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# Circular Faster Than Nyquist: Transmitter and Iterative Receiver Design

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**Abstract**—In this article, we propose low complexity transmitter and receiver architectures for Faster Than Nyquist communications. Both architectures are based on circular convolutions rather than linear convolutions. These circular architectures allow us to consider a turbo-equalization scheme based on linear MMSE equalization. We can consider low-values for the time compression factor without increasing the complexity of the equalizer.

## I. INTRODUCTION

Most waveforms for modern communication systems satisfy the Nyquist criterion to remove Inter Symbol Interference (ISI) at the receiver. Systems that aim to increase their throughput by having a data rate above Nyquist rate are called Faster Than Nyquist (FTN) systems. Introduced by Salz [1] in the early seventies, it is shown that FTN can improve the data rate but the generated ISI may be prohibitive compared to the increase of the modulation order. Later, Mazo [2] showed that uncoded FTN is possible without altering the Bit Error Rate (BER) performance of a binary communication with a cardinal sine pulse as long as the ratio of the Nyquist rate and the actual symbol rate is above a threshold (now called Mazo bound). More recently, FTN has known a regain of interest as in [3], [4], [5], [6], [7], [8] and references therein. In particular, [7] has shown that FTN achieves the maximal information rate for independent and identically distributed Gaussian codebooks. In fact [7] also considers modulation constrained inputs. Information rates for modulation constrained alphabets are numerically computed using the method proposed in [9]; this method requires to implement the forward part of a BCJR (Bahl Cocke Jelinek Raviv) algorithm on the trellis accounting for the channel memory.

Most works on FTN consider receivers that are trellis based detectors following a matched filtering, derived either from the Forney receiver when noise whitening is possible (ie. for compression factors above a given threshold) or, if not applicable, from the Ungerboeck receiver [10]. Both receiver architectures are based on trellises whose number of states is exponential in the equivalent ISI channel length. For FTN, the exact implementation of these receivers is restricted to symbol rates close to the Nyquist rate since these rates create ISI with small length. Moreover, the modulation order is often kept low to avoid a complexity increase. Reduced complexity trellis based detectors, such as  $M$ -BCJR [11], have been also

considered to reduce the number of states and to handle higher compression factors. But it often comes with a significant loss in performance.

In this paper, we consider an iterative Minimum Mean Square Error (MMSE) linear equalizer for mitigating ISI introduced by FTN signaling. As an efficient implementation, we propose to use a circular pulse shaping which is achieved through circular convolution at the emitter. We propose an efficient implementation of this circular pulse shaping in the frequency domain. It then allows us to perform low complexity equalization in the frequency domain at the receiver. The corresponding transmitter is based on Fast Fourier Transform (FFT) operations that allows a low complexity and that avoids the introduction of a cyclic prefix as done in [12] and [13]. Compared to [12], [13], the proposed architecture will automatically have a better spectral efficiency, in particular for high data rates that create ISI with a long delay spread (and so a long cyclic prefix). The proposed architecture is straightforwardly extendable to frequency selective channels by adding a cyclic prefix accounting for the length of the frequency selective channel only. The proposed MMSE equalizer happens before the matched filter, hence we do not have to handle correlated noise as it is the case in [12], [14], [13]. This structure prevents us from making strong assumptions on the noise (as an AR assumption in [14] or white noise assumption in [12] or [13]).

In [15] an efficient OFDM/OQAM multicarrier FTN architecture is proposed. As this architecture is OFDM-based, it is also based on a FFT step, however in our case, IFFT and FFT steps are used for an efficient computation of a circular convolution so that we consider single carrier FTN.

The rest of this paper is organized as follows. The proposed circular FTN architecture is detailed in Section II. In Section III, the iterative MMSE is derived. In Section IV simulation results that show the performances of our approach are presented.

## II. CIRCULAR PULSE SHAPING

### A. Transmitted signal

We consider a frame based transmission, where the frames have duration  $PT$ ,  $P$  being the number of subframes composing a frame. We will consider that codewords always span all subframes. The transmission of  $PK$  information

