns with lower kurtosis were forecasted better by using EGARCH. Skewness is -0.05748, this indicates that return volatiles are negatively skewed (left skewness). Negative skewness indicates that the upper tail of distribution is thinner than the lower tail. That indicates that in the SGE market, prices fall more often than they rise. The ADF-test, as well as the PP-test, are used to get confirmation regarding whether the return series is stationary or not. The value of the ADF test statistic, - 62.283, is less than its test critical value, -3.410, at 5%, level of significance, which implies that the gold price return series is stationary. The findings of the PP test also confirm that the return series is stationary, since the values of the PP test statistic is less than its test critical value. The plotted autocorrelation and partial autocorrelation of squared returns indicate dependence and hence imply a time-varying volatility (Figures A3 and A4 in the Appendix).

#### **3.2 Specification of the Models used in this study**

In order to model the volatility of the returns, we need to determine their mean equation. The return for today will depend on returns in previous periods (autoregressive component) and the surprise terms in previous periods (moving order component). In this empirical study, plotting the autocorrelation and partial autocorrelation of the returns series can help determine the order of the mean equation.

Like most financial time series, the returns series exhibits what is referred to as "volatility clustering" (Figures A5 and A6 in the Appendix). In order to model such patterns of behavior, the variance of the error term is allowed to depend on its history. The classic model of such behavior is the ARCH model introduced by Engle (1982), which simultaneously models the mean and variance of a series.

#### **3.2.1 ARCH(q) model**

Autoregressive conditional heteroscedasticity (ARCH) models are used when the error terms will have a characteristic size or variance. The ARCH models assume the variance of the current error term to be a function of the actual sizes of the previous time period's error terms. The ARCH model is a non-linear model which does not assume the variance is constant. The basic linear ARCH model may be presented as:

$$
Y_t = \phi X_t + \varepsilon. \tag{2}
$$

The error terms are split into a stochastic piece and a time dependent standard deviation:

$$
\epsilon_t = \sigma_t z_t \tag{3}
$$

The random variable is a white noise process, the series  $\sigma_t^2$  is the volatility of the time series which changes over time, and  $\sigma_t^2$  (variances of the residual) is modelled by:

$$
\sigma_t^2 = a_0 + a_1 \epsilon_{t-1}^2 + \dots + a_q \epsilon_{t-q}^2 = a_0 + \sum_{i=1}^q a_i \epsilon_{t-i}^2 \tag{4}
$$

Where  $a_0 > 0$  and  $a_i > 0$ .

#### **3.2.2 GARCH(p, q) model**

The GARCH model is a generalized ARCH model, developed by Bollerslev (1986) and Taylor (1986) independently. They introduced a moving average term into the ARCH estimation. A fixed lag structure is imposed. The GARCH(p, q) model, where p is the order of the GARCH terms  $\sigma^2$  and q is the order of the ARCH terms  $\varepsilon^2$ .

$$
\sigma_t^2 = w + a_1 \epsilon_{t-1}^2 + \dots + a_q \epsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \dots + \beta_p \sigma_{t-p}^2
$$
  
= 
$$
w + \sum_{i=1}^q a_i \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2
$$
 (5)

The form of  $GARCH(1,1)$  is given below, the equation  $(5)$  is the conditional variance equation,  $\sigma_t^2$  is the one period forecast variance based on past information,  $\sigma_t^2$  is determined by  $a_0$ ,  $\varepsilon_{t-1}^2$  and  $\sigma_{t-1}^2$ . Where the news about volatility from the previous period is  $\varepsilon_{t-1}^2$ , measured as the lag of the squared residual from the equation (2), and  $\sigma_{t-1}^2$  is used to be considered as the last period's (forecast) variance.

$$
\sigma_t^2 = a_0 + a\varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{6}
$$

Engle and Bollerslev (1986) indicated that the coefficient  $(\alpha + \beta)$  measures the volatility shock, when  $(\alpha + \beta)$  is closed to one, the volatility shocks are persistent, this means that the volatility may take long time to return to a quieter phase.

