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Branch-and-price algorithms for the Bi-Objective Vehicle Routing Problem with Time Windows

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This paper presents generic exact methods for the bi-objective vehicle routing problems (BOVRP). The first part explains the BOVRP. Then, an efficient mono-objective solution method used as a component of bi-objective methods is introduced. After that, the methods are presented. The computational evaluation is performed on a specific variant of the VRP, the VRP with time windows (VRPTW).

1 Context

The Vehicle Routing Problem was first introduced in 1959 by Dantzig and Ramser [3] which aimed to minimize the cost of gasoline transportation between a bulk terminal and several service stations.

The bi-objective VRP with time windows (BOVRPTW) is a variant of the VRP. Let \(G = (V, E)\) be a non-oriented graph. A node \(i \in V \setminus v₀\) is called a customer and has a demand \(q_i\). These demands are satisfied by a fleet of \(K\) vehicles of capacity \(Q\). A vehicle \(k\) starts and returns at a node \(v₀\) called the depot and performs a route \(r_k\) by passing through a set of customers. The route \(r_k\) is said to be feasible if each customer \(i\) is served by the route between the due time \(dt_i\) and the end time \(et_i\) and if the total capacity of a vehicle is not exceeded.

An edge of the graph has two costs \(c_1^e\) and \(c_2^e\). Each route \(r_k\) provides two costs \(c_1^e_k\) and \(c_2^e_k\) representing the sum of the two costs on the used edges. The aim of the BOVRP is to minimize the sum of each cost of the routes used.

Let \(Ω\) be the set of feasible routes \(r_k, k \in [1,..,K]\) and \(a_{ik}\) be equal to 1 if the customer \(i\) belongs to the route \(r_k\). The Set Partitioning formulation of the BOVRPRTW [2] can be stated as follows:

\[
\begin{aligned}
\text{minimize} \quad & \left( \sum_{r_k \in Ω} c_1^k \theta_k, \sum_{r_k \in Ω} c_2^k \theta_k \right) \\
\text{subject to} \quad & \sum_{r_k \in Ω} a_{ik} \theta_k \geq 1 \quad (v_i \in V \setminus \{v₀\}), \\
& \sum_{r_k \in Ω} \theta_k \leq K, \\
& \theta_k \in \{0,1\} \quad (r_k \in Ω).
\end{aligned}
\]

where \(\theta_k\) is a variable that indicates if the route \(r_k, k \in [1,..,K]\), is selected in the solution \((\theta_k = 1)\) or not \((\theta_k = 0)\).

This formulation has an exponential number of variables and a polynomial number of constraints. It is therefore suitable for a column-generation-based solution method such as the method of Baldacci et al. [1] presented in section 2.
The $\epsilon$-constraint method, introduced in section 3, is one of the most efficient exact method
for bi-objective integer problems as for the bi-objective prize-collecting steiner tree problem[4].
To compare our methods, the reference method (3.1) reproduced a classical $\epsilon$-constraint.

2 Solution in mono-objective vehicle routing problems

Solving the BOVRP requires a method that optimally solve the mono-objective VRP (MOVPRP).
One of the state-of-the-art methods is the one of Baldacci et al. [1]: an exact method to solve
the MOVPRP, based on the set partitioning formulation (1) with only one cost to minimize.
This method allows to restrict the number of routes to obtain the optimal integer solution with
a three steps algorithm:

1. Compute a good lower bound $LB$ with column generation and a good upper bound $UB$
   via a heuristic. Compute the gap $\gamma = UB - LB$.
2. Generate all routes with a reduced cost less or equal to $\gamma$. Indeed, it can be proven that
   routes with a reduced cost higher than $\gamma$ can not be in the optimal integer solution [4].
3. Solve the initial integer problem on the restricted set of routes from step 2 to obtain the
   optimal (integer) solution.

As previously mentioned, this method will be used to solve the BOVRP. In the following, we
will refer to the step 2 by $B\text{ALD}(UB, LB)$ with $UB$ and $LB$ the upper bound and the lower
bound previously computed. It returns a restricted set of routes $\Omega$.

3 Methods

This section presents three methods that compute the minimum complete Pareto front of the
BOVRPTW introduced in section 1. So, all supported and non-supported non-dominated
points are given, but only one efficient solution is provided for each one. They are based on
the Baldacci et al. method, presented in section 2.

3.1 Reference method

The first method is called the reference method as it is the more direct way to use the Baldacci
et al. method in a BOVRP with an $\epsilon$-constraint method.