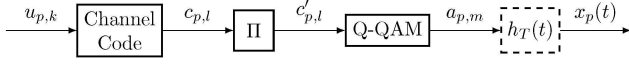


Figure 1. Circularly shaped BICM

bits  $\mathbf{u} = (u_0, u_1, \dots, u_{PK-1})^t$  follows the steps presented in FIGURE 1. First,  $\mathbf{u}$  is encoded into  $PM$  symbols using a  $Q$ -ary Bit Interleaved Coded Modulation (BICM):  $\mathbf{u}$  is encoded into a binary codeword  $\mathbf{c} = (c_0, c_1, \dots, c_{PL-1})^t$  using a channel code with rate  $R = \frac{K}{L}$ , then  $\mathbf{c}$  is interleaved so that  $c'_l = c_{\Pi(l)}$  and finally  $\mathbf{c}'$  is modulated using the  $Q$ -ary modulation into  $\mathbf{a} = (a_0, \dots, a_{PM-1})^t$ . This modulated packet is split into  $P$  parts  $\mathbf{a}_p = (a_{p,0}, \dots, a_{p,M-1})^t$ , each part being circularly shaped independently of the others.

The shaping filter is a Root Raised Cosine (RRC) filter  $h(t)$  with rolloff coefficient  $\beta$  and Nyquist bandwidth  $\frac{1}{T_h}$ . Symbols  $a_{p,m}$  are sent with a symbol period  $T_s = \tau T_h$  (with  $\tau \in [0, 1]$ ) and the circularly pulse shaped as follows

$$x_p(t) = \sqrt{\tau} \sum_{m=0}^{M-1} a_{p,m} h_T(t - mT_s) \quad (1)$$

where  $h_T(t)$  is the periodic summation of  $h(t)$  defined as

$$h_T(t) = \sum_{n=-\infty}^{+\infty} h(t - nT). \quad (2)$$

Finally, the complete frame is obtained as the concatenation of all subframes:

$$x(t) = \sum_{p=0}^{P-1} x_p(t) W_T(t - pT), \quad (3)$$

where  $W_T(\cdot)$  represents a window of size  $T$ .

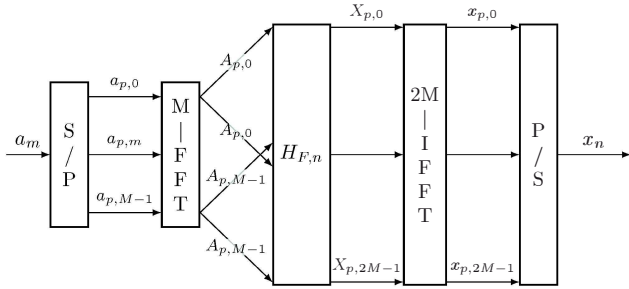


Figure 2. Transmitter architecture, the filter  $H_n$  block corresponds to a scalar multiplication of each stream by  $\frac{1}{T_x} H(\frac{n}{T})$

In practice, we consider a digital version of  $x(t)$  that is synthesized from its samples  $x_n = x(nT_x)$  with  $T_x = \frac{T_s}{2}$ :

$$x_n = \sum_{p=0}^{P-1} x_{p,n} W_N(n - pN) \quad (4)$$

where  $N = 2M$ , and  $x_{p,n}$  is given by

$$x_{p,n} = \sqrt{\tau} \sum_{m=0}^{M-1} a_{p,m} \sqrt{\frac{T_s}{2}} h_T(n \frac{T_s}{2} - mT_s). \quad (5)$$

$x_{p,n}$  is obtained as the discrete time circular convolution between  $a_{p,m}$  upsampled by a factor 2 and the sampled filter  $h_T(nT_x)$ . hence,  $x_{p,n}$  can be computed using the architecture proposed in FIGURE 2 where

$$H_{F,n} = \sqrt{T_x} \sum_{k=0}^{N-1} h_T(kT_x) e^{-j2\pi k \frac{n}{N}}. \quad (6)$$

It can be noted that our solution is different from the one proposed in [12] since the circular convolution is a direct consequence of the FFT/IFFT steps and does not require to transmit any cyclic prefix. Consequently, our solution has a better spectral efficiency than the one proposed in [12]. In this paper, we only consider a flat channel, if a frequency selective channel is considered, a cyclic prefix should be added to ensure a circular convolution of the channel and a simplified equalization.

For the rest of this article, we consider that  $T$  is sufficiently large so that the PSD (Power Spectral Density) of  $x(t)$  can be considered as zero outside a bandwidth  $B_h = \frac{1+\beta}{T_h}$ , consequently, the spectral efficiency of the proposed communication system is

$$\eta(\tau) = \frac{1}{T_s B_h} R \log_2(Q) = \frac{R \log_2(Q)}{\tau(1+\beta)} \quad (7)$$

where  $R$  is the binary code rate,  $Q$  is the modulation order,  $\tau = \frac{T_s}{T_h}$ , and  $\beta$  is the rolloff factor of the pulse shaping RRC.