#### **3.2.3 GARCH-M model**

The M in GARCH-M stands for "in the mean". This model is an alternative ARCH model developed by Engle and Bollerslev (1986). GARCH-M is used when the expected financial time series return is related to the expected asset risk. The conditional variance is included in the conditional mean equation. The GARCH-M  $(1,1)$  is written as:

$$
y_t = [g(x_{t-1})] + f(\sigma_{t-1}^2) + \varepsilon_t \qquad (7)
$$

$$
\varepsilon_t = z_t \sigma_t
$$

The equation shows that the returns  $(y_t)$  has a positive relation to its own volatility. Engle and Bollerslev proposed to introduce the logarithm of conditional variance into the mean equation; the equation (7) can be expressed as:

$$
Y_t = \phi X_t + \delta \log(\sigma_t^2) + \varepsilon_t \tag{8}
$$

#### **3.2.4 IGARCH**

Consider the GARCH model in ARMA equation:

$$
\varepsilon_{t} = z_{t}\sigma_{t}
$$

$$
\sigma_{t}^{2} = a_{0} + \sum_{i=1}^{q} a_{i}\varepsilon_{t-i}^{2} + \sum_{j=1}^{p} \beta_{j}\sigma_{t-j}^{2} \qquad (9)
$$

Where the parameter  $\alpha$  is the ARCH parameter and  $\beta$  is the GARCH parameter,  $\alpha(L)$  and  $\beta(L)$ is defined polynomials, and  $a_0 > 0$ ,  $a_i \ge 0$ ,  $\beta_i \ge 0$  and  $\sum_{i=1}^{\max(p,q)} (a_i + \beta_i) < 1$ .

#### **3.2.5 EGARCH model**

The EGARCH model was developed by Nelson (1991) as a solution to avoid problems with negative variance parameter estimates. If "bad news" has a more pronounced effect on volatility than "good news" of the same magnitude, then a symmetric specification such as ARCH or GARCH is not appropriate since in standard ARCH/GARCH models the conditional variance is unaffected by the sign of the past periods' errors (it depends only on squared errors). The logarithmic function ensures that the conditional variance is positive and, therefore, the parameters can be allowed to take negative values. The form of  $GARCH(1,1)$  is given below:

$$
\log \sigma_t^2 = w + \beta \log \sigma_{t-1}^2 + \gamma \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + a \left( \frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{2/\pi} \right) (10)
$$

According to Engle and Ng. (1993), the EGARCH model allows positive return shocks and negative return shocks so that there are different effects on volatility; the coefficient  $\gamma$ measures the asymmetry effect. Gokan (2000) indicated that when  $\gamma = 0$ , the goods news (positives return shocks) has the same effect on volatility as bad news represents the negative return shocks.

In equation (10), positive return shocks, namely good news, has  $(w + \gamma)$  impacts on return volatility while the negative return shocks, bad news, has a  $(w - \gamma)$  impacts. If w and  $\gamma$  are positive, the good news will have more effect on return than bad news. According to previous studies in this subject, the coefficient  $\gamma$  is often negative; this suggests negative return shocks have more impact on return volatility than good news.

#### **3.2.6 PARCH model**

The PARCH model is a GARCH model with an additional term to account for the asymmetries effect. It employs an indicator function as follows (PARCH(1, 1)):

$$
\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} \qquad (11)
$$

The indicator function takes a value of 1 if the error  $> 0$ , and 0 otherwise. For the effect of the previous period's bad news to be greater than the effect of good news of the same magnitude, γ should be significant and have a negative sign.