The algorithm, summarized in Algorithm 1, is based on the $\epsilon$-constraint formulation that
aims to minimize the first cost $c^1$ under the constraint that the second cost has to be lower
than a certain value $\epsilon$. This formulation CMP($\epsilon$) can be stated as follows:

$$
\begin{align*}
\text{minimize} & \quad \sum_{r_k \in \Omega} c^1_k \theta_k \\
\sum_{r_k \in \Omega} c^2_k \theta_k & \leq \epsilon \\
\sum_{r_k \in \Omega} a_{ik} \theta_k & \geq 1 \quad (v_i \in V \setminus \{v_0\}), \\
\sum_{r_k \in \Omega} \theta_k & \leq K, \\
\theta_k & \in \mathbb{N} \quad (r_k \in \Omega).
\end{align*}
$$

(2)

where all notations are the same as in the formulation (1).

The reference method consists in finding $LB$, the optimal value of the linear relaxation of the
problem (2) (LCMP) and $UB$, a feasible solution of the integer problem (2) (RCMP). Then, it
applies $B\text{ALD}(UB, LB)$ to obtain $\overline{\Omega}$, the restricted set of routes with their reduced cost
within the gap $\gamma$. And finally, the algorithm optimally solves the $\epsilon$-constraint formulation (2).

This process is repeated from $\epsilon = +\infty$ to a certain value for which there is no more solution.
Algorithm 1 Algorithm of the reference method
\[
\begin{align*}
\epsilon &\leftarrow +\infty \\
\textbf{while } \exists \text{ a solution do} & \\
& \quad \text{Solve LCMP}(\epsilon) \text{ to obtain } LB \\
& \quad \text{Solve RCMP}(\epsilon) \text{ to obtain } UB \\
& \quad \Omega \leftarrow \text{BALD}(UB, LB) \\
& \quad \text{Solve CMP}(\epsilon) \text{ on } \Omega \text{ to obtain } S_{OPT} \\
& \quad \text{Set } \epsilon \leftarrow S_{OPT}^2 - 1 \\
\end{align*}
\]
end while

3.2 Generate-one-time

The second method requires as input:

- \( LB \): the cost \( c^1 \) of the optimal solution of the linear relaxation of the problem which minimizes the cost \( c^1 \).
- \( UB \): the cost \( c^1 \) of the optimal solution of the linear relaxation of the problem which minimizes the cost \( c^2 \).

The method applies BALD\((UB, LB)\) and performs an \( \epsilon \)-constraint method of the formulation (2) on this new set of routes. It is summarized in the Algorithm 2.

Algorithm 2 Algorithm of generate-one-time
\[
\begin{align*}
\Omega &\leftarrow \text{BALD}(UB, LB) \\
\epsilon &\leftarrow +\infty \\
\textbf{while } \exists \text{ a solution do} & \\
& \quad \text{Solve CMP}(\epsilon) \text{ on } \Omega \text{ to obtain } S_{OPT} \\
& \quad \text{Set } \epsilon \leftarrow S_{OPT}^2 - 1 \\
\end{align*}
\]
end while

3.3 Two-steps

This method is based on the parametrized formulation that optimizes a weighted-sum of the two objectives. The formulation is noted PMP(\( \lambda \)):
\[
\begin{align*}
\text{minimize } \lambda & \left( \sum_{r_k \in \Omega} c^1_k \theta_k + (1 - \lambda) \sum_{r_k \in \Omega} c^2_k \theta_k \right) \\
\sum_{r_k \in \Omega} a_{ik} \theta_k & \geq 1 \quad (v_i \in V \setminus \{v_0\}) \\
\sum_{r_k \in \Omega} \theta_k & \leq K, \\
\theta_k & \in \mathbb{N} \quad (r_k \in \Omega).
\end{align*}
\]
(3)

We also need to introduce the call of the second step of Baldacci et al. method for the direction \( \lambda : \text{BALD}(UB, LB, \lambda) \), where the cost of a solution is computed for the weight \( \lambda \).

It requires as input the list of \( N \) points \( LB \) (the front of the linear relaxation of the BOVRPTW obtained by the parametric simplex) and their associated dual solutions. It also needs a list of \( N+1 \) directions \( \lambda \) in which the points \( LB_i \) and \( LB_{i+1}, i \in [1, .., N-1] \) are optimal.