### B. On the matrix form of the received signal

Considering a complex Additive White Gaussian Noise (AWGN) channel with variance  $\sigma_z^2$ , the received signal corresponding to the reception of the packet  $p$  can be expressed matrixially as

$$\mathbf{y}_p = \sqrt{\tau} H \mathbf{a}_p + \mathbf{z}_p \quad (8)$$

where  $\mathbf{z}_p \sim \mathcal{CN}(\mathbf{0}_N, \sigma_z^2 I_N)$  and  $H$  is the  $N \times M$  matrix defined as

$$H = (\mathbf{h}_0, \mathbf{h}_{T_s}, \dots, \mathbf{h}_{(M-1)T_s}) \quad (9)$$

where  $\mathbf{h}_t$  is the following column vector

$$\mathbf{h}_t = \sqrt{T_x} (h_T(t), h_T(t + T_x), \dots, h_T(t + (N-1)T_x))^T$$

where  $T_x = \frac{T_s}{2}$ . The matrix  $H$  represents the joint effect of the upsampling with factor 2 and the circular convolution given by equation (1).

### III. RECEIVER ARCHITECTURE

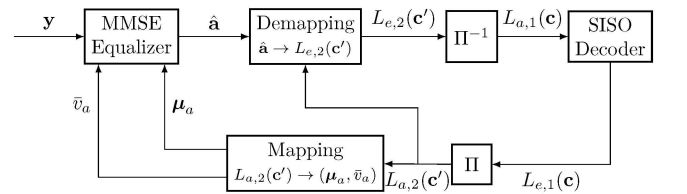


Figure 3. Iterative receiver architecture.

In this paper, we consider an iterative receiver based on the turbo equalization technique described in [16]. The receiver

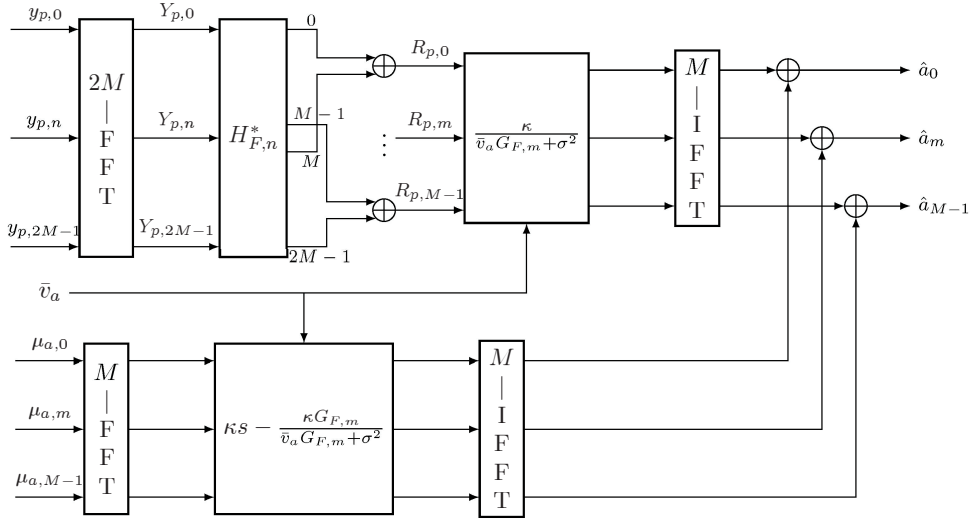


Figure 4. IFFT implementation of the linear MMSE equalizer with *a priori*.

architecture is depicted in FIGURE 3, it is based on a linear Minimum Mean Square Error (MMSE) equalizer using *a priori* information from a SISO decoder.

#### A. Expression of the mapper

The MMSE equalizer inputs are observations from the channel  $\mathbf{y}$ , mean  $\boldsymbol{\mu}_a$  and variance  $\bar{v}_a$  representing the *a priori* distribution on  $\mathbf{a}$ .  $\boldsymbol{\mu}_a$  and  $\bar{v}_a$  are computed from the Log Likelihood Ratio (LLR)  $L_{a,2}(\mathbf{c}')$  by the mapper. The mapper is identical to the one described in [16]; means and variances of symbols are given by the following set of equations:

$$\mu_{a,n} = \sum_{a \in \mathcal{A}} a p(a_n = a | L_{a,2}(\mathbf{c}')) \quad (10)$$