#### **3.2.7 TARCH model**

The Threshold GARCH (TARCH) model by Zakoïan (1994) is one on conditional standard deviation instead of conditional variance. The effect of good and bad news is captured separately through the two coefficients,  $\alpha$  and  $\gamma$ , respectively. The TARCH model adds a separate variable for negative shocks.

$$
\sigma_t^2 = w + \sum_{i=1}^q a_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{l=1}^m \gamma_l \varepsilon_{t-l}^2 \overline{l_{t-l}} \qquad (12)
$$

 $\overline{I_{t-l}}$  equals one if  $\varepsilon$  is less than zero and zero if else. The form of TARCH(1,1) is given below:

$$
\sigma_t^2 = w + a\varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 \overline{I_{t-1}}
$$
 (13)

In equation (12) and (13), good news (positive return shocks) and bad news ( $\varepsilon_t$  < 0) have different impacts on the conditional variance. Positive return shocks have an  $\alpha$ -effect on volatility, while bad news has an  $(\alpha + \gamma)$ -effect on the conditional variance (volatility). If  $\gamma$ equals zero, the TARCH model becomes a linear GARCH (symmetric) model. If  $\gamma \neq 0$ , then that suggests an asymmetric effect.

#### **3.2.8 NGARCH model**

Nonlinear GARCH (NGARCH) is known as nonlinear asymmetric GARCH(1,1).

$$
\sigma_t^2 = a_0 + a_1(\epsilon_{t-1} - \varphi \sigma_{t-1})^2 + \beta \sigma_{t-1}^2 \qquad (14)
$$

$$
a_1, \beta \ge 0; a_0 > 0
$$

For the return series, parameter  $\varphi$  is positive, it means a leverage effect: negative returns increase future volatility by a larger amount than positive returns of the same magnitude.

#### **3.2.9 Student's t distribution**

One of the problems with forecasting volatility is the distribution: there are periods with low movements and periods with high movements. Köksal (2009) indicates that the Student's t distribution characterizes better the heavy-tailed returns than the Gaussian distribution. It seems that stocks with higher kurtosis were better predicted by using GARCH with student's distribution and stocks with lower kurtosis were better forecasted by using EGARCH model (see Grek A., 2014).

In the ARCH models, the first part is  $\epsilon_t = \sigma_t z_t$  with  $Z_t \sim N(0,1)$ . In the GARCH model with student distribution, the first part is written as part  $\epsilon_t = \sigma_t y_t$  with  $Y_t \sim t(d)$  and d is the degrees of freedom.

#### **3.2.10 Extension to Stable Models**

The most famous stochastic model for self-similar phenomena is certainly the Fractional Brownian Motion (fBm) popularized by Benoit Mandelbrot in the 60's. The fractional Brownian motion is the only H-self-similar centered Gaussian process with stationary increments in dimension 1. There exist several generalizations of the fractional Brownian motion, such as the multifractional Brownian motion (mBm) defined independently by Peltier and Lévy-Véhel, and by Benassi et al.

Such Gaussian models are well known and applied in different fields. Nevertheless, as soon as the observed phenomenon has large fluctuations or high variability, they are no longer appropriate. Such limitations lead us to propose stable non-Gaussian models as those data present large fluctuations and variability on the collected values.

The stable non-gaussian framework is however much more complicated than the Gaussian one. It is richer as there exist at least two distinct models of H-self-similar symmetric  $\alpha$ -stable (SαS) family of processes: the so-called Moving Average and the Harmonizable stable process. Actually, they are distinct since they form two disjoint classes of processes, as proved in Cambanis et al. and in the reference book of Samorodnitsky and Taqqu. In the Gaussian case where  $\alpha = 2$ , the two classes describe in fact the same, since they have the same law, up to a constant. Let us then consider stable non-gaussian stochastic processes; they are defined in the following way:

For  $x, y \in \mathbb{R}^d$ 

$$
g_{\beta,H}(x; y) = |x - y|^{H - \frac{d}{\beta}} - |y|^{H - \frac{d}{\beta}}
$$

$$
f_{\beta,H}(x;y) = \frac{exp(ix,y) - 1}{|y|^{H + \frac{d}{\beta}}}
$$

for some  $\beta > 0$ . Next we take  $\beta = \alpha$  the stability index, or  $\beta = \alpha^*$  such that

$$
\frac{1}{\alpha} + \frac{1}{\alpha^*} = 1.
$$

If  $\alpha = \alpha^* = 2$ , we come back to the Gaussian framework. Then  $\alpha$ -stable stochastic processes are defined as the integral, for a random measure $M_\alpha$ , of the previous kernels:

$$
X(x) = \int_{\mathbb{R}^d} g_{\beta, H}(x; y) M_{\alpha}(dy) \tag{15}
$$

and

$$
Z(x) = \int_{\mathbb{R}^d} f_{\beta,H}(x; y) M_\alpha(dy) \qquad (16)
$$

where process  $X$  is the Moving Average and process  $Z$  is the harmonizable representation.