The method is composed of two steps. First, a non-complete Pareto front of \( S \) points is computed and then, the area in which non dominated point could be are explored. The first step iteratively takes a point \( LB_i, i \in [1, .., N] \), its dual solutions and its direction \( \lambda_i \). In this direction, a feasible solution \( UB_i \) is computed and \( \text{BALD}(UB_i, LB_i, \lambda_i) \) is applied to obtain a non-dominated point. The second step searches all non-dominated points between two successive points found in the first step. The search between two points \( S_i \) and \( S_{i+1}, i \in [1, .., S-1] \), can be summarized as follows:
Algorithm 3 Second step of two-steps method

Require: Point $S_i$ and $S_{i+1}$

Compute direction $\lambda'_i$, gradient between $S_i$ and $S_{i+1}$

Solve LPMP($\lambda'_i$) to obtain $LB'_i$ (in practice $LB'_i$ is in the initial list of $N$ points $LB$)

Solve RPMP($\lambda'_i$) to obtain $UB'_i$ (in practice $UB'_i$ is the nadir point)

$\Omega \leftarrow BACD(LB'_i, UB'_i, \lambda'_i)$

Solve PMP($\lambda'_i$) on $\Omega$ to obtain $S'_i$

if PMP($\lambda'_i$) is feasible then

Repeat algorithm for $S_i$ and $S'_i$ and for $S'_i$ and $S_{i+1}$

end if

4 Results

Each method returns the minimum complete set of non-dominated points. To compare the three methods introduced in Section 3, pairs of Solomon’s instances have been merged to create BOVRPTW. The implementation is in C++ and the linear problems and the integer problems are solved with Gurobi 7.1. The following table shows the results of the execution with the CPU time in seconds and on a graph with 25 customers. The column called BALD represents the number of calls to procedure $BACD$ and the one called col. is the average number of generated columns per call.

<table>
<thead>
<tr>
<th>Instance</th>
<th>Ref.</th>
<th>time</th>
<th>BALD</th>
<th>col.</th>
<th>time</th>
<th>BALD</th>
<th>col.</th>
<th>time</th>
<th>BALD</th>
<th>col.</th>
</tr>
</thead>
<tbody>
<tr>
<td>R101_C101</td>
<td>10</td>
<td>24</td>
<td>735</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1192</td>
<td>5</td>
<td>3</td>
<td>420</td>
</tr>
<tr>
<td>R101_C201</td>
<td>8</td>
<td>22</td>
<td>745</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1224</td>
<td>3</td>
<td>25</td>
<td>438</td>
</tr>
<tr>
<td>R105_RC105</td>
<td>43</td>
<td>37</td>
<td>3736</td>
<td>22</td>
<td>1</td>
<td>1</td>
<td>7119</td>
<td>119</td>
<td>66</td>
<td>2505</td>
</tr>
<tr>
<td>R105_C201</td>
<td>47</td>
<td>35</td>
<td>3911</td>
<td>30</td>
<td>1</td>
<td>1</td>
<td>6225</td>
<td>108</td>
<td>57</td>
<td>2471</td>
</tr>
<tr>
<td>R109_C101</td>
<td>750</td>
<td>26</td>
<td>33149</td>
<td>318</td>
<td>1</td>
<td>1</td>
<td>62 072</td>
<td>119</td>
<td>66</td>
<td>2505</td>
</tr>
<tr>
<td>R109_RC105</td>
<td>112</td>
<td>48</td>
<td>30559</td>
<td>259</td>
<td>1</td>
<td>1</td>
<td>53 513</td>
<td>119</td>
<td>66</td>
<td>2505</td>
</tr>
<tr>
<td>RC101_C101</td>
<td>15</td>
<td>10</td>
<td>5128</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>4850</td>
<td>15</td>
<td>17</td>
<td>3423</td>
</tr>
<tr>
<td>RC105_R101</td>
<td>248</td>
<td>18</td>
<td>26797</td>
<td>137</td>
<td>1</td>
<td>1</td>
<td>42 602</td>
<td>189</td>
<td>32</td>
<td>8298</td>
</tr>
<tr>
<td>RC106_C101</td>
<td>373</td>
<td>14</td>
<td>45113</td>
<td>35</td>
<td>1</td>
<td>1</td>
<td>18 280</td>
<td>243</td>
<td>18</td>
<td>11 985</td>
</tr>
</tbody>
</table>

It demonstrates that the generate-one-time is more efficient than the two others and also that two-steps method is a lot less efficient. Regarding the number of calls to the procedure $BACD$, we can notice that two-steps method has a very high number of calls, but the number of generated routes is lower than for generate-one-time method.

5 Conclusion and perspectives

The results in Section 4 show that two-steps method is less efficient, but the resulting integer problems are easier to solve than for the generate-one-time method. For larger graphs, the generate-one-time method could produce a prohibitively high number of routes. Therefore, the two-steps can not be disregarded based only on the results on small graphs, although it remains to be improved, for example using parallelism or a hybridization of the two methods.

References