$$\bar{v}_{a,n} = \sum_{a \in \mathcal{A}} |a - \mu_{a,n}|^2 p(a_n = a | L_{a,2}(\mathbf{c}')) \quad (11)$$

$$\bar{v}_a = \frac{1}{N} \sum_{n=0}^{N-1} \bar{v}_{a,n}. \quad (12)$$

To compute these equations, one needs  $p(a_n = a | L_{a,2}(\mathbf{c}'))$  which is defined as

$$p(a_n = a | L_{a,2}(\mathbf{c}')) = \prod_{k=0}^{q-1} \frac{\exp(-m_k L_{a,2}(c'_{nq+k}))}{1 + \exp(-L_{a,2}(c'_{nq+k}))}, \quad (13)$$

where  $a = \text{mapping}(m_0, \dots, m_{q-1})$ .

#### B. Expression of the MMSE equalizer with *a priori*

The channel model of equation (8) does not correspond to a circular convolution since the received signal is sampled at time  $\frac{T_s}{2}$ , however, the MMSE with *a priori* can still be computed using the same steps as in [16]. The output soft estimate of  $\mathbf{a}_p$  is denoted by  $\hat{\mathbf{a}}_p$  and is given by

$$\hat{\mathbf{a}}_p = \kappa \Sigma^{-1} (H^H \mathbf{y}_p - H^H H \boldsymbol{\mu}_a) + \kappa s \boldsymbol{\mu}_a. \quad (14)$$

Let  $\mathbf{e}_0$  be the length  $M$  defined as  $\mathbf{e}_0 = (1, 0, \dots, 0)^T$ ;  $s$ ,  $\kappa$  and the matrix  $\Sigma$  are given by

$$s = \mathbf{e}_0^H \Sigma^{-1} H^H H \mathbf{e}_0 \quad (15)$$

$$\kappa = \frac{\tau \sigma_a^2}{1 + (\tau \sigma_a^2 - \bar{v}_a) s} \quad (16)$$

$$\Sigma = \bar{v}_a H^H H + \sigma_z^2 I_N. \quad (17)$$

It can be argued that no matched filter has been done to obtain the channel model of equation (8). In fact, multiplications by  $H^H$  in equation (14) correspond to the application of a circular matched filter followed by a downsampling of factor 2 corresponding to sampling at rate  $R_s$ . This step is considered by default in [12] and [13] however the correlation of the noise at the output of the matched filter is not considered to design the MMSE equalizer. It can be further noted that  $\Sigma^{-1}$  equation (14) corresponds to the MMSE equalizer expression when a correlated noise with correlation matrix  $\sigma_z^2 G$  is considered.

Now we show that equations (14)-(17) can be efficiently implemented in the frequency domain. Indeed, it can be noted that the matrix  $G = H^H H$  is a circulant matrix of size  $M \times M$  with first line  $(g_T(0), g_T(T_s), \dots, g_T((M-1)T_s))$ . Hence  $\Sigma^{-1}$  is also a circulant matrix and multiplications by  $G$  and  $\Sigma^{-1}$  are equivalent to circular convolutions that can be implemented using FFT and IFFT. In particular,  $\hat{\mathbf{a}}_p$  can be rewritten as

$$\hat{\mathbf{a}}_p = \kappa \Sigma^{-1} H^H \mathbf{y}_p + (\kappa s I_M - \kappa \Sigma^{-1} H^H H) \boldsymbol{\mu}_a \quad (18)$$

Equation (18) can be implemented following the block diagram given in FIGURE 4, where  $H_{F,n}$  is given by equation (6) and  $G_{F,n}$  is

$$G_{F,n} = \sum_{m=0}^{M-1} g_T(mT_s) e^{-j2\pi m \frac{n}{M}}. \quad (19)$$

Using the fact that the matrix  $\Sigma^{-1}H^{\dagger}H$  is a circulant matrix,  $s$  can also be expressed using  $G_{F,n}$  as

$$s = \frac{1}{M} \sum_{m=0}^{M-1} \frac{G_{F,m}}{\sigma_z^2 + \bar{v}_a G_{F,m}} \quad (20)$$