#### **3.3 Measures of the Statistical Performance of the Model**

After producing the forecasts, they are evaluated by comparing out-of-sample forecasts with historical volatility. To identify the best-performing model in both the in-sample data set and the out-of-sample data set of this study, statistical performance measures are applied. Measures such as MAE (mean absolute error), MAPE (mean absolute percentage error) and RMSE (root mean squared error) are used. Their definition and way of computation is:

$$
MAE = \left(\frac{1}{n}\right)\sum_{i=1}^{n} \left|\hat{\sigma}_i^2 - \sigma_i^2\right| \tag{17}
$$

$$
MAPE = (1/n) \sum_{i=1}^{n} |(\hat{\sigma}_i^2 - \sigma_i^2)/\sigma_i^2|
$$
 (18)

$$
RMSE = \frac{1}{n} \sqrt{\sum_{i=1}^{n} (\hat{\sigma}_i^2 - \sigma_i^2)^2}
$$
 (19)

We detail the empirical findings in next Section. Then, from this study, we think that Stable non-Gaussian processes would fit better the heavy-tailed returns of such data than the

Gaussian distribution. Actually, consider a non-Gaussian  $\alpha$ -Stable process  $X_s$  with distribution  $S_\alpha(\sigma, \beta, \mu)$ , where  $0 < \alpha < 2$  is the stability index,

- $\sigma \geq 0$  is the scale parameter,
- $-1 \leq \beta \leq 1$  is the skewness parameter,

 $\mu \in \mathbb{R}$  is the shift parameter.

The case  $\alpha = 2$  can be added, but it comes back to the Gaussian framework as when  $\beta = 0$ then this Stable distribution corresponds to the Normal law  $S_2(\sigma, 0, \mu) \coloneqq N(\mu, 2\sigma^2)$ .

A symmetric  $\alpha$ -stable non-Gaussian process  $X_{S\alpha S}$  with distribution  $S_\alpha(\sigma, 0, 0)$  for  $\alpha < 2$ satisfies an important characteristic property

$$
P(|X| > x) \sim \frac{1}{2} c_{\alpha} \sigma^{\alpha} x^{-\alpha} \text{ when } x \to +\infty \qquad (20)
$$

Namely, the tails of the distribution decay like a power function while it is an exponential decay for a Gaussian process. Consequently stable non-Gaussian processes exhibits more variability than Gaussian ones. Thus, the smaller α, the highest is the variability: as described in the books of Taqqu et al., a Stable process is likely to take values far away from the median, which is called the Noah Effect (defined by Mandelbrot for "very severe flood"). This high variability is given by the following property of the order-p moments

$$
E(|X|^p) = +\infty, \,\forall p \ge \alpha. \quad (21)
$$

Property (21) will entail in particular an infinite variance. This high variability of Stable distributions is one of the reasons they play an important role in modeling. Stable non-Gaussian processes appear as good candidates to take into account heavy-tailed data.

#### 4. **Empirical Findings (Analysis and Results)**

Our aim is to determine how well these different models perform in terms of forecasting volatility. The forecasting approach used is such that the last 100 observations of the sample are used to assess out-of-sample forecasts. The study period contains 3, 279 trading days. The in-sample data set covers from October 30, 2002 to December 7, 2015 and includes 3, 179

observations, whereas the out-of-sample data set covers from December 8, 2015 to May 6, 2016 and incorporates 100 observations.

Our first step is to identify the mean equation for the returns. The autocorrelation and partial autocorrelation function for the returns shows that autocorrelations and partial autocorrelations up to the fifth lag are significant. We, therefore, propose using an autoregressive moving average ARMA(1,1) mean equation to model volatility in the ARCH models. The main criterion to choose our mean equation is the log likelihood. We noticed that the AR model and the ARMA model provide the best quality of regression in terms of log likelihood. The rest of the models obtained the same level of likelihood, of around 9862 against 9874 for  $AR(1)$  model and  $ARMA(1,1)$  model. For our mean equation, we chose arbitrarily the  $ARMA(1,1)$  model for forecasting. The estimated  $ARMA(1,1)$  equation for the mean is found to be a significant t-value for the coefficients. The residuals of the mean equation indicate the absence of autocorrelation (Figures A7 and A8 in the Appendix).

# **Table 2: Mean equation estimated**



p-values are given in parentheses

There is second-order dependence in the squared residuals of the mean equation and, hence, the presence of conditional heteroscedasticity in the returns (Figures A7 and A8 in the Appendix). Further, the ARCH-Lagrange Multiplier (LM) test confirms the presence of ARCH effects and the need to model this conditional heteroscedasticity using the ARCH family models (Table A3 in the Appendix).

The ARCH and GARCH models are easy to identify and estimate. But if « bad news » has a more pronounced effect on volatility than « good news » of the same magnitude, the symmetric specification such as ARCH or GARCH is not appropriate, because their conditional variance is unaffected by the sign of the past period's errors. It depends only on squared errors. Before applying the asymmetric models, we can test the presence of asymmetric effects. Engle and Ng (1993) propose various tests to detect the asymmetric effects. After the GARCH regression, the squared residual is given by:

$$
\varepsilon_t^2 = a_0 + a_1 l_{t-1}^- + error \qquad (22)
$$

 $I_{t-1}^- = 1$  where  $\varepsilon_{t-1} < 0$ , and 0 otherwise. If the dummy coefficient is significant and positive, this suggests the presence of asymmetric effects. Then we can determine whether the size of the negative shock also affects the impact on conditional variance by equation (23). For the existence of a size effect, the coefficient must be negative and significant.

$$
\varepsilon_t^2 = a_2 + a_3 I_{t-1}^- \varepsilon_{t-1} + error \tag{23}
$$

The positive sign bias test determines if the size of a positive shock impacts its conditional variance, the regression is given by equation (24). For the size effect to be present, the coefficient of  $I_{t-1}^- \varepsilon_{t-1}$  and  $I_{t-1}^+ \varepsilon_{t-1}$  must be significant.

$$
\varepsilon_t^2 = a_2 + a_3 I_{t-1}^+ \varepsilon_{t-1} + error \tag{24}
$$

Tables 3 and 4 present the results of the models fitted to the data on returns with Gaussian distribution and Student's t distribution. The outputs on returns show that the constant is statistically significant in the mean equation. The ARMA(1,1) term is also statistically significant for all models except the ARCH(1). The variance equation illustrates that all the terms are statistically significant at 1% level of significance which implies that the volatility of risk is influenced by past square residual terms. It can be mentioned that the past volatility of returns is significantly, influencing the current volatility. The EGARCH variance

equation also signifies that an asymmetric effect in volatility exists, which means that positive shocks are affecting volatility differently to negative shocks.

Table 3 presents linear and non-linear GARCH parameter estimations for Chinese gold market returns with Gaussian distribution. The coefficients of ARCH and GARCH models are statistically significant, and GARCH (1,1) presents better likelihood at 10324. For the GARCH model, the coefficient α+β =0.9762251 *(0.1211033+0.8551218)*, indicates the volatility shocks of SGE returns are persistent. The coefficients of IGARCH's mean equation are not significant. For the EGARCH model, the asymmetry parameter equals 0.2571219, it is significantly positive, the coefficient of the ARCH term is positive at 10%, this indicates the positive return shocks will have greater impacts on volatility. TARCH is created to capture the negative movements of the volatility that usually is bigger than the positive movements. In our case, the coefficient  $\gamma$  is 0.0277673 but significant at 1%.