### C. Expression of the demapper

In [17], it is shown that  $\hat{\mathbf{a}}_p$  is a biased estimator of  $\mathbf{a}_p$  because

$$\hat{a}_{p,n} = \kappa s a_{p,n} + w_n \quad (21)$$

where  $w_n$  is a noise term with variance  $\sigma_w^2 = \kappa s(1 - \kappa s)\sigma_a^2$ . Taking into account this bias term into the demapper expression, we have

$$L_{e,2}(c'_{nq+k}) = \frac{\sum_{\mathbf{m} \in \{0,1\}^N : m_k=0} e^{-\frac{|\hat{a}_n - \kappa s \text{map}(\mathbf{m})|}{\kappa s(1-\kappa s)\sigma_a^2}} \prod_{j \neq k} e^{-m_j L_{a,2}(c'_{nq+j})}}{\sum_{\mathbf{m} \in \{0,1\}^N : m_k=1} e^{-\frac{|\hat{a}_n - \kappa s \text{map}(\mathbf{m})|}{\kappa s(1-\kappa s)\sigma_a^2}} \prod_{j \neq k} e^{-m_j L_{a,2}(c'_{nq+j})}}$$

where  $\text{map}(\mathbf{m})$  represents the symbol associated with label  $\mathbf{m}$ .

## IV. SIMULATION RESULTS

In this section, we highlight the performances of the proposed transmitter/receiver architectures for the Convolutional Code  $(1, \frac{5}{7})_8$  and for Low Density Parity Check (LDPC) codes. The filter  $h(t)$  is a Root Raised Cosine (RRC) filter with a rolloff coefficient of 0.35 and considering a 4-QAM modulation. We consider codewords spanning  $P = 8$  subcodewords of duration  $M = 1024$  symbols, giving codewords of size  $L = 16384$ .

First, we propose to analyze the performances of the proposed method for the CC  $(1, \frac{5}{7})_8$ . The associated Soft Input Soft Output (SISO) decoder is a bit-wise Maximum A Posteriori (MAP) Log-BCJR (Bahl Cocke Jelinek Raviv) algorithm. In FIGURE 5, we give the EXtrinsic Information Transfer (EXIT) charts for the proposed equalizer for  $\tau \in \{0.4, 0.375, 0.35, 0.325, 0.3\}$  for a Signal to Noise Ratio  $\frac{E_b}{N_0} = 5dB$ . The EXIT chart of the SISO decoder of the CC is also provided. We observe that for a SNR of  $\frac{E_b}{N_0} = 5dB$ , values of  $\tau \geq 0.325$  the iterative decoding should converge while for  $\tau = 0.3$  it is not the case. This behavior is corroborated by the Bit Error Rate curves given in FIGURE 6.

In FIGURE 7 we compare spectral efficiency of different coding schemes. The curve with no marker represents the achievable spectral efficiency for independent and identically distributed Gaussian codebooks. The curve with cross markers is the information rate for Uniformly and Independently Distributed (UID) 4-QAM input alphabets on the ISI channel corresponding to  $\tau = 0.35$ . This curve has been computed using the Shamai Laroia Conjecture (SLC, see [18]). The curve with triangle markers represents the information rate obtained by computing the area under the EXIT chart of the proposed SISO equalizer.

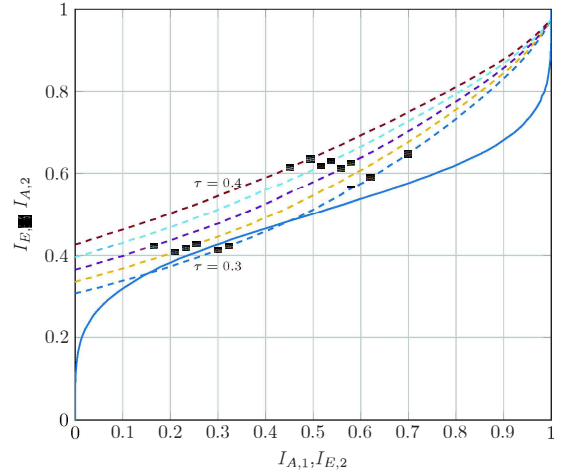


Figure 5. EXIT charts for  $\tau \in \{0.4, 0.375, 0.35, 0.325, 0.3\}$  and for the Log-BCJR decoder of the convolutional code with polynomial  $(1, \frac{5}{7})_8$  for  $\frac{E_b}{N_0} = 5dB$  considering an inner code of rate  $R = \frac{1}{2}$ .