Table 4 presents the coefficients obtained with student's distribution. The likelihood parameter is greater than regression with Gaussian distribution, but the coefficients of mean equation are not significant. The result of GARCH regression confirms the persistent volatility shocks,  $\alpha + \beta$  equals 0.9801919. The asymmetry parameter of the EGARCH model is 0.1760426 (significant at 1%), this result confirms the asymmetry return shocks. The coefficient  $\gamma$  of TARCH model is significant by negative (-0.1333102 at 1%). This is an unusual outcome; the result is contradictory in relation to our finding by using Gaussian distribution. The negative coefficient means good news has a greater impact than bad news. In a general way, the quality of the student's distribution is not satisfactory, because of poor significant coefficients of the mean equation. According to recent research, the financial return time series with high kurtosis was better predicted by using the student's distribution, we will study the forecasting quality of the GARCH type models in the next section.

#### **Table 3: Estimated coefficients for the ARCH models**



p-values are given in parentheses.



#### **Table 4: Estimated coefficients for the ARCH models (Student distribution)**

p-values are given in parentheses.

We test for the presence of asymmetric effects. The sign bias test yields the following results:

$$
\varepsilon_t^2 = 0.0001273 + 0.000305S_{t-1}^- + error
$$
  
p-value (0.000) (0.120)  

$$
\varepsilon_t^2 = 0.0001579 - 0.000305S_{t-1}^+ + error
$$
  
p-value (0.000) (0.120)

A positive but not significant coefficient indicates the presence of leverage effects, implying the positive and negative shocks may have a different effect on the conditional variance. Estimating the negative and positive sign bias tests yields the following results:

$$
\varepsilon_t^2 = 0.0000751 - 0.0170669S_{t-1}^-\varepsilon_{t-1} + error
$$
  
p-value (0.000) (0.000)  

$$
\varepsilon_t^2 = 0.0001372 + 0.0012538S_{t-1}^+\varepsilon_{t-1} + error
$$
  
p-value (0.000) (0.352)

Significant coefficients on the negative sign bias-test imply that there are sign effects and size effects. Positive and negative shocks do have a different effect on the conditional variance and the negative effect on the variance depends on the size of the shocks.

# **4.1 In-Sample Statistical Performance**

The following tables present the comparison of the in-sample statistical performance results of the selected models. For the models with Gaussian distribution, they reveal that the EGARCH has the lowest MAE and the lowest MAPE, the GARCH model and the IGARCH model have the lowest RMSE at 3.53e-07. Based on the Student's t distribution, the GARCH and ARCHM model have the lowest MAE and the lowest MAPE. The IGARH model has the lowest RMSE.

So, it can be said that, based on the outputs of in-sample statistical performance the GARCH and EGARCH models are the best models for Gaussian distribution and the ARCHM model is the best model for Student distribution. The ARCH, PARCH and NARCH models have the worst MAE, MAPE and RMSE.

Model	<b>MAE</b>	<b>MAPE</b>	<b>RMSE</b>
ARCH(1)	0.000193	8173.961	$3.60^{\circ}$ -07
GARCH(1,1)	0.0001914	5421.735	$3.53^{\circ} - 07$
EGARCH(1,1)	0.0001905	5156.383	$3.55^{\circ} - 07$
PARCH(1,1)	0.0001964	5437.983	$3.55^{\circ} - 07$
TARCH(1,1)	0.0001944	5345.467	$3.55^{\circ} - 07$
ARCHM(1)	0.0001925	8077.519	$3.64^{\circ} - 07$
IGARCH(1,1)	0.0002	5589.213	$3.53^{\circ}$ -07
NARCH(1)	0.0001932	8195.746	$3.61^{\circ} - 07$

**Table 5: In-sample statistical performance results (Gaussian distribution)** 

p-values are given in parentheses.





# **4.2 Out-Sample Statistical Performance**

Having estimated the models, our next step is to assess their forecasts. We use the models to make dynamic forecasts of volatility for the next 100 observations. For the Gaussian distribution, the GARCH model has the lowest MAE at 0.000126 and the lowest MAPE at 129.4682, the ARCH model has the lowest RMSE at 6.34e-08. Using Student's t distribution, the GARCH and ARCHM models have the lowest MAE, MAPE and RMSE.



# **Table 7: Out of sample statistical performance results (Gaussian distribution)**

p-values are given in parentheses.



# **Table 8: Out of sample statistical performance results (Student distribution)**

# **5. Conclusion**

This study has implications for investors who wish to price, hedge and speculate in the Chinese gold market. Gold is an underlying asset in which volatility is an important factor when pricing options. Secondly, the investor may use the Gold futures contracts to hedge their underlying Gold position. Forecasting volatility may be important for investors for adjusting their hedging strategies. In a general way, understanding how information (bad news and good news) impacts on return volatility improves the performance of portfolio management. For example, this knowledge allows the investors to have a better portfolio selection in asset pricing and allows them a more efficient risk management. In addition, the Chinese gold market offers an opportunity for international diversification.

Our study has attempted to model gold prices at the Shanghai Gold Exchange (SGE) return and assess the forecasting ability of the ARCH family of models. We have used historical volatility for modeling purposes through the ARCH family of models and made dynamic forecasts of future volatility. The analysis covers from October 30, 2002 to December 7, 2015 and from December 8, 2015 to May 6, 2016 as in-sample and out-of-sample sets respectively. For the Gaussian distribution, the results of the in-sample statistical performance show that the GARCH and EARCH models are selected as the best performing models for the returns. According to the outcomes of MAE and MAPT, the EGARCH model is better than the GARCH model in explaining the return (modeling the volatility), furthermore, using t-student distribution, the EGARCH model gets the best MAPE estimation of all models. This finding confirms our asymmetric effect analysis. Outcomes of the out-of-sample statistical performance demonstrate that the GARCH is considered to be the best model. Based on Student's t distribution, the GARCH and the ARCHM models are the best models for in and out-of-samples. This result confirms the power of the GARCH model in prediction volatility by using t-student distribution. At the same time, the ARCHM model shows good predicting capacity, in our study, volatility shocks appear persistent for the Gold return series as the sum of the coefficients  $\alpha$  and  $\beta$  is close to unity for all estimated models. According to Engle and Bollerslev (1986, 1987), including conditional variance in the conditional mean equation is more appropriate for financial time series when the expected return on an asset is related to the expected asset risk. At the same time, the ARCHM model shows good predicting capacity, this result corroborates the finding of Zou, Rose and Pinflod (2007), in which they conclude the EGARCH and ARCHM models outperform all other models in capturing the dynamic return volatility for Australian Three-Year T-Bond futures contracts. Two other facts have caught our attention: firstly, the specification of mean equation by using t-student distribution is not satisfactory; this study demonstrates that it is necessary to improve the quality of the mean equation estimates. Secondly, the quality of PARCH regression is not satisfactory, according to the PARCH model, the power term allows the capture volatility by changing the influence of the extreme value. But, according to new literature review, when the return is non-normally distributed, the use of a power transformation is not appropriate. We think using stable process distribution would be a solution to this issue, the extension to Stable Models opens up new paths for this research, this approach allows us to represent high

volatility. Next step is the simulation of the model for which we have to develop the computer programming.

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# **Appendix:**

### **Tables**

# **Table A1: Summary statistics for SGE Index**



# **Table A2: Dickey-Fuller test for SGE index**



# **Table A3: Phillips-Perron test for SGE index**



MacKinnon approximate p-value for  $Z(t) = 0.5364$ 

#### **Table A4: Dickey-Fuller test for returns**



# **Table A5: Phillips-Perron test for returns**



MacKinnon approximate p-value for  $Z(t) = 0.0000$ 

# **Table A6: LM test for autoregressive conditional heteroscedasticity**



**Figures** 









**Figure A3: AC of squared returns Figure A4: PAC of squared returns** 





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Figure A1: AC of SGE index **Figure A2: PAC of SGE index** 



Figure A9: AC of res.<sup>2</sup> mean equation Figure A10: PAC of res.<sup>2</sup> mean **equation** 