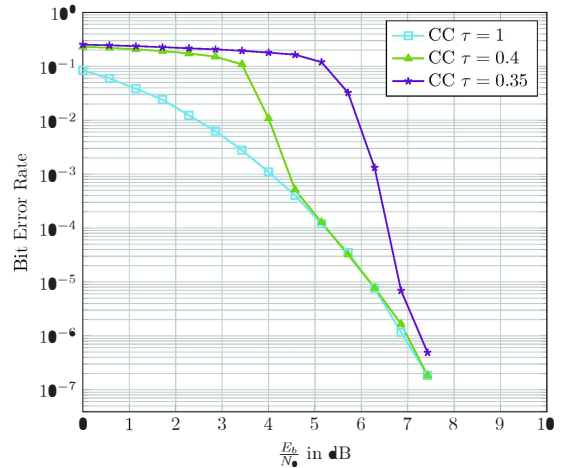


Figure 6. BER vs  $\frac{E_b}{N_0}$  for  $\tau \in \{1, 0.4, 0.35\}$  after 10 iterations

We observe that the proposed scheme suffers of a performance degradation for high values of  $\frac{E_b}{N_0}$  compared to the SLC. We suppose that this performance loss is due to the imposed linear structure of our equalizer. For every value of  $\frac{P}{N_0 B}$ , we have computed the EXIT charts of our SISO equalizer and we have found the optimal Low Density Parity Check code profile using the method proposed in [19]. From these profiles, we have computed the corresponding information rates using equation (7). The resulting curve has square markers. We note that the performance loss compared to the area under the EXIT chart is low. We also provide the UID information rate for 4-QAM modulation at a Nyquist rate ( $\tau = 1$  with round markers) and observe that for equivalent  $\frac{E_b}{N_0}$  FTN allows significant gains in spectral efficiency. Two coordinates corresponding to the CC for  $\tau = 0.35$  and  $\tau = 0.4$  are provided for a target BER of  $10^{-5}$ .

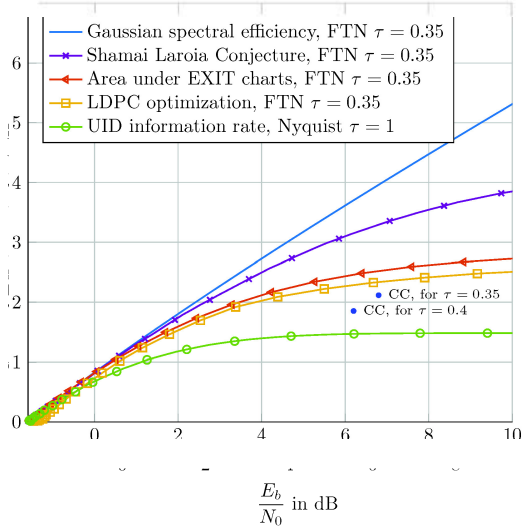


Figure 7. Comparison of several spectral efficiencies for  $\tau = 0.35$  and  $\beta = 0.35$ .

## V. CONCLUSION

To conclude, in this paper we consider a circular pulse shaped FTN signaling. An architecture based on FFT/IFFT operations is proposed that avoids the use of a cyclic prefix at the transmitter. We then propose an iterative receiver architecture based on the sampling of the received signal at twice the symbol rate. Finally, we propose simulation results to show the performances of the proposed transmitter/receiver architectures and observe that significant gains can be achieved using FTN. Furthermore, with the proposed iterative receiver scheme, we observe that for target BER below  $10^{-5}$  compression factors as low as 0.35 can be used with a performance loss less than  $0.5\text{dB}$  compared to the Nyquist case with  $\tau = 1$ .

## APPENDIX A

### AUTOCORRELATION OF $x(t)$

In order to derive the PSD of  $x(t)$ , we start by computing its autocorrelation function  $R_x(t, \delta) = \mathbb{E}[x(t)x^*(t - \delta)]$ . From equation (3), after some calculation steps (using the fact that symbols  $a_{p,m}$  are iid) and after remarking that  $x(t)$  is wide sense cyclo-stationary for a period of  $T$ , the average autocorrelation function of  $x(t)$  is equal to

$$\bar{R}_x(\delta) = \frac{\tau\sigma_a^2}{T} \sum_{m=0}^{M-1} \int_0^T h_T(t - mT_s) h_T^*(t - \delta - mT_s) W_T(t - \delta) dt \quad (22)$$

For  $\delta = 0$  we obtain

$$\begin{aligned} P_x &= \frac{\tau\sigma_a^2}{MT_s} \sum_{m=0}^{M-1} \int_0^T h_T(t - mT_s) h_T^*(t - mT_s) dt \\ &= \frac{\tau\sigma_a^2}{T_s} \int_{-\infty}^{+\infty} |h(t)|^2 dt \\ &= \frac{\sigma_a^2 g(0)}{T_h} \end{aligned} \quad (23)$$

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